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# The plethora of explicit solutions of the fractional KS equation through liquid–gas bubbles mix under the thermodynamic conditions via Atangana–Baleanu derivative operator

Chen Yue<sup>1</sup>, Mostafa M.A. Khater<sup>1\*</sup>, Raghda A.M. Attia<sup>1,2</sup> and Dianchen Lu<sup>1</sup>

\*Correspondence:

[mostafa.khater2024@yahoo.com](mailto:mostafa.khater2024@yahoo.com)

<sup>1</sup>Department of Mathematics,  
Faculty of Science, Jiangsu  
University, Zhanjiang, China  
Full list of author information is  
available at the end of the article

## Abstract

Novel explicit wave solutions are constructed for the Kudryashov–Sinelshchikov (KS) equation through liquid–gas bubbles mix under the thermodynamic conditions. A new fractional definition (Atangana–Baleanu derivative operator) is employed through the modified Khater method to get new wave solutions in distinct types of this model that is used to describe the phenomena of pressure waves through liquid–gas bubbles mix under the thermodynamic conditions. The stability property of the obtained solutions is tested to show the ability of our obtained solutions through the physical experiments. The novelty and advantage of the proposed method are illustrated by applying to this model. Some sketches are plotted to show more about the dynamical behavior of this model.

**MSC:** 35C08; 34K20; 35Q53

**Keywords:** Fractional nonlinear Kudryashov–Sinelshchikov (KS) equation; *ABR* fractional operator; Modified Khater (mK) method; Stability property

## 1 Introduction

Nowadays, many natural phenomena have been derived in nonlinear partial differential equations with an integer order. These models are included in various and distinct branches of science such as chemistry, physics, biology, engineering, economy, etc. However, using the integer order of these models is not sufficient where the nonlocal property does not appear in these formulas so that many models have been formulated in fractional nonlinear partial differential equations specially to discover that kind of property. Studying these models gives more novel properties of them specially by using the computational and numerical schemes. For using most of these schemes, one needs fractional operators to convert the fractional formulas to nonlinear ordinary differential equations with integer order such as Caputo, Caputo–Fabrizio definition, fractional Riemann–Liouville derivatives, conformable fractional derivative, and so on [1–15]. These fractional opera-

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tors have been employed to investigate the exact and numerical solutions of many phenomena. These solutions have been obtained in explicit formulas by using different analytical schemes such as [16–25].

Recently, the mK method has been formulated and applied to distinct physical models such as the (2 + 1)-dimensional KD equation, the complex Ginzburg–Landau model, KdV equation, the fractional (N + 1) Sinh–Gordon, biological population, equal width, modified equal width, Duffing equations, and so on [26–35].

This method depends on a new auxiliary equation, which is equal to the auxiliary equation of the main future of the modified mathematical technique [36]. The auxiliary equation of the mK method is given by

$$\mathcal{E}'(\varphi) = \frac{1}{\ln(Q)} [\delta Q^{\mathcal{E}(\varphi)} + \varrho Q^{-\mathcal{E}(\varphi)} + \chi], \tag{1}$$

where  $\delta, \varrho, \chi, Q$  are arbitrary constants; whereas the auxiliary equation of the extended exponential-expansion function method is given by

$$\Psi'^2(\wp) = \beta_1 \Psi^2(\wp) + \beta_2 \Psi^3(\wp) + \beta_3 \Psi^4(\wp), \tag{2}$$

where  $\beta_i$  ( $i = 1, 2, 3$ ) are arbitrary constants. So Eqs. (1) and (2) are equal when  $[Q^{\mathcal{E}(\varphi)} = \Psi(\wp), \varrho = 0, \beta_1 = \chi^2, \beta_2 = 2\delta\chi, \beta_3 = \delta^2]$ . Using this technique leads to the equality of the mK auxiliary equation with many other analytical methods, but the mK method can obtain more solutions than most of them. This equivalence shows superiority, power, and productivity of the mK method.

In this context, the mK method is employed to construct new formulas of solutions for the fractional nonlinear KS equation, which is given by [37–43]:

$$D_x^\vartheta S_t + \lambda S S_x + S_{xxx} - \alpha (S S_{xx})_x - \mu S_x S_{xx} - \beta S_{xx} - \sigma (S S_x)_x = 0, \tag{3}$$

where  $[S = S(x, t)]$  is the function that is used to describe the dynamical behavior of the nonlinear wave processes in a liquid containing gas bubbles. Additionally,  $[\lambda, \alpha, \mu, \beta, \sigma]$  are arbitrary constants while  $[\vartheta \in ]0, 1[$ ]. This equation was defined by Kudryashov and Sinelshchikov in 2010 to describe the nonlinear wave processes in a liquid containing gas bubbles. This equation is also considered as a general form of the well-known models KdV and KdV–Burger equations under the following conditions:

- For  $[\mu = \alpha = \sigma = \beta = 0]$ , Eq. (3) equals the well-known Korteweg–de Vries equation.
- For  $[\alpha = \mu = \sigma = 0]$ , Eq. (3) equals the well-known Korteweg–de Vries Burgers equation.
- For  $[\lambda = \alpha = 1, \beta = \sigma = 0]$ , Eq. (3) equals the generalized Korteweg–de Vries equation.

### 1.1 Fractional $\mathcal{ABR}$ operator

The  $\mathcal{ABR}$  fractional operator is given by [44–48]

$${}^{\mathcal{ABR}}D_{a^+}^\vartheta \mathcal{F}(t) = \frac{\mathcal{B}(\vartheta)}{1 - \vartheta} \frac{d}{dt} \int_a^t \mathcal{F}(x) \mathcal{G}_\vartheta \left( \frac{-\vartheta(t - \vartheta)^\vartheta}{1 - \vartheta} \right) dx, \tag{4}$$

where  $\mathcal{G}_\vartheta$  is the Mittag-Leffler function, defined by the following formula:

$$\mathcal{G}_\vartheta\left(\frac{-\vartheta(t-\vartheta)^\vartheta}{1-\vartheta}\right) = \sum_{n=0}^\infty \frac{\left(\frac{-\vartheta}{1-\vartheta}\right)^n (t-\vartheta)^{\vartheta n}}{\Gamma(\vartheta n + 1)},$$

and  $\mathcal{B}(\vartheta)$  is a normalization function. Thus

$${}^{ABR}\mathcal{D}_{a^+}^\vartheta \mathcal{F}(x) = \frac{\mathcal{B}(\vartheta)}{1-\vartheta} \sum_{n=0}^\infty \left(\frac{-\vartheta}{1-\vartheta}\right)^n {}^{RL}\mathcal{I}_a^{\vartheta n} \mathcal{F}(x). \tag{5}$$

For further properties of this fractional operator, you can see [44, 49, 50]. This leads to

$$\mathcal{S}(x, t) = \mathcal{S}(\wp), \quad \wp = x + \frac{c(1-\vartheta)t^{-\vartheta n}}{\mathcal{B}(\vartheta) \sum_{n=0}^\infty \left(\frac{-\vartheta}{1-\vartheta}\right)^n \Gamma(1-\vartheta n)},$$

where  $c$  is an arbitrary constant. This wave transformation converts Eq. (3) to ODE. Integration of the obtained ODEs once with zero constant of the integration gives

$$c\mathcal{S} + \frac{\lambda}{2}\mathcal{S}^2 + \mathcal{S}'' - \alpha\mathcal{S}\mathcal{S}'' - \frac{\mu}{2}\mathcal{S}'^2 - \beta\mathcal{S}' - \sigma\mathcal{S}\mathcal{S}' = 0. \tag{6}$$

Calculating the homogeneous balance value in Eq. (6) yields  $N = 2$ . Thus, the general formula of solution according to the mK method is given by

$$\begin{aligned} \mathcal{S}(\wp) &= \sum_{i=1}^N a_i \mathcal{Q}^{i\mathcal{E}(\wp)} + \sum_{i=1}^N b_i \mathcal{Q}^{-i\mathcal{E}(\wp)} + a_0 \\ &= a_2 \mathcal{Q}^{2\mathcal{E}(\wp)} + a_1 \mathcal{Q}^{\mathcal{E}(\wp)} + a_0 + b_1 \mathcal{Q}^{-\mathcal{E}(\wp)} + b_2 \mathcal{Q}^{-2\mathcal{E}(\wp)}, \end{aligned} \tag{7}$$

where  $a_0, a_1, a_2, b_1, b_2$  are arbitrary constants.

The rest of this article is arranged in the following order. In Sect. 2 we apply the mK method to the nonlinear fractional nonlinear  $(2 + 1)$ -BLMP equation. Moreover, some sketches are given to show more physical properties of both models. Section 4 discusses the stability property of the obtained solutions. Section 5 gives the conclusion of the whole research.

### 2 Abundant wave solutions of the fractional KS equation

Applying the mK method with its auxiliary equation and the suggested general solutions for the fractional KS equation leads to a system of algebraic equations. Using Mathematica 11.2 to find the values of the parameters in this system leads to the following:

*Family I*

$$\left[ \begin{aligned} a_1 &\rightarrow \frac{a_2(\mu\chi + 3\sigma)}{\delta\mu}, b_1 \rightarrow 0, b_2 \rightarrow 0, \\ \lambda &\rightarrow -\frac{4\delta^2(a_0\mu + 3)}{a_2} + \frac{8\delta\mu\varrho}{3} + \frac{15\sigma^2}{\mu} + \frac{\mu\chi^2}{3} + 6\sigma\chi, \alpha \rightarrow -\frac{\mu}{3}, \\ &\text{where } (\delta \neq 0, \mu \neq 0, a_2 \neq 0) \end{aligned} \right].$$

Consequently, the closed forms of solutions for the fractional KS model are given by:

When  $[\chi^2 - 4\delta\varrho < 0 \ \& \ \delta \neq 0]$

$$\begin{aligned}
 \mathcal{S}_1 = & a_0 - \frac{1}{4\delta^2\mu} \left[ a_2 \left( \mu(\chi^2 - 4\delta\varrho) \right. \right. \\
 & \times \tan^2 \left( \frac{1}{2} \sqrt{4\delta\varrho - \chi^2} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & - 6\sigma \sqrt{4\delta\varrho - \chi^2} \tan \left( \frac{1}{2} \sqrt{4\delta\varrho - \chi^2} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \left. \left. + \chi(\mu\chi + 6\sigma) \right) \right], \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S}_2 = & a_0 - \frac{1}{4\delta^2\mu} \left[ a_2 \left( \mu(\chi^2 - 4\delta\varrho) \right. \right. \\
 & \times \cot^2 \left( \frac{1}{2} \sqrt{4\delta\varrho - \chi^2} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & - 6\sigma \sqrt{4\delta\varrho - \chi^2} \cot \left( \frac{1}{2} \sqrt{4\delta\varrho - \chi^2} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \left. \left. + \chi(\mu\chi + 6\sigma) \right) \right]. \tag{9}
 \end{aligned}$$

When  $[\chi^2 - 4\delta\varrho > 0 \ \& \ \delta \neq 0]$

$$\begin{aligned}
 \mathcal{S}_3 = & a_0 - \frac{1}{4\delta^2\mu} \left[ a_2 \left( \mu(4\delta\varrho - \chi^2) \right. \right. \\
 & \times \tanh^2 \left( \frac{1}{2} \sqrt{\chi^2 - 4\delta\varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & + 6\sigma \sqrt{\chi^2 - 4\delta\varrho} \tanh \left( \frac{1}{2} \sqrt{\chi^2 - 4\delta\varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \left. \left. + \chi(\mu\chi + 6\sigma) \right) \right], \tag{10}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{S}_4 = & a_0 - \frac{1}{4\delta^2\mu} \left[ a_2 \left( \mu(4\delta\varrho - \chi^2) \right. \right. \\
 & \times \coth^2 \left( \frac{1}{2} \sqrt{\chi^2 - 4\delta\varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & + 6\sigma \sqrt{\chi^2 - 4\delta\varrho} \coth \left( \frac{1}{2} \sqrt{\chi^2 - 4\delta\varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \left. \left. + \chi(\mu\chi + 6\sigma) \right) \right]. \tag{11}
 \end{aligned}$$

When  $[\delta\varrho > 0 \ \& \ \varrho \neq 0 \ \& \ \delta \neq 0 \ \& \ \chi = 0]$

$$\begin{aligned}
 \mathcal{S}_5 = & a_0 + \frac{1}{\delta^2\mu} \left[ a_2 \tan \left( \sqrt{\delta\varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \right. \\
 & \left. \times \left( \delta\mu\varrho \tan \left( \sqrt{\delta\varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) + 3\sigma \sqrt{\delta\varrho} \right) \right], \tag{12}
 \end{aligned}$$

$$\begin{aligned}
 S_6 = a_0 + \frac{1}{\delta^2 \mu} & \left[ a_2 \cot \left( \sqrt{\delta \varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \right. \\
 & \left. \times \left( \delta \mu \varrho \cot \left( \sqrt{\delta \varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) - 3\sigma \sqrt{\delta \varrho} \right) \right]. \tag{13}
 \end{aligned}$$

When  $[\delta \varrho < 0 \ \& \ \varrho \neq 0 \ \& \ \delta \neq 0 \ \& \ \chi = 0]$

$$\begin{aligned}
 S_7 = a_0 - \frac{a_2}{\delta^2 \mu} & \tanh \left( \sqrt{-\delta \varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \times \left( \delta \mu \varrho \tanh \left( \sqrt{-\delta \varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) + 3\sigma \sqrt{-\delta \varrho} \right), \tag{14}
 \end{aligned}$$

$$\begin{aligned}
 S_8 = a_0 - \frac{a_2}{\delta^2 \mu} & \coth \left( \sqrt{-\delta \varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \times \left( \delta \mu \varrho \coth \left( \sqrt{-\delta \varrho} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) + 3\sigma \sqrt{-\delta \varrho} \right). \tag{15}
 \end{aligned}$$

When  $[\chi = 0 \ \& \ \varrho = -\delta]$

$$\begin{aligned}
 S_9 = (\mu \varrho)^{-1} & \left( \exp \left( 2\varrho \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) - 1 \right)^{-2} \\
 & \times \left[ (a_2(\mu \varrho - 3\sigma) + a_0 \mu \varrho) \exp \left( 4\varrho \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \right. \\
 & - 2(a_0 - a_2)\mu \varrho \exp \left( 2\varrho \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \\
 & \left. + a_0 \mu \varrho + a_2 \mu \varrho + 3a_2 \sigma \right]. \tag{16}
 \end{aligned}$$

When  $[\chi = \delta = \kappa \ \& \ \varrho = 0]$

$$\begin{aligned}
 S_{10} = a_0 + \frac{a_2}{4} & \left[ \frac{12\sigma}{\kappa \mu} \left( \frac{1}{1 - \exp \left( \kappa \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right)} - 1 \right) \right. \\
 & \left. + \operatorname{csch}^2 \left( \frac{\kappa}{2} \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) \right]. \tag{17}
 \end{aligned}$$

When  $[\varrho = 0 \ \& \ \chi \neq 0 \ \& \ \delta \neq 0]$

$$\begin{aligned}
 S_{11} = a_0 + \frac{a_2 \chi}{\delta^2} & \left[ \frac{2(\mu \chi - 3\sigma)}{\mu \left( \delta \exp \left( \chi \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) - 2} \right. \\
 & \left. + \frac{4\chi}{\left( \delta \exp \left( \chi \left( x - \frac{(\vartheta - 1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left(-\frac{\vartheta}{1-\vartheta}\right)^n \Gamma(1 - \vartheta n)} \right) \right) - 2 \right)^2} - \frac{3\sigma}{\mu} \right]. \tag{18}
 \end{aligned}$$

When  $[\chi = \varrho = 0 \ \& \ \delta \neq 0]$

$$S_{12} = \frac{a_2 \left( 1 - \frac{3\sigma \left( x - \frac{(\vartheta-1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n)} \right)}{\mu} \right)}{\delta^2 \left( x - \frac{(\vartheta-1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n)} \right)^2} + a_0. \tag{19}$$

When  $[\chi = 0 \ \& \ \varrho = \delta]$

$$S_{13} = a_0 + \frac{1}{\mu\varrho} \left[ a_2 \tan \left( -\frac{(\alpha-1)c\varrho t^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n)} + C + x\varrho \right) \right. \\ \left. \times \left( \mu\varrho \tan \left( \varrho \left( x - \frac{(\vartheta-1)ct^{-2\vartheta}}{B(\vartheta) \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n)} \right) + C \right) + 3\sigma \right) \right]. \tag{20}$$

When  $[\chi^2 - 4\delta\varrho = 0]$

$$S_{14} = a_0 + \frac{1}{\delta\mu\chi^4 \left( xB(\vartheta)t^{2\vartheta} \left( \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n) \right) - \alpha c + c \right)^2} \\ \times \left[ 2a_2\varrho \left( (\alpha-1)c\chi - B(\vartheta)t^{2\vartheta}(\chi x + 2) \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n) \right) \right. \\ \times \left( B(\vartheta)t^{2\vartheta} \left( \sum_{n=0}^{\infty} \left( -\frac{\vartheta}{1-\vartheta} \right)^n \Gamma(1-\vartheta n) \right) \left( -(2\delta\mu\varrho(\chi x + 2) - \chi^2 x(\mu\chi + 3\sigma)) \right) \right. \\ \left. \left. - (\alpha-1)c\chi(\chi(\mu\chi + 3\sigma) - 2\delta\mu\varrho) \right) \right]. \tag{21}$$

*Family II*

$$\left[ a_0 \rightarrow \frac{b_2\sigma(\mu(2\delta\mu\varrho + \chi(\mu\chi - 9\sigma)) + 18\sigma^2) - \mu\varrho^2(\beta\mu + 15\sigma)}{6\mu^2\sigma\varrho^2}, a_1 \rightarrow 0, a_2 \rightarrow 0, \right. \\ \left. b_1 \rightarrow \frac{b_2(\mu\chi - 3\sigma)}{\mu\varrho}, \lambda \rightarrow \frac{1}{3} \left( \frac{2\varrho^2(\beta\mu - 3\sigma)}{b_2\sigma} + 4\delta\mu\varrho + \frac{9\sigma^2}{\mu} - \mu\chi^2 \right), \alpha \rightarrow -\frac{\mu}{3}, \right. \\ \left. \text{where } \left( b_2 \neq 0, \sigma \neq 0, \mu \neq 0, \mu \neq \frac{3\sigma}{\chi}, \beta \neq \frac{3\sigma}{\mu} \right) \right].$$

Consequently, the closed forms of solutions for the fractional KS model are given by:

When  $[\chi^2 - 4\delta\varrho < 0 \ \& \ \delta \neq 0]$

$$S_{15} = \frac{4b_2\delta^2}{(\chi - \sqrt{4\delta\varrho - \chi^2} \tan(\frac{1}{2}\Delta\sqrt{4\delta\varrho - \chi^2}))^2} + \frac{b_2(\mu(2\delta\mu\varrho + \chi(\mu\chi - 9\sigma)) + 18\sigma^2)}{6\mu^2\varrho^2} \\ + \frac{6b_2\delta\sigma - 2b_2\delta\mu\chi}{\mu\chi\varrho - \mu\varrho\sqrt{4\delta\varrho - \chi^2} \tan(\frac{1}{2}\Delta\sqrt{4\delta\varrho - \chi^2})} - \frac{\beta}{6\sigma} - \frac{5}{2\mu}, \tag{22}$$

$$S_{16} = \frac{4b_2\delta^2}{(\chi - \sqrt{4\delta\varrho - \chi^2} \cot(\frac{1}{2}\Delta\sqrt{4\delta\varrho - \chi^2}))^2} + \frac{b_2(\mu(2\delta\mu\varrho + \chi(\mu\chi - 9\sigma)) + 18\sigma^2)}{6\mu^2\varrho^2} \\ + \frac{6b_2\delta\sigma - 2b_2\delta\mu\chi}{\mu\chi\varrho - \mu\varrho\sqrt{4\delta\varrho - \chi^2} \cot(\frac{1}{2}\Delta\sqrt{4\delta\varrho - \chi^2})} - \frac{\beta}{6\sigma} - \frac{5}{2\mu}. \tag{23}$$

When  $[\chi^2 - 4\delta\varrho > 0 \ \& \ \delta \neq 0]$

$$S_{17} = \frac{4b_2\delta^2}{(\sqrt{\chi^2 - 4\delta\varrho} \tanh(\frac{1}{2}\Delta\sqrt{\chi^2 - 4\delta\varrho}) + \chi)^2} + \frac{b_2(\mu(2\delta\mu\varrho + \chi(\mu\chi - 9\sigma)) + 18\sigma^2)}{6\mu^2\varrho^2} + \frac{6b_2\delta\sigma - 2b_2\delta\mu\chi}{\mu\varrho\sqrt{\chi^2 - 4\delta\varrho} \tanh(\frac{1}{2}\Delta\sqrt{\chi^2 - 4\delta\varrho}) + \mu\chi\varrho} - \frac{\beta}{6\sigma} - \frac{5}{2\mu}, \tag{24}$$

$$S_{18} = \frac{4b_2\delta^2}{(\sqrt{\chi^2 - 4\delta\varrho} \coth(\frac{1}{2}\Delta\sqrt{\chi^2 - 4\delta\varrho}) + \chi)^2} + \frac{b_2(\mu(2\delta\mu\varrho + \chi(\mu\chi - 9\sigma)) + 18\sigma^2)}{6\mu^2\varrho^2} + \frac{6b_2\delta\sigma - 2b_2\delta\mu\chi}{\mu\varrho\sqrt{\chi^2 - 4\delta\varrho} \coth(\frac{1}{2}\Delta\sqrt{\chi^2 - 4\delta\varrho}) + \mu\chi\varrho} - \frac{\beta}{6\sigma} - \frac{5}{2\mu}. \tag{25}$$

When  $[\delta\varrho > 0 \ \& \ \varrho \neq 0 \ \& \ \delta \neq 0 \ \& \ \chi = 0]$

$$S_{19} = \frac{1}{6\mu^2} \left[ \frac{2b_2(\delta\mu^2\varrho(3 \cot^2(\Delta\sqrt{\delta\varrho}) + 1) - 9\mu\sigma\sqrt{\delta\varrho} \cot(\Delta\sqrt{\delta\varrho}) + 9\sigma^2)}{\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right], \tag{26}$$

$$S_{20} = \frac{1}{6\mu^2} \left[ \frac{2b_2(\delta\mu^2\varrho(3 \tan^2(\Delta\sqrt{\delta\varrho}) + 1) + 9\mu\sigma\sqrt{\delta\varrho} \tan(\Delta\sqrt{\delta\varrho}) + 9\sigma^2)}{\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right]. \tag{27}$$

When  $[\delta\varrho < 0 \ \& \ \varrho \neq 0 \ \& \ \delta \neq 0 \ \& \ \chi = 0]$

$$S_{21} = \frac{1}{6\mu^2} \left[ \frac{2b_2(3\delta\mu^2\varrho \cot^2(\sqrt{\delta}\Delta\sqrt{\varrho}) + \delta\mu^2\varrho - 9\mu\sigma\sqrt{-\delta\varrho} \coth(\Delta\sqrt{-\delta\varrho}) + 9\sigma^2)}{\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right], \tag{28}$$

$$S_{22} = \frac{1}{6\mu^2} \left[ \frac{2b_2(3\delta\mu^2\varrho \tan^2(\sqrt{\delta}\Delta\sqrt{\varrho}) + \delta\mu^2\varrho - 9\mu\sigma\sqrt{-\delta\varrho} \tanh(\Delta\sqrt{-\delta\varrho}) + 9\sigma^2)}{\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right]. \tag{29}$$

When  $[\chi = 0 \ \& \ \varrho = -\delta]$

$$S_{23} = \frac{1}{6\mu^2} \left[ \frac{2b_2(-\mu^2\varrho^2 + 3\mu\varrho \tanh(\Delta\varrho)(\mu\varrho \tanh(\Delta\varrho) - 3\sigma) + 9\sigma^2)}{\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right]. \tag{30}$$

When  $[\chi = \frac{\varrho}{2} = \kappa \ \& \ \delta = 0]$

$$S_{24} = \frac{1}{6} \left( b_2 \left( \frac{(\kappa\mu - 3\sigma)(\kappa(\frac{12\mu}{e^{\kappa\Delta} - 2} + \mu) - 6\sigma)}{4\kappa^2\mu^2} + \frac{6}{(e^{\kappa\Delta} - 2)^2} \right) - \frac{\beta}{\sigma} - \frac{15}{\mu} \right). \tag{31}$$

When  $[\chi = \delta = 0 \ \& \ \varrho \neq 0]$

$$S_{25} = \frac{1}{6\mu^2} \left[ \frac{6b_2(\mu^2 - 3\mu\Delta\sigma + 3\Delta^2\sigma^2)}{\Delta^2\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right]. \tag{32}$$

When  $[\chi = 0 \ \& \ \varrho = \delta]$

$$S_{26} = \frac{1}{6\mu^2} \left[ \frac{2b_2(3\mu\varrho \cot(C + \Delta\varrho)(\mu\varrho \cot(C + \Delta\varrho) - 3\sigma) + \mu^2\varrho^2 + 9\sigma^2)}{\varrho^2} - \frac{\mu(\beta\mu + 15\sigma)}{\sigma} \right]. \tag{33}$$

When  $[\delta = 0 \ \& \ \chi \neq 0 \ \& \ \varrho \neq 0]$

$$S_{27} = \frac{1}{6} \left( b_2 \left( \frac{(\mu\chi - 6\sigma)(\mu\chi - 3\sigma)}{\mu^2\varrho^2} + \frac{18\sigma\chi - 6\mu\chi^2}{\mu\varrho^2 - \mu\chi\varrho e^{\Delta\chi}} + \frac{6\chi^2}{(\varrho - \chi e^{\Delta\chi})^2} \right) - \frac{\beta}{\sigma} - \frac{15}{\mu} \right). \tag{34}$$

When  $[\chi^2 - 4\delta\varrho = 0]$

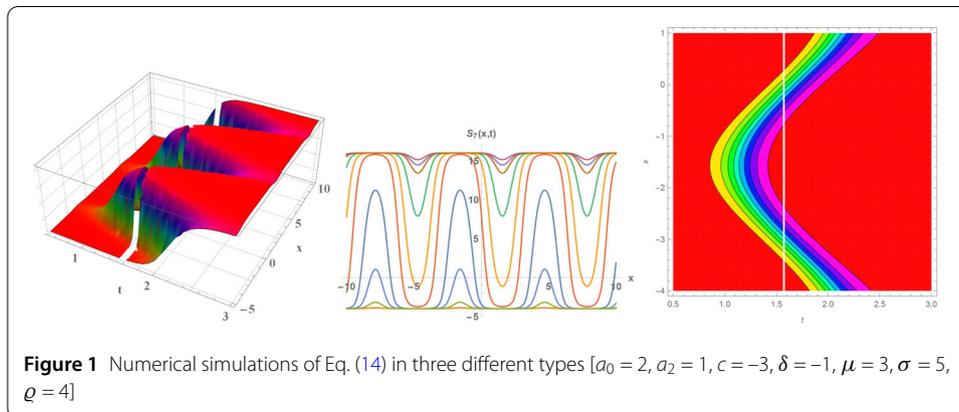
$$S_{28} = \frac{1}{12\mu^2\varrho^2} \left[ b_2 \left( \mu \left( 4\delta\mu\varrho - \frac{\chi(\mu\chi(\Delta\chi(\Delta\chi + 4) - 8) + 36\sigma(\Delta\chi + 2))}{(\Delta\chi + 2)^2} \right) + 36\sigma^2 \right) - \frac{2\mu\varrho^2(\beta\mu + 15\sigma)}{\sigma} \right]. \tag{35}$$

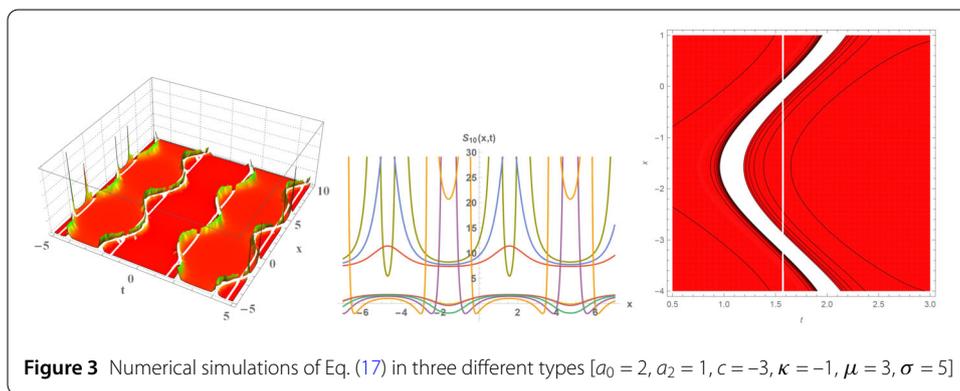
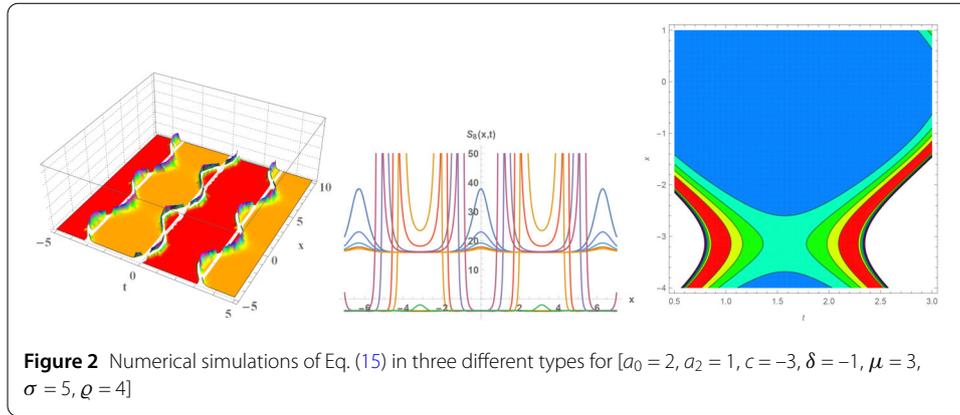
Here  $[\Delta = x - \frac{(\vartheta-1)ct^{-2\vartheta}}{B(\vartheta)\sum_{n=0}^{\infty} (-\frac{\vartheta}{1-\vartheta})^n \Gamma(1-\vartheta n)}]$ .

### 3 Figure interpretation

This section gives a physical interpretation of the shown figures as follows:

- Fig. 1 explains periodic breathes waves equations for Eq. (14) when  $[a_0 = 2, a_2 = 1, c = -3, \delta = -1, \mu = 3, \sigma = 5, \varrho = 4]$ .
- Fig. 2 explains periodic solitary waves equations for Eq. (15) when  $[a_0 = 2, a_2 = 1, c = -3, \delta = -1, \mu = 3, \sigma = 5, \varrho = 4]$ .
- Fig. 3 explains periodic solitary waves equations for Eq. (17) when  $[a_0 = 2, a_2 = 1, c = -3, \kappa = -1, \mu = 3, \sigma = 5]$ .





### 4 Stability

This section of our research paper investigates one of the basic properties of any model. It examines the stability property for the fractional nonlinear KS equation by using a Hamiltonian system. The momentum in the Hamiltonian system is given by the following formula:

$$M = \frac{1}{2} \int_{-\vartheta}^{\vartheta} \mathcal{S}^2(\wp) d\wp, \tag{36}$$

where  $\vartheta$  is an arbitrary constant. Thus, the condition for stability is given in the following condition:

$$\left. \frac{\partial M}{\partial c} \right|_{c=\bar{h}} > 0, \tag{37}$$

where  $c, \bar{h}$  are arbitrary constants.

For an example of studying the stability of the solution of Eq. (3) by using (30) with the following values of the constants  $[a_0 = 1, a_2 = -1, \delta = -5, \mu = \frac{1}{25}, \sigma = \frac{1}{15}, \varrho = 5]$  yields

$$M = \frac{1}{15} \int_{-5}^5 \frac{1}{(2e^{50} \cosh(\frac{5c}{t-\sqrt{\pi}t}) + e^{100} + 1)^3} \left[ 6e^{50} \left( 2(e^{200} - 1) \sinh\left(\frac{5c}{t-\sqrt{\pi}t}\right) + 2e^{50}(e^{100} - 1) \sinh\left(\frac{10c}{t-\sqrt{\pi}t}\right) + 75(1 + 3e^{100} + e^{200}) \cosh\left(\frac{5c}{t-\sqrt{\pi}t}\right) \right) \right]$$

$$\begin{aligned}
& + 2e^{50}(38 + 37e^{100}) \cosh\left(\frac{10c}{t - \sqrt{\pi}t}\right) + 25e^{100} \cosh\left(\frac{15c}{t - \sqrt{\pi}t}\right) \\
& + 74e^{300} + 684e^{200} + 666e^{100} + 76 \Big] dt. \tag{38}
\end{aligned}$$

Thus, we obtain

$$\left. \frac{\partial M}{\partial c} \right|_{c=2} < 0. \tag{39}$$

This means that this solution is unstable and, by applying the same steps to other obtained solutions, the stability property of each one of them can be determined.

## 5 Conclusion

This research has successfully applied the modified Khater method with a new fractional operator to the fractional nonlinear KS equation that is arising in the nonlinear wave processes in a liquid containing gas bubbles. This new operator is used to avoid the disadvantage of the other fractional operator. Distinct, solitary wave solutions have been obtained for this equation. For more illustrations of the dynamical behavior of this kind of fluid, some solutions have been sketched (Figs. 1, 2, 3) in three different formulas of each figure (two, three-dimensional, and contour plots).

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### Availability of data and materials

Not applicable.

### Competing interests

The authors declare that they have no competing interests.

### Authors' contributions

All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

### Author details

<sup>1</sup>Department of Mathematics, Faculty of Science, Jiangsu University, Zhanjiang, China. <sup>2</sup>Department of Basic Science, Higher Technological Institute, 10th of Ramadan City, Egypt.

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