Meena et al. *Advances in Difference Equations* (2020) 2020:562
https://doi.org/10.1186/s13662-020-03022-z

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

# Some expansion formulas for incomplete $H$- and $\overline{H}$-functions involving Bessel functions

Sapna Meena[1], Sanjay Bhatter[1], Kamlesh Jangid[2*] and Sunil Dutt Purohit[2]

*Correspondence:
jangidkamlesh7@gmail.com
[2]Department of HEAS (Mathematics), Rajasthan Technical University, Kota, India
Full list of author information is available at the end of the article

**Abstract**

In this paper, we assess an integral containing incomplete $H$-functions and utilize it to build up an expansion formula for the incomplete $H$-functions including the Bessel function. Next, we evaluate an integral containing incomplete $\overline{H}$-functions and use it to develop an expansion formula for the incomplete $\overline{H}$-functions including the Bessel function. The outcomes introduced in this paper are general in nature, and several particular cases can be acquired by giving specific values to the parameters engaged with the principle results. As particular cases, we derive expansions for the incomplete Meijer ${}^{(\Gamma)}G$-function, Fox–Wright ${}_p\Psi_q^{(\Gamma)}$-function, and generalized hypergeometric ${}_p\Gamma_q$ function.

**MSC:** Primary 33B20; 33C60; secondary 33C10

**Keywords:** Fox's $H$-function; Incomplete $H$-functions; Incomplete $\overline{H}$-functions; Mellin–Barnes contour integral; Bessel function

## 1 Introduction and preliminaries

The topic of special functions is very rich and is constantly increasing with the advent of new problems in the field of applications in engineering and applied sciences. In addition, applications to $H$-functions was already documented in a large range of response-related topics, such as diffusion, reaction–diffusion, electronics and communication, fractional differential and additive equations, other fields of theoretical physics, biology, and mathematical probability theory. Therefore, due to the overwhelming demand, a number of papers on these functions and their possible applications have been made available in the literature. For further information, see the research monographs [18, 19] and recent work [1–3, 16]. The main object of this paper is to build up integrals including incomplete $H$-functions and incomplete $\overline{H}$-functions and utilize them to get expansions for incomplete $H$-functions and incomplete $\overline{H}$-functions involving the Bessel function with the help of the orthogonal properties of Bessel functions.

The $H$-function, $\overline{H}$-function, and incomplete type functions such as gamma functions, $H$-functions, $\overline{H}$-functions that are to be used further are described below.

Fox [12] investigated and defined a new function during his study of symmetrical Fourier kernels in terms of the Mellin–Barnes-type contour integral, known as Fox's $H$-function

$$H_{p,q}^{m,n}(z) = H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,A_j)_{1,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_{\mathcal{L}}\Theta(\vartheta)z^{-\vartheta}\,d\vartheta, \tag{1}$$

where

$$\Theta(\vartheta) = \frac{\prod_{j=1}^{m}\Gamma(b_j+B_j\vartheta)\prod_{j=i}^{n}\Gamma(1-a_j-A_j\vartheta)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-B_j\vartheta)\prod_{j=n+1}^{p}\Gamma(a_j+A_j\vartheta)}, \tag{2}$$

$\mathcal{L}$ is a convenient contour that detached the poles. $m,n,p,q$ are positive integers with constraints $0\leq n\leq p$, $1\leq m\leq q$, the coefficients $A_j$ $(j=1,\dots,p)$ and $B_j$ $(j=1,\dots,q)\in\mathbb{R}^+$, and $a_j$ and $b_j$ are complex parameters. The $H$-function is absolutely convergent and defines an analytic function under the set of conditions described in [12] (see also [15, 18, 19]).

In 1987, Inayat-Hussain [13] introduced a generalization of the $H$-functions, known as the $\overline{H}$-functions:

$$\overline{H}_{p,q}^{m,n}(z) = \overline{H}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,A_j;\zeta_j)_{1,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_{\mathcal{L}\prime}\overline{\Theta}(\vartheta)z^{-\vartheta}\,d\vartheta, \tag{3}$$

where

$$\overline{\Theta}(\vartheta) = \frac{\prod_{j=1}^{m}\Gamma(b_j+B_j\vartheta)\prod_{j=1}^{n}[\Gamma(1-a_j-A_j\vartheta)]^{\zeta_j}}{\prod_{j=m+1}^{q}[\Gamma(1-b_j-B_j\vartheta)]^{\eta_j}\prod_{j=n+1}^{p}\Gamma(a_j+A_j\vartheta)}, \tag{4}$$

$m,n,p,q\in N_0$ with constraints $0\leq n\leq p$, $1\leq m\leq q$, $A_j$ $(j=1,\dots,p)$, $B_j$ $(j=1,\dots,q)\in\mathbb{R}^+$, $a_j$ $(j=1,\dots,p)$, and $b_j$ $(j=1,\dots,q)$ are complex numbers. The exponents $\zeta_j$ $(j=1,\dots,n)$ and $\eta_j$ $(j=m+1,\dots,q)$ take noninteger values, and $\mathcal{L}\prime$ is a reasonable contour that detaches the poles. The $\overline{H}$-function is absolutely convergent under the arrangement of conditions described by Buschman and Srivastava [10].

We next recollect and define the lower and upper incomplete gamma functions $\gamma(\vartheta,z)$ and $\Gamma(\vartheta,z)$ as follows:

$$\gamma(\vartheta,z) = \int_0^z y^{\vartheta-1}e^{-y}\,dy \quad \bigl(\Re(\vartheta)>0; z\geqq 0\bigr) \tag{5}$$

and

$$\Gamma(\vartheta,z) = \int_z^\infty y^{\vartheta-1}e^{-y}\,dy \quad \bigl(z\geqq 0; \Re(\vartheta)>0 \text{ if } z=0\bigr). \tag{6}$$

These functions fulfill the following relation:

$$\gamma(\vartheta,z)+\Gamma(\vartheta,z) = \Gamma(z) \quad \bigl(\Re(\vartheta)>0\bigr). \tag{7}$$

Using the incomplete gamma functions defined above, Srivastava et al. [23] presented and researched the incomplete $H$-functions as follows:

$$\gamma_{p,q}^{m,n}(z) = \gamma_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_1,A_1,y),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_{\mathcal{L}}g(\vartheta,y)z^{-\vartheta}\,d\vartheta \tag{8}$$

and

$$\Gamma_{p,q}^{m,n}(z) = \Gamma_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_1,A_1,y),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_{\mathcal{L}} G(\vartheta,y)z^{-\vartheta}\,d\vartheta, \tag{9}$$

where

$$g(\vartheta,y) = \frac{\gamma(1-a_1-A_1\vartheta,y)\prod_{j=1}^{m}\Gamma(b_j+B_j\vartheta)\prod_{j=2}^{n}\Gamma(1-a_j-A_j\vartheta)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-B_j\vartheta)\prod_{j=n+1}^{p}\Gamma(a_j+A_j\vartheta)} \tag{10}$$

and

$$G(\vartheta,y) = \frac{\Gamma(1-a_1-A_1\vartheta,y)\prod_{j=1}^{m}\Gamma(b_j+B_j\vartheta)\prod_{j=2}^{n}\Gamma(1-a_j-A_j\vartheta)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-B_j\vartheta)\prod_{j=n+1}^{p}\Gamma(a_j+A_j\vartheta)}, \tag{11}$$

with the arrangement of conditions setout in [23].

These incomplete $H$-functions fulfill the following relation (known as decomposition formula):

$$\gamma_{p,q}^{m,n}(z) + \Gamma_{p,q}^{m,n}(z) = H_{p,q}^{m,n}(z). \tag{12}$$

The incomplete $H$-functions $\gamma_{p,q}^{m,n}(z)$ and $\Gamma_{p,q}^{m,n}(z)$ defined in (8) and (9) exist for $x \geq 0$ under the set of conditions given by Srivastava et al. [23], with

$$\Omega > 0, \qquad |\arg(z)| < \frac{\Omega\pi}{2} \quad \text{and} \quad \Delta > 0,$$

where

$$\begin{aligned}
\Omega &= \sum_{i=1}^{m} B_i - \sum_{i=m+1}^{q} B_i + \sum_{i=1}^{n} A_i - \sum_{i=n+1}^{p} A_i,\\
\delta &= \sum_{i=1}^{q} b_i - \sum_{i=1}^{p} a_i + \frac{p-q}{2} \quad \text{and} \quad \Delta = \sum_{i=1}^{q} B_i - \sum_{i=1}^{p} A_i.
\end{aligned}$$

Srivastava et al. [23] introduced a generalization of the incomplete $H$-functions, known as the incomplete $\overline{H}$-functions defined in the following way:

$$\begin{aligned}
\overline{\gamma}_{p,q}^{m,n}(z) &= \overline{\gamma}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right.\right]\\
&= \frac{1}{2\pi i}\int_{\mathcal{L}} \overline{g}(\vartheta,y)z^{-\vartheta}\,d\vartheta,
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\overline{\Gamma}_{p,q}^{m,n}(z) &= \overline{\Gamma}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right.\right]\\
&= \frac{1}{2\pi i}\int_{\mathcal{L}} \overline{G}(\vartheta,y)z^{-\vartheta}\,d\vartheta,
\end{aligned} \tag{14}$$

where

$$\overline{g}(\vartheta, y) = \frac{[\gamma(1-a_1-A_1\vartheta, y)]^{\zeta_1}\prod_{j=1}^{m}\Gamma(b_j+B_j\vartheta)\prod_{j=2}^{n}[\Gamma(1-a_j-A_j\vartheta)]^{\zeta_j}}{\prod_{j=m+1}^{q}[\Gamma(1-b_j-B_j\vartheta)]^{\eta_j}\prod_{j=n+1}^{p}\Gamma(a_j+A_j\vartheta)} \tag{15}$$

and

$$\overline{G}(\vartheta, y) = \frac{[\Gamma(1-a_1-A_1\vartheta, y)]^{\zeta_1}\prod_{j=1}^{m}\Gamma(b_j+B_j\vartheta)\prod_{j=2}^{n}[\Gamma(1-a_j-A_j\vartheta)]^{\zeta_j}}{\prod_{j=m+1}^{q}[\Gamma(1-b_j-B_j\vartheta)]^{\eta_j}\prod_{j=n+1}^{p}\Gamma(a_j+A_j\vartheta)}, \tag{16}$$

with the arrangements of conditions setout in [23] with

$$\overline{\Omega} = \sum_{i=1}^{m}|B_i| - \sum_{i=m+1}^{q}|\eta_i B_i| + \sum_{i=1}^{n}|\zeta_i A_i| - \sum_{i=n+1}^{p}|A_i| > 0 \quad \text{and} \quad \left|\arg(z)\right| < \frac{\pi\overline{\Omega}}{2}. \tag{17}$$

Several authors currently work on a wide variety of applications for these incomplete functions. See, for example, recent works [6–9, 14, 20–22] and references therein.

The paper is organized in the following way. In Sect. 2, we evaluate the improper integrals involving the Bessel function, incomplete $H$-functions, and incomplete $\overline{H}$-functions. In Sect. 3, we derive expansions for incomplete $H$-functions and incomplete $\overline{H}$-functions involving the Bessel function with the help of integrals presented in Sect. 2 and the orthogonal properties of Bessel functions. In Sect. 4, we obtain particular cases.

## 2 The integrals

In this section, we derive improper integrals involving the Bessel function, incomplete $H$-functions, and incomplete $\overline{H}$-functions. These integrals will be used in Sect. 3 to prove the expansions for incomplete $H$-functions and incomplete $\overline{H}$-functions.

**Theorem 2.1** *Let* $\Re(\vartheta) > 0$, $\Omega > 0$, $h > 0$, $|\arg(z)| < \frac{\pi\Omega}{2}$, $\Re(u+v+h\frac{b_i}{B_i}) > -1$, *and* $\Re(u) < -\frac{1}{2}$. *Then for* $y \geq 0$,

$$\begin{aligned}
&\int_0^\infty e^{ix}x^u J_v(x)\Gamma_{p,q}^{m,n}\left[z\,x^h \,\middle|\, \begin{matrix}(a_1,A_1;y),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right]dx\\
&\quad= \frac{e^{\frac{1}{2}(u+v+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}\\
&\qquad\times \Gamma_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{\frac{i\pi}{2}}}{2}\right)^h \,\middle|\, \begin{matrix}(a_1,A_1;y),(-u-v,h),(a_j,A_j)_{2,p},(v-u,h)\\(-u-\frac{1}{2},h),(b_j,B_j)_{1,q}\end{matrix}\right].
\end{aligned}\tag{18}$$

*Proof* To demonstrate (18), consider its the left-hand side. Expressing the incomplete $H$-function in terms of the Mellin–Barnes-type integral defined in (9), we have

$$\text{L.H.S} = \int_0^\infty e^{ix}x^u J_v(x)\frac{1}{2\pi i}\int_{\mathcal{L}} G(\vartheta,y)\left(z\,x^h\right)^{-\vartheta}\,d\vartheta\,dx.$$

Change the integration order, we have

$$\text{L.H.S} = \frac{1}{2\pi i}\int_{\mathcal{L}} G(\vartheta,y)z^{-\vartheta}\int_0^\infty e^{ix}x^{u-h\vartheta}J_v(x)\,dx\,d\vartheta.$$

To assess the above internal integral, we will use the following formula [17, p. 106, (1)]:

$$\int_0^\infty e^{ix}x^u J_\nu(x)\,dx = \frac{e^{\frac{1}{2}(u+\nu+1)i\pi}\Gamma(u+\nu+1)\Gamma(-u-\frac{1}{2})}{2^{u+1}\Gamma(\frac{1}{2})\Gamma(\nu-u)},$$
$$\Re(u+\nu) < -1, \Re(u) < -\frac{1}{2}. \tag{19}$$

Then we get

$$\text{L.H.S} = \frac{1}{2\pi i}\int_{\mathcal{L}} G(\vartheta,y)\frac{e^{\frac{1}{2}(u+\nu+1-h\vartheta)i\pi}\Gamma(u+\nu+1-h\vartheta)\Gamma(-u-\frac{1}{2}+h\vartheta)}{2^{u+1-h\vartheta}\Gamma(\frac{1}{2})\Gamma(\nu-u+h\vartheta)}z^{-\vartheta}\,d\vartheta. \tag{20}$$

Using (11), we obtain the required right-hand side of (18). □

**Theorem 2.2** *Let* $\Re(\vartheta) > 0$, $\Omega > 0$, $h > 0$, $|\arg z| < \frac{\pi\Omega}{2}$, $\Re(u+\nu+h\frac{b_i}{B_i}) > -1$, *and* $\Re(u) < -\frac{1}{2}$, *Then for* $y \geq 0$,

$$\int_0^\infty e^{ix}x^u J_\nu(x)\gamma_{p,q}^{m,n}\left[z\,x^h\left|\begin{matrix}(a_1,A_1;y),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right]dx$$
$$= \frac{e^{\frac{1}{2}(u+\nu+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}$$
$$\times\,\gamma_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{\frac{i\pi}{2}}}{2}\right)^h\left|\begin{matrix}(a_1,A_1;y),(-u-\nu,h),(a_j,A_j)_{2,p},(\nu-u,h)\\(-u-\frac{1}{2},h),(b_j,B_j)_{1,q}\end{matrix}\right.\right]. \tag{21}$$

**Theorem 2.3** *Let* $\Re(\vartheta) > 0$, $\overline{\Omega} > 0$, $h > 0$, $|\arg(z)| < \frac{\pi\overline{\Omega}}{2}$, $\Re(u+\nu+h\frac{b_i}{B_i}) > -1$, *and* $\Re(u) < -\frac{1}{2}$. *Then for* $y \geq 0$,

$$\int_0^\infty e^{ix}x^u J_\nu(x)\overline{\Gamma}_{p,q}^{m,n}\left[z\,x^h\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right.\right]dx$$
$$= \frac{e^{\frac{1}{2}(u+\nu+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}$$
$$\times\,\overline{\Gamma}_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{\frac{i\pi}{2}}}{2}\right)^h\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(-u-\nu,h;1),(a_j,A_j;\zeta_j)_{2,n},\\(-u-\frac{1}{2},h),(b_j,B_j)_{1,m},\end{matrix}\right.\right.$$
$$\left.\begin{matrix}(a_j,A_j)_{n+1,p},(\nu-u,h)\\(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right]. \tag{22}$$

*Proof* To demonstrate (22), consider its left-hand side. Expressing the incomplete $\overline{H}$-function in terms of the Mellin–Barnes-type integral defined in (14), we have

$$\text{L.H.S} = \int_0^\infty e^{ix}x^u J_\nu(x)\frac{1}{2\pi i}\int_{\mathcal{L}}\overline{G}(\vartheta,y)\left(z\,x^h\right)^{-\vartheta}\,d\vartheta\,dx.$$

Changing the integration order, we have

$$\text{L.H.S} = \frac{1}{2\pi i}\int_{\mathcal{L}}\overline{G}(\vartheta,y)z^{-\vartheta}\int_0^\infty e^{ix}x^{u-h\vartheta}J_\nu(x)\,dx\,d\vartheta.$$

From this by means of formula (19) we obtain

$$\text{L.H.S} = \frac{1}{2\pi i}\int_{\mathcal{L}} \overline{G}(\vartheta, y)\frac{e^{\frac{1}{2}(u+v+1-h\vartheta)i\pi}\Gamma(u+v+1-h\vartheta)\Gamma(-u-\frac{1}{2}+h\vartheta)}{2^{u+1-h\vartheta}\Gamma(\frac{1}{2})\Gamma(v-u+h\vartheta)}z^{-\vartheta}\,d\vartheta, \tag{23}$$

and using (16), we obtain the required right-hand side of (22). □

**Theorem 2.4** *Let* $\Re(\vartheta) > 0$, $\overline{\Omega} > 0$, $h > 0$, $|\arg(z)| < \frac{\pi\overline{\Omega}}{2}$, $\Re(u+v+h\frac{b_i}{B_i}) > -1$, *and* $\Re(u) < -\frac{1}{2}$. *Then for* $y \geq 0$,

$$\begin{aligned}
&\int_0^\infty e^{ix}x^u J_v(x)\overline{\gamma}_{p,q}^{m,n}\left[z\,x^h\middle|\begin{matrix}(a_1, A_1; \zeta_1; y), (a_j, A_j; \zeta_j)_{2,n}, (a_j, A_j)_{n+1,p}\\(b_j, B_j)_{1,m}, (b_j, B_j; \eta_j)_{m+1,q}\end{matrix}\right]dx\\
&\quad= \frac{e^{\frac{1}{2}(u+v+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}\\
&\qquad\times \overline{\gamma}_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{\frac{i\pi}{2}}}{2}\right)^h\middle|\begin{matrix}(a_1, A_1; \zeta_1; y), (-u-v, h; 1), (a_j, A_j; \zeta_j)_{2,n}, (a_j, A_j)_{n+1,p}, (v-u, h)\\(-u-\frac{1}{2}, h), (b_j, B_j)_{1,m}, (b_j, B_j; \eta_j)_{m+1,q}\end{matrix}\right].
\end{aligned}\tag{24}$$

*Remark* If we set $\zeta_j = 1$ ($j = 1, \ldots, n$) and $\eta_j = 1$ ($j = m+1, \ldots, q$) into Theorems 2.3 and 2.4, we obtained the results of Theorems 2.1 and 2.2, respectively.

## 3 Expansion formulas

In this section, we present expansions for incomplete $H$-functions and incomplete $\overline{H}$-functions involving the Bessel function with the help of integrals presented in Sect. 2 and derive the orthogonality of Bessel functions.

**Theorem 3.1** *Let* $h > 0$, $\sum_{i=1}^{p} A_i - \sum_{i=1}^{q} B_i \leq 0$, $\Omega > 0$, $|\arg(z)| < \frac{\Omega\pi}{2}$, $r = \mu + 2\vartheta + 1$, $\Re(u + v + 2h\frac{b_i}{B_i}) > 0$ $(i = 1, \ldots, m)$, *and* $\Re(u) < \frac{1}{2}$. *The for* $y \geq 0$,

$$\begin{aligned}
&e^{ix}x^u\Gamma_{p,q}^{m,n}\left[z\,x^h\middle|\begin{matrix}(a_1, A_1; y), (a_j, A_j)_{2,p}\\(b_j, B_j)_{1,q}\end{matrix}\right]\\
&\quad= \frac{1}{2^{u-1}\Gamma(\frac{1}{2})}\sum_{\vartheta=0}^{\infty} re^{\frac{1}{2}(u+r)i\pi}J_r(x)\\
&\qquad\times \Gamma_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{i\pi/2}}{2}\right)^h\middle|\begin{matrix}(a_1, A_1; y), (-u-r+1, h), (a_j, A_j)_{2,p}, (r-u+1, h)\\(-u+\frac{1}{2}, h), (b_j, B_j)_{1,q}\end{matrix}\right].
\end{aligned}\tag{25}$$

*Proof* To demonstrate (25), let

$$f(x) = e^{ix}x^u\Gamma_{p,q}^{m,n}\left[z\,x^h\middle|\begin{matrix}(a_1, A_1; y), (a_j, A_j)_{2,p}\\(b_j, B_j)_{1,q}\end{matrix}\right] = \sum_{\vartheta=0}^{\infty} C_\vartheta J_{\mu+2\vartheta+1}, \tag{26}$$

where $f(x)$ is ceaseless and bounded in the interval $(0, \infty)$ when $u \geq 0$. Hence statement (25) is substantial.

Multiplying both sides of (26) by $x^{-1}J_{\mu+2t+1}(x)$ and integrating with respect to $x$ from 0 to $\infty$, we get

$$\begin{aligned}&\int_0^\infty e^{ix}x^{u-1}J_{\mu+2t+1}(x)\Gamma_{p,q}^{m,n}\left[z\,x^h\left|\begin{matrix}(a_1,A_1;y),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right]dx\\&\quad=\sum_{\vartheta=0}^{\infty}C_\vartheta\int_0^\infty x^{-1}J_{\mu+2t+1}(x)J_{\mu+2\vartheta+1}(x)\,dx.\end{aligned}$$

Now applying statement (18) and the orthogonality of the Bessel functions [17, p. 291, (6)], we obtain

$$\begin{aligned}C_t&=\frac{\nu e^{\frac{1}{2}(u+\nu)i\pi}}{2^{u-1}\Gamma(\frac{1}{2})}\\&\quad\times\Gamma_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{i\pi/2}}{2}\right)^h\left|\begin{matrix}(a_1,A_1;y),(-u-\nu+1,h),(a_j,A_j)_{2,p},(\nu-u+1,h)\\(-u+\frac{1}{2},h),(b_j,B_j)_{1,q}\end{matrix}\right.\right],\end{aligned}\tag{27}$$

where $\nu=\mu+2t+1$. From equations (26) and (27) we obtain the desired result (22). □

**Theorem 3.2** *Let $h>0$, $\sum_{i=1}^{p}A_i-\sum_{i=1}^{q}B_i\le 0$, $\Omega>0$, $|\arg(z)|<\frac{\Omega\pi}{2}$, $r=\mu+2\vartheta+1$, $\Re(u+\nu+2h\frac{b_i}{B_i})>0$ $(i=1,\dots,m)$, and $\Re(u)<\frac{1}{2}$, Then for $y\ge 0$,*

$$\begin{aligned}&e^{ix}x^u\gamma_{p,q}^{m,n}\left[z\,x^h\left|\begin{matrix}(a_1,A_1;y),(a_j,A_j)_{2,p}\\(b_j,B_j)_{1,q}\end{matrix}\right.\right]\\&\quad=\frac{1}{2^{u-1}\Gamma(\frac{1}{2})}\sum_{\vartheta=0}^{\infty}re^{\frac{1}{2}(u+r)i\pi}J_r(x)\\&\qquad\times\gamma_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{i\pi/2}}{2}\right)^h\left|\begin{matrix}(a_1,A_1;y),(-u-r+1,h),(a_j,A_j)_{2,p},(r-u+1,h)\\(-u+\frac{1}{2},h),(b_j,B_j)_{1,q}\end{matrix}\right.\right].\end{aligned}\tag{28}$$

**Theorem 3.3** *Let $h>0$, $\overline{\Omega}>0$, $|\arg(z)|<\frac{\pi\overline{\Omega}}{2}$, $r=\mu+2\vartheta+1$, $\Re(u+\nu+2h\frac{b_i}{B_i})>0$ $(i=1,\dots,m)$, and $\Re(u)<\frac{1}{2}$. Then for $y\ge 0$,*

$$\begin{aligned}&e^{ix}x^u\overline{\Gamma}_{p,q}^{m,n}\left[z\,x^h\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right.\right]\\&\quad=\frac{1}{2^{u-1}\Gamma(\frac{1}{2})}\sum_{\vartheta=0}^{\infty}re^{\frac{1}{2}(u+r)i\pi}J_r(x)\\&\qquad\times\overline{\Gamma}_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{i\pi/2}}{2}\right)^h\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(-u-r+1,h;1),(a_j,A_j;\zeta_j)_{2,n},\\(-u+\frac{1}{2},h),(b_j,B_j)_{1,m},\end{matrix}\right.\right.\\&\qquad\qquad\left.\begin{matrix}(a_j,A_j)_{n+1,p},(r-u+1,h)\\(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right].\end{aligned}\tag{29}$$

*Proof* To demonstrate (29), let

$$g(x)=e^{ix}x^u\overline{\Gamma}_{p,q}^{m,n}\left[z\,x^h\left|\begin{matrix}(a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p}\\(b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q}\end{matrix}\right.\right]$$

$$= \sum_{\vartheta=0}^{\infty} C_{\vartheta} J_{\mu+2\vartheta+1}, \tag{30}$$

where $g(x)$ is ceaseless and bounded in the interval $(0,\infty)$ when $u \geq 0$. Hence statement (29) is substantial.

Multiplying both sides of (30) by $x^{-1}J_{\mu+2t+1}(x)$ and integrating with respect to $x$ from 0 to $\infty$, we have

$$\begin{aligned} &\int_0^{\infty} e^{ix} x^{u-1} J_{\mu+2t+1}(x) \overline{\Gamma}_{p,q}^{m,n}\left[ z\,x^h \,\middle|\, \begin{matrix} (a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p} \\ (b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q} \end{matrix} \right] dx \\ &= \sum_{\vartheta=0}^{\infty} C_{\vartheta} \int_0^{\infty} x^{-1} J_{\mu+2t+1}(x) J_{\mu+2\vartheta+1}(x)\,dx. \end{aligned}$$

Now applying (22) and the orthogonality property of the Bessel functions [17, p. 291, (6)], we obtain

$$\begin{aligned} C_t = {}& \frac{v e^{\frac{1}{2}(u+v)i\pi}}{2^{u-1}\Gamma(\frac{1}{2})} \\ &\times \overline{\Gamma}_{p+2,q+1}^{m+1,n+1}\left[ z\left(\frac{e^{i\pi/2}}{2}\right)^h \,\middle|\, \begin{matrix} (a_1,A_1;\zeta_1;y),(-u-v+1,h;1)(a_j,A_j;\zeta_j)_{2,n}, \\ (-u+\frac{1}{2},h),(b_j,B_j)_{1,m}, \end{matrix} \right. \\ &\qquad \left. \begin{matrix} (a_j,A_j)_{n+1,p},(v-u+1,h) \\ (b_j,B_j;\eta_j)_{m+1,q} \end{matrix} \right], \end{aligned} \tag{31}$$

where $v = \mu + 2t + 1$. From equations (30) and (31) we obtain the desired result (29). □

**Theorem 3.4** *Let* $h > 0$, $\overline{\Omega} > 0$, $|\arg(z)| < \frac{\pi\overline{\Omega}}{2}$, $r = \mu + 2\vartheta + 1$, $\Re(u+v+2h\frac{b_i}{B_i}) > 0$ $(i = 1,\ldots,m)$, *and* $\Re(u) < \frac{1}{2}$. *Then for* $y \geq 0$,

$$\begin{aligned} &e^{ix} x^u \overline{\gamma}_{p,q}^{m,n}\left[ z\,x^h \,\middle|\, \begin{matrix} (a_1,A_1;\zeta_1;y),(a_j,A_j;\zeta_j)_{2,n},(a_j,A_j)_{n+1,p} \\ (b_j,B_j)_{1,m},(b_j,B_j;\eta_j)_{m+1,q} \end{matrix} \right] \\ &= \frac{1}{2^{u-1}\Gamma(\frac{1}{2})} \sum_{\vartheta=0}^{\infty} r e^{\frac{1}{2}(u+r)i\pi} J_r(x) \\ &\quad \times \overline{\gamma}_{p+2,q+1}^{m+1,n+1}\left[ z\left(\frac{e^{i\pi/2}}{2}\right)^h \,\middle|\, \begin{matrix} (a_1,A_1;\zeta_1;y),(-u-r+1,h;1),(a_j,A_j;\zeta_j)_{2,n}, \\ (-u+\frac{1}{2},h),(b_j,B_j)_{1,m}, \end{matrix} \right. \\ &\qquad \left. \begin{matrix} (a_j,A_j)_{n+1,p},(r-u+1,h) \\ (b_j,B_j;\eta_j)_{m+1,q} \end{matrix} \right]. \end{aligned} \tag{32}$$

*Remark* Setting $\zeta_j = 1$ $(j = 1,\ldots,n)$ and $\eta_j = 1$ $(j = m+1,\ldots,q)$ in Theorems 3.3 and 3.4, we obtain Theorems 3.1 and 3.2, respectively.

## 4 Particular cases

The results presented in this paper are of a very general nature, and their particular cases are scattered throughout the literature. Particular cases of expansion are mentioned only for the incomplete $\Gamma_{p,q}^{m,n}$ function.

If we assign specific values to the parameters of the incomplete $\Gamma_{p,q}^{m,n}$ function, then this function converts into the incomplete Meijer ${}^{(\Gamma)}G$-function, incomplete Fox–Wright ${}_p\Psi_q^{(\Gamma)}$-function, and incomplete generalized hypergeometric ${}_p\Gamma_q$ function. In this Section, we establish integral formulas and expansion formulas for these incomplete functions as particular cases of Theorem 2.1 and Theorem 3.1.

(1) Letting $A_j = 1$ ($j = 1,\dots,p$), $B_j = 1$ ($j = 1,\dots,q$), and $h = 1$, the function (9) reduces into the incomplete Meijer ${}^{(\Gamma)}G$-function as follows:

$$\Gamma_{p,q}^{m,n}\left[z\,x\,\middle|\begin{matrix}(a_1,1,y),(a_j,1)_{2,p}\\(b_j,1)_{1,q}\end{matrix}\right] = {}^{(\Gamma)}G_{p,q}^{m,n}\left[z\,x\,\middle|\begin{matrix}(a_1,y),(a_j)_{2,p}\\(b_j)_{1,q}\end{matrix}\right]. \tag{33}$$

Using relation (33) in (18) and (25), respectively, we obtain the following corollaries.

**Corollary 1** *Let* $\Re(\vartheta) > 0$, $2(m+n) > p+q$, $|\arg(z)| < (m+n-\frac{p}{2}-\frac{q}{2})\pi$, $\Re(u+v+b_i) > -1$, $i = 1,\dots,m$, *and* $\Re(u) < -\frac{1}{2}$. *Then*

$$\begin{aligned}&\int_0^\infty e^{ix}x^u J_v(x)\,{}^{(\Gamma)}G_{p,q}^{m,n}\left[z\,x\,\middle|\begin{matrix}(a_1,y),(a_j)_{2,p}\\(b_j)_{1,q}\end{matrix}\right]dx\\&\quad= \frac{e^{\frac{1}{2}(u+v+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}\,{}^{(\Gamma)}G_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{\frac{i\pi}{2}}}{2}\right)\middle|\begin{matrix}(a_1;y),(-u-v),(a_j)_{2,p},(v-u)\\(-u-\frac{1}{2}),(b_j)_{1,q}\end{matrix}\right].\end{aligned} \tag{34}$$

**Corollary 2** *Let* $\Re(\vartheta) > 0$, $2(m+n) > p+q$, $|\arg(z)| < (m+n-\frac{p}{2}-\frac{q}{2})\pi$, $\Re(u+b_i) > 0$, $i = 1,\dots,m$, $\Re(u) < \frac{1}{2}$, *and* $r = \mu + 2\vartheta + 1$. *Then*

$$\begin{aligned}&e^{ix}x^{u}\,{}^{(\Gamma)}G_{p,q}^{m,n}\left[z\,x\,\middle|\begin{matrix}(a_1;y),(a_j)_{2,p}\\(b_j)_{1,q}\end{matrix}\right]\\&\quad= \frac{1}{2^{u-1}\Gamma(\frac{1}{2})}\sum_{\vartheta=0}^{\infty} r e^{\frac{1}{2}(u+r)i\pi}J_r(x)\\&\qquad\times {}^{(\Gamma)}G_{p+2,q+1}^{m+1,n+1}\left[z\left(\frac{e^{i\pi/2}}{2}\right)\middle|\begin{matrix}(a_1;y),(-u-r+1),(a_j)_{2,p},(r-u+1)\\(-u+\frac{1}{2}),(b_i)_{1,q}\end{matrix}\right].\end{aligned} \tag{35}$$

(2) Taking $m = 1$ and $n = p$, replacing $q$ with $q+1$, and taking appropriate parameters such as $z = -z$, $a_j \to (1-a_j)$ ($j = 1,\dots,p$), and $b_j \to (1-b_j)$ ($j = 1,\dots,q$), the incomplete $H$-function (9) converts to the incomplete Fox–Wright ${}_p\Psi_q^{(\Gamma)}$-function (see [23]):

$$\begin{aligned}&\Gamma_{p,q+1}^{1,p}\left[-z\middle|\begin{matrix}(1-a_1,A_1,y),(1-a_j,A_j)_{2,p}\\(0,1),(1-b_j,B_j)_{1,q}\end{matrix}\right]\\&\quad= {}_p\Psi_q^{(\Gamma)}\left[\begin{matrix}(a_1,A_1,y),(a_j,A_j)_{2,p};\\ \\(b_j,B_j)_{1,q};\end{matrix}\;z\right].\end{aligned} \tag{36}$$

Using relation (36) in (18) and (25), respectively, we get the following corollaries.

**Corollary 3** *Let* $\Re(\vartheta) > 0, h > 0, |\arg(-z)| < \pi, \Re(u+v+h\frac{(1-b_i)}{B_i}) > -1$, *and* $\Re(u) < -\frac{1}{2}$. *Then*

$$\begin{aligned}
&\int_0^\infty e^{ix}x^u J_v(x)_p\Psi_q^{(\Gamma)}\left[\begin{matrix}(a_1,A_1;y),(a_j,A_j)_{2,p}; \\ \\ (b_j,B_j)_{1,q};\end{matrix}\quad z\,x^h\right]dx \\
&= \frac{e^{\frac{1}{2}(u+v+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}\;{}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{matrix}(a_1,A_1;y),(-u-v,h),(a_j,A_j)_{2,p},(v-u,h); \\ \\ (-u-\frac{1}{2},h),(b_j,B_j)_{1,q};\end{matrix}\quad z\Big(\frac{e^{\frac{i\pi}{2}}}{2}\Big)^h\right]. \qquad (37)
\end{aligned}$$

**Corollary 4** *Let* $\Re\vartheta > 0$, $h > 0$, $|\arg(-z)| < \pi$, $\Re(u + v + 2h\frac{(1-b_i)}{B_i}) > 0$, $\Re(u) < \frac{1}{2}$, *and* $r = \mu + 2\vartheta + 1$. *Then*

$$\begin{aligned}
&e^{ix}x^u{}_p\Psi_q^{(\Gamma)}\left[\begin{matrix}(a_1,A_1;y),(a_j,A_j)_{2,p}; \\ \\ (b_j,B_j)_{1,q};\end{matrix}\quad z\,x^h\right] \\
&= \frac{1}{2^{u-1}\Gamma(\frac{1}{2})}\sum_{\vartheta=0}^{\infty} re^{\frac{1}{2}(u+r)i\pi}J_r(x) \\
&\times {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{matrix}(a_1,A_1;y),(-u-r+1,h),(a_j,A_j)_{2,p},(r-u+1,h); \\ \\ (-u+\frac{1}{2},h),(b_j,B_j)_{1,q};\end{matrix}\quad z\Big(\frac{e^{i\pi/2}}{2}\Big)^h\right]. \qquad (38)
\end{aligned}$$

(3) Further, substituting $h = 1$, $A_j = 1$ $(j = 1,\dots,p)$, and $B_j = 1$ $(j = 1,\dots,q)$ into (37) and (38) and using the relation (see [23])

$$\begin{aligned}
&\Gamma_{p,q+1}^{1,p}\left[-z\,x\left|\begin{matrix}(1-a_1,1,y),(1-a_i,1)_{2,p} \\ (0,1),(1-b_i,1)_{1,q}\end{matrix}\right.\right] \\
&= \mathcal{C}_q^p\,{}_p\Gamma_q\left[\begin{matrix}(a_1,y),a_2,\dots,a_p; \\ \\ b_1,\dots,b_q;\end{matrix}\quad z\,x\right], \qquad (39)
\end{aligned}$$

where $\mathcal{C}_q^p = \frac{\prod_{j=1}^p\Gamma(a_j)}{\prod_{j=1}^q\Gamma(b_j)}$, we obtain the subsequent corollaries.

**Corollary 5** *Let* $\Re(\vartheta) > 0, |\arg(-z)| < \pi, \Re(u+v+(1-b_i)) > -1$, *and* $\Re(u) < -\frac{1}{2}$. *Then*

$$\begin{aligned}
&\int_0^\infty e^{ix}x^u J_v(x)_p\Gamma_q\left[\begin{matrix}(a_1,y),a_2,\dots,a_p; \\ \\ b_1,\dots,b_q;\end{matrix}\quad z\,x\right]dx \\
&= \frac{e^{\frac{1}{2}(u+v+1)i\pi}}{2^{u+1}\Gamma(\frac{1}{2})}\frac{\Gamma(-u-v)\Gamma(v-u)}{\Gamma(-u-\frac{1}{2})} \\
&\times {}_{p+2}\Gamma_{q+1}\left[\begin{matrix}(a_1,y),(-u-v),a_2,\dots,a_p,(v-u); \\ \\ (-u-\frac{1}{2}),b_1,\dots,b_q;\end{matrix}\quad z\Big(\frac{e^{i\pi/2}}{2}\Big)\right]. \qquad (40)
\end{aligned}$$

**Corollary 6** *Let* $\Re(\vartheta) > 0$, $|\arg(-z)| < \pi$, $\Re(u+v+2(1-b_i)) > 0$, $\Re(u) < \frac{1}{2}$, *and* $r = \mu + 2\vartheta + 1$. *Then*

$$\begin{aligned} & e^{ix} x^u {}_p\Gamma_q \left[ \begin{matrix} (a_1, y), a_2, \dots, a_p; & \\ & z\,x \\ b_1, \dots, b_q; & \end{matrix} \right] \\ & = \frac{1}{2^{u-1}\Gamma(\frac{1}{2})} \sum_{\vartheta=0}^{\infty} r e^{\frac{1}{2}(u+r)i\pi} \frac{\Gamma(1-u-r)\Gamma(r-u+1)}{\Gamma(-u+\frac{1}{2})} J_r(x) \\ & \quad \times {}_{p+2}\Gamma_{q+1} \left[ \begin{matrix} (a_1, y), (-u-r+1), a_2, \dots, a_p, (r-u+1); & \\ & z(\frac{e^{i\pi/2}}{2}) \\ (-u+\frac{1}{2}), b_1, \dots, b_q; & \end{matrix} \right]. \end{aligned} \tag{41}$$

(4) If we put $y = 0$ in (9), then the incomplete $H$-function $\Gamma_{p,q}^{m,n}(z)$ converts into the generally known Fox $H$-function:

$$\Gamma_{p,q}^{m,n} \left[ z \left| \begin{matrix} (a_1, A_1; 0), (a_j, A_j)_{2,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right. \right] = H_{p,q}^{m,n} \left[ z \left| \begin{matrix} (a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right. \right].$$

Using this relation in (25), we obtain the result previously derived by Bajpai [5, p. 44, (3.1)],

$$\begin{aligned} & e^{ix} x^u H_{p,q}^{m,n} \left[ z\,x^h \left| \begin{matrix} (a_j, A_j)_{1,p} \\ (b_j, B_j)_{1,q} \end{matrix} \right. \right] \\ & = \frac{1}{2^{u-1}\Gamma(\frac{1}{2})} \sum_{\vartheta=0}^{\infty} r e^{\frac{1}{2}(u+r)i\pi} J_r(x) \\ & \quad \times H_{p+2,q+1}^{m+1,n+1} \left[ z \left( \frac{e^{i\pi/2}}{2} \right)^h \left| \begin{matrix} (-u-r+1, h), (a_j, A_j)_{1,p}, (r-u+1, h) \\ (-u+\frac{1}{2}, h), (b_j, B_j)_{1,q} \end{matrix} \right. \right]. \end{aligned} \tag{42}$$

(5) In (25), setting $A_j = 1$ $(j = 1, \dots, p)$, $B_j = 1$ $(j = 1, \dots, q)$, and $y = 0$, assuming $h$ to be a positive integer, using the relation

$$\Gamma_{p,q}^{m,n} \left[ z \left| \begin{matrix} (a_1, 1; 0), (a_i, 1)_{2,p} \\ (b_i, 1)_{1,q} \end{matrix} \right. \right] = G_{p,q}^{m,n} \left[ z \left| \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right. \right],$$

and solving with the assistance of [11, p. 4, (11)] and [11, p. 207, (1)], we get the result obtained by Bajpai [4, p. 287, (3.1)]:

$$\begin{aligned} & e^{ix} x^u G_{p,q}^{m,n} \left[ z\,x^h \left| \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \right. \right] \\ & = \frac{h^{u-1(2\pi)^{\frac{1-h}{2}}}}{2^{u-1}\Gamma(\frac{1}{2})} \sum_{\vartheta=0}^{\infty} r e^{\frac{1}{2}(u+r)i\pi} J_r(x) \\ & \quad \times G_{p+2h,q+h}^{m+h,n+h} \left[ z \left( \frac{h e^{i\pi/2}}{2} \right)^h \left| \begin{matrix} \Delta(h, 1-u-r), (a_j)_{1,p}, \Delta(h, 1-u+r) \\ \Delta(h, \frac{1}{2}-u), (b_j)_{1,q} \end{matrix} \right. \right], \end{aligned} \tag{43}$$

where $\Delta(h,a)$ represents the set of parameters $(\frac{a}{h}), (\frac{a+1}{h}), \ldots, (\frac{a+h-1}{h})$, and $2(m+n) > p+q$, $|\arg(z)| < (m+n-\frac{1}{2}p-\frac{1}{2}q)\pi$, $\Re(u+hb_i) > 0$ $(i=1,\ldots,m)$, $\Re(u) < \frac{1}{2}$, $r = \mu + 2\vartheta + 1$.

We summarize this paper by stating that by using the orthogonality of the Bessel function, we discussed the integral formulas and expansion formulas of incomplete $H$-functions and incomplete $\overline{H}$-functions. In Sect. 4, we also obtained some particular cases of our main results and some known ones. The results introduced in this paper are new and can be used to subsidiary different new and known outcomes having applications in science and engineering.

**Acknowledgements**
The authors would like to express their appreciation to the referees for their valuable suggestions, which helped to achieve better presentation of this paper.

**Funding**
Not applicable.

**Availability of data and materials**
Not applicable.

**Competing interests**
The authors declare that they have no competing interests.

**Authors' contributions**
The authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

**Author details**
[1]Department of Mathematics, Malaviya National Institute of Technology, Jaipur, India. [2]Department of HEAS (Mathematics), Rajasthan Technical University, Kota, India.

## Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 1 July 2020 Accepted: 30 September 2020 Published online: 07 October 2020

**References**

1. Al-Omari, S.K.: On a class of generalized Meijer–Laplace transforms of Fox function type kernels and their extension to a class of Boehmians. Georgian Math. J. **24**(1), 1–13 (2017)
2. Al-Omari, S.K.: Estimation of a modified integral associated with a special function kernel of Fox's *H*-function type. Commun. Korean Math. Soc. **35**(1), 125–136 (2020)
3. Al-Omari, S.K., Jumah, H., Al-Omari, J., Saxena, D.: A new version of the generalized Krätzel–Fox integral operators. Mathematics **6**, 222 (2018)
4. Bajpai, S.D.: Some expansion formulae for *G*-function involving Bessel functions. Proc. Ind. Acad. Sci. **58**, 285–290 (1968)
5. Bajpai, S.D.: Some results involving Fox's *H*-function and Bessel function. Proc. Ind. Acad. Sci. **72**, 42–46 (1970)
6. Bansal, M.K., Choi, J.: A note on pathway fractional integral formulas associated with the incomplete *H*-functions. Int. J. Appl. Comput. Math. **5**(5), 133 (2019)
7. Bansal, M.K., Kumar, D., Khan, I., Singh, J., Nisar, K.S.: Certain unified integrals associated with product of *M*-series and incomplete *H*-functions. Mathematics **7**(12), 1191 (2019)
8. Bansal, M.K., Kumar, D., Singh, J., Nisar, K.S.: On the solutions of a class of integral equations pertaining to incomplete *H*-function and incomplete $\overline{H}$-function. Mathematics **8**(5), 819 (2020)
9. Bansal, M.K., Kumar, D., Singh, J., Tassaddiq, A., Nisar, K.S.: Some new results for the Srivastava–Luo–Raina $\mathbb{M}$-transform pertaining to the incomplete *H*-functions. AIMS Math. **5**(1), 717–722 (2020)
10. Buschman, R.G., Srivastava, H.M.: The $\overline{H}$-function associated with certain class of Feynman integrals. J. Phys. A, Math. Gen. **23**, 4707–4710 (1990)
11. Erdelyi, A.: Higher Transcendental Functions. McGraw-Hill, New York (1953)
12. Fox, C.: The *G* and *H*-functions as symmetrical Fourier kernels. Trans. Am. Math. Soc. **98**, 395–429 (1961)
13. Inayat-Hussain, A.A.: New properties of hypergeometric series derivable from Feynman integrals. II: a generalisations of the *H*-function. J. Phys. A **20**, 4119–4128 (1987)
14. Jangid, K., Bhatter, S., Meena, S., Baleanu, D., Qurashi, M.A., Purohit, S.D.: Some fractional calculus findings associated with the incomplete *I*-functions. Adv. Differ. Equ. **2020**, 265 (2020)
15. Kilbas, A.A., Srivastava, H.M., Trujillo, J.J.: Theory and Applications of Fractional Differential Equations. North-Holland Mathematical Studies, vol. 204. Elsevier, Amsterdam (2006)
16. Kumar, D., Ayant, F.Y., Purohit, S.D., Uçar, F.: On partial derivatives of the *I*-function of *r*-variables. Azerb. J. Math. **10**(2), 49–61 (2020)

17. Luke, Y.L.: Integrals of Bessel Functions. MacGraw-Hill, New York (1962)
18. Mathai, A.M., Saxena, R.K.: The *H*-Function with Applications in Statistics and Other Disciplines. Wiley, New York (1978)
19. Mathai, A.M., Saxena, R.K., Haubold, H.J.: The *H*-Functions: Theory and Applications. Springer, New York (2010)
20. Nisar, K.S., Purohit, S.D., Abouzaid, M.S., Al-Qurashi, M., Baleanu, D.: Generalized *k*-Mittag-Leffler function and its composition with pathway integral operators. J. Nonlinear Sci. Appl. **9**, 3519–3526 (2016)
21. Parmar, R.K., Saxena, R.K.: Incomplete extended Hurwitz–Lerch zeta functions and associated properties. Commun. Korean Math. Soc. **32**, 287–304 (2017)
22. Purohit, S.D., Khan, A.M., Suthar, D.L., Dave, S.: The impact on raise of environmental pollution and occurrence in biological populations pertaining to incomplete *H*-function. Nat. Acad. Sci. Lett. (2020). https://doi.org/10.1007/s40009-020-00996-y
23. Srivastava, H.M., Saxena, R.K., Parmar, R.K.: Some families of the incomplete *H*-functions and the incomplete $\overline{H}$-functions and associated integral transforms and operators of fractional calculus with applications. Russ. J. Math. Phys. **25**(1), 116–138 (2018)