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Stabilization for uncertain stochastic T–S fuzzy system driven by Lévy noise

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Abstract

Different from the other work, the almost sure asymptotic stability of an uncertain stochastic T–S fuzzy system driven by Lévy noise has been investigated. However, the Lévy noise caused the càdlàg paths in the system, and the uncertainty was the linear fractional form, which made difference to the general norm-bounded type. Using the special stochastic techniques and new matrix decomposition method, we deal with the càdlàg paths and uncertainty of the system. As the main results, the sufficient conditions of almost sure asymptotic stability for stochastic T–S fuzzy system driven by Lévy noise have been presented. On this basis, the closed-loop system is robustly almost surely asymptotically stable with fuzzy state-feedback controller. Furthermore, our stabilization criteria are based on linear matrix inequalities (LMIs), whence the feedback controller could be designed more easily in practice.

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Keywords: Almost sure asymptotic stability; Uncertain; Lévy noise; T–S fuzzy; LMIs

1 Introduction

With the improvement of linear system theory, the research of nonlinear system has become a difficult problem. Since the Takagi–Sugeno (T–S) fuzzy system was first introduced [24], it provided a general framework to represent a nonlinear plant by using a set of local linear models, and the models were smoothly connected through nonlinear fuzzy membership functions, which have received considerable attention, the basis for developing of systematic approaches has been used to stability analysis and controller design of fuzzy control (Chang et al. [5]; Chen et al. [6]; Ding [7]; Gassara et al. [9]; Senthilkumar&Balasubramaniam [20]; Sheng et al. [21]; Song et al. [22]; Tseng [26]; Uang [27]; Xu et al. [28]). The T–S fuzzy model is able to approximate any smooth nonlinear function to any degree of accuracy in any convex compact region (Tanaka and Wang [25]), it was regarded as a powerful solution to bridge the gap between the fruitful linear control and the fuzzy logic control targeting complex nonlinear systems (Feng [8]), and expanded to uncertain stochastic systems with fuzzy formation (Huo et al. [11]). Based on the results of the above literature, we have extended the T–S fuzzy system theory to a more complex stochastic system.

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On the other hand, with the development of science and technology, practical problems require more and more accurate description. A wide range of uncertainties were added to the stochastic controlled system, which produced uncertain controlled system, uncertain fuzzy controlled system, and uncertain stochastic fuzzy controlled system. The uncertainty of the system was general norm-bounded type, but Ghaoui and Scorletti [10] and Zhou et al. [31] presented the linear fractional form, which has a broader meaning. In order to make the uncertainty approach to the application, much difficulty and challenge appeared in this problem, and the existing techniques cannot deal with the above-mentioned problems directly. However, on the basis of Zhou et al. [31], we have expanded the new matrix decomposition method to deal with linear fractional type directly.

As for the research of the system's noise source, there were a number of studies on the topic using different types of driving noise and the following paper list is far from exhaustive: Arnold and Crauel [3], Bellman et al. [4], Liu et al. [12], Mao [13, 14, 16, 18], Stojanovic et al. [23], and Zhai et al. [29]. In these sources of driving noise, the multidimensional Brownian motion has been recognized as the general theory. Furthermore, the Lévy process as the more general source of driving noise has been employed to stabilize the unstable dynamical system, so it is the more general theory and builds extensively on Mao's results in the Brownian motion case. For the theoretical development and applications of the Lévy processes, there has recently been extensive activity (Applebaum [1] and Applebaum&Siakalli [2]), but there were less studies that seemed to be timely as the source of noise in the controlled systems, because the Lévy process was more complex. Compared with the Brownian motion, the different point is that the path of the systems driven by Lévy process was technically much more challenging, which was a càd-làg path. In this paper, we present the new method, which generalizes the application scope of the classical nonnegative semi-martingale convergence theorem, and search for special techniques to deal with this difficulty by the semi-martingale and stopping time theory.

Based on the above discussion, this paper considers the problems of almost sure asymptotic stability analysis and controller synthesis for a class of uncertain stochastic T–S fuzzy systems driven by a multi-dimensional Lévy process. Following the same idea as in dealing with the stabilization problem, linear state feedback controllers are designed so that the closed-loop systems are almost surely asymptotically stable. Furthermore, in order to design easily in practice, the explicit expressions for the desired state feedback controllers are given with LMIs. At last, the main contributions of this paper are mainly two aspects: *(1) The source of driving noise for almost sure asymptotic stability is the Lévy process, and the stable form is almost surely asymptotically stable; (2) The uncertainty description is linear fractional in nature, and the explicit expressions for the desired state feedback controllers are clear with LMIs.*

The rest of the paper is organized as follows. In Sect. 2, we present some basic preliminaries, the uncertain stochastic T–S fuzzy system driven by Lévy noise, and necessary lemmas. In Sect. 3, the sufficient conditions for almost sure asymptotic stability of stochastic systems driven by Lévy noise are given. In Sect. 4, the stability analysis of uncertain stochastic system is presented. In Sect. 5, the desired state feedback controllers are designed for the uncertain stochastic fuzzy close-loop systems with LMIs. In Sect. 6, the related discussion on the main results is presented.

2 Preliminaries

Let $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\}$ be a complete probability space with a filtration satisfying the usual conditions, i.e., where Ω is the sample space and $\mathbb{P}$ is the probability measure, the filtration is continuous on the right and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets. Let $D([-\tau, 0]; R^n)$ denote the family of functions φ from $[-\tau, 0]$ to R^n that are right-continuous and have limits on the left, where $\tau \in R$ is the constant and R^n is the n-dimensional vector space. $D([-\tau, 0]; R^n)$ is equipped with the norm $\|\varphi\| = \sup_{-\tau \leq s \leq 0} |\varphi(s)|$ and $|x| = \sqrt{x^T x}$ for any $x \in R^n$. If A is a vector or matrix, its trace norm is denoted by $|A| = \sqrt{\text{trace}(A^T A)}$, while its operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\}$. Denote by $D^b_{\mathcal{F}_0}([-\tau, 0]; R^n)$ the family of all bounded, $\mathcal{F}_0$ measurable, $D([-\tau, 0]; R^n)$-valued random variables. We denote by $L^1(R_+; R_+)$ the family of all functions $\lambda(t)$ such that $\int_0^\infty \lambda(t)\, dt < \infty$, where R_+ is the set of positive real numbers, and $\mathcal{K}(R_+; R_+)$ is the family of all functions $\gamma(x)$, if it is continuous, strictly increasing, and $\gamma(0) = 0$. It is said to belong to the family $\mathcal{K}_\infty$ if $\gamma \in \mathcal{K}$ and $\gamma(x) \to \infty$ as $x \to \infty$. The real symmetric matrix $P > 0$ ($P \geq 0$) denotes P being a positive definite (or positive semidefinite) matrix, and $A > (\geq)B$ means $A - B > (\geq)0$. Let I denote an identity matrix with proper dimension. The trace of A is denoted by $\text{tr}\,A$, the notation A^T represents the transpose of the matrix A and the notation $*$ represents an ellipsis for a block matrix that is induced by symmetry. The Kronecker product of two matrices is denoted by $\otimes$.

Let us consider the following uncertain stochastic T–S fuzzy system driven by Lévy process described by the following IF-THEN rules:

Plant rule k: if $\theta_1(t)$ is η_{k1}, and, ..., $\theta_r(t)$ is η_{kr}, then

$$dx(t) = \left[(A_k + \Delta A_k)x(t-) + (B_k + \Delta B_k)u\big(x(t-)\big)\right] dt$$
$$+ \sum_{i=1}^{m} \left[(C_{ki} + \Delta C_{ki})x(t-) + (F_{ki} + \Delta F_{ki})u\big(x(t-)\big)\right] dY_i(t), \quad k = 1, 2, \ldots, L, \tag{2.1}$$

where $\theta_1(t), \ldots, \theta_r(t)$ are the premise variables, $\eta_{k1}, \ldots, \eta_{kr}$ are the fuzzy sets and L is the number of rules. $A_k(\Delta A_k)$, $B_k(\Delta B_k)$, $C_{ki}(\Delta C_{ki})$, and $F_{ki}(\Delta F_{ki})$ are $n \times n$ real matrices, $x(t-) \in R^n$ is the lift limit of state, and $Y(t) = (Y_i(t), t \geq 0, i = 1, 2, \ldots, m)$ is a Lévy process taking values in R^m, $u(x(t))$ is the feedback control. The parametric uncertainties are assumed to take the following linear fractional form:

$$[\Delta A_k, \Delta B_k, \Delta C_{ki}, \Delta F_{ki}] = G_k \Lambda \left[H_1^k, H_2^k, H_{3i}^k, H_{4i}^k\right], \tag{2.2}$$

$$\Lambda = \left[I - Q(t)J\right]^{-1} Q(t), \tag{2.3}$$

$$I - JJ^T > 0,$$

where $G_k, H_1^k, H_2^k, H_{3i}^k, H_{4i}^k$, and J are known real constant matrices of appropriate dimensions, and $Q(t)$ is an uncertain time-varying matrix satisfying $I - Q(t)Q^T(t) \geq 0$.

The uncertain stochastic T–S fuzzy systems driven by Lévy noise can be represented by

$$dx(t) = \sum_{k=1}^{L} h_k\big(\theta(t)\big)\left\{\left[(A_k + \Delta A_k)x(t-) + (B_k + \Delta B_k)u\big(x(t-)\big)\right] dt\right.$$
$$\left. + \sum_{i=1}^{m} \left[(C_{ki} + \Delta C_{ki})x(t-) + (F_{ki} + \Delta F_{ki})u\big(x(t-)\big)\right] dY_i(t)\right\}, \tag{2.4}$$

where

$$h_k\big(\theta(t)\big) = \frac{\prod_{l=1}^r \mu_{kl}(\theta_l(t))}{\sum_{q=1}^L \prod_{l=1}^r \mu_{ql}(\theta_l(t))}, \quad k = 1, 2, \ldots, L,$$

$\mu_{kl}(\theta_l(t))$ is the grade of membership of $\theta_l(t)$ in η_{kl}, and $h_k(\theta(t))$ satisfies $\sum_{k=1}^L h_k(\theta(t)) = 1$, $h_k(\theta(t)) \geq 0$, which have been short for h_k in this paper.

In order to give the Lévy–Itô decomposition of $(Y_i(t), t \geq 0)$, we present the following assumption, which holds in the rest of this paper.

Assumption A1

$$\int_{R^m \setminus \{0\}} \big(|y|^2 \wedge 1\big)\pi(dy) < \infty.$$

So the Lévy process $(Y(t), t \geq 0)$ has the following decomposition:

$$Y_i(t) = b_i t + \sigma B_i(t) + \int_{|y|<1} y_i \widetilde{N}(t, dy) + \int_{|y| \geq 1} y_i N(t, dy), \tag{2.5}$$

where $b_i, \sigma \in R$, $B(t)$ is an m-dimensional Brownian motion, N is an independent $\mathcal{F}_t$-adapted Poisson random measure defined on $R_+ \times R^m \setminus \{0\}$ with compensator $\widetilde{N}$ of the form $\widetilde{N}(dt, dy) = N(dt, dy) - \pi(dy)\,dt$, where π is a Lévy measure. Note that if for some $p \geq 1$, $E|Y(\cdot)|^p < \infty$, then $\int_{|y| \geq 1} |y|^p \pi(dy) < \infty$ and hence the Lévy process $(Y(t), t \geq 0)$ admits the following decomposition:

$$Y_i(t) = \tilde{b}_i t + \sigma B_i(t) + \int_{R^m \setminus \{0\}} y_i \widetilde{N}(t, dy),$$

where $\tilde{b}_i = b_i + \int_{|y| \geq 1} y_i \pi(dy)$. Therefore, (2.4) can be rewritten as

$$
\begin{aligned}
dx(t) = \sum_{k=1}^L h_k \Bigg\{ & \Bigg[\Bigg(A_k + \sum_{i=1}^m \tilde{b}_i C_{ki} + \Delta A_k + \sum_{i=1}^m \tilde{b}_i \Delta C_{ki} \Bigg) x(t-) \\
& + \Bigg(B_k + \sum_{i=1}^m \tilde{b}_i F_{ki} + \Delta B_k + \sum_{i=1}^m \tilde{b}_i \Delta F_{ki} \Bigg) u\big(x(t-)\big) \Bigg] dt \\
& + \sum_{i=1}^m \big[(\sigma C_{ki} + \sigma \Delta C_{ki}) x(t-) + (\sigma F_{ki} + \sigma \Delta F_{ki}) u\big(x(t-)\big) \big] dB_i(t) \\
& + \sum_{i=1}^m \int_{R^m \setminus \{0\}} \big[(C_{ki} + \Delta C_{ki}) x(t-) + (F_{ki} + \Delta F_{ki}) u\big(x(t-)\big) \big] y_i \widetilde{N}(dt, dy) \Bigg\}.
\end{aligned}
\tag{2.6}
$$

Obviously, the drift and diffusion terms satisfy the usual linear growth and local Lipschitz condition, which guarantees the uniqueness and existence of the local solution for system (2.6). It is readily to see that system (2.6) has a trivial solution $x(t) \equiv 0$ for all $t \geq 0$ with the initial condition $x_0 = 0$. And we point that the perturbation of noise preserves the equilibrium of system (2.6).

At the end of this section, let us present the definition of the almost sure asymptotic stability for system (2.6).

Definition 2.1 System (2.6) is said to be almost surely asymptotically stable if, for any $x_0 \in R^n$, $\lim_{t\to\infty} x(t, x_0) = 0$, a.s.

Before the main results have been established, some lemmas should be given for the following theorems.

Lemma 2.1 ([30]) *If any matrix $P > 0$, the inequality*

$$M^T PN + N^T PM \le M^T PM + N^T PN$$

holds.

Lemma 2.2 ([31]) *Suppose that Λ is given by (2.2) and (2.3). With matrices $M^T = M$, S, and N of appropriate dimensions, the inequality*

$$M + S\Lambda N + N^T \Lambda^T S^T \le 0$$

holds for all $Q(t)$ such that $Q(t)Q^T(t) \le I$, if and only if, for some $\delta > 0$,

$$\begin{bmatrix} \delta M & S & \delta N^T \\ S^T & -I & J^T \\ \delta N & J & -I \end{bmatrix} < 0.$$

3 Almost sure asymptotic stability of uncertain system driven by Lévy noise

In this section, its main duty is to consider almost sure asymptotic stability of system (2.6). The form of system (2.6) is very complex, which is simplified with corresponding symbols for convenience as follows:

$$dx(t) = f\big(x(t-)\big)\,dt + g\big(x(t-)\big)\,dB(t) + \int_{R^m \setminus \{0\}} h\big(x(t-), y\big)\widetilde{N}(dt, dy), \tag{3.1}$$

where $f(x(t-))$, $g(x(t-))$, and $h(x(t-), y)$ represent the corresponding parts of system (2.6). For the above simple form, we give the differential operator L in the following assumption.

Assumption A2 Assume that $V \in C^{1,2}(R_+ \times R^n; R_+)$, $\lambda \in L^1(R_+; R_+)$, and $\mu : R^n \to R_+$ are continuous and nonnegative. For any $(t, x) \in R_+ \times R^n$,

$$LV(t, x) \le \lambda(t) - \mu(x), \quad \mu(0) = 0,$$

where L is the differential operator associated with equation (3.1). And L acts on a V function in the following form:

$$LV(t, x) = V_t(t, x) + V_x(t, x)f(x) + \frac{1}{2}\operatorname{tr}\big[g^T(x)V_{xx}(t, x)g(x)\big]$$

$$+ \int_{R^m \setminus \{0\}} \big[V\big(t, x + h(x, y)\big) - V(t, x) - V_x(t, x)h(x, y)\big]\pi(dy),$$

where

$$V_t(t,x) = \frac{\partial V(t,x)}{\partial t}, \qquad V_x(t,x) = \left(\frac{\partial V(t,x)}{\partial x_1}, \dots, \frac{\partial V(t,x)}{\partial x_n}\right)$$

$$V_{xx}(t,x) = \left(\frac{\partial^2 V(t,x)}{\partial x_i \partial x_j}\right)_{n \times n}.$$

In the rest of this section, the almost surely stable result reads as follows.

Theorem 3.1 *Let Assumption* A2 *hold. Further, suppose that there is a function* $V \in C^{1,2}(R_+ \times R^n; R_+)$ *such that, for all* $x \in R^n$, $t \geq 0$,

$$\alpha_1(t, |x|) \leq V(t,x) \leq \alpha_2(t, |x|), \tag{3.2}$$

where $\alpha_1(t,x)$, $\alpha_2(t,x)$ *belong to* $\mathcal{K}_\infty$ *with respect to* x. *Then, for any initial data* x_0, *the trivial solution* $x(t; x_0)$ *of system* (3.1) *is almost surely asymptotically stable, i.e.,*

$$\lim_{t \to \infty} x(t; x_0) = 0, \quad a.s.$$

Proof For notation simplicity, we write $x(t)$ instead of $x(t, x_0)$. It is clear that this theorem holds because of the solution $x(t) \equiv 0$ a.s. for $x_0 = 0$. So, for $x_0 \neq 0$, we have the following proof. Due to the complexity of the proof, we divide the proof into three steps as follows.

Step 1: In this step, we show that system (3.1) is stable in probability and the sample space is divided. Applying the Itô formula to $V(t,x)$ and by Assumption A2, for any $t > 0$, we have

$$V(t, x(t)) = V(0, x_0) + \int_0^t LV(s, x(s-)) \, ds + \int_0^t V_x(s, x(s-)) g(x(s-)) \, dB_j(s)$$

$$+ \int_0^t \int_{R^m \setminus \{0\}} [V(s, x(s-) + h(x(s-), y)) - V(s, x(s-))] \widetilde{N}(ds, dy)$$

$$\leq V(0, x_0) + \int_0^t \lambda(s) \, ds - \int_0^t \mu(x(s-)) \, ds + \int_0^t V_x(s, x(s-)) g(x(s-)) \, dB_j(s)$$

$$+ \int_0^t \int_{R^m \setminus \{0\}} [V(s, x(s-) + h(x(s-), y)) - V(s, x(s-))] \widetilde{N}(ds, dy)$$

$$\leq V(0, x_0) + \int_0^t \lambda(s) \, ds + \int_0^t V_x(s, x(s-)) g(x(s-)) \, dB_j(s)$$

$$+ \int_0^t \int_{R^m \setminus \{0\}} [V(s, x(s-) + h(x(s-), y)) - V(s, x(s-))] \widetilde{N}(ds, dy)$$

$$\doteq V_t(x_0) + M(t),$$

where

$$V_t(x_0) = V(0, x_0) + \int_0^t \lambda(s) \, ds$$

and

$$M(t) = \int_0^t V_x\big(s, x(s-)\big) g\big(x(s-)\big)\, dB_j(s)$$
$$+ \int_0^t \int_{R^m \setminus \{0\}} \big[V\big(s, x(s-) + h\big(x(s-), y\big)\big) - V\big(s, x(s-)\big) \big] \widetilde{N}(ds, dy).$$

Due to $x_0 \in D_{\mathcal{F}_0}^b([-\tau, 0]; R^n)$ and $\int_0^\infty \lambda(t)\, dt < \infty$, $V_t(x_0)$ is bounded. $M(t)$ is a supermartingale with respect to the filtration $\{\mathcal{F}_t\}_{t \geq 0}$ generated by $B(\cdot)$ and $\widetilde{N}(\cdot, \cdot)$. By the supermartingale inequality (Rogers&Williams [19], p. 154, (54.5)), for any function $\delta(\cdot) \in \mathcal{K}_\infty$, one has

$$\mathbb{P}\left\{ \sup_{0 \leq s \leq t} V\big(s, x(s)\big) < \delta\big(V_t(x_0)\big) \right\} \geq 1 - \frac{21 V_t(x_0)}{\delta(V_t(x_0))}, \quad t \geq 0. \tag{3.3}$$

It follows from $\sup_{0 \leq s \leq t} V(s, x(s)) < \delta(V_t(x_0))$ that $\sup_{0 \leq s \leq t} |x| < \upsilon_t(V_t(x_0))$, where $\upsilon_t = \alpha_1^{-1} \circ \delta$, and α_1^{-1} is the inverse function of α_1 with respect to x. For any given $\epsilon > 0$, choosing an appropriate function $\delta(\cdot)$ and by (3.3), we obtain $\frac{21 V_t(x_0)}{\delta(V_t(x_0))} \leq \epsilon$. Then, for $t > 0$,

$$\mathbb{P}\left\{ \sup_{0 \leq s \leq t} |x(s)| < \upsilon_t\big(V_t(x_0)\big) \right\} \geq 1 - \epsilon, \quad t \geq 0,$$

which yields

$$\mathbb{P}\big\{ |x(s)| < \upsilon_t\big(V_t(x_0)\big) \big\} \geq 1 - \epsilon, \quad t \geq 0.$$

Let us decompose the sample space

$$\Omega_1 = \left\{ \omega : \limsup_{t \to \infty} \mu\big(x(t, \omega)\big) = 0 \right\},$$
$$\Omega_2 = \left\{ \omega : \liminf_{t \to \infty} \mu\big(x(t, \omega)\big) > 0 \right\},$$
$$\Omega_3 = \left\{ \omega : \liminf_{t \to \infty} \mu\big(x(t, \omega)\big) = 0 \text{ and } \limsup_{t \to \infty} \mu\big(x(t, \omega)\big) > 0 \right\}.$$

In order to obtain the results, we will show that $\mathbb{P}(\Omega_2) = \mathbb{P}(\Omega_3) = 0$, which implies that $\mathbb{P}(\Omega_1) = 1$.

Step 2: In this step, we prove $\mathbb{P}(\Omega_2) = 0$. By the Itô formula and Assumption A2, we have

$$EV\big(t, x(t)\big) = V(0, x_0) + E\left\{ \int_0^t LV\big(s, x(s-)\big)\, ds \right\}$$
$$\leq V_t(x_0) - E\left\{ \int_0^t \mu\big(x(s)\big)\, ds \right\}, \tag{3.4}$$

where $E(\cdot)$ is the mathematical expectation. This yields $E\{ \int_0^t \mu(x(s))\, ds \} \leq V_t(x_0)$, since $V(t, x) \geq 0$. Letting $t \to \infty$ and using Fatou's lemma give $E\{ \int_0^\infty \mu(x(s))\, ds \} \leq C_{V_t}$, where C_{V_t} is the upper bound of $V_t(x_0)$. Due to the nonnegative function μ, one has $\int_0^\infty \mu(x(s))\, ds \leq C_{V_t}$, which implies that $\mathbb{P}(\Omega_2) = 0$.

Step 3: In this step, we prove $\mathbb{P}(\Omega_3) = 0$ by contradiction. That is, there exist $\epsilon_0 > 0$ and $\epsilon_1 > 0$ such that

$$\mathbb{P}\big\{\mu\big(x(\cdot)\big) \text{ cross from below } \epsilon_1 \text{ to above } 2\epsilon_1 \\ \text{and back infinitely many times}\big\} \geq \epsilon_0. \tag{3.5}$$

For $r > 0$, let $\rho_r = \inf\{t > 0 : |x(t; x_0)| \geq r, x_0 \neq 0\}$, and recall the local boundedness of $f(x)$, $g(x)$, and $\int_{R^m \backslash \{0\}} h(x, y)\pi(dy)$. Then there exist constants $C_f, C_g, C_h \in R_+$ such that $\sup_{|x|<r} |f(x)| \leq C_f$, $\sup_{|x|<r} |g(x)| \leq C_g$, and $\sup_{|x|<r} \int_{R^m \backslash \{0\}} |h(x, y)|^2 \pi(dy) < C_h^2$. By directly calculating, we get that

$$E\Big\{ \sup_{0 \leq s \leq t} \big|x(t \wedge \rho_r) - x_0\big|^2 \Big\}$$
$$= E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} f\big(x(q-)\big)\,dq + \int_0^{s \wedge \rho_r} g\big(x(q-)\big)\,dB_j(q)$$
$$+ \int_0^{s \wedge \rho_r} \int_{R^m \backslash \{0\}} h\big(x(q-), y\big)\widetilde{N}(dq, dy) \Big|^2 \Big\}$$
$$\leq 3E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} f\big(x(q-)\big)\,dq \Big|^2 \Big\} + 3E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} g\big(x(q-)\big)\,dB_j(q) \Big|^2 \Big\} \tag{3.6}$$
$$+ 3E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} \int_{R^m \backslash \{0\}} h\big(x(q-), y\big)\widetilde{N}(dq, dy) \Big|^2 \Big\}$$
$$\leq 3C_f^2 t^2 + 3E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} g\big(x(q-)\big)\,dB_j(q) \Big|^2 \Big\}$$
$$+ 3E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} \int_{R^m \backslash \{0\}} h\big(x(q-), y\big)\widetilde{N}(dq, dy) \Big|^2 \Big\}.$$

Combining Burkholder's inequality (Applebaum [1], Chap. 4, Theorem 4.4.21) with Applebaum [1], Chap. 4, Theorem 4.4.22, Doob's martingale inequality, we obtain

$$E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} g\big(x(q-)\big)\,dB_j(q) \Big|^2 \Big\}$$
$$\leq 4E\Big\{ \int_0^{t \wedge \rho_r} \big|g\big(x(q-)\big)\big|^2\,dq \Big\} \tag{3.7}$$
$$\leq 4C_g^2 t.$$

Applying Kunita's first inequality (Applebaum [1], Chap. 4, Theorem 4.4.23), we get

$$E\Big\{ \sup_{0 \leq s \leq t} \Big| \int_0^{s \wedge \rho_r} \int_{R^m \backslash \{0\}} h\big(x(q-), y\big)\widetilde{N}(dq, dy) \Big|^2 \Big\}$$
$$\leq 4E\Big\{ \sum_{i=1}^m \int_0^{t \wedge \rho_r} \int_{R^m \backslash \{0\}} \big|h\big(x(q-), y\big)\big|^2 \pi(dy)\,dq \Big\} \tag{3.8}$$
$$\leq 4C_h^2 t.$$

Substituting (3.7) and (3.8) into (3.6), we derive

$$E\left\{\sup_{0\leq s\leq t}\left|x(s\wedge\rho_r)-x_0\right|^2\right\}\leq 3C_f^2 t^2+12C_g^2 t+12C_h^2 t,$$

and further by Chebyshev's inequality, for any $\vartheta>0$,

$$\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|x(s\wedge\rho_r)-x_0\right|>\vartheta\right\}$$

$$\leq\frac{E\{\sup_{0\leq s\leq t}|x(s\wedge\rho_r)-x_0|^2\}}{\vartheta^2}$$

$$\leq\frac{3C_f^2 t^2+12C_g^2 t+12C_h^2 t}{\vartheta^2}.$$

Since $\mu(\cdot)$ is continuous, it must be uniformly continuous in the closed ball $\mathcal{O}:=\{x\in R^n: |x|\leq\upsilon_t(k)\}$, where $\upsilon_t=\alpha_1^{-1}\circ\delta$. For given $\varrho>0$, choose a function $\gamma\in\mathcal{K}$ such that, for any $x,y\in\mathcal{O}$, $|x-y|\leq\gamma(\varrho)$, which implies $|\mu(x)-\mu(y)|\leq\varrho$. Then, for $|x_0|\leq r$ and $\epsilon_2>0$,

$$\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|\mu\big(x(s)\big)-\mu(x_0)\right|>\epsilon_2\right\}$$

$$\leq\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|x(s)-x_0\right|>\gamma(\epsilon_2)\text{ and }\sup_{0\leq s\leq t}\left|x(s)\right|<\upsilon_t(r)\right\}+\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|x(s)\right|\geq\upsilon_t(r)\right\}$$

$$\leq\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|x(s\wedge\rho_{\upsilon_t(r)})-x_0\right|>\gamma(\epsilon_2)\right\}+\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|x(s)\right|\geq\upsilon_t(r)\right\}$$

$$\leq\frac{3C_f^2 t^2+12C_g^2 t+3C_h^2 t}{\gamma(\epsilon_2)^2}+\epsilon.$$

Set $\epsilon=\frac{1}{2}$. For any $\epsilon_2>0$, there exists $t^*=t^*(k,\epsilon_2)$ such that

$$\mathbb{P}\left\{\sup_{0\leq s\leq t}\left|\mu\big(x(s)\big)-\mu(x_0)\right|\leq\epsilon_2\right\}\geq\frac{1}{4},\quad\forall t\in(0,t^*].$$

Define a sequence of stopping times

$$\mathcal{T}_1:=\inf\big\{t\geq 0:\mu\big(x(t)\big)<\epsilon_1\big\},$$

$$\mathcal{T}_{2n}:=\inf\big\{t\geq\mathcal{T}_{2n-1}:\mu\big(x(t)\big)>2\epsilon_1\big\},\quad n=1,2,\ldots,$$

$$\mathcal{T}_{2n+1}:=\inf\big\{t\geq\mathcal{T}_{2n}:\mu\big(x(t)\big)<\epsilon_1\big\},\quad n=1,2,\ldots,$$

and set $\inf\emptyset=\infty$. By (3.4), it is easy to get

$$\infty>E\int_0^\infty\mu\big(x(s)\big)\,ds$$

$$\geq\sum_{n=1}^\infty E\left[I_{\{\mathcal{T}_{2n}<\rho_r\}}\int_{\mathcal{T}_{2n}}^{\mathcal{T}_{2n+1}}\mu\big(x(s)\big)\,ds\right]\tag{3.9}$$

$$\geq \epsilon_1 \sum_{n=1}^{\infty} E\big[I_{\{\mathcal{T}_{2n} < \rho_r\}} (\mathcal{T}_{2n+1} - \mathcal{T}_{2n}) \big]$$

$$= \epsilon_1 \sum_{n=1}^{\infty} E\big[I_{\{\mathcal{T}_{2n} < \rho_r\}} E(\mathcal{T}_{2n+1} - \mathcal{T}_{2n} | \mathcal{F}_{\mathcal{T}_{2n}}) \big].$$

By the strong Markov property and setting $\epsilon_1 = 2\epsilon_2$, we obtain

$$E(\mathcal{T}_{2n+1} - \mathcal{T}_{2n} | \mathcal{F}_{\mathcal{T}_{2n}})$$

$$\geq E\left[(\mathcal{T}_{2n+1} - \mathcal{T}_{2n}) I_{\{\sup_{0 \leq s \leq t^*} |\mu(\tilde{x}(s)) - \mu(\tilde{x}_0)| \leq \frac{\epsilon_1}{2}\}} \Big| \mathcal{F}_{\mathcal{T}_{2n}} \right]$$

$$\geq t^* P\left\{ \sup_{0 \leq s \leq t^*} |\mu(\tilde{x}(s)) - \mu(\tilde{x}_0)| \leq \frac{\epsilon_1}{2} \Big| \mathcal{F}_{\mathcal{T}_{2n}} \right\} \tag{3.10}$$

$$\geq \frac{t^*}{4},$$

where $\{\mathcal{T}_{2n} < \rho_r\}$ and $t^* = t^*(k, \epsilon_1/2)$ and $\tilde{x} = x(\cdot + \mathcal{T}_{2n})$. Substituting (3.10) into (3.9), we have

$$\frac{t^* \epsilon_1}{4} \sum_{n=1}^{\infty} P\{\mathcal{T}_{2n} < \rho_r\} < \infty.$$

This, together with the Borel–Cantelli lemma, yields

$$\mathbb{P}\{\mathcal{T}_{2n} < \rho_r \text{ for infinitely many } n\} = 0.$$

Since

$$\{\mathcal{T}_{2n} < \rho_r \text{ for infinitely many } n\} = \{\mathcal{T}_{2n} < \rho_r \text{ for infinitely many } n \text{ and } \rho_r = \infty\}$$

$$\cup \{\mathcal{T}_{2n} < \rho_r \text{ for infinitely many } n \text{ and } \rho_r < \infty\},$$

then

$$\mathbb{P}\{\mathcal{T}_{2n} < \infty \text{ for infinitely many } n \text{ and } \rho_r = \infty\} = 0. \tag{3.11}$$

By (3.2), for any $k > 0$, one has

$$\mathbb{P}\{\rho_r = \infty\} \geq \mathbb{P}\left\{ \sup_{t \geq 0} |x(t)| < k \right\} \geq \mathbb{P}\left\{ \sup_{t \geq 0} |V(t, (x(t))| < \alpha_1(t, k) \right\}$$

$$\geq 1 - \frac{V(0, x_0)}{\alpha_1(t, k)}. \tag{3.12}$$

Letting $k \to \infty$, we obtain $\mathbb{P}\{\rho_r = \infty\} \to 1$, which, together with (3.11), yields

$$\mathbb{P}\{\mathcal{T}_{2n} < \infty \text{ for infinitely many } n\} = 0. \tag{3.13}$$

This does contradict (3.5). Hence $\mathbb{P}(\Omega_3) = 0$, which implies $\lim_{t \to \infty} \mu(x(t)) = 0$ a.s. This, together with the property of the function $\mu(0) = 0$, yields $\lim_{t \to \infty} x(t) = 0$ a.s. The proof is completed. $\qquad\square$

Before concluding this section, we present an example to illustrate Theorem 3.1.

Example 3.1 Consider a scalar stochastic differential equation with jumps in the form

$$dx(t) = k_1 x(t-)\,dt + k_2 x(t-)\,dB(t) + \int_0^\infty k_3 x(t-)y\widetilde{N}(dt, dy), \quad t > 0,$$

$$x(0) = x_0,$$

(3.14)

where $k_i \in R$, $i = 1, 2, 3$, are constants, $B(t)$ is a scalar standard Brownian motion, and $\widetilde{N}(\cdot, \cdot)$ is a compensated Poisson random measure.

Let $V(t, x) = x^2$ for any $x \in R$, we obtain

$$LV(t, x) \le \left[2k_1 + k_2^2 + k_3^2 \int_0^\infty y^2 \pi(dy) \right] x^2,$$

then, by Theorem 3.1, the solution of system (3.14) is almost surely asymptotically stable with choosing the appropriate constant K as the feedback control part, such that

$$2k_1 + k_2^2 + k_3^2 \int_0^\infty y^2 \pi(dy) < 0.$$

4 Stability analysis of uncertain system without feedback

In this section, we present the stability conditions for the uncertain system without feedback, the unforced fuzzy system with $\Lambda = 0$ is presented as

$$dx(t) = \sum_{k=1}^L h_k \left[\left(A_k + \sum_{i=1}^m \tilde{b}_i C_{ki} \right) x(t-)\,dt + \sum_{i=1}^m \sigma C_{ki} x(t-)\,dB_i(t) \right.$$

$$\left. + \sum_{i=1}^m \int_{R^m \setminus \{0\}} C_{ki} x(t-) y_i \widetilde{N}(dt, dy) \right],$$

(4.1)

and the corresponding result is also given in the following.

Theorem 4.1 *If there exists a positive definite matrix $Z = Z^T > 0$ such that*

$$\Xi_1 = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5 & \Phi_6 & \Phi_7 \\ * & -I_m \otimes Z & 0 & 0 & 0 & 0 & 0 \\ * & 0 & -I_m \otimes Z & 0 & 0 & 0 & 0 \\ * & 0 & 0 & -I_m \otimes Z & 0 & 0 & 0 \\ * & 0 & 0 & 0 & -I_m \otimes Z & 0 & 0 \\ * & 0 & 0 & 0 & 0 & -I_m \otimes I & 0 \\ * & 0 & 0 & 0 & 0 & 0 & -I_m \otimes I, \end{bmatrix}$$

$$< 0,$$

(4.2)

where

$$\Phi_1 = A_k Z + Z A_k^T + A_j Z + Z A_j^T + \sum_{i=1}^m \tilde{b}_i C_{ki} Z + \sum_{i=1}^m \tilde{b}_i Z C_{ki}^T + \sum_{i=1}^m \tilde{b}_i C_{ji} Z + \sum_{i=1}^m \tilde{b}_i Z C_{ji}^T,$$

$$\Phi_2 = \left[\sigma Z C_{k1}^T, \sigma Z C_{k2}^T, \ldots, \sigma Z C_{km}^T\right],$$

$$\Phi_3 = \left[\sigma Z C_{j1}^T, \sigma Z C_{j2}^T, \ldots, \sigma Z C_{jm}^T\right],$$

$$\Phi_4 = \left[\sqrt{c_1} Z C_{k1}^T, \sqrt{c_1} Z C_{k2}^T, \ldots, \sqrt{c_1} Z C_{km}^T\right],$$

$$\Phi_5 = \left[\sqrt{c_1} Z C_{j1}^T, \sqrt{c_1} Z C_{j2}^T, \ldots, \sqrt{c_1} Z C_{jm}^T\right],$$

$$\Phi_6 = \left[\sqrt{2c_1} Z C_{k1}^T, \sqrt{2c_1} Z C_{j2}^T, \ldots, \sqrt{2c_1} Z C_{jm}^T\right],$$

$$\Phi_7 = \left[\sqrt{2c_1} I, \sqrt{2c_1} I, \ldots, \sqrt{2c_1} I\right],$$

$$k, j = 1, 2, \ldots, L.$$

Then system (4.1) *is almost surely asymptotically stable.*

Proof Let $V(x) = x^T P x$. Using Lemma 2.1, we obtain that

$$
\begin{aligned}
&LV(t,x) \\
&= 2x^T P \left[\sum_{k=1}^{L} h_k \left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki}\right)\right] x + \sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T \left(\sum_{i=1}^{m} \sigma C_{ki}^T P \sigma C_{ji}\right) x \\
&\quad + \sum_{k=1}^{L} h_k \left\{\sum_{i=1}^{m} \int_{R^m \setminus \{0\}} \left[(C_{ki} x y_i)^T P(C_{ki} x y_i) - x^T P C_{ki} x y_i + (C_{ki} x y_i)^T P x\right] \pi\,(dy)\right\} \\
&\leq \sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T \left[2P\left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki}\right) + \sum_{i=1}^{m} \sigma C_{ki}^T P \sigma C_{ji}\right. \\
&\quad \left. + c_1 \sum_{i=1}^{m}\left(C_{ki}^T P C_{ki} - P C_{ki} + C_{ki}^T P\right)\right] x \\
&\leq \frac{1}{2}\sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T \left[2\left(P A_k + A_k^T P\right) + \sum_{i=1}^{m} 2\tilde{b}_i\left(P C_{ki} + C_{ki}^T P\right)\right. \\
&\quad \left. + \sum_{i=1}^{m}\left(\sigma C_{ki}^T P \sigma C_{ki} + \sigma C_{ji}^T P \sigma C_{ji}\right) + 2c_1 \sum_{i=1}^{m}\left(C_{ki}^T P C_{ki} - P C_{ki} + C_{ki}^T P\right)\right] x \\
&\leq \frac{1}{2}\sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T \left[\left(P A_k + A_k^T P + P A_j + A_j^T P\right)\right. \\
&\quad + \sum_{i=1}^{m} \tilde{b}_i\left(P C_{ki} + C_{ki}^T P + P C_{ji} + C_{ji}^T P\right) \\
&\quad + \sum_{i=1}^{m}\left(\sigma C_{ki}^T P \sigma C_{ki} + \sigma C_{ji}^T P \sigma C_{ji}\right) + \sum_{i=1}^{m}\left(\sqrt{c_1} C_{ki}^T P \sqrt{c_1} C_{ki} + \sqrt{c_1} C_{ji}^T P \sqrt{c_1} C_{ji}\right) \\
&\quad \left. + \sum_{i=1}^{m} 2c_1\left(C_{ki}^T C_{ki} + P^2\right)\right] x \\
&\doteq \frac{1}{2}\sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T (-\Theta_1) x,
\end{aligned}
\tag{4.3}
$$

where $c_1 \in R_+$ depends on Assumption A1, and Θ_1 represents the corresponding matrix. We pre-multiply and post-multiply (4.2) by the matrix $\mathrm{diag}(Z^{-1}, I_{mn}, I_{mn}, I_{mn}, I_{mn}, I_{mn}, I_{mn})$. Then we can have the following inequality by the Schur complements lemma:

$$
\begin{aligned}
&\left(Z^{-1}A_k + A_k^T Z^{-1} + Z^{-1}A_j + A_j^T Z^{-1}\right) + \sum_{i=1}^{m} \tilde{b}_i\left(Z^{-1}C_{ki} + C_{ki}^T Z^{-1} + Z^{-1}C_{ji} + C_{ji}^T Z^{-1}\right) \\
&+ \sum_{i=1}^{m} \sigma C_{ki}^T Z^{-1}\sigma C_{ki} + \sum_{i=1}^{m} \sigma C_{ji}^T Z^{-1}\sigma C_{ji} + \sum_{i=1}^{m} \sqrt{c_1}C_{ki}^T Z^{-1}\sqrt{c_1}C_{ki} \\
&+ \sum_{i=1}^{m} \sqrt{c_1}C_{ji}^T Z^{-1}\sqrt{c_1}C_{ji} + \sum_{i=1}^{m} 2c_1\left(C_{ki}^T C_{ki} + \left(Z^{-1}\right)^2\right) < 0.
\end{aligned}
\tag{4.4}
$$

Letting $Z^{-1} = P$ and substituting (4.4) into (4.3), for any $x \neq 0$, we have

$$
LV(t,x) \le \frac{1}{2}\sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T(-\Theta_1)x < 0.
$$

Set $\mu(x) = 1/2\sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j x^T\Theta_1 x$ and $\lambda(t) = 0$. According to Theorem 3.1 with a form of $V(t,x)$, system (4.1) is almost surely asymptotically stable. The proof is completed. $\square$

On the other hand, the unforced fuzzy system with $\Lambda \neq 0$ is expressed as

$$
\begin{aligned}
dx(t) = \sum_{k=1}^{L} h_k \Bigg[&\left(A_k + \sum_{i=1}^{m}\tilde{b}_i C_{ki} + \Delta A_k + \sum_{i=1}^{m}\tilde{b}_i\Delta C_{ki}\right)x(t-)\,dt \\
&+ \sum_{i=1}^{m}(\sigma C_{ki} + \sigma\Delta C_{ki})x(t-)\,dB_i(t) \\
&+ \sum_{i=1}^{m}\int_{R^m\backslash\{0\}}(C_{ki} + \Delta C_{ki})x(t-)y_i\widetilde{N}(dt,dy)\Bigg],
\end{aligned}
\tag{4.5}
$$

and the corresponding theorem is given for this system.

Corollary 4.1 *If there exists a positive definite matrix $Z = Z^T > 0$ such that*

$$
\begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3^T \\ * & -I & \bar{J}^T \\ * & * & -I \end{bmatrix} < 0,
\tag{4.6}
$$

where

$$
\Xi_1 = \begin{bmatrix}
\Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 & \Phi_5 & \Phi_6 & \Phi_7 \\
* & -I_m \otimes Z & 0 & 0 & 0 & 0 & 0 \\
* & 0 & -I_m \otimes Z & 0 & 0 & 0 & 0 \\
* & 0 & 0 & -I_m \otimes Z & 0 & 0 & 0 \\
* & 0 & 0 & 0 & -I_m \otimes Z & 0 & 0 \\
* & 0 & 0 & 0 & 0 & -I_m \otimes I & 0 \\
* & 0 & 0 & 0 & 0 & 0 & -I_m \otimes I
\end{bmatrix},
$$

$\Phi_1, \Phi_2, \ldots, \Phi_7$ *have been defined in Theorem* 4.1, *and*

$$\Xi_2 = \begin{bmatrix} G_k & G_j & G_k & G_j & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_m \otimes G_k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_m \otimes G_j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_m \otimes G_k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_m \otimes G_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_m \otimes G_k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_m \otimes I \end{bmatrix},$$

$$\Xi_3 = \begin{bmatrix} H_1^k Z & 0_{n \times 6mn} \\ H_1^j Z & 0_{n \times 6mn} \\ \sum_{i=1}^m \tilde{b}_i H_{3i}^k Z & 0_{n \times 6mn} \\ \sum_{i=1}^m \tilde{b}_i H_{3i}^j Z & 0_{n \times 6mn} \\ \sigma H_3^k Z & 0_{mn \times 6mn} \\ \sigma H_3^j Z & 0_{mn \times 6mn} \\ c_1 H_3^k Z & 0_{mn \times 6mn} \\ c_1 H_3^j Z & 0_{mn \times 6mn} \\ \sqrt{2c_1} H_3^k Z & 0_{mn \times 6mn} \\ \sqrt{2c_1} I & 0_{mn \times 6mn} \end{bmatrix},$$

and $\bar{J} = I_{6m+4} \otimes J$, $H_3^k = [(H_{31}^k)^T, (H_{32}^k)^T, \ldots, (H_{3m}^k)^T]^T$, $H_3^j = [(H_{31}^j)^T, (H_{32}^j)^T, \ldots, (H_{3m}^j)^T]^T$. *Then system* (4.5) *is almost surely asymptotically stable.*

Proof In order to seek the appropriate dimension for the matrix calculations, we present the following definitions and verification. Let $\overline{Q}(t) = I_{6m+4} \otimes Q(t)$, then

$$\begin{aligned} \left[I - \overline{Q}(t)\bar{J}\right]^{-1}\overline{Q}(t) &= \left\{I_{6m+4} \otimes \left[(I - Q(t)J)\right]\right\}^{-1}\left[I_{6m+4} \otimes Q(t)\right] \\ &= \left\{I_{6m+4} \otimes \left[(I - Q(t)J)\right]^{-1}\right\}\left[I_{6m+4} \otimes Q(t)\right] \\ &= I_{6m+4} \otimes \left\{\left[(I - Q(t)J)\right]^{-1}Q(t)\right\} = I_{6m+4} \otimes \Lambda = \overline{\Lambda}. \end{aligned}$$

Carrying out a similar calculation, we obtain

$$I - \overline{J}\overline{J}^T = I_{6m+4} \otimes \left(I - JJ^T\right) > 0,$$
$$I - \overline{Q}(t)\overline{Q}(t) = I_{6m+4} \otimes \left(I - Q(t)Q^T(t)\right) \geq 0.$$

Combining the above definitions with (4.6), for positive definite matrix Z and any $\delta > 0$, and

$$\overline{Z} = \delta^{-1} Z, \qquad \overline{\Xi}_1 = \delta^{-1}\Xi_1, \qquad \overline{\Xi}_3 = \delta^{-1}\Xi_3, \tag{4.7}$$

substituting (4.7) into (4.6), yields

$$\begin{bmatrix} \delta\overline{\Xi}_1 & \Xi_2 & \delta\overline{\Xi}_3^T \\ * & -I & \bar{J}^T \\ * & * & -I \end{bmatrix} < 0. \tag{4.8}$$

Using Lemma 2.2 and (4.8), we have

$$\overline{\Xi}_1 + \Xi_2 \overline{\Lambda}\,\overline{\Xi}_3 + \overline{\Xi}_3^T \overline{\Lambda}^T \Xi_2^T < 0. \tag{4.9}$$

Reviewing the definitions of matrices in (4.9), one has

$$
\begin{bmatrix}
\widehat{\Phi}_1 & \widehat{\Phi}_2 & \widehat{\Phi}_3 & \widehat{\Phi}_4 & \widehat{\Phi}_5 & \widehat{\Phi}_6 & \widehat{\Phi}_7 \\
* & -I_m \otimes \overline{Z} & 0 & 0 & 0 & 0 & 0 \\
* & 0 & -I_m \otimes \overline{Z} & 0 & 0 & 0 & 0 \\
* & 0 & 0 & -I_m \otimes \overline{Z} & 0 & 0 & 0 \\
* & 0 & 0 & 0 & -I_m \otimes \overline{Z} & 0 & 0 \\
* & 0 & 0 & 0 & 0 & -I_m \otimes I & 0 \\
* & 0 & 0 & 0 & 0 & 0 & -I_m \otimes I
\end{bmatrix} < 0,
$$

where

$$
\begin{aligned}
\widehat{\Phi}_1 ={}& A_k \overline{Z} + G_k \Lambda H_1^k \overline{Z} + \overline{Z} A_k^T + \overline{Z}\big(H_1^k\big)^T \Lambda^T G_k^T \\
&+ A_j \overline{Z} + G_j \Lambda H_1^j \overline{Z} + \overline{Z} A_j^T + \overline{Z}\big(H_1^j\big)^T \Lambda^T G_j^T \\
&+ \sum_{i=1}^m \tilde{b}_i C_{ki} \overline{Z} + \sum_{i=1}^m \tilde{b}_i G_k \Lambda H_{3i}^k \overline{Z} + \sum_{i=1}^m \tilde{b}_i \overline{Z} C_{ki}^T + \sum_{i=1}^m \tilde{b}_i \overline{Z}\big(H_{3i}^k\big)^T \Lambda^T G_k^T \\
&+ \sum_{i=1}^m \tilde{b}_i C_{ji} \overline{Z} + \sum_{i=1}^m \tilde{b}_i G_j \Lambda H_{3i}^j \overline{Z} + \sum_{i=1}^m \tilde{b}_i \overline{Z} C_{ji}^T + \sum_{i=1}^m \tilde{b}_i \overline{Z}\big(H_{3i}^j\big)^T \Lambda^T G_j^T, \\
\widehat{\Phi}_2 ={}& \Big[\sigma \overline{Z} C_{k1}^T + \sigma \overline{Z}\big(H_{31}^k\big)^T \Lambda^T G_k^T, \sigma \overline{Z} C_{k2}^T + \sigma \overline{Z}\big(H_{32}^k\big)^T \Lambda^T G_k^T, \ldots, \\
& \sigma \overline{Z} C_{km}^T + \sigma \overline{Z}\big(H_{3m}^k\big)^T \Lambda^T G_k^T \Big], \\
\widehat{\Phi}_3 ={}& \Big[\sigma \overline{Z} C_{j1}^T + \sigma \overline{Z}\big(H_{32}^j\big)^T \Lambda^T G_j^T, \sigma \overline{Z} C_{j2}^T + \sigma \overline{Z}\big(H_{32}^j\big)^T \Lambda^T G_j^T, \ldots, \\
& \sigma \overline{Z} C_{jm}^T + \sigma \overline{Z}\big(H_{3m}^j\big)^T \Lambda^T G_j^T \Big], \\
\widehat{\Phi}_4 ={}& \Big[\sqrt{c_1}\,\overline{Z} C_{k1}^T + \sqrt{c_1}\,\overline{Z}\big(H_{31}^k\big)^T \Lambda^T G_k^T, \sqrt{c_1}\,\overline{Z} C_{k2}^T + \sqrt{c_1}\,\overline{Z}\big(H_{32}^k\big)^T \Lambda^T G_k^T, \ldots, \\
& \sqrt{c_1}\,\overline{Z} C_{km}^T + \sqrt{c_1}\,\overline{Z}\big(H_{3m}^k\big)^T \Lambda^T G_k^T \Big], \\
\widehat{\Phi}_5 ={}& \Big[\sqrt{c_1}\,\overline{Z} C_{j1}^T + \sqrt{c_1}\,\overline{Z}\big(H_{31}^j\big)^T \Lambda^T G_j^T, \sqrt{c_1}\,\overline{Z} C_{j2}^T + \sqrt{c_1}\,\overline{Z}\big(H_{32}^j\big)^T \Lambda^T G_j^T, \ldots, \\
& \sqrt{c_1}\,\overline{Z} C_{jm}^T + \sqrt{c_1}\,\overline{Z}\big(H_{3m}^j\big)^T \Lambda^T G_j^T \Big], \\
\widehat{\Phi}_6 ={}& \Big[\sqrt{2c_1}\big(\overline{Z} C_{k1}^T + \overline{Z}\big(H_{31}^k\big)^T \Lambda^T G_k^T\big), \sqrt{2c_1}\big(\overline{Z} C_{k2}^T + \overline{Z}\big(H_{32}^k\big)^T \Lambda^T G_k^T\big), \ldots, \\
& \sqrt{2c_1}\big(\overline{Z} C_{km}^T + \overline{Z}\big(H_{3m}^k\big)^T \Lambda^T G_k^T\big) \Big], \\
\widehat{\Phi}_7 ={}& \Big[\sqrt{2c_1}\,\Lambda^T, \sqrt{2c_1}\,\Lambda^T, \ldots, \sqrt{2c_1}\,\Lambda^T \Big].
\end{aligned}
$$

The rest of the proof is the same as Theorem 4.1, and then the proof is completed. □

5 Stability analysis of uncertain system with feedback

In this section, the stability conditions for the uncertain system with feedback will be obtained. First of all, we have to give the fuzzy control design of feedback as follows.

Control rule k: if $\theta_1(t)$ is η_{k1}, and, ..., $\theta_r(t)$ is η_{kr}, then $u = K_k x$, $k = 1, 2, \ldots, L$. And the overall state feedback fuzzy controller is represented by

$$u = \sum_{k=1}^{L} h_k K_k x.$$

In view of the above control law, the overall closed-loop system is obtained with $\Lambda = 0$ as follows:

$$
\begin{aligned}
dx(t) = \sum_{k=1}^{L}\sum_{j=1}^{L} h_k h_j &\left\{ \left[\left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \right) x(t-) + \left(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \right) K_j x(t-) \right] dt \right. \\
&+ \sum_{i=1}^{m} \left[\sigma C_{ki} x(t-) + \sigma F_{ki} K_j x(t-) \right] dB_i(t) \\
&\left. + \sum_{i=1}^{m} \int_{R^m \setminus \{0\}} \left[C_{ki} x(t-) + F_{ki} K_j x(t-) \right] y_i \widetilde{N}(dt, dy) \right\}.
\end{aligned}
\tag{5.1}
$$

The following theorem is obtained for system (5.1).

Theorem 5.1 *If there exist a sequence of $\{Y_j\}$ and a positive definite matrix $Z = Z^T > 0$ such that*

$$
\Upsilon_1 = \begin{bmatrix}
\Psi_1 & \Psi_2 & \Psi_3 & \Psi_4 & \Psi_5 & \Psi_6 \\
* & -I_m \otimes Z & 0 & 0 & 0 & 0 \\
* & 0 & -I_m \otimes Z & 0 & 0 & 0 \\
* & 0 & 0 & -I_m \otimes Z & 0 & 0 \\
* & 0 & 0 & 0 & -I_m \otimes I & 0 \\
* & 0 & 0 & 0 & 0 & -I_m \otimes I
\end{bmatrix} < 0,
\tag{5.2}
$$

where

$$
\begin{aligned}
\Psi_1 = 2 &\left[\left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \right) Z + Z \left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \right)^T + \left(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \right) Y_j \right. \\
&\left. + Y_j^T \left(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \right)^T \right],
\end{aligned}
$$

$$\Psi_2 = \left[\sigma Z C_{k1}^T + \sigma Y_j^T F_{k1}^T, \sigma Z C_{k2}^T + \sigma Y_j^T F_{k2}^T, \ldots, \sigma Z C_{km}^T + \sigma Y_j^T F_{km}^T \right],$$

$$\Psi_3 = \left[\sigma Z C_{l1}^T + \sigma Y_q^T F_{l1}^T, \sigma Z C_{l2}^T + \sigma Y_q^T F_{l2}^T, \ldots, \sigma Z C_{lm}^T + \sigma Y_q^T F_{lm}^T \right],$$

$$
\begin{aligned}
\Psi_4 = &\left[\sqrt{2c_1} Z C_{k1}^T + \sqrt{2c_1} Y_j^T F_{k1}^T, \sqrt{2c_1} Z C_{k2}^T + \sqrt{2c_1} Y_j^T F_{k2}^T, \ldots, \right. \\
&\left. \sqrt{2c_1} Z C_{km}^T + \sqrt{2c_1} Y_j^T F_{km}^T \right],
\end{aligned}
$$

$$\Psi_5 = \left[\sqrt{2c_1} \left(Z C_{k1}^T + Y_j^T F_{k1}^T \right), \sqrt{2c_1} \left(Z C_{k2}^T + Y_j^T F_{k2}^T \right), \ldots, \sqrt{2c_1} \left(Z C_{km}^T + Y_j^T F_{km}^T \right) \right],$$

$$\Psi_6 = \left[\sqrt{2c_1} I, \sqrt{2c_1} I, \ldots, \sqrt{2c_1} I \right],$$

$$k, j, l, q = 1, 2, \ldots, L.$$

Then system (5.1) *is almost surely asymptotically stable. Moreover, the fuzzy feedback controller is designed as follows*:

$$u = \sum_{j=1}^{L} h_j K_j x, \quad K_j = Y_j Z^{-1}, j = 1, 2, \dots, L.$$

Proof Define $V(x) = x^T P x$. By Lemma 2.1, one has

$$
\begin{aligned}
LV(t,x) = {}& 2x^T P \Bigg\{ \sum_{k=1}^{L} \sum_{j=1}^{L} h_k h_j \Bigg[\Bigg(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \Bigg) x + \Bigg(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \Bigg) K_j x \Bigg] \Bigg\} \\
& + \sum_{k=1}^{L} \sum_{l=1}^{L} \sum_{j=1}^{L} \sum_{q=1}^{L} h_k h_l h_j h_q \Bigg\{ \sum_{i=1}^{m} \big[(\sigma C_{ki} x + \sigma F_{ki} K_j x)^T \\
& \times P (\sigma C_{li} x + \sigma F_{li} K_q x) \big] \Bigg\} \\
& + \sum_{k=1}^{L} \sum_{j=1}^{L} h_k h_j \Bigg\{ \sum_{i=1}^{m} \int_{R^m \setminus \{0\}} \big[(C_{ki} x + F_{ki} K_j x) y_i \big]^T P \big[(C_{ki} x + F_{ki} K_j x) y_i \big] \\
& + \big[(C_{ki} x + F_{ki} K_j x) y_i \big]^T P x - x^T P \big[(C_{ki} x + F_{ki} K_j x) y_i \big] \pi(dy) \Bigg\} \\
\leq {}& \frac{1}{2} \sum_{k=1}^{L} \sum_{l=1}^{L} \sum_{j=1}^{L} \sum_{q=1}^{L} h_k h_l h_j h_q \\
& \times x^T \Bigg\{ 2 \Bigg[P \Bigg(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \Bigg) + P \Bigg(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \Bigg) K_j \\
& + \Bigg(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \Bigg)^T P + K_j^T \Bigg(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \Bigg)^T P \Bigg] \\
& + \sum_{i=1}^{m} \big[(\sigma C_{ki} + \sigma F_{ki} K_j)^T P (\sigma C_{ki} + \sigma F_{ki} K_j) + (\sigma C_{li} + \sigma F_{li} K_q)^T \\
& \times P (\sigma C_{li} + \sigma F_{li} K_q) \big] \\
& + \sum_{i=1}^{m} 2 c_1 \big[(C_{ki} + F_{ki} K_j)^T P (C_{ki} + F_{ki} K_j) + (C_{ki} + F_{ki} K_j)^T \\
& \times (C_{ki} + F_{ki} K_j) + P^2 \big] \Bigg\} x \\
\doteq {}& \frac{1}{2} \sum_{k=1}^{L} \sum_{l=1}^{L} \sum_{j=1}^{L} \sum_{q=1}^{L} h_k h_l h_j h_q x^T (-\Theta_2) x,
\end{aligned}
\tag{5.3}
$$

where $c_1 \in R_+$ was defined in Theorem 4.1, and Θ_2 represents the corresponding matrix. We pre-multiply and post-multiply (5.2) by the matrix $\mathrm{diag}(Z^{-1}, I_{mn}, I_{mn}, I_{mn}, I_{mn}, I_{mn})$, then

we can have the following inequality by the Schur complements lemma:

$$
\begin{aligned}
&2\left[Z^{-1}\left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \right) + Z^{-1}\left(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \right) Y_j Z^{-1} \right.\\
&\quad \left. + \left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \right)^T Z^{-1} + \left(Y_j Z^{-1} \right)^T \left(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \right)^T Z^{-1} \right]\\
&\quad + \sum_{i=1}^{m} \left[\left(\sigma C_{ki} + \sigma F_{ki} Y_j Z^{-1} \right)^T Z^{-1} \left(\sigma C_{ki} + \sigma F_{ki} Y_j Z^{-1} \right) \right.\\
&\quad \left. + \left(\sigma C_{li} + \sigma F_{li} Y_q Z^{-1} \right)^T Z^{-1} \left(\sigma C_{li} + \sigma F_{li} Y_q Z^{-1} \right) \right]\\
&\quad + \sum_{i=1}^{m} 2c_1 \left[\left(C_{ki} + F_{ki} Y_j Z^{-1} \right)^T Z^{-1} \left(C_{ki} + F_{ki} Y_j Z^{-1} \right) \right.\\
&\quad \left. + \left(C_{ki} + F_{ki} Y_j Z^{-1} \right)^T \left(C_{ki} + F_{ki} Y_j Z^{-1} \right) + \left(Z^{-1} \right)^2 \right] < 0.
\end{aligned}
\tag{5.4}
$$

Letting $Z^{-1} = P$, together with $K_j = Y_j Z^{-1}$, substituting (5.4) into (5.3), for any $x \neq 0$, we arrive at

$$
LV(t,x) \leq \frac{1}{2} \sum_{k=1}^{L} \sum_{l=1}^{L} \sum_{j=1}^{L} \sum_{q=1}^{L} h_k h_l h_j h_q x^T (-\Theta_2) x < 0.
\tag{5.5}
$$

Let $\mu(x) = 1/2 \sum_{k=1}^{L} \sum_{l=1}^{L} \sum_{j=1}^{L} \sum_{q=1}^{L} h_k h_l h_j h_q x^T(-\Theta_2)x$ and $\lambda(t) = 0$. In view of Theorem 3.1, system (5.1) is almost surely asymptotically stable. The proof is completed. □

Under the above results, the overall closed-loop system (2.6) is represented as follows:

$$
\begin{aligned}
dx(t) = \sum_{k=1}^{L} \sum_{j=1}^{L} h_k h_j &\left\{ \left[\left(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} + \Delta A_k + \sum_{i=1}^{m} \tilde{b}_i \Delta C_{ki} \right) x(t-) \right. \right.\\
&\quad \left. + \left(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} + \Delta B_k + \sum_{i=1}^{m} \tilde{b}_i \Delta F_{ki} \right) K_j x(t-) \right] dt\\
&\quad + \sum_{i=1}^{m} \left[(\sigma C_{ki} + \sigma \Delta C_{ki}) x(t-) + (\sigma F_{ki} + \sigma \Delta F_{ki}) K_j x(t-) \right] dB_i(t)\\
&\quad \left. + \sum_{i=1}^{m} \int_{R^m \setminus \{0\}} \left[(C_{ki} + \Delta C_{ki}) x(t-) + (F_{ki} + \Delta F_{ki}) K_j x(t-) \right] y_i \widetilde{N}(dt,dy) \right\}.
\end{aligned}
\tag{5.6}
$$

The following theorem is given for system (5.6).

Corollary 5.1 *If there exist a sequence of $\{Y_j\}$ and positive definite matrix $Z = Z^T > 0$ such that*

$$
\begin{bmatrix} \Upsilon_1 & \Upsilon_2 & \Upsilon_3^T \\ * & -I & \widehat{J}^T \\ * & * & -I \end{bmatrix} < 0,
\tag{5.7}
$$

where

$$
\Upsilon_1 = \begin{bmatrix}
\Psi_1 & \Psi_2 & \Psi_3 & \Psi_4 & \Psi_5 & \Psi_6 \\
* & -I_m \otimes Z & 0 & 0 & 0 & 0 \\
* & 0 & -I_m \otimes Z & 0 & 0 & 0 \\
* & 0 & 0 & -I_m \otimes Z & 0 & 0 \\
* & 0 & 0 & 0 & -I_m \otimes I & 0 \\
* & 0 & 0 & 0 & 0 & -I_m \otimes I
\end{bmatrix},
$$

$\Psi_1, \Psi_2, \ldots, \Psi_6$ *defined in Theorem* 5.1, *and*

$$
\Upsilon_2 = \begin{bmatrix}
2G_k & 0 & 0 & 0 & 0 & 0 \\
0 & I_m \otimes G_k & 0 & 0 & 0 & 0 \\
0 & 0 & I_m \otimes G_l & 0 & 0 & 0 \\
0 & 0 & 0 & I_m \otimes G_k & 0 & 0 \\
0 & 0 & 0 & 0 & I_m \otimes G_k & 0 \\
0 & 0 & 0 & 0 & 0 & I_m \otimes I
\end{bmatrix},
$$

$$
\Upsilon_3 = \begin{bmatrix}
H_1^k Z + \sum_{i=1}^m \tilde{b}_i H_{3i}^k Z + H_2^k Y_j + \sum_{i=1}^m \tilde{b}_i H_{4i}^k Y_j & 0_{n \times 5mn} \\
\sigma H_3^k Z + \sigma H_4^k Y_j & 0_{mn \times 5mn} \\
\sigma H_3^l Z + \sigma H_4^l Y_q & 0_{mn \times 5mn} \\
\sqrt{2c_1} H_3^k Z + \sqrt{2c_1} H_4^k Y_j & 0_{mn \times 5mn} \\
\sqrt{2c_1} H_3^k Z + \sqrt{2c_1} H_4^k Y_j & 0_{mn \times 5mn} \\
\sqrt{2c_1} I & 0_{mn \times 5mn}
\end{bmatrix},
$$

and $\widehat{J} = I_{5m+1} \otimes J$, $H_3^k = [(H_{31}^k)^T, (H_{32}^k)^T, \ldots, (H_{3m}^k)^T]^T$, $H_3^l = [(H_{31}^l)^T, (H_{32}^l)^T, \ldots, (H_{3m}^l)^T]^T$, $H_4^k = [(H_{41}^k)^T, (H_{42}^k)^T, \ldots, (H_{4m}^k)^T]^T$, $H_4^l = [(H_{41}^l)^T, (H_{42}^l)^T, \ldots, (H_{4m}^l)^T]^T$. *Then system* (5.6) *is almost surely asymptotically stable, and moreover, the feedback controller is designed as follows:*

$$
u = \sum_{j=1}^L h_j K_j x, \quad K_j = Y_j Z^{-1}, j = 1, 2, \ldots, L.
$$

Proof Let $\widehat{Q}(t) = I_{5m+1} \otimes Q(t)$, then

$$
\begin{aligned}
\left[I - \widehat{Q}(t)\widehat{J}\right]^{-1}\widehat{Q}(t) &= \left\{I_{5m+1} \otimes \left[(I - Q(t)J)\right]\right\}^{-1}\left[I_{5m+1} \otimes Q(t)\right] \\
&= \left\{I_{5m+1} \otimes \left[(I - Q(t)J)\right]^{-1}\right\}\left[I_{5m+1} \otimes Q(t)\right] \\
&= I_{5m+1} \otimes \left\{\left[(I - Q(t)J)\right]^{-1}Q(t)\right\} = I_{5m+1} \otimes \Lambda = \widehat{\Lambda}.
\end{aligned}
$$

Carrying out an argument analogous to that of the above equality, one has

$$
I - \widehat{J}\widehat{J}^T = I_{5m+1} \otimes \left(I - JJ^T\right) > 0,
$$
$$
I - \widehat{Q}(t)\widehat{Q}(t) = I_{5m+1} \otimes \left(I - Q(t)Q^T(t)\right) \geq 0.
$$

Using the above definitions and (5.7), for the positive definite matrix Z and any $\delta > 0$, letting

$$\widehat{Z} = \delta^{-1}Z, \quad \widehat{\Upsilon}_1 = \delta^{-1}\Upsilon_1, \quad \widehat{\Upsilon}_3 = \delta^{-1}\Upsilon_3, \quad \widehat{Y}_j = \delta^{-1}Y_j, \tag{5.8}$$

and substituting (5.8) into (5.7), we have

$$\begin{bmatrix} \delta\widehat{\Upsilon}_1 & \Upsilon_2 & \delta\widehat{\Upsilon}_3^T \\ * & -I & \widehat{J}^T \\ * & * & -I \end{bmatrix} < 0. \tag{5.9}$$

Using Lemma 2.2 and (5.9), we have

$$\widehat{\Upsilon}_1 + \Upsilon_2\widehat{\Lambda}\widehat{\Upsilon}_3 + \widehat{\Upsilon}_3^T\widehat{\Lambda}^T\Upsilon_2^T < 0. \tag{5.10}$$

According to the definitions of matrices in (5.10), it follows that

$$\begin{bmatrix} \widetilde{\Psi}_1 & \widetilde{\Psi}_2 & \widetilde{\Psi}_3 & \widetilde{\Psi}_4 & \widetilde{\Psi}_5 & \widetilde{\Psi}_6 \\ * & -I_m \otimes \widehat{Z} & 0 & 0 & 0 & 0 \\ * & 0 & -I_m \otimes \widehat{Z} & 0 & 0 & 0 \\ * & 0 & 0 & -I_m \otimes \widehat{Z} & 0 & 0 \\ * & 0 & 0 & 0 & -I_m \otimes I & 0 \\ * & 0 & 0 & 0 & 0 & -I_m \otimes I \end{bmatrix} < 0, \tag{5.11}$$

where

$$\begin{aligned} \widetilde{\Psi}_1 = {} & 2\Bigg\{ \Bigg(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \Bigg)\widehat{Z} + \Bigg(G_k\Lambda H_1^k + G_k\Lambda \sum_{i=1}^{m} \tilde{b}_i H_{3i}^k \Bigg)\widehat{Z} + \widehat{Z}\Bigg(A_k + \sum_{i=1}^{m} \tilde{b}_i C_{ki} \Bigg)^T \\ & + \widehat{Z}\Bigg[(H_1^k)^T\Lambda^T G_k^T + \sum_{i=1}^{m} \tilde{b}_i(H_{3i}^k)^T\Lambda^T G_k^T \Bigg] + \Bigg(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \Bigg)\widehat{Y}_j \\ & + \widehat{Y}_j^T\Bigg(B_k + \sum_{i=1}^{m} \tilde{b}_i F_{ki} \Bigg)^T + \Bigg(G_k\Lambda H_2^k + G_k\Lambda \sum_{i=1}^{m} \tilde{b}_i H_{4i}^k \Bigg)\widehat{Y}_j \\ & + \widehat{Y}_j^T\Bigg[(H_2^k)^T\Lambda^T G_k^T + \sum_{i=1}^{m} \tilde{b}_i(H_{4i}^k)^T\Lambda^T G_k^T \Bigg] \Bigg\}, \end{aligned}$$

$$\begin{aligned} \widetilde{\Psi}_2 = {} & \Big[\sigma\widehat{Z}C_{k1}^T + \sigma\widehat{Z}(H_{31}^k)^T\Lambda^T G_k^T + \sigma Y_j^T F_{k1}^T + \sigma Y_j^T(H_{41}^k)^T\Lambda^T G_k^T, \\ & \sigma\widehat{Z}C_{k2}^T + \sigma\widehat{Z}(H_{32}^k)^T\Lambda^T G_k^T + \sigma Y_j^T F_{k2}^T + \sigma Y_j^T(H_{42}^k)^T\Lambda^T G_k^T, \\ & \dots, \sigma\widehat{Z}C_{km}^T + \sigma\widehat{Z}(H_{3m}^k)^T\Lambda^T G_k^T + \sigma Y_j^T F_{km}^T + \sigma Y_j^T(H_{4m}^k)^T\Lambda^T G_k^T \Big], \end{aligned}$$

$$\begin{aligned} \widetilde{\Psi}_3 = {} & \Big[\sigma\widehat{Z}C_{l1}^T + \sigma\widehat{Z}(H_{31}^l)^T\Lambda^T G_l^T + \sigma Y_q^T F_{l1}^T + \sigma Y_q^T(H_{41}^l)^T\Lambda^T G_l^T, \\ & \sigma\widehat{Z}C_{l2}^T + \sigma\widehat{Z}(H_{32}^l)^T\Lambda^T G_l^T + \sigma Y_q^T F_{l2}^T + \sigma Y_q^T(H_{42}^l)^T\Lambda^T G_l^T, \\ & \dots, \sigma\widehat{Z}C_{lm}^T + \sigma\widehat{Z}(H_{3m}^k)^T\Lambda^T G_l^T + \sigma Y_q^T F_{lm}^T + \sigma Y_q^T(H_{4m}^l)^T\Lambda^T G_l^T \Big], \end{aligned}$$

$$\widetilde{\Psi}_4 = \Big[\sqrt{2c_1}\widehat{Z}C_{k1}^T + \sqrt{2c_1}\widehat{Z}(H_{31}^k)^T \Lambda^T G_k^T + \sqrt{2c_1}Y_j^T F_{k1}^T + \sqrt{2c_1}Y_j^T (H_{41}^k)^T \Lambda^T G_k^T,$$
$$\sqrt{2c_1}\widehat{Z}C_{k2}^T + \sqrt{2c_1}\widehat{Z}(H_{32}^k)^T \Lambda^T G_k^T + \sqrt{2c_1}Y_j^T F_{k2}^T + \sqrt{2c_1}Y_j^T (H_{42}^k)^T \Lambda^T G_k^T,$$
$$\dots, \sqrt{2c_1}\widehat{Z}C_{km}^T + \sqrt{2c_1}\widehat{Z}(H_{3m}^k)^T \Lambda^T G_k^T + \sqrt{2c_1}Y_j^T F_{km}^T + \sqrt{2c_1}Y_j^T (H_{4m}^k)^T \Lambda^T G_k^T \Big],$$
$$\widetilde{\Psi}_5 = \Big[\sqrt{2c_1}\widehat{Z}C_{k1}^T + \sqrt{2c_1}\widehat{Z}(H_{31}^k)^T \Lambda^T G_k^T + \sqrt{2c_1}Y_j^T F_{k1}^T + \sqrt{2c_1}Y_j^T (H_{41}^k)^T \Lambda^T G_k^T,$$
$$\sqrt{2c_1}\widehat{Z}C_{k2}^T + \sqrt{2c_1}\widehat{Z}(H_{32}^k)^T \Lambda^T G_k^T + \sqrt{2c_1}Y_j^T F_{k2}^T + \sqrt{2c_1}Y_j^T (H_{42}^k)^T \Lambda^T G_k^T,$$
$$\dots, \sqrt{2c_1}\widehat{Z}C_{km}^T + \sqrt{2c_1}\widehat{Z}(H_{3m}^k)^T \Lambda^T G_k^T + \sqrt{2c_1}Y_j^T F_{km}^T + \sqrt{2c_1}Y_j^T (H_{4m}^k)^T \Lambda^T G_k^T \Big],$$
$$\widetilde{\Psi}_6 = \Big[\sqrt{2c_1}\Lambda^T, \sqrt{2c_1}\Lambda^T, \dots, \sqrt{2c_1}\Lambda^T \Big],$$

and the desired result is obtained. □

6 Discussion of the main results

The paper has considered the almost sure asymptotic stability of an uncertain stochastic T–S fuzzy system driven by Lévy noise. Since the Lévy process has a càdlàg path, we provided the special stochastic techniques. On the other hand, the uncertainty of the system was the linear fractional form, which was different from the previous norm-bounded uncertainty, so the new matrix decomposition method has been used to deal with it.

In the main results, the proofs for almost sure asymptotic stability of stochastic systems with càd-làg path have been presented, which generalized the application scope of the nonnegative semi-martingale convergence theorem. Under this results, we have designed the fuzzy state-feedback controller, so the closed-loop system was robustly almost surely asymptotically stable, and the stabilization criteria were in terms of linear matrix inequalities (LMIs). Therefore, the fuzzy state-feedback controller for stabilization was easier to design and apply. Of course, the above research have also made some problems to be easier and more feasible, such as H_2/H_∞, output feedback control, design of observer, and so on.

Availability of data and materials
All the data and material of this manuscript can be available.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors have achieved equal contributions to each part of this paper. All the authors read and approved the final manuscript.

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