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Classical stabilities of multiplicative inverse difference and adjoint functional equations

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Abstract

The aim of this present article is to investigate various classical stability results of the multiplicative inverse difference and adjoint functional equations

$$m_d\left(\frac{rs}{r+s}\right) - m_d\left(\frac{2rs}{r+s}\right) = \frac{1}{2}[m_d(r) + m_d(s)]$$

and

$$m_a\left(\frac{rs}{r+s}\right) + m_a\left(\frac{2rs}{r+s}\right) = \frac{3}{2}[m_a(r) + m_a(s)]$$

in the framework of non-zero real numbers. A proper counter-example is illustrated to prove the failure of the stability results for control cases. The relevance of these functional equations in optics is also discussed.

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1 Introduction

The investigation of an approximate solution near to the exact solution of many mathematical equations such as functional, differential, difference and algebraic equations is a fascinating concept in the research domain of pure and applied mathematics. This theory emanated from an inquisitive query proposed in [24] and was developed through different versions, namely Ulam–Hyers Stability (UHS), Ulam–Hyers–T. Rassias Stability (UHTRS), Ulam–Hyers–J. Rassias Stability (UHJRS) and Ulam–Hyers–Rassias–Gavruta Stability (HURGS) by many mathematicians [1, 9, 10, 13, 14]. Recently, this theory has got a lot of momentum in dealing with different types of rational and multiplicative inverse functional equations with their applications in diversified fields, such as physics, electrical engineering, digital image processing. There are more instigating and novel results concerning this topic. For more information, refer to [2–7, 11, 12, 15, 18–22].

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The rational form of the reciprocal, reciprocal difference and reciprocal adjoint functional equations

$$m(r + s) = \frac{m(r)m(s)}{m(r) + m(s)}, \tag{1.1}$$

$$m\left(\frac{r + s}{2}\right) - m(r + s) = \frac{m(r)m(s)}{m(r) + m(s)}, \tag{1.2}$$

and

$$m\left(\frac{r + s}{2}\right) + m(r + s) = \frac{3m(r)m(s)}{m(r) + m(s)}, \tag{1.3}$$

were introduced and obtained their classical stability results in [16, 17].

The multiplicative inverse functional equation

$$m\left(\frac{rs}{r + s}\right) = m(r) + m(s), \tag{1.4}$$

was dealt with in [23] to prove its non-Archimedean stabilities. For the first time in stability theory, a functional equation involving function of a rational argument is considered. It is easy to examine that the reciprocal mapping $m(r) = \frac{1}{r}$ is a solution of (1.4). Induced by the solution of (1.4) and since the power of a thin lens is the multiplicative inverse of its focal length, we focus on the following multiplicative inverse difference and adjoint functional equations, respectively, to investigate their solutions and analytical stabilities:

$$m_d\left(\frac{rs}{r + s}\right) - m_d\left(\frac{2rs}{r + s}\right) = \frac{1}{2}[m_d(r) + m_d(s)] \tag{1.5}$$

and

$$m_a\left(\frac{rs}{r + s}\right) + m_a\left(\frac{2rs}{r + s}\right) = \frac{3}{2}[m_a(r) + m_a(s)], \tag{1.6}$$

where $m_d, m_a : \mathbb{R}^* \rightarrow \mathbb{R}$ are mappings. We solve Eqs. (1.5) and (1.6). Then we show that there exists an approximate solution near the exact solution of these equations via the Ulam stability idea by considering different upper bounds. We present a suitable example to prove the failure of stability results for singular cases. We also discuss the situations in optics where Eqs. (1.5) and (1.6) could be utilized.

To avoid a singularity in the main results, throughout this paper, let us assume that $\frac{rs}{r+s} \neq 0$ for all $r, s \in \mathbb{R}^*$.

2 Equivalency of Eqs. (1.5) and (1.6)

In the ensuing result, we will prove that Eqs. (1.5) and (1.6) are equivalent and we obtain their solutions.

Theorem 2.1 *Assume a mapping $m : \mathbb{R}^* \rightarrow \mathbb{R}$. Then the following statements are equivalent.*

- (a) m satisfies (1.4).

(b) m satisfies (1.5).

(c) m satisfies (1.6).

Hence, the solution of Eqs. (1.5) and (1.6) is a multiplicative inverse mapping.

Proof Initially, assume that m satisfies (1.4). Now, reinstating (r, s) by $(2r, 2r)$ in (1.4) and then dividing by 2, one finds that

$$m(2r) = \frac{1}{2}m(r) \tag{2.1}$$

for all $r \in \mathbb{R}^*$. Now, replacing (r, s) by $(2r, 2s)$ in (1.4) and in lieu of (2.1) in the resultant, one obtains

$$m\left(\frac{2rs}{r+s}\right) = \frac{1}{2}[m(r) + m(s)] \tag{2.2}$$

for all $r, s \in \mathbb{R}^*$. Now, subtracting (2.2) from (1.4), one arrives at (1.5). Next, let us assume that m satisfies (1.5). If (r, s) is replaced by $(2r, 2r)$ in (1.5), then the resultant produces (1.6). Finally, let m satisfy (1.6). By a similar argument as above, substituting $(2r, 2r)$ in place of (r, s) in (1.6), and then employing the resulting outcome leads to (1.4). \square

3 Stabilities of Eqs. (1.5) and (1.6)

In this fragment, we investigate various analytical stabilities associated with (1.5) and (1.6) in the domain of non-zero real numbers. To obtain the main results in a simple approach, consider the difference operators $\Delta_{m_d}, \Delta_{m_a} : \mathbb{R}^* \times \mathbb{R}^* \rightarrow \mathbb{R}$, defined respectively by

$$\begin{aligned} \Delta_{m_d}(r, s) &= m_d\left(\frac{rs}{r+s}\right) - m_d\left(\frac{2rs}{r+s}\right) - \frac{1}{2}[m_d(r) + m_d(s)], \\ \Delta_{m_a}(r, s) &= m_a\left(\frac{rs}{r+s}\right) + m_a\left(\frac{2rs}{r+s}\right) - \frac{3}{2}[m_a(r) + m_a(s)], \end{aligned}$$

for all $r, s \in \mathbb{R}^*$.

Theorem 3.1 Assume a fixed constant $\mu = \pm 1$ and a mapping $m_d : \mathbb{R}^* \rightarrow \mathbb{R}$. Suppose $\zeta : \mathbb{R}^* \times \mathbb{R}^* \rightarrow [0, \infty)$ is a function satisfying

$$|\Delta_{m_d}(r, s)| \leq \zeta(r, s) \tag{3.1}$$

for all $r, s \in \mathbb{R}^*$, where ζ satisfies

$$\sum_{p=0}^{\infty} \frac{1}{2^{\mu p}} \zeta\left(\frac{r}{2^{\mu(p+1)}}, \frac{s}{2^{\mu(p+1)}}\right) < +\infty \tag{3.2}$$

for all $r, s \in \mathbb{R}^*$. Then a unique multiplicative inverse mapping $M_d : \mathbb{R}^* \rightarrow \mathbb{R}$ exists and satisfies (1.5) with the result that

$$|m_d(r) - M_d(r)| \leq \sum_{p=0}^{\infty} \frac{1}{2^{\mu p}} \zeta\left(\frac{r}{2^{\mu(p+1)}}, \frac{r}{2^{\mu(p+1)}}\right) \tag{3.3}$$

for all $r \in \mathbb{R}^*$.

Proof Firstly, let us prove it for the case $\mu = 1$. Switching (r, s) to (r, r) in (3.1), we obtain

$$\left| \frac{1}{2^\mu} m_d \left(\frac{r}{2^\mu} \right) - m_d(r) \right| \leq |2|^{\frac{|\mu-1|}{2}} \zeta \left(\frac{r}{2^{\frac{\mu+1}{2}}}, \frac{r}{2^{\frac{\mu+1}{2}}} \right) \tag{3.4}$$

for all $r \in \mathbb{R}^*$. Now, replacing r with $\frac{r}{2^{\ell\mu}}$ in (3.4) and then multiplying by $|\frac{1}{2}|^{\ell\mu}$, we get

$$\left| \frac{1}{2^{\ell\mu}} m_d \left(\frac{r}{2^{\ell\mu}} \right) - \frac{1}{2^{(\ell+1)\mu}} m_d \left(\frac{r}{2^{(\ell+1)\mu}} \right) \right| \leq \frac{1}{2^{\ell\mu}} \zeta \left(\frac{r}{2^{(\ell+1)\mu}}, \frac{r}{2^{(\ell+1)\mu}} \right). \tag{3.5}$$

Letting $\ell \rightarrow \infty$ in (3.5) and using (3.2), we see that $\{\frac{1}{2^{\ell\mu}} m_d(\frac{r}{2^{(\ell+1)\mu}})\}$ is a Cauchy sequence. In lieu of completeness of \mathbb{R} , the sequence $\{\frac{1}{2^{\ell\mu}} m_d(\frac{r}{2^{(\ell+1)\mu}})\}$ converges to a mapping M_d defined by

$$M_d(r) = \lim_{n \rightarrow \infty} \frac{1}{2^{\ell\mu}} m_d(2^{(\ell+1)\mu} r). \tag{3.6}$$

Now, we claim that M_d satisfies (1.5). Plugging (r, s) into $(\frac{r}{2^{\ell\mu}}, \frac{s}{2^{\ell\mu}})$ in (3.1) and then in the resultant, dividing by $2^{\ell\mu}$ on its both sides, we obtain

$$\left| \frac{1}{2^{\ell\mu}} \Delta_{m_d} \left(\frac{r}{2^{\ell\mu}}, \frac{s}{2^{\ell\mu}} \right) \right| \leq \frac{1}{2^{\ell\mu}} \zeta \left(\frac{r}{2^{\ell\mu}}, \frac{s}{2^{\ell\mu}} \right) \tag{3.7}$$

for all $r, s \in \mathbb{R}^*$ and for all positive integers ℓ . Now, using (3.2), (3.6) in (3.7), we find that M_d satisfies (1.5) for all $r, s \in \mathbb{R}^*$. For each $r \in \mathbb{R}^*$ and each integer ℓ , we have

$$\begin{aligned} \left| \frac{1}{2^{\ell\mu}} m_d \left(\frac{r}{2^{\ell\mu}} \right) - m_d(r) \right| &\leq \sum_{k=0}^{\ell-1} \left| \frac{1}{2^{k\mu}} m_d \left(\frac{r}{2^{(k+1)\mu}} \right) - \frac{1}{2^{(k-1)\mu}} m_d \left(\frac{r}{2^{k\mu}} \right) \right| \\ &\leq \sum_{k=0}^{\ell-1} \frac{1}{2^{k\mu}} \zeta \left(\frac{r}{2^{(k+1)\mu}}, \frac{r}{2^{(k+1)\mu}} \right). \end{aligned}$$

Applying (3.6) and letting $\ell \rightarrow \infty$, we obtain (3.3). Next we prove that M_d is unique. For this, let us consider $M'_d : \mathbb{R}^* \rightarrow \mathbb{R}$ to be an alternative multiplicative inverse mapping satisfying (1.5) and (3.3). Therefore

$$M'_d \left(\frac{r}{2^{\ell\mu}} \right) = 2^{\ell\mu} M'_d(r) \quad \text{and} \quad M_d \left(\frac{r}{2^{\ell\mu}} \right) = 2^{\ell\mu} M_d(r)$$

for all $r \in \mathbb{R}^*$. By the application of (3.3), we obtain

$$\begin{aligned} |M'_d(r) - M_d(r)| &= 2^{-\ell\mu} |M'_d(2^{-\ell\mu} r) - m_d(2^{-\ell\mu} r)| \\ &\leq 2^{-\ell\mu} |m'_d(2^{-\ell\mu} r) - m_d(2^{-\ell\mu} r)| + 2^{-\ell\mu} |m_d(2^{-\ell\mu} r) - m_d(2^{-\ell\mu} r)| \\ &\leq 2 \sum_{p=0}^{\infty} \frac{1}{2^{(\ell+p)\mu}} \zeta \left(\frac{r}{2^{(\ell+p+1)\mu}}, \frac{r}{2^{(\ell+p+1)\mu}} \right) \\ &\leq 2 \sum_{p=\ell}^{\infty} \frac{1}{2^{p\mu}} \zeta \left(\frac{r}{2^{(p+1)\mu}}, \frac{r}{2^{(p+1)\mu}} \right) \end{aligned} \tag{3.8}$$

for all $r \in \mathbb{R}^*$. It can be noticed that M_d is unique by allowing ℓ to ∞ in (3.8). This completes the proof for $\mu = 1$. The proof for the case $\mu = -1$ is similar to the above arguments. \square

The subsequent results are various stabilities pertinent to HUS, HUTRS and HUIRS of Eq. (1.5). The proofs directly follow from Theorem 3.1 for $\mu = -1$. Hence, we omit the proofs. In the ensuing results, let $m_d : \mathbb{R}^* \rightarrow \mathbb{R}$ be a mapping.

Corollary 3.2 *Suppose there exists a constant $\eta \geq 0$ (independent of r and s) such that the inequality*

$$|\Delta_{m_d}(r, s)| \leq \frac{\eta}{2}$$

holds for all $r, s \in \mathbb{R}^$. Then the limit*

$$M_d(r) = \lim_{\ell \rightarrow \infty} \frac{1}{2^{\ell\mu}} m_d\left(\frac{r}{2^{\ell\mu}}\right)$$

exists for all $r \in \mathbb{R}^$, $\ell \in \mathbb{N}$ and $M_d : \mathbb{R}^* \rightarrow \mathbb{R}$ is the unique multiplicative inverse mapping exists and satisfies (1.5) with the result that*

$$|m_d(r) - M_d(r)| \leq \eta$$

for all $r \in \mathbb{R}^$.*

Corollary 3.3 *For any fixed $\eta_1 \geq 0$ and $\theta \neq -1$, let the mapping m_d satisfy*

$$|\Delta_{m_d}(r, s)| \leq \eta_1 (|r|^\theta + |s|^\theta)$$

for all $r, s \in \mathbb{R}^$. Then a unique multiplicative inverse mapping $M_d : \mathbb{R}^* \rightarrow \mathbb{R}$ exists and satisfies (1.5) with the result that*

$$|m_d(r) - M_d(r)| \leq \frac{2^{\mu+1} \eta_1}{|2^{\mu(\theta+1)} - 1|} |r|^\theta \quad \text{for } \theta \neq -1$$

for all $r \in \mathbb{R}^$.*

Corollary 3.4 *Suppose the constants γ, δ exist such that $\theta = \gamma + \delta \neq -1$ and $\eta_2 \geq 0$. Assume the mapping m_d satisfies*

$$|\Delta_{m_d}(r, s)| \leq \eta_2 |r|^\gamma |s|^\delta$$

for all $r, s \in \mathbb{R}^$. Then there exists a unique multiplicative inverse mapping $M_d : \mathbb{R}^* \rightarrow \mathbb{R}$ satisfying (1.5) with the result that*

$$|m_d(r) - M_d(r)| \leq \frac{2^\mu \eta_2}{|2^{\mu(\theta+1)} - 1|} |r|^\theta \quad \text{for } \theta \neq -1$$

and satisfies (1.5) for all $r, s \in \mathbb{R}^$.*

Inspired by the counter-example in [8], we prove that Eq. (1.5) fails to be stable for $\theta = -1$ in Corollary 3.3 in the domain of non-zero real numbers. For this, the function defined below is used to present a suitable counter-example.

Let a function $v : \mathbb{R}^* \rightarrow \mathbb{R}$ be defined as

$$v(r) = \begin{cases} \frac{\alpha}{r} & \text{for } r \in (1, \infty), \\ \alpha & \text{elsewhere.} \end{cases} \tag{3.9}$$

Let $m_d : \mathbb{R}^* \rightarrow \mathbb{R}$ be defined by

$$m_d(r) = \sum_{k=0}^{\infty} 2^{-k} v(2^{-k}r) \tag{3.10}$$

for all $r \in \mathbb{R}^*$. Then the function m_d turns into a suitable example to illustrate that (1.5) is unstable for $\theta = -1$ in Corollary 3.3 in the subsequent theorem.

Theorem 3.5 *Assume that the function $m_d : \mathbb{R}^* \rightarrow \mathbb{R}$ described in (3.9) satisfies the inequality*

$$|\Delta_{m_d}(r, s)| \leq 12\alpha(|r|^{-1} + |s|^{-1}) \tag{3.11}$$

for all $r, s \in \mathbb{R}^*$. Then a multiplicative inverse mapping $M_d : \mathbb{R}^* \rightarrow \mathbb{R}$ and a constant $K > 0$ do not exist such that

$$|m_d(r) - M_d(r)| \leq K|r|^{-1} \tag{3.12}$$

for all $r \in \mathbb{R}^*$.

Proof Firstly, let us prove that the mapping m_d satisfies (3.11). Using the definition of m_d , we find $|m_d(r)| = |\sum_{k=0}^{\infty} 2^{-k} v(2^{-k}r)| \leq \sum_{k=0}^{\infty} \frac{\alpha}{2^k} = 2\alpha$, which indicates that m_d is bounded by 2α on \mathbb{R} . If $|r|^{-1} + |s|^{-1} \geq 1$, then the expression on the left hand side of (3.11) is less than 12α . On the other hand, assume that $0 < |r|^{-1} + |s|^{-1} < 1$. Hence, there exists an integer $j > 0$ such that

$$\frac{1}{2^{j+1}} \leq |r|^{-1} + |s|^{-1} < \frac{1}{2^j}. \tag{3.13}$$

As a result, Eq. (3.13) leads to $2^j(|r|^{-1} + |s|^{-1}) < 1$, or equivalently; $2^j r^{-1} < 1, 2^j s^{-1} < 1$. So, $\frac{r}{2^j} > 1, \frac{s}{2^j} > 1$. From the last inequalities, we observe that $\frac{r}{2^{j-1}} > 2 > 1, \frac{s}{2^{j-1}} > 2 > 1$ and as a consequence, we have

$$\frac{1}{2^{j-1}}(r) > 1, \quad \frac{1}{2^{j-1}}(s) > 1, \quad \frac{1}{2^{j-1}}(rs) > 1, \quad \frac{1}{2^{j-1}}(r + s) > 1.$$

Thus, for every $k = 0, 1, 2, \dots, j - 1$, we acquire

$$\frac{1}{2^k}(r) > 1, \quad \frac{1}{2^k}(s) > 1, \quad \frac{1}{2^k}(rs) > 1, \quad \frac{1}{2^k}(r + s) > 1,$$

$$\frac{1}{2^k} \left(\frac{rs}{r+s} \right) > 1, \quad \frac{1}{2^k} \left(\frac{2rs}{r+s} \right) > 1$$

and $\Delta_{m_d}(2^{-k}r, 2^{-k}s) = 0$ for $k = 0, 1, 2, \dots, j - 1$. By the application of (3.9) and using the definition of the mapping m_d , we obtain

$$\begin{aligned} |\Delta_{m_d}(r, s)| &\leq \sum_{k=j}^{\infty} \frac{\alpha}{2^k} + \sum_{k=j}^{\infty} \frac{\alpha}{2^k} + \frac{1}{2} \sum_{k=j}^{\infty} \frac{\alpha}{2^k} + \frac{1}{2} \sum_{k=j}^{\infty} \frac{\alpha}{2^k} \\ &\leq 3\alpha \sum_{k=j}^{\infty} \frac{1}{2^k} \leq \frac{12\alpha}{2^{k+1}} \leq 12\alpha(|r|^{-1} + |s|^{-1}) \end{aligned}$$

for all $r, s \in \mathbb{R}^*$, which signifies that the inequality (3.11) holds. Now, we claim that Eq. (1.5) fails to be stable for $\theta = -1$ in Corollary 3.3. On the other hand, let us assume that a multiplicative inverse mapping $m_d : \mathbb{R}^* \rightarrow \mathbb{R}$ exists and satisfies (3.12). Then we arrive at

$$|m_d(r)| \leq (K + 1)|r|^{-1}. \tag{3.14}$$

But we can find a positive integer m with $m\alpha > K + 1$. If $r \in (1, 2^{m-1})$ then $2^{-k}r \in (1, \infty)$ for all $k = 0, 1, 2, \dots, m - 1$ and thus

$$|m_d(r)| = \sum_{k=0}^{\infty} \frac{v(2^{-k}r)}{2^k} \geq \sum_{k=0}^{m-1} \frac{2^k\alpha}{2^k} = \frac{m\alpha}{r} > (K + 1)r^{-1},$$

which contradicts (3.14). Hence we conclude that Eq. (1.5) fails to be stable for $\theta = -1$ in Corollary 3.3. □

Remark 3.6 We omit the proof of propositions needed for investigating various stabilities of Eq. (1.6), since they are obtained by similar arguments as in the proof of stability results concerning Eq. (1.5).

4 Occurrences of (1.5) and (1.6)

In manufacturing optical instruments, we come across compound lenses. Let the focal lengths of two thin lenses be r and s . Then the combined focal length F is given by $F = \frac{rs}{r+s}$. But the power P of a lens is the reciprocal of the focal length F . Suppose $p_1 = \frac{1}{r}$ and $p_2 = \frac{1}{s}$ are powers of the above lenses, then the combined power P of the lenses is given by $P = r + s$. In view of these theories, we can associate Eqs. (1.5) and (1.6) with the combined focal length and power of lenses. A comparative study of Eqs. (1.1)–(1.6) in various situations of optics is presented below.

- Equation (1.4) interprets the combined power of lenses with forcal length r and s to be equal to half of arithmetic mean of powers of individual lenses; whereas Eq. (1.1) shows that the power of a lens of focal length $r + s$ is equal to half of the harmonic mean of focal lengths r and s of individual lenses.
- Equation (1.5) implies that the difference between the combined power of lenses of focal lengths r and s and the combined power of lenses of focal lengths $2r$ and $2s$ is equal to the arithmetic mean of powers of individual lenses with focal lengths r and s ; while Eq. (1.2) indicates the difference between 2 times the power of a lens of focal

length $r + s$ and the power of a lens of focal length $r + s$ is equal to half of the harmonic mean of the focal lengths r and s of the individual lenses.

- Equation (1.6) indicates that the sum of the combined power of lenses of focal lengths r and s and the combined power of lenses of focal lengths $2r$ and $2s$ is equal to 3 times the arithmetic mean of the powers of the individual lenses with focal lengths r and s . But Eq. (1.3) shows the sum of 2 times the power of a lens of focal length $r + s$ and the power of a lens of focal length $r + s$ is equal to three-second of the harmonic mean of the focal lengths r and s of the individual lenses.

5 Conclusion

In this study, we have proved that Eqs. (1.4), (1.5) and (1.6) are equivalent, which indicates that the multiplicative inverse mapping is a solution of Eqs. (1.5) and (1.6). In this investigation, zero is omitted to avoid singular cases in obtaining the stability results. We conclude that the stability results of the multiplicative inverse difference and adjoint functional equations hold good in the domain of non-zero real numbers. We have illustrated a suitable example that the stability result of (1.5) fails to hold good for a critical case. We have also discussed the occurrence of Eqs. (1.5) and (1.6) arising in various situations where focal lengths and powers of thin lenses in optics are related.

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Authors' contributions

All authors contributed equally to this paper, participating from drafting to checking and rechecking the manuscript. All authors read and approved the final manuscript.

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