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Dynamics of the rumor-spreading model with hesitation mechanism in heterogenous networks and bilingual environment

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Abstract

In this paper, a novel rumor-spreading model is proposed under bilingual environment and heterogenous networks, which considers that exposures may be converted to spreaders or stiflers at a set rate. Firstly, the nonnegativity and boundedness of the solution for rumor-spreading model are proved by reductio ad absurdum. Secondly, both the basic reproduction number and the stability of the rumor-free equilibrium are systematically discussed. Whereafter, the global stability of rumor-prevailing equilibrium is explored by utilizing Lyapunov method and LaSalle's invariance principle. Finally, the sensitivity analysis and the numerical simulation are respectively presented to analyze the impact of model parameters and illustrate the validity of theoretical results.

Keywords: Rumor spreading; Hesitation mechanism; Heterogenous networks; Bilingual environment; Sensitivity analysis

1 Introduction

Rumors have been generally described as the unconfirmed comprehension and interpretation of social events, natural phenomena, and other issues, which can be spread through interpersonal communication [1, 2]. In addition, the veracity of rumors could be interpreted and demonstrated by exerting several exterior interferences as time goes on. Actually, rumors which seriously twist the facts will lead some negative repercussions, such as anxious state of mind, social panic, serious financial losses, and so on [3]. In February 2011, for instance, it was rumored in Jiangsu Province that a large explosion had happened in Daiwa chemical enterprise of Chenjiagang chemical park, which caused a sense of panic and terror, six people were dead and many others wounded. Therefore, it is quite necessary and meaningful to investigate the dynamics of rumor spreading.

Starting from the complex environment, some researchers have discussed the dynamics for rumor spreading based on the peculiarities and diversities of complex networks recently. In 2002, Zanette [4, 5] firstly combined rumor propagation with complex networks to construct ISR (ignoramuses–spreaders–stiflers) model on small world networks, and the threshold was obtained on the homogeneous networks. In 2007, Nekovee et al.

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[6] studied the rumor-spreading model with a forgetting mechanism on complex networks, and the threshold of rumor propagation was discussed by simulation experiments. Some researchers presented a novel ILSR (ignoramus–lurker–spreader–removal) model to study the stability and control of rumors in case of an emergency [7, 8]. Note that the aforementioned rumor-spreading models are concentrated on homogeneous networks, which include only a single type of node and edge that will certainly lead some problems with incomplete or loss of information [9–14]. In contrast to homogeneous networks, heterogeneous networks (i.e., the degree distribution of nodes is uneven) blend more types of nodes and their complex interactions. Obviously, some practical phenomena can be described more precisely by utilizing heterogeneous networks. Consequently, it is necessary to discuss rumor propagation in such networks. The heterogeneity of models has an important impact on the dynamics of rumor spreading, and some results of rumor spreading in heterogeneous networks have been acquired recently [15–17].

It is worth noting that a hesitation mechanism is introduced to explore the dynamics of rumor spreading for the reason that people cannot immediately spread or deny rumors received from the complex environment. At present, few scholars studied the dynamic behaviors of a rumor-propagation model with a hesitation mechanism [18–21]. For example, the dynamics of SEIR (ignoramuses–exposures–spreaders–stiflers) rumor-spreading model in heterogeneous networks was studied in [18–20], where exposures were converted to stiflers with a certain rate. Hosseini et al. [21] investigated an SEIRS propagation model with vaccinations and quarantine strategies by considering the effects of user awareness, network delay, and diverse configuration of nodes. However, almost all of results about SEIR model were studied in monolingual environment and few researchers concentrated on bilingual or even multilingual environment based on heterogeneous networks.

Note that rumors are widely spread in an increasingly complicated environment with the rapid development of market economy and globalization. That is, the area of rumor spreading includes people of different nationalities and languages [22–25]. In such a social environment, the dissemination of rumors, the ways and means of receiving rumors, and the guidance of public opinion of rumors are more complicated than those in a monolingual environment. In addition, people of different nationalities and languages have different propagation characteristics for the same information due to different focus. Therefore, it has profound practical significance and application value to study the discipline and dynamics of rumor spreading.

From the above analysis, the purpose of this paper is to address the global stability of the rumor-spreading model in heterogeneous networks and bilingual environment. Compared with the existing results, the main contributions of this paper can be summarized as follows:

- (1) By extending the existing results [22–25], a novel IE2S2R model is proposed by adding a hesitation mechanism. Especially, exposures may be translated into spreaders or stiflers with a certain rate in heterogeneous networks and bilingual environment.
- (2) The existence and stability of rumor-free and rumor-prevailing equilibrium points are proved mainly based on the Lyapunov function, graph theoretic approach, and LaSalle's invariance principle.

(3) In order to be better at guiding and controlling the spread of rumors, sensitivity analysis which can indicate the relative importance of several different factors in IE2S2R model is introduced in this paper.

The rest of this paper is organized as follows. In Sect. 2, we present an IE2S2R model with a hesitation mechanism in heterogenous networks. The dynamics of the rumor-free and rumor-prevailing equilibria for an IE2S2R rumor-spreading model is explored in Sect. 3. The sensitivity analysis is performed in Sect. 4. In Sect. 5, some numerical simulations are provided to validate the correctness and effectiveness of the obtained results. Finally, the conclusion of this paper is brought in Sect. 6.

2 Description of the rumor-spreading model

In this section, we start with a description of the proposed rumor spreading in a bilingual environment and then explore rumor spreading in heterogeneous networks.

During the rumor propagation in the whole population, the individuals are often in the following states: ignoramuses ($I(t)$), who have never heard the rumor but are vulnerable to be infected; exposures ($E(t)$), who have been infected but they are hesitant to start spreading rumors; spreaders-1 ($S_1(t)$), who have known and spread the rumor by their first language; spreaders-2 ($S_2(t)$), who have known and spread the rumor via the second language; stiflers-1 ($R_1(t)$), who have heard the rumor through the first language and then do not transmit the rumor again for certain reasons (For instance, the exposures are not interested in rumors; the spreaders stop the propagation of rumors due to the impacts of education or forgetting mechanism.); stiflers-2 ($R_2(t)$), who have heard the rumor through the second language but couldn't disseminate the rumor. Moreover, the populations are divided into different groups on the basis of their different connectivity degree (the scope of human's social circle), which enables the addressed networks to describe the spreading of rumors more accurately.

The population has been divided into groups of $k_{\max}$ individuals based on the different connectivity degree in the follow-up research, where $k_{\max}$ is the maximum number of connections to each person. Thus let $I_k(t)$, $E_k(t)$, $S_{1k}(t)$, $S_{2k}(t)$, $R_{1k}(t)$, and $R_{2k}(t)$ be the densities of ignoramuses, exposures, spreaders-1, spreaders-2, stiflers-1, and stiflers-2 with degree k at time t , respectively. Hence, at any time t , the density of the whole population with degree k satisfies

$$N_k(t) = I_k(t) + E_k(t) + S_{1k}(t) + S_{2k}(t) + R_{1k}(t) + R_{2k}(t),$$

and

$$N(t) = \sum_{k=1}^{k_{\max}} N_k(t).$$

As shown in Fig. 1, the rules of the IE2S2R rumor-spreading model and their expressions can be summarized as follows:

(1) Only when ignoramuses contact with spreaders-1 or spreaders-2 will they be converted to exposures with a certain probability.

(2) As time goes by, exposures may be converted to spreaders (spreaders-1 or spreaders-2) with a certain probability since they believe and spread rumors, or they could also be translated into stiflers (stiflers-1 or stiflers-2) with a specified probability because exposures are not interested in spreading rumors.

(3) Because the rumor exists or the truth continues to spread for a long time, spreaders-1 (spreaders-2) will forget the existing rumor or know the truth with a certain probability. Ultimately, they don't transmit the rumor again and will convert to stiflers-1 (stiflers-2).

In view of the above mentioned rules, the rumor-spreading model in a bilingual environment and heterogeneous networks can be described by

$$\begin{cases} \frac{dI_k(t)}{dt} = b - kI_k(t)(\Theta_1(t) + \Theta_2(t)) - \mu I_k(t), \\ \frac{dE_k(t)}{dt} = kI_k(t)(\Theta_1(t) + \Theta_2(t)) - hE_k(t) - \mu E_k(t), \\ \frac{dS_{1k}(t)}{dt} = \alpha\beta hE_k(t) - \delta S_{1k}(t) - \mu S_{1k}(t), \\ \frac{dS_{2k}(t)}{dt} = (1-\alpha)\gamma hE_k(t) - \delta S_{2k}(t) - \mu S_{2k}(t), \\ \frac{dR_{1k}(t)}{dt} = \alpha(1-\beta)hE_k(t) + \delta S_{1k}(t) - \mu R_{1k}(t), \\ \frac{dR_{2k}(t)}{dt} = (1-\alpha)(1-\gamma)hE_k(t) + \delta S_{2k}(t) - \mu R_{2k}(t), \end{cases} \quad (1)$$

where $b, \mu, h, \delta \geq 0$ and $\alpha, \beta, \gamma \in (0, 1)$; $\Theta_1(t)$ and $\Theta_2(t)$ are described as

$$\Theta_1(t) = \sum_i \lambda_i \frac{\varphi(i)}{i} P(i|k) S_{1i}(t), \quad \Theta_2(t) = \sum_i \lambda_i \frac{\varphi(i)}{i} P(i|k) S_{2i}(t).$$

Here, $\varphi(i)$ represents the infectivity of spreaders-1 or spreaders-2 with degree i , λ_i denotes the acceptability of i degree individuals, $\frac{\lambda_i \varphi(i)}{i}$ denotes the probability of converting to exposures when ignoramuses contact with spreaders-1 or spreaders-2 in unit time, $P(i|k)$ is the probability that individuals with degree i connect with degree k . In this paper, we focus on degree-uncorrelated networks. Then, substituting $P(i|k) = \frac{iP(i)}{\langle k \rangle}$ into $\Theta_1(t)$ and $\Theta_2(t)$, we

Table 1 The meaning of the parameters in model (1)

Parameters	Meaning
b	the coming rate of ignoramuses
μ	the removal rate of the different compartment
α	the probability of exposed with language-1 based
βh	the transformation rate from exposed to spreaders-1
γh	the transformation rate from exposed to spreaders-2
δ	the transfer rate from spreaders to stiflers due to forgetting or educational mechanism

can obtain

$$\Theta_1(t) = \frac{1}{\langle k \rangle} \sum_i \lambda_i \varphi(i) P(i) S_{1i}(t), \quad \Theta_2(t) = \frac{1}{\langle k \rangle} \sum_i \lambda_i \varphi(i) P(i) S_{2i}(t).$$

Moreover, the meaning of the parameters in model (1) is presented in Table 1.

Remark 1 Note that the process of rumor spreading in this paper is different from the previous papers [22, 23] in that the hesitation mechanism is considered and the exposures may be translated into spreaders or stiflers with a certain rate. In addition, both cross-propagation and contact transfer are considered in the rumor-spreading model (1). Obviously, the model proposed in this paper is more general and practical.

In view of the biological background of system (1), in this paper, we only consider the solutions of system (1) starting at $t = 0$ with initial values:

$$\begin{aligned} I_k(0) \geq 0, \quad E_k(0) \geq 0, \quad S_{1k}(0) \geq 0, \quad S_{2k}(0) \geq 0, \\ R_{1k}(0) \geq 0, \quad R_{2k}(0) \geq 0, \quad k = 1, \dots, k_{\max}. \end{aligned} \quad (2)$$

3 Main results

For the nonnegativeness of the solution and the feasible region for IE2S2R model (1), the following conclusion can be derived.

Theorem 1 Let $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ be the solution of model (1) with initial conditions (2), then we have

- (i) $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ is also positive for all $t > 0$.
- (ii) The feasible region Ω is a positively invariant set of system (1), which is defined as

$$\begin{aligned} \Omega = \left\{ (I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))^T \in \mathbb{R}_+^6 \mid I_k(t) + E_k(t) + S_{1k}(t) \right. \\ \left. + S_{2k}(t) + R_{1k}(t) + R_{2k}(t) \leq \frac{b}{\mu}, k = 1, 2, \dots, k_{\max}, t > 0 \right\}. \end{aligned} \quad (3)$$

Proof (i) Suppose that $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ is the solution of system (1) with (2) for all $t \in [0, T)$, where $T > 0$. Let

$$F(t) = \min_t \{I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t)\}.$$

Here, we will prove that the solution of system (1) is positive, which implies that we only need to prove $F(t) > 0$ for all $t \in [0, T]$. Assume that there exists $t^* \in (0, T)$ such that

$$F(t^*) = \min_t \{I_k(t^*), E_k(t^*), S_{1k}(t^*), S_{2k}(t^*), R_{1k}(t^*), R_{2k}(t^*)\} = 0.$$

According to the initial condition (2), we can further assume $F(t) > 0$ for all $t \in (0, t^*)$. Next, we discuss the above function $F(t) > 0$ in six cases.

If $F(t^*) = I_k(t^*)$, then from the first equation of system (1) and $S_{1k}(t), S_{2k}(t) \geq 0$, we can get

$$\frac{dI_k(t)}{dt} > -(k(\Theta_1(t) + \Theta_2(t)) + \mu)I_k(t), \quad t \in [0, t^*]. \quad (4)$$

Integrating both sides of (4) from 0 to t^* , one has

$$0 = I_k(t^*) > I_k(0) \exp\left(-\int_0^{t^*} [k(\Theta_1(t) + \Theta_2(t)) + \mu] dt\right) \geq 0,$$

which leads to a contradiction. Further, the other five cases can be discussed in a similar way. This shows that $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ is positive on the existence interval (i.e., for $t \in (0, T)$).

Then, we prove that the interval of existence of $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ is $[0, \infty)$. In fact, if the interval of existence is a finite interval $t \in (0, T)$, we know that $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ is unbounded on $[0, T)$. From system (1) and $N_k(t) = I_k(t) + E_k(t) + S_{1k}(t) + S_{2k}(t) + R_{1k}(t) + R_{2k}(t)$, one has

$$\frac{dN_k(t)}{dt} = b - \mu N_k(t).$$

Integrating the above equation from 0 to t , we obtain

$$N_k(t) = \frac{b}{\mu} + \left(N_k(0) - \frac{b}{\mu}\right) \exp(-\mu t).$$

Hence, $N_k(t)$ is bounded on $[0, T)$, which implies that $I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t)$ are bounded on $[0, T)$. This leads to a contradiction. Therefore, we finally have that $(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t))$ is positive for all $t \in (0, \infty)$.

(ii) According to the initial condition (2) and the result of (i), we have

$$\limsup_{t \rightarrow \infty} N_k(t) = \frac{b}{\mu},$$

which implies that the feasible region Ω is positively invariant with respect to system (1). The proof of Theorem 1 is completed. $\square$

According to the methodology of infectious diseases, our aim is to explore the rumor-free equilibrium $\tilde{E}_0$ and the basic reproduction number $\mathcal{R}_0$ of system (1). One can check that system (1) has the rumor-free equilibrium

$$\tilde{E}_0 = \left(\frac{b}{\mu}, 0, 0, 0, 0, 0\right).$$

When we choose

$$\chi = (I_1(t), \dots, I_{k_{\max}}(t), E_1(t), \dots, E_{k_{\max}}(t), S_{11}(t), \dots, S_{1k_{\max}}(t), \\ S_{21}(t), \dots, S_{2k_{\max}}(t), R_{11}(t), \dots, R_{1k_{\max}}(t), R_{21}(t), \dots, R_{2k_{\max}}(t)),$$

the following model can be obtained:

$$\frac{d\chi}{dt} = \mathcal{F}(t) - \mathcal{V}(t),$$

where

$$\mathcal{F}(\chi) = \begin{pmatrix} \frac{I_1(t)}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) [S_{1i}(t) + S_{2i}(t)] \\ \vdots \\ \frac{k_{\max} I_{k_{\max}}(t)}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) [S_{1i}(t) + S_{2i}(t)] \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

and

$$\mathcal{V}(\chi) = \begin{pmatrix} (h + \mu)E_1(t) \\ \vdots \\ (h + \mu)E_{k_{\max}}(t) \\ (\delta + \mu)S_{11}(t) - \alpha\beta hE_1(t) \\ \vdots \\ (\delta + \mu)S_{2k_{\max}}(t) - (1 - \alpha)\gamma hE_{k_{\max}}(t) \\ \mu R_{11}(t) - \delta S_{11}(t) - \alpha(1 - \beta)hE_1(t) \\ \vdots \\ \mu R_{21}(t) - \delta S_{21}(t) - (1 - \alpha)(1 - \gamma)hE_{k_{\max}}(t) \\ \frac{I_1(t)}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) [S_{1i}(t) + S_{2i}(t)] + \mu I_1(t) - b \\ \vdots \\ \frac{k_{\max} I_{k_{\max}}(t)}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) [S_{1i}(t) + S_{2i}(t)] + \mu I_{k_{\max}}(t) - b \end{pmatrix}.$$

The Jacobian matrices of $\mathcal{F}(\chi)$ and $\mathcal{V}(\chi)$ at $\tilde{E}_0 = (\frac{b}{\mu}, 0, 0, 0, 0, 0)$ are written as

$$D\mathcal{F}(\tilde{E}_0) = \begin{pmatrix} F & \mathbf{0}_{3k_{\max} \times 3k_{\max}} \\ \mathbf{0}_{3k_{\max} \times 3k_{\max}} & \mathbf{0}_{3k_{\max} \times 3k_{\max}} \end{pmatrix}, \quad D\mathcal{V}(E_0) = \begin{pmatrix} V & \mathbf{0}_{3k_{\max} \times 3k_{\max}} \\ J_1 & J_2 \end{pmatrix},$$

where

$$F = \begin{pmatrix} A & A & \mathbf{0}_{k_{\max} \times k_{\max}} \\ \mathbf{0}_{k_{\max} \times k_{\max}} & \mathbf{0}_{k_{\max} \times k_{\max}} & \mathbf{0}_{k_{\max} \times k_{\max}} \\ \mathbf{0}_{k_{\max} \times k_{\max}} & \mathbf{0}_{k_{\max} \times k_{\max}} & \mathbf{0}_{k_{\max} \times k_{\max}} \end{pmatrix},$$

$$V = \begin{pmatrix} \mathbf{0}_{k_{\max} \times k_{\max}} & \mathbf{0}_{k_{\max} \times k_{\max}} & (h + \mu)\mathbf{I}_{k_{\max}} \\ (\delta + h)\mathbf{I}_{k_{\max}} & \mathbf{0}_{k_{\max} \times k_{\max}} & -\alpha\beta h\mathbf{I}_{k_{\max}} \\ \mathbf{0}_{k_{\max} \times k_{\max}} & (\delta + h)\mathbf{I}_{k_{\max}} & -(1 - \alpha)\gamma h\mathbf{I}_{k_{\max}} \end{pmatrix},$$

and

$$\begin{aligned} A &= \frac{b}{\mu \langle k \rangle} \begin{pmatrix} \lambda_1 \varphi(1)P(1) & \cdots & \lambda_1 \varphi(k_{\max})P(k_{\max}) \\ \vdots & \ddots & \vdots \\ k_{\max} \lambda_{k_{\max}} \varphi(1)P(1) & \cdots & k_{\max} \lambda_{k_{\max}} \varphi(k_{\max})P(k_{\max}) \end{pmatrix} \\ &= \frac{b}{\mu \langle k \rangle} \begin{pmatrix} \sum_{i=1}^{k_{\max}} i \lambda_i \varphi(i)P(i) & \cdots & \lambda_1 \varphi(k_{\max})P(k_{\max}) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}. \end{aligned}$$

It is clearly shown that V is a nonsingular matrix and F is a nonnegative matrix. Based on the concept of the next generation matrix and reproduction number given in [26, 27], the basic reproduction number is defined by

$$\mathcal{R}_0 = \rho(FV^{-1}) = \frac{bh \sum_{i=1}^{k_{\max}} i \lambda_i \varphi(i)P(i) [\alpha\beta + (1 - \alpha)\gamma]}{\langle k \rangle \mu (h + \mu)(\delta + h)}.$$

Moreover, the following result can be derived by combining the above analysis with references [26, 27].

Theorem 2 *When $\mathcal{R}_0 < 1$, the rumor-free equilibrium $\tilde{E}_0$ of system (1) is locally asymptotically stable, and when $\mathcal{R}_0 > 1$, it is unstable.*

Proof Since the local stability of $\tilde{E}_0$ is related to the eigenvalues of the corresponding Jacobian matrix $J(\tilde{E}_0)$, we firstly derive the Jacobian matrix $J(\tilde{E}_0)$ of system (1) as follows:

$$A = \begin{pmatrix} M_1 & N_{12} & \cdots & N_{1k_{\max}} \\ \vdots & \vdots & \ddots & \vdots \\ N_{k_{\max}1} & N_{k_{\max}2} & \cdots & M_{k_{\max}} \end{pmatrix},$$

where

$$M_i = \begin{pmatrix} -\mu & 0 & -\frac{b}{\mu \langle k \rangle} i \lambda_i \varphi(i)P(i) & -\frac{b}{\mu \langle k \rangle} i \lambda_i \varphi(i)P(i) & 0 & 0 \\ 0 & -(h + \mu) & -\frac{b}{\mu \langle k \rangle} i \lambda_i \varphi(i)P(i) & -\frac{b}{\mu \langle k \rangle} i \lambda_i \varphi(i)P(i) & 0 & 0 \\ 0 & \alpha\beta h & -(\delta + \mu) & 0 & 0 & 0 \\ 0 & (1 - \alpha)\gamma h & 0 & -(\delta + \mu) & 0 & 0 \\ 0 & \alpha(1 - \beta)h & \delta & 0 & -\mu & 0 \\ 0 & (1 - \alpha)(1 - \gamma)h & 0 & \delta & 0 & -\mu \end{pmatrix},$$

and for $i \neq j$,

$$N_{ij} = \begin{pmatrix} 0 & 0 & -\frac{b}{\mu\langle k \rangle} j \lambda_j \varphi(j) P(j) & -\frac{b}{\mu\langle k \rangle} j \lambda_j \varphi(j) P(j) & 0 & 0 \\ 0 & 0 & -\frac{b}{\mu\langle k \rangle} j \lambda_j \varphi(j) P(j) & -\frac{b}{\mu\langle k \rangle} j \lambda_j \varphi(j) P(j) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

By a simple calculation, the characteristic equation of $J(\tilde{E}_0)$ is written as

$$\left\{ (\lambda + h + \mu)(\lambda + \delta + \mu) - \frac{bh[\alpha\beta + (1 - \alpha)\gamma] \sum_{i=1}^{k_{\max}} i \lambda_i \varphi(i) P(i)}{\mu\langle k \rangle} \right\} \\ \times (\lambda + \mu)^{3k_{\max}} (\lambda + \delta + \mu)^{2k_{\max}} (\lambda + h + \mu)^{k_{\max}-1} = 0.$$

A simple calculation shows that the stability of $\tilde{E}_0$ is only dependent of the solution of the following equation:

$$\lambda^2 + (h + 2\mu + \delta)\lambda + (h + \mu)(\mathcal{R}_0 - 1) = 0.$$

Let

$$G(\lambda) = \lambda^2 + (h + 2\mu + \delta)\lambda + (h + \mu)(1 - \mathcal{R}_0),$$

then

$$\frac{dG(\lambda)}{d\lambda} = 2\lambda + h + 2\mu + \delta.$$

Obviously, $\frac{dG(\lambda)}{d\lambda} > 0$ when $\lambda > 0$. That is, $G(\lambda)$ is a monotonically increasing function when $\lambda > 0$. And if $\mathcal{R}_0 < 1$,

$$G(0) > 0 \quad \text{and} \quad \lim_{\lambda \rightarrow \infty} G(\lambda) = \infty > 0,$$

which shows that the solutions of $G(\lambda) = 0$ are negative when $\mathcal{R}_0 < 1$ and at least one solution for $G(\lambda) = 0$ is positive when $\mathcal{R}_0 > 1$. Hence, from Routh–Hurwitz criterion [28], the rumor-free equilibrium $\tilde{E}_0$ of system (1) is locally asymptotically stable for $\mathcal{R}_0 < 1$, and it is unstable when $\mathcal{R}_0 > 1$. The proof of Theorem 2 is completed. $\square$

Theorem 3 When $\mathcal{R}_0 < 1$, the rumor-free equilibrium $\tilde{E}_0$ of system (1) is globally asymptotically stable.

Proof Consider the following Lyapunov function:

$$V(t) = \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \left\{ E_k(t) + \frac{h + \mu}{\alpha\beta h + (1 - \alpha)\gamma h} (S_{1k}(t) + S_{2k}(t)) \right\}.$$

Calculating the derivative of $V(t)$ along the solution of (1), we can obtain

$$\begin{aligned}\frac{dV(t)}{dt} &= \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \left\{ k I_k(t) (\Theta_1(t) + \Theta_2(t)) - \frac{h + \mu}{\alpha \beta h + (1 - \alpha) \gamma h} \right. \\ &\quad \times (S_{1k}(t) + S_{2k}(t)) \Big\} \\ &\leq (\mathcal{R}_0 - 1) \frac{(h + \mu)(\delta + \mu)(\Theta_1(t) + \Theta_2(t))}{\alpha \beta h + (1 - \alpha) \gamma h}.\end{aligned}$$

When $\mathcal{R}_0 < 1$, we can easily find that $\frac{dV(t)}{dt} \leq 0$. Also $\frac{dV(t)}{dt} = 0$ if and only if $\Theta_1(t) + \Theta_2(t) = 0$, i.e., $S_{1k}(t) = S_{2k}(t) = 0$. Thus, by LaSalle's invariance principle [29], the rumor-free equilibrium $\tilde{E}_0$ of system (1) is globally asymptotically stable. The proof of Theorem 3 is completed. $\square$

Remark 2 When $R_{1k}(t) + R_{2k}(t) = R_k(t)$, the fifth and sixth equations in model (1) can be rewritten as

$$\frac{dR_k(t)}{dt} = h[\alpha(1 - \beta) + (1 - \alpha)(1 - \gamma)]E_k(t) + \delta(S_{1k}(t) + S_{2k}(t)) - \mu R_k(t).$$

Based on this, the propagation of rumor in model (1) is similar to that of [23] and the following corollaries are obtained.

Corollary 1 When $\mathcal{R}_0 < 1$, the rumor-free equilibrium $\hat{E}_0 = (\frac{b}{\mu}, 0, 0, 0, 0)$ of system (1) is locally asymptotically stable, and when $\mathcal{R}_0 > 1$, it is unstable.

Corollary 2 When $\mathcal{R}_0 < 1$, the rumor-free equilibrium $\hat{E}_0$ of system (1) is globally asymptotically stable.

Proof The proofs of Corollaries 1–2 are similar to those of Theorems 2–3, which are omitted here. $\square$

Next, we will prove the uniqueness of the rumor-prevailing equilibrium $\tilde{E}^*$.

Theorem 4 When $\mathcal{R}_0 > 1$, system (1) has a unique rumor-prevailing equilibrium $\tilde{E}^* = (I_k^*, E_k^*, S_{1k}^*, S_{2k}^*, R_{1k}^*, R_{2k}^*)$.

Proof Assuming that $\tilde{E}^* = (I_k^*, E_k^*, S_{1k}^*, S_{2k}^*, R_{1k}^*, R_{2k}^*)$ is an equilibrium of system (1), we can obtain

$$\begin{cases} b - k I_k^* (\Theta_1^* + \Theta_2^*) - \mu I_k^* = 0, \\ k I_k^* (\Theta_1^* + \Theta_2^*) - h E_k^* - \mu E_k^* = 0, \\ \alpha \beta h E_k^* - \delta S_{1k}^* - \mu S_{1k}^* = 0, \\ (1 - \alpha) \gamma h E_k^* - \delta S_{2k}^* - \mu S_{2k}^* = 0, \\ \alpha(1 - \beta) h E_k^* + \delta S_{1k}^* - \mu R_{1k}^* = 0, \\ (1 - \alpha)(1 - \gamma) h E_k^* + \delta S_{2k}^* - \mu R_{2k}^* = 0, \end{cases} \quad (5)$$

where $\Theta_1^* = \frac{1}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) S_{1i}^*$ and $\Theta_2^* = \frac{1}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) S_{2i}^*$. By (5), one has

$$\begin{cases} I_k^* = \frac{b}{k(\Theta_1^* + \Theta_2^*) + \mu}, \\ E_k^* = \frac{kb(\Theta_1^* + \Theta_2^*)}{(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)}, \\ S_{1k}^* = \frac{\alpha\beta hkb(\Theta_1^* + \Theta_2^*)}{(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)(\delta + \mu)}, \\ S_{2k}^* = \frac{(1 - \alpha)\gamma hkb(\Theta_1^* + \Theta_2^*)}{(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)(\delta + \mu)}, \\ R_{1k}^* = \frac{(1 - \beta + \beta\delta)\alpha hkb(\Theta_1^* + \Theta_2^*)}{\mu(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)(\delta + \mu)}, \\ R_{2k}^* = \frac{(1 - \gamma + \gamma\delta)(1 - \alpha)hkb(\Theta_1^* + \Theta_2^*)}{\mu(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)(\delta + \mu)}. \end{cases} \quad (6)$$

From (6), we can get the self-consistency equation as follows:

$$\begin{aligned} \Theta &= \Theta_1^* + \Theta_2^* \\ &= \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \frac{(\alpha\beta + (1 - \alpha)\gamma)hkb(\Theta_1^* + \Theta_2^*)}{(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)(\delta + \mu)} = f(\Theta). \end{aligned} \quad (7)$$

Obviously, $\Theta = 0$ is a solution of (7). Then, $I_k^* = \frac{b}{\mu}$ and $E_k^* = S_{1k}^* = S_{2k}^* = R_{1k}^* = R_{2k}^* = 0$, which is the rumor-free equilibrium of system (1). Notice that

$$\begin{aligned} f(0) &= 0, \\ \frac{df(\Theta)}{d\Theta} &= \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \frac{(\alpha\beta + (1 - \alpha)\gamma)\mu hkb}{(k(\Theta_1^* + \Theta_2^*) + \mu)^2 (h + \mu)^2 (\delta + \mu)^2} > 0, \end{aligned}$$

which shows that $f(\Theta)$ is a monotonously increasing function and

$$\lim_{\Theta \rightarrow \infty} f(\Theta) = \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \frac{(\alpha\beta + (1 - \alpha)\gamma)hb}{(h + \mu)(\delta + \mu)}.$$

If $\mathcal{R}_0 > 1$, then

$$\left. \frac{df(\Theta)}{d\Theta} \right|_{\Theta=0} = \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \frac{(\alpha\beta + (1 - \alpha)\gamma)khb}{\mu(h + \mu)(\delta + \mu)} = \mathcal{R}_0 > 1$$

and

$$\frac{d^2 f(\Theta)}{d\Theta^2} = -\frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \frac{2k^2 \mu bh(\alpha\beta + (1 - \alpha)\gamma)}{(k(\Theta_1^* + \Theta_2^*) + \mu)^3 (h + \mu)^2 (\delta + \mu)^2} < 0.$$

Thus, system (1) has a unique positive solution if and only if $\mathcal{R}_0 > 1$. That is, system (1) has a unique $\tilde{E}^* = (I_k^*, E_k^*, S_{1k}^*, S_{2k}^*, R_{1k}^*, R_{2k}^*)$ for $\mathcal{R}_0 > 1$. The proof of Theorem 4 is completed. $\square$

Theorem 5 *The unique rumor-prevailing equilibrium $\tilde{E}^*$ is globally asymptotically stable, when $\mathcal{R}_0 > 1$.*

Proof Because the first four equations of system (1) do not depend on $R_{1k}(t)$ and $R_{2k}(t)$, we need only to consider a Lyapunov function in the following form:

$$W(t) = \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) W_k(t),$$

where

$$\begin{cases} W_k(t) = I_k^* g\left(\frac{I_k(t)}{I_k^*}\right) + E_k^* g\left(\frac{E_k(t)}{E_k^*}\right) + \frac{h+\mu}{\alpha\beta h + (1-\alpha)\gamma h} \\ \quad \times \{S_{1k}^* g\left(\frac{S_{1k}(t)}{S_{1k}^*}\right) + S_{2k}^* g\left(\frac{S_{2k}(t)}{S_{2k}^*}\right)\}, \\ g(x) = x - 1 - \ln x \geq 0, \quad \text{for } x > 0. \end{cases} \quad (8)$$

Put

$$x_k = \frac{I_k(t)}{I_k^*}, \quad y_k = \frac{E_k(t)}{E_k^*}, \quad u_{1k} = \frac{S_{1k}(t)}{S_{1k}^*}, \quad u_{2k} = \frac{S_{2k}(t)}{S_{2k}^*}.$$

First, system (1) and the equations in (5) yield that

$$\begin{aligned} \frac{dI_k(t)}{dt} &= kI_k^* (\Theta_1^* + \Theta_2^*) - kI_k(t) (\Theta_1(t) + \Theta_2(t)) + \mu (I_k^* - I_k(t)) \\ &= \frac{k}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) I_k^* \{S_{1i}^* (1 - u_{1i} x_k) + S_{2i} (1 - u_{2i} x_k)\} \\ &\quad + \mu I_k^* (1 - x_k), \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{dE_k(t)}{dt} &= kI_k(t) (\Theta_1(t) + \Theta_2(t)) - kI_k^* (\Theta_1^* + \Theta_2^*) + (\alpha + \mu) (E_k^* - E_k(t)) \\ &= \frac{k}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) I_k^* \{S_{1i}^* (u_{1i} x_k - 1) + S_{2i} (u_{2i} x_k - 1)\} \\ &\quad + (\alpha + \mu) E_k^* (1 - y_k), \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{dS_{1k}(t)}{dt} &= \alpha\beta h (E_k(t) - E_k^*) + (\delta + \mu) (S_{1k}^* - S_{1k}(t)) \\ &= \alpha\beta h E_k^* (y_k - 1) + (\delta + \mu) S_{1k}^* (1 - u_{1k}), \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{dS_{2k}(t)}{dt} &= (1 - \alpha)\gamma h (E_k(t) - E_k^*) + (\delta + \mu) (S_{2k}^* - S_{2k}(t)) \\ &= (1 - \alpha)\gamma h E_k^* (y_k - 1) + (\delta + \mu) S_{2k}^* (1 - u_{2k}). \end{aligned} \quad (12)$$

From (8)–(12), the derivative of $W(t)$ is

$$\begin{aligned} \frac{dW_k(t)}{dt} &= \left\{ \frac{kI_k^*}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) (S_{1i}^* (1 - u_{1i} x_k) + S_{2i} (1 - u_{2i} x_k)) + \mu (1 - x_k) I_k^* \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left(1 - \frac{1}{x_k}\right) + \left\{ \frac{k}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) I_k^* (S_{1i}^* (u_{1i} x_k - 1) + S_{2i}^* (u_{2i} x_k - 1)) \right. \\
& \left. + E_k^* (\delta + \mu) (1 - u_{2k}) \right\} \times \left(1 - \frac{1}{y_k}\right) + \{ S_{1k}^* (\delta + \mu) (1 - u_{1k}) + \alpha \beta h E_k^* \\
& \times (y_k - 1) \} \times \left(1 - \frac{1}{u_{1k}}\right) + \{ (1 - \alpha) \gamma h E_k^* (y_k - 1) + S_{2k}^* (\delta + \mu) (1 - u_{2k}) \} \\
& \times \left(1 - \frac{1}{u_{2k}}\right) \\
& = \frac{k I_k^*}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi(i) P(i) \{ S_{1i}^* g(u_{1i}) + S_{2i}^* g(u_{2i}) \} - \{ S_{1k}^* g(u_{1k}) + S_{2k}^* g(u_{2k}) \} \\
& \times \frac{(h + \mu)(\delta + \mu)}{\alpha \beta h + (1 - \alpha) \gamma h} - \frac{k I_k^*}{\langle k \rangle} \sum_{i=1}^{k_{\max}} \lambda_i \varphi_i P(i) \left\{ S_{1k}^* \left[g\left(\frac{1}{x_k}\right) + g\left(\frac{u_{1i} x_k}{y_k}\right) \right] + S_{2k}^* \right. \\
& \times \left[g\left(\frac{1}{x_k}\right) + g\left(\frac{u_{2i} x_k}{y_k}\right) \right] \left. \right\} - \mu I_k^* \left[g\left(\frac{1}{x_k}\right) + g(x_k) \right] - \frac{(h + \mu) \alpha \beta E_k^*}{\alpha \beta + (1 - \alpha) \gamma} g\left(\frac{y_k}{u_{1k}}\right) \\
& - \frac{\gamma (1 - \alpha) (h + \mu) E_k^*}{\alpha \beta + (1 - \alpha) \gamma} g\left(\frac{y_k}{u_{2k}}\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{dW(t)}{dt} & \leq \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} \lambda_k \varphi(k) P(k) \{ S_{1k}^* g(u_{1k}) + S_{2k}^* g(u_{2k}) \} \\
& \times \left\{ \frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} k \lambda_k \varphi(k) P(k) I_k^* - \frac{(h + \mu)(\delta + \mu)}{\alpha \beta h + (1 - \alpha) \gamma h} \right\}.
\end{aligned}$$

From the third and forth equations of (5), we can obtain

$$E_k^* = \frac{\delta + \mu}{\alpha \beta h + (1 - \alpha) \gamma h} (S_{1k}^* + S_{2k}^*).$$

Then, from the second equation of (5), one has

$$\frac{1}{\langle k \rangle} \sum_{k=1}^{k_{\max}} k \lambda_k \varphi(k) P(k) I_k^* - \frac{(h + \mu)(\delta + \mu)}{\alpha \beta h + (1 - \alpha) \gamma h} = 0,$$

which implies that $\frac{dW(t)}{dt} \leq 0$ and $\frac{dW(t)}{dt} = 0$ if and only if

$$(I_k(t), E_k(t), S_{1k}(t), S_{2k}(t), R_{1k}(t), R_{2k}(t)) = (I_k^*, E_k^*, S_{1k}^*, S_{2k}^*, R_{1k}^*, R_{2k}^*).$$

Therefore, $\tilde{E}^*$ of system (1) is globally asymptotically stable for $\mathcal{R}_0 > 1$. The proof of Theorem 5 is completed. $\square$

Remark 3 At present, the Lyapunov direct method is still one of the most effective methods to investigate the stability of some complex systems including nonlinear systems and rumor-spreading model in networks. By employing such theory and referring to the

Table 2 Sensitivity index of $\mathcal{R}_0$ for parameter values of model (1)

Variable	Parameter	Sensitivity index	Variable	Parameter	Sensitivity index
$\mathcal{R}_0$	$\sum i\lambda_i \varphi(i)P(i)$	1	$\mathcal{R}_0$	h	0.2
	b	1		δ	-0.5
	α	0.03		μ	-1
	β	0.3488		$\langle k \rangle$	-1
	γ	0.6512			

method of constructing Lyapunov function in [30], two appropriate Lyapunov functions are proposed to explore the stability of equilibrium points for model (1) in Theorems 3 and 5. Subsequently, the main results related to the global asymptotic stability have been established with a combination of LaSalle's invariance principle in [27].

Corollary 3 *The unique rumor-prevailing equilibrium $\widehat{E}^*$ is globally asymptotically stable, when $\mathcal{R}_0 > 1$ and $R_{1k}(t) + R_{2k}(t) = R_k(t)$.*

Proof From Remark 2 and Theorem 4, the unique rumor-prevailing equilibrium $\widehat{E}^* = (I_k^*, E_k^*, S_{1k}^*, S_{2k}^*, \widehat{R}_k^*)$ is obtained, where

$$\widehat{R}_k^* = \frac{\{(\delta + \mu)[\alpha(1 - \beta) + (1 - \alpha)(1 - \gamma)] + [\alpha\beta + (1 - \alpha)\gamma]\}hkb(\Theta_1^* + \Theta_2^*)}{\mu(k(\Theta_1^* + \Theta_2^*) + \mu)(h + \mu)(\delta + \mu)}.$$

The following proof is similar to that of Theorem 5, which is omitted here. $\square$

Remark 4 In [18, 19], different SEIR rumor-spreading models were presented and of the rumor-prevailing equilibrium was explored by utilizing a monotone iterative technique. It is worth noting that the global stability of rumor equilibrium for an SEIR model on heterogeneous network has not been discussed, as far as we know. Different from those, a more general SEIR model (4) is proposed based on a bilingual environment and heterogeneous complex network and the global stability of the equilibrium points of model (1) is investigated by mainly employing the Lyapunov stability theory and LaSalle's invariance principle.

4 Sensitivity analysis

In this section, we will discuss the effect of parameters in model (1) on the basic reproduction number by means of sensitivity analysis. First, we introduce the definition of sensitivity analysis as follows:

Definition 1 ([28, 31]) The normalized forward sensitivity index of the variable $\mathcal{R}_0$, which depends on a differentiable parameter q , is defined as

$$\Upsilon_q^{\mathcal{R}_0} := \frac{\partial \mathcal{R}_0}{\partial q} \times \frac{q}{\mathcal{R}_0}.$$

By a simple calculation, the sensitivity index of $\mathcal{R}_0$ for parameters in model (1) is presented in Table 2, where some parameters are fixed as follows: $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.4$, $h = 0.8$, $\delta = 0.2$, and $\mu = 0.2$.

Next, the influence extent of parameters related to $\mathcal{R}_0$ on IE2S2R rumor-spreading model (1) can be evaluated quantitatively by sensitivity analysis in Table 2. The relative

variation of parameters related to $\mathcal{R}_0$ are shown in Table 2 if we alter the parameters by 1%. Whereupon, a reduction of 1% in forgetting or educational mechanism δ may lead to an increase of $\mathcal{R}_0$ by 0.5%; a reduction of 1% in leaving rate μ may lead to an increase of $\mathcal{R}_0$ by 1%; a reduction of 1% in the average degree $\langle k \rangle$ may lead to an increase of $\mathcal{R}_0$ by 1%. Otherwise, a reduction of 1% in $\sum i\lambda_i\varphi(i)P(i)$ or b results in a decrease of $\mathcal{R}_0$ by 1%; a reduction of 1% in the probability of exposures with language-1 based α results in a decrease of $\mathcal{R}_0$ by 0.03%; a reduction of 1% in the transmission rate $\beta(\gamma)$ decreases $\mathcal{R}_0$ by 0.3488% (0.6512%); a reduction of 1% in h decreases $\mathcal{R}_0$ by 0.2%.

These results clearly show that the most effective tactics in providing $\mathcal{R}_0$ reduction would be to increase the leaving rate μ or strengthening education so as to increase the value of δ (through the government punishment mechanism, releasing official information timely, and developing ideology education activity). Obviously, the transmission rates β, γ can also be reduced due to the strengthening of education. Hence, educational mechanisms play a major role in controlling the spread of rumors.

5 Numerical simulations

In this section, some numerical simulations are presented to demonstrate the validity of our proposed theoretical results with different parameters.

A new study in this paper is based on a heterogenous network with a power law degree distribution:

$$P(k) = \frac{2m(m+1)}{k(k+1)(k+2)} \propto 2m^2k^{-3},$$

where m is the minimum degree of model (1). In this paper, we choose $\varphi(k) = k^{1.5}$, $\lambda_i = 0.02$, $m = 1$, and $k_{\max} = 100$. From a simple calculation, the average degree $\langle k \rangle = 3.27$ and $\sum_{i=1}^{100} i\lambda_i\varphi(i)P(i) = 0.7436$.

Example 1 At first, we verify the stability of rumor-free equilibrium $\tilde{E}_0$ in Theorems 2 and 3. Choose $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.4$, $h = 0.8$, $\delta = 0.2$, $b = 0.05$, and $\mu = 0.05$. By a simple computation, it is derived that the basic reproduction number $\mathcal{R}_0 \approx 0.3681 < 1$ for rumor spreading model (1). From Theorem 2, IE2S2R rumor-spreading model (1) has a unique locally asymptotically stable rumor-free equilibrium $\tilde{E}_0$, which is verified in Figs. 2(a)–(b) with $k = 30$ and Figs. 3(a)–(b) with $k = 80$. According to Theorem 3, the rumor-free equilibrium $\tilde{E}_0$ is globally asymptotically stable, which is verified in Figs. 2(c)–(d) with $k = 30$ and Figs. 3(c)–(d) with $k = 80$.

Example 2 The globally asymptotical stability of rumor-prevailing equilibrium $\tilde{E}^*$ in Theorem 5 needs to be verified. Select $\alpha = 0.7$, $\beta = \gamma = 0.9$, $h = 0.8$, $\delta = 0.01$, $b = 0.05$, and $\mu = 0.05$. By a simple computation, it is derived that the basic reproduction number $\mathcal{R}_0 \approx 3.2104 > 1$ for the above chosen parameters for model (1). Hence, $\tilde{E}^*$ is globally asymptotically stable, which is verified in Fig. 4 with $k = 40$ and Fig. 5 with $k = 70$.

Remark 5 From Figs. 2–5, it is easy to check how different connection degree affects the densities of ignoramuses, exposures, spreaders, and stiflers. When $\mathcal{R}_0 < 1$, the peak of

$S_{ik}(t)$ ($i = 1, 2$) is increased and the convergence speed of rumor-free equilibrium $\tilde{E}_0$ is accelerated with increasing k . When $\mathcal{R}_0 > 1$, the value of $S_{ik}(t)$ ($I_k(t)$) is increased (decreased) by increasing the connection degree k .

From sensitivity analysis in Sect. 4, it is shown that β, γ, h, δ have an important influence on rumor spreading in model (1). In order to more clearly understand the impact of these parameters for $\mathcal{R}_0 > 1$, an example with some different situations is given.

Example 3 Choose $\alpha = 0.5$, $\gamma = 0.9$, $h = 0.8$, $\delta = 0.05$, $b = 0.04$, and $\mu = 0.05$. By a simple computation, it is derived that $0.253 \leq \beta \leq 1$ when $\mathcal{R}_0 > 1$. Hence, we can select $\beta = 0.3$, $\beta = 0.5$, $\beta = 0.7$, and $\beta = 0.9$ to describe the influence of β in rumor spreading. The influence extent of β on $S_{1,90}(t)$ and $S_{2,90}(t)$ is shown in Fig. 6, which implies that the value of $S_{1,90}(t)$ is increased with the increase of β , and β has a little effect on $S_{2,90}(t)$.

Fix $\alpha = 0.2$, $\beta = 0.8$, $h = 0.6$, $\delta = 0.02$, $b = 0.08$, and $\mu = 0.1$. By a simple computation, it is derived that $0.762 \leq \gamma \leq 1$ when $\mathcal{R}_0 > 1$. Hence, we can choose $\gamma = 0.8$, $\gamma = 0.85$, $\gamma = 0.9$, and $\gamma = 0.95$. The impact extent of γ on $S_{1,90}(t)$ and $S_{2,90}(t)$ is shown in Fig. 7, which implies that γ has a little effect on $S_{1,90}(t)$, and the value of $S_{2,90}(t)$ is increased with the increase of γ .

Choose $\alpha = 0.5$, $\beta = \gamma = 0.8$, $h = 0.9$, $b = 0.05$, and $\mu = 0.05$. By a simple computation, it is derived that $0 \leq \delta \leq 0.173$ when $\mathcal{R}_0 > 1$. Hence, we can select $\delta = 0.05$, $\delta = 0.1$, $\delta = 0.15$, and $\delta = 0.17$ to describe the influence of δ in model (1). The impact extent of δ on $S_{1,90}(t)$ and $S_{2,90}(t)$ is shown in Fig. 8, which just indicates that the value of $S_{1,90}(t)$ and $S_{2,90}(t)$ is decreased with the increase of δ .

Choose $\alpha = 0.2$, $\beta = 0.6$, $\gamma = 0.8$, $\delta = 0.05$, $b = 0.1$, and $\mu = 0.1$. By a simple computation, it is derived that $0.028 \leq h \leq 1$ when $\mathcal{R}_0 > 1$. Hence, we can select $h = 0.05$, $h = 0.35$, $h = 0.65$, and $h = 0.95$ to describe the influence of h in rumor spreading. Figure 9 shows that the value of $S_{1,90}(t)$ and $S_{2,90}(t)$ will increase if h increases.

Remark 6 This paper studies the dynamics of IE2S2R model under heterogeneous networks, in which time delays are not considered. It is generally appreciated that time de-

lays are unavoidable in real life, especially with regard to the process of rumor spreading. Until now, there are few articles on the subject of rumor spreading in which time delays were taken into account [32–34]. These results, however, were only established in homogeneous networks and a single-language environment. Therefore, it is necessary to take time delays into model (1) of this paper in the future work.

6 Conclusion

By adding the hesitation mechanism, the dynamics of a new IE2S2R rumor-spreading model in a bilingual environment and heterogeneous networks is studied in this paper. Firstly, the threshold condition of $\mathcal{R}_0$ is given to determine whether the rumor lives or dies. Next, theoretical results obtained in this paper indicate that the rumor-free equilibrium $\tilde{E}_0$ is globally asymptotically stable if $\mathcal{R}_0 < 1$ and the rumor-prevailing equilibrium $\tilde{E}^*$ is globally asymptotically stable if $\mathcal{R}_0 > 1$. In addition, sensitivity analysis is given to clarify the influence extent for rumor spreading of different parameters in (1). Finally, the effectiveness of theoretical results has been shown through some numerical examples. Further work will be to study the dynamics of IE2S2R rumor-spreading model with time delays, which is more complicated.

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Availability of data and materials

Data sharing not applicable to this paper as no datasets were generated or analyzed during the current study.

Competing interests

The authors have declared that no competing interests exist.

Authors' contributions

SY established the mathematical model, theoretical analysis, and wrote the original draft; HJJ provided modeling ideas and analysis methods; CH checked the correctness of theoretical results, JY collected relevant literature and modified manuscript; JRL performed the simulation experiments. All authors read and approved the final manuscript.

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