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Hopf bifurcation control for the main drive delay system of rolling mill

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Abstract

In this work, the Hopf bifurcation of the main drive delay system of rolling mill is controlled and analyzed by designing a nonlinear controller. The time-delay is selected as a bifurcation parameter, and the following conclusions are obtained through analysis: (1) in the absence of state feedback control, the system will generate the Hopf bifurcation at the expense of its stability when the bifurcation parameter exceeds the threshold value; (2) in the state under feedback control, the occurrence of Hopf bifurcation is effectively delayed and the stable region of the system is also well extended. More importantly, we can change the properties of bifurcation periodic solutions by selecting the appropriate gain parameters. Some numerical simulations reveal that under the nonlinear feedback control, the vibration amplitude of the system can be effectively reduced.

Keywords: Main drive delay system of rolling mill; Hopf bifurcation; Bifurcation control; Nonlinear state feedback; Delay

1 Introduction

For decades, we have witnessed the development and improvement of nonlinear dynamics theory. The rolling mill vibration model [1–3] is a class of nonlinear dynamic systems that evolves from a simple linear model to a complex nonlinear dynamic model. According to the structure and load characteristics of the actual equipment, the nonlinear torsional vibration model is established with time delay [4], nonlinear stiffness [5, 6], and damping as nonlinear characteristic parameters to reveal more vibration mechanism and phenomena.

Many studies have shown that frictional vibrations encountered in actual production often cause bifurcations, which can seriously threaten the stable operation of the system. The Hopf bifurcation [7–11] is a common and important bifurcation phenomenon. The so-called Hopf bifurcation refers to the phenomenon that a closed orbit will occur in the vicinity equilibrium point when the stability of the singularity of the system is reversed, which can be used to explain many of the vibration problems in engineering.

In view of the system instability caused by Hopf bifurcation, some literature works [12–16] have studied the various bifurcation control methods to modify the bifurcation characteristics, which can obtain some expected dynamical behaviors, for example, in order to postpone the onset of Hopf bifurcation [17], to change the properties of Hopf bifurcation [18], and to reduce the amplitude of vibration [19]. Xiao et al. [20] controlled unstable or

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steady states and periodic orbits for a novel congestion control model by using the state feedback method. Xu [21] et al. used the polynomial function as a state feedback controller to realize the control of Hopf bifurcation for an internet congestion system.

In 2014, Zhang et al. established the main drive delay system of rolling mill [4]:

$$\ddot{\theta} + \zeta \dot{\theta} + \xi \dot{\theta}^2 + \eta \dot{\theta}^3 + \beta' \dot{\theta}^3 = p' \theta(t - \tau), \quad (1)$$

where $\zeta, \xi, \eta, \tau, c'$, and p' are the real parameters. The specific meaning of parameters can be seen in [4].

Zhang et al. only gave the conditions for Hopf bifurcation to exist and some properties of bifurcating periodic solutions were not discussed. Furthermore, works seldom pay attention to the problem of Hopf bifurcation control for the main drive delay system of rolling mill. Our work will adopt the state feedback to control Hopf bifurcations. The main contributions of this paper are as follows:

- (1) A nonlinear controller is established to control the Hopf bifurcation of the main drive delay system of rolling mill;
- (2) The conditions for the existence of Hopf bifurcation in the main drive delay system of the rolling mill without control and with control are given, respectively;
- (3) In the state under feedback control, the occurrence of Hopf bifurcation is effectively delayed and the nature of bifurcating periodic solutions can be changed by selecting proper gain parameters of the nonlinear part of the controller.

2 Hopf bifurcation without control

Firstly, let $x_1 = \theta, x_2 = \dot{\theta}, a_1 = \xi, a_2 = \eta, a_3 = \beta', a_4 = \zeta, a_5 = p'$, and system (1) becomes

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -a_1 x_2 - a_2 x_2^2 - a_3 x_2^3 - a_4 x_1 + a_5 x_1(t - \tau). \end{cases} \quad (2)$$

The characteristic equation of system (2) at the zero equilibrium point $E_0 = (0, 0)^T$ is

$$\lambda^2 + a_1 \lambda + a_4 - a_5 e^{-\lambda \tau} = 0. \quad (3)$$

If $a_1 > 0, a_4 > a_5, \tau = 0$, then we can obtain that the equilibrium point E_0 is locally asymptotically stable.

If $\lambda = i\omega_1 (\omega_1 > 0)$ is a solution of Eq. (3), then

$$\begin{aligned} -\omega_1^2 + a_4 - a_5 \cos \omega_1 \tau &= 0 \\ a_1 \omega_1 + a_5 \sin \omega_1 \tau &= 0, \end{aligned} \quad (4)$$

which can lead to

$$\omega_1^4 + (a_1 - 2a_4)\omega_1^2 + (a_4^2 - a_5^2) = 0. \quad (5)$$

On substituting, let $\omega_1^2 = z, P_1 = a_1^2 - 2a_4, P_2 = a_4^2 - a_5^2$, Eq. (5) can be rewritten as

$$z^2 + P_1 z + P_2 = 0, \quad (6)$$

and we define $h(z) = z^2 + P_1 z + P_2, z > 0$.

If $P_2 < 0$, then Eq. (6) has only a positive root $z_{1,2} = \frac{-P_1 + \sqrt{\Delta}}{2}$, where $\Delta = P_1^2 - 4P_2$. Thus, if $P_2 < 0$ holds, $\pm i\omega_1^c$ is a pair of purely imaginary roots of Eq. (3) with $\tau_{k1} (k = 0, 1, \dots)$, where $\tau_{k1} = \frac{1}{\omega_1^c}(\theta + 2k\pi)$, $\theta = \arcsin(\frac{a_1\omega_1^c}{-a_5})$, $\omega_1^c = \sqrt{z_{1,2}} = \sqrt{\frac{-P_1 + \sqrt{\Delta}}{2}}$. Suppose that $\lambda(\tau) = \alpha(\tau) + i\omega(\tau)$ is the root of (3) satisfying $\alpha(\tau_{k1}) = 0$, $\omega(\tau_{k1}) = \omega_1^c (k = 0, 1, 2, \dots)$.

Theorem 1 *If $P_1 < 0$, $z \in (-\frac{P_1}{2}, +\infty)$ holds, then $\text{sign}(\frac{d\text{Re}\lambda(\tau)}{d\tau}|_{\tau=\tau_{k1}}) = \text{sign}(h'(z)) > 0$; if $P_1 < 0$, $z \in (0, -\frac{P_1}{2})$ holds, then $\text{sign}(\frac{d\text{Re}\lambda(\tau)}{d\tau}|_{\tau=\tau_{k1}}) = \text{sign}(h'(z)) < 0$.*

Proof Differentiating Eq. (3) with respect to τ , we have

$$\begin{aligned}\frac{d\lambda}{d\tau} &= \frac{-a_5\lambda e^{-\lambda\tau}}{a_1 + 2\lambda + a_5\tau e^{-\lambda\tau}} \\ &= \frac{-\lambda(\lambda^2 + a_1\lambda + a_4)}{a_1 + 2\lambda + a_5\tau e^{-\lambda\tau}}\end{aligned}$$

and

$$\frac{d\text{Re}(\lambda(\tau_k))}{d\tau} = \frac{\omega^2}{\Lambda} [a_1^2 - 2a_4 + 2\omega_1^2] = \frac{\omega^2}{\Lambda} [h'(z)].$$

Then

$$\text{sign}\left(\frac{d\text{Re}\lambda(\tau)}{d\tau}\bigg|_{\tau=\tau_{k1}}\right) = \text{sign}(h'(z)),$$

where $\Lambda = (a_1 + a_5\tau \cos(\omega_1\tau_{k1}))^2 + (2\omega - a_5\tau \sin(\omega_1\tau_{k1}))^2$. By analyzing, we have the following results: (1) if $P_1 < 0$, $z \in (0, -\frac{P_1}{2})$ holds, then $h'(z) = 2z + P_1 < 0$, $z > 0$; (2) if $P_1 < 0$, $z \in (-\frac{P_1}{2}, +\infty)$ holds, then $h'(z) = 2z + P_1 > 0$, $z > 0$. It completes the proof. $\square$

According to the above analysis, we have the following results.

Theorem 2 *For system (2), suppose that $P_2 < 0$ holds.*

- (i) *If $P_1 < 0$, $z \in (-\frac{P_1}{2}, +\infty)$ holds, E_0 is locally asymptotically stable whenever $\tau \in [0, \tau_{01})$ and E_0 is unstable whenever $\tau \in (\tau_{01}, +\infty)$. Moreover, it generates a Hopf bifurcation at E_0 when $\tau = \tau_{k1}$.*
- (ii) *If $P_1 < 0$, $z \in (0, -\frac{P_1}{2})$ holds, E_0 is locally asymptotically stable whenever $\tau \in [0, \tau_{01}) \cup (\bigcup_{i=1}^{\infty} (\tau_{i,1}, \tau_{i+1,1}))$. Furthermore, it generates a Hopf bifurcation at E_0 when $\tau = \tau_{k1}$.*

3 Hopf bifurcation under control

In this section, we design a nonlinear controller to control the Hopf bifurcation in the main drive delay system of rolling mill. The nonlinear state feedback controller is as follows:

$$u = -b_1x_1 - b_2x_1^2 - b_3x_1^3, \quad (7)$$

where b_1 , b_2 , and b_3 are positive feedback parameters. The rolling mill main drive system Eq. (2) with state feedback controller can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 - b_1x_1 - b_2x_1^2 - b_3x_1^3, \\ \dot{x}_2 = -a_1x_2 - a_2x_2^2 - a_3x_2^3 - a_4x_1 + a_5x_1(t - \tau). \end{cases} \quad (8)$$

By calculating, system (8) has the characteristic equation

$$\lambda^2 + (a_1 + b_1)\lambda + a_4 + a_1b_1 - a_5e^{-\lambda\tau} = 0. \quad (9)$$

Let $\lambda = i\omega_2$ ($\omega_2 > 0$) be a root of Eq. (9), then

$$\begin{aligned} -\omega_2^2 + a_4 + a_1b_1 - a_5 \cos \omega_2\tau &= 0, \\ (a_1 + b_1)\omega_2 + a_5 \sin \omega_2\tau &= 0 \end{aligned} \quad (10)$$

and

$$\omega_2^4 + (a_1^2 + b_1^2 - 2a_4)\omega_2^2 + (a_4 + a_1b_1)^2 - a_5^2 = 0. \quad (11)$$

Let $\omega_2^2 = \nu$, $Q_1 = a_1^2 + b_1^2 - 2a_4$, $Q_2 = (a_4 + a_1b_1)^2 - a_5^2$, then Eq. (11) can be rewritten as

$$\nu^2 + Q_1\nu + Q_2 = 0, \quad (12)$$

and we define $h(\nu) = \nu^2 + Q_1\nu + Q_2$, $\nu > 0$.

By analyzing, if $Q_2 < 0$, then Eq. (12) has only a positive root $\nu_{1,2} = \frac{-Q_1 \pm \sqrt{\Delta}}{2}$, where $\Delta = Q_1^2 - 4Q_2$. Thus, if $Q_2 < 0$ holds, $\pm i\omega_2^\varepsilon$ is a pair of purely imaginary roots of Eq. (9) with $\tau_{k2}(k = 0, 1, \dots)$, where $\tau_{k2} = \frac{1}{\omega_2^\varepsilon}(\theta + 2k\pi)$, $\theta = \arcsin(\frac{(a_1+b_1)\omega_2^\varepsilon}{-a_5})$, $\omega_2^\varepsilon = \sqrt{\nu_{1,2}} = \sqrt{\frac{-Q_1 \pm \sqrt{\Delta}}{2}}$. Let $\lambda(\tau) = \alpha(\tau) + i\omega(\tau)$ be the root of (3) satisfying $\alpha(\tau_{k2}) = 0$, $\omega(\tau_{k2}) = \omega_2^\varepsilon(k = 0, 1, 2, \dots)$.

Similar to the analysis in the second part, the following conclusions can be drawn.

Theorem 3 *If $Q_1 < 0$, $\nu \in (-\frac{Q_1}{2}, +\infty)$ holds, then $\text{sign}(\frac{d\text{Re}\lambda(\tau)}{d\tau}|_{\tau=\tau_{k2}}) = \text{sign}(h'(\nu)) > 0$; if $Q_1 < 0$, $\nu \in (0, -\frac{Q_1}{2})$ holds, then $\text{sign}(\frac{d\text{Re}\lambda(\tau)}{d\tau}|_{\tau=\tau_{k2}}) = \text{sign}(h'(\nu)) < 0$.*

Theorem 4 *For system (8), suppose that $Q_2 < 0$ holds.*

- If $Q_1 < 0$, $\nu \in (-\frac{Q_1}{2}, +\infty)$ holds, E_0 is locally asymptotically stable whenever $\tau \in [0, \tau_{02})$ and E_0 is unstable whenever $\tau \in (\tau_{02}, +\infty)$. Moreover, it generates a Hopf bifurcation at E_0 when $\tau = \tau_{k2}$.*
- If $Q_1 < 0$, $\nu \in (0, -\frac{Q_1}{2})$ holds, E_0 is locally asymptotically stable whenever $\tau \in [0, \tau_{02}) \cup (\bigcup_{i=1}^{\infty}(\tau_{i2}, \tau_{i+1,2}))$. Furthermore, it generates a Hopf bifurcation at E_0 when $\tau = \tau_{k2}$.*

In the following, we will explore the nature of the Hopf bifurcation for controlled system (8) by implementing the normal form (NF) and the center manifold reduction (CMR) [22].

Let $t \rightarrow \frac{t}{\tau}$, $\tau = \tau_{02} + \mu$, $\mu \in \mathbb{R}$, then Eq. (8) can be written in a functional differential equation in $C = C([0, 1], \mathbb{R}^2)$ as follows:

$$\dot{x}(t) = L_\mu(\phi) + F(\phi, \mu), \quad (13)$$

where

$$L_\mu(\phi) = (\tau_{02} + \mu) \begin{pmatrix} -b_1 & 1 \\ -a_4 & -a_1 \end{pmatrix} \begin{pmatrix} \phi_1(0) \\ \phi_2(0) \end{pmatrix}$$

$$\begin{aligned}
& + (\tau_{02} + \mu) \begin{pmatrix} 0 & 0 \\ a_5 & 0 \end{pmatrix} \begin{pmatrix} \phi_1(-1) \\ \phi_2(-1) \end{pmatrix}, \\
F(\mu, \phi) &= (\tau_{02} + \mu) \begin{pmatrix} -b_2\phi_1(0)^2 - b_3\phi_1(0)^3 \\ -a_2\phi_2(0)^2 - a_3\phi_2(0)^3 \end{pmatrix}.
\end{aligned}$$

By Riesz representation theorem, let

$$\eta(\theta, \mu) = (\tau_{02} + \mu) \begin{pmatrix} -b_1 & 1 \\ -a_4 & -a_1 \end{pmatrix} \delta(\theta) - (\tau_{02} + \mu) \begin{pmatrix} 0 & 0 \\ a_5 & 0 \end{pmatrix} \delta(\theta + 1),$$

where δ represents the Dirac delta function. Define

$$A(\mu)\varphi = \begin{cases} \frac{d\varphi(\theta)}{d\theta}, & \theta \in [-1, 0), \\ \int_{-1}^0 d\eta(\theta, \mu)\varphi(\theta), & \theta = 0, \end{cases}$$

and

$$R(\mu)\varphi = \begin{cases} 0, & \theta \in [-1, 0), \\ F(\mu, \varphi), & \theta = 0, \end{cases}$$

where $\varphi \in C^1([-1, 0], R^2)$. For $\psi \in C^1([0, 1], R^2)$, define the adjoint operator of $A(\mu)$ as follows:

$$A^*\psi(s) = \begin{cases} \int_{-1}^0 d\eta(t, 0)\psi(-t), & s = 0, \\ -\frac{d\psi(s)}{ds}, & s \in (0, 1], \end{cases}$$

and

$$\langle \psi, \varphi \rangle = \bar{\psi}(0)\varphi(0) - \int_{-1}^0 \int_{\xi=0}^{\theta} \bar{\psi}(\xi - \theta) d\eta(\theta)\varphi(\xi) d\xi.$$

$A(0)$ and A^* are adjoint operators, and $A(0)$ has a pair of purely imaginary eigenvalues $\pm i\omega_2^c \tau_{02}$.

Define $q(\theta) = (1, q_2(0))^T e^{i\omega_2^c \tau_{02}\theta}$, $q^*(s) = D(1, q_2^*(0))^T e^{i\omega_2^c \tau_{02}s}$. By calculating, we obtain $D = (1 + \overline{q_2(0)}q_2^*(0) - \tau_{02}q_2^*(0)a_5e^{i\omega_0\tau_{02}})^{-1}$, $q_2(0) = i\omega_2^c \tau_{02} + b_1$ and $q_2^*(0) = \frac{1}{a_1 - i\omega_2^c \tau_{02}}$.

On the center manifold Σ_0 ,

$$w(z(t), \bar{z}(t), \theta) = w_{20}(\theta) \frac{z^2}{2} + w_{11}(\theta) z\bar{z} + w_{02}(\theta) \frac{\bar{z}^2}{2} + \cdots,$$

we can get

$$\begin{aligned}
\dot{z} &= i\omega \tau_{02} z + \langle q^*(\theta), F(W + 2\operatorname{Re} z(t)q(\theta)) \rangle \\
&= i\omega \tau_{02} z + \bar{q}^*(0)F(W(z, \bar{z}, 0) + 2\operatorname{Re} z(t)q(\theta)) \\
&\doteq i\omega \tau_{02} z + \bar{q}_0^*F(z, \bar{z}),
\end{aligned}$$

and this equation is rewritten as

$$\dot{z}(t) = i\omega\tau_{02}z(t) + g(z, \bar{z}), \quad (14)$$

where

$$\begin{aligned} g(z, \bar{z}) &= \bar{q}_0^* F(W(z, \bar{z}, 0) + 2 \operatorname{Re} z(t) q(\theta)) \\ &= g_{20} \frac{z^2}{2} + g_{11} z \bar{z} + g_{02} \frac{\bar{z}^2}{2} + g_{21} \frac{z^2 \bar{z}}{2} + \cdots \end{aligned}$$

The following coefficients are obtained by using a computation similar to that of [12, 13], which are used for determining the important qualities

$$\begin{aligned} g_{20} &= -2\bar{D}e^{2i\tau_{02}\omega_2^c\theta} [a_2 q_2^2(0) \overline{q_2^*(0)} + b_2]; \\ g_{11} &= -\bar{D} [q_2^*(0) a_2 q_2(0) \overline{q_2(0)} + b_2]; \\ g_{20} &= -2\bar{D}e^{-2i\tau_{02}\omega_2^c\theta} [a_2 \overline{q_2^2(0)} \overline{q_2^*(0)} + b_2]; \\ g_{21} &= -2\bar{D} [b_2 (e^{i\tau_{02}\omega_2^c\theta} + e^{-i\tau_{02}\omega_2^c\theta}) w_{20}^1 + a_2 \overline{q_2^*(0)} (q_2(0) w_{11}^2(\theta) e^{i\tau_{02}\omega_2^c\theta} \\ &\quad + \overline{q_2(0)} w_{20}^2(\theta) e^{-i\tau_{02}\omega_2^c\theta})], \end{aligned}$$

where w_{20} and w_{11} satisfy

$$\begin{aligned} w_{20}(\theta) &= \frac{g_{20}}{i\omega_2^c \tau_{02}} q(0) e^{i\tau_{02}\omega_2^c\theta} - \frac{\bar{g}_{02}}{3i\omega_2^c \tau_{02}} \bar{q}(0) e^{-i\tau_{02}\omega_2^c\theta} + E_1 e^{2i\omega\tau_{02}\theta}, \\ w_{11}(\theta) &= \frac{g_{11}}{i\omega_2^c \tau_{02}} q(0) e^{i\tau_{02}\omega_2^c\theta} - \frac{\bar{g}_{11}}{i\omega_2^c \tau_{02}} \bar{q}(0) e^{-i\tau_{02}\omega_2^c\theta} + E_2. \end{aligned}$$

Note that

$$\begin{aligned} E_1 &= \left(2i\omega_2^c I - \int_{-1}^0 e^{i\tau_{02}\omega_2^c\theta} d\eta(\theta, 0) \right)^{-1} F_{z^2} \\ E_2 &= - \left(\int_{-1}^0 d\eta(\theta, 0) \right)^{-1} F_{z\bar{z}}, \end{aligned}$$

we can get

$$\begin{aligned} E_1 &= \begin{pmatrix} 2i\omega_2^c & -1 \\ a_4 - a_5 e^{-i\tau_{02}\omega_2^c} & 2i\omega_2^c + a_1 \end{pmatrix}^{-1} \begin{pmatrix} 2b_2 e^{2i\tau_{02}\omega_2^c\theta} \\ 2a_2 q_2(0)^2 e^{2i\tau_{02}\omega_2^c\theta} \end{pmatrix} \\ E_2 &= \begin{pmatrix} 0 & -1 \\ a_4 - a_5 & a_1 \end{pmatrix}^{-1} \begin{pmatrix} b_2 \\ a_2 q_2(0) \overline{q_2(0)} \end{pmatrix}. \end{aligned}$$

Therefore, by our previous analysis, we can obtain the following parameters that determine the nature of Hopf bifurcation:

$$c_1(0) = \frac{i}{2\omega_2^c \tau_{02}} \left(g_{20} g_{11} - 2|g_{11}|^2 - \frac{1}{3}|g_{02}|^2 \right) + \frac{g_{21}}{2},$$

Figure 1 The equilibrium of system (2) with $\tau = 0.5 < \tau_{01}$ is stable

Figure 2 The bifurcation periodic solution of uncontrolled system (2) with $\tau = 0.7 > \tau_{01}$ is stable

Figure 3 The waveform of system (2) without control at $\tau = 2.3, 2.7, 3.1$, respectively

$$\mu_2 = -\frac{\operatorname{Re} c_1(0)}{\operatorname{Re} \lambda'(\tau_{02})},$$

$$\beta_2 = 2 \operatorname{Re} c_1(0),$$

$$T_2 = -\frac{\operatorname{Im} c_1(0) + \mu_2 \operatorname{Im} \lambda'(\tau_{02})}{\omega_2^c}.$$

Theorem 5 For system (8) with τ_{k2} , the following results hold:

- (i) If $\mu_2 > 0$ ($\mu_2 < 0$), then the Hopf bifurcation is supercritical (subcritical).
- (ii) If $\beta_2 < 0$ ($\beta_2 > 0$), then the bifurcating periodic solutions are stable (unstable).
- (iii) If $T_2 > 0$ ($T_2 < 0$), then the period increases (decreases).

Figure 4 Phase portrait of system (2) without control at $\tau = 2.3, 2.7, 3.1$, respectively

Figure 5 The equilibrium of system (8) with $\tau = 1 < \tau_{02}$ is stable

Figure 6 The bifurcation periodic solution of controlled system (8) with $\tau = 1.2 > \tau_{02}$ is stable

4 Numerical simulation

In this section, we implement some simulations to corroborate the obtained conclusions in the previous section.

Case 1: The system without control.

Fix parameters $a_1 = 1, a_2 = a_3 = a_4 = 1, a_5 = -2$. From Theorem 2, we can get $P_1 = -1 < 0, P_2 = -3 < 0, \omega_1^c = 1.517, \tau_{01} = 0.568$.

We choose $\tau = 0.5 < \tau_{01} = 0.568, \tau \in [0, \tau_{01})$ and note that the equilibrium of system (2) is stable, as shown in Fig. 1. As τ exceeds the critical value τ_{01} , system (2) generates a Hopf bifurcation at the expense of its stability. By choosing $\tau = 0.7 > \tau_{01} = 0.568$, Fig. 2 reveals that there exists a stable periodic solution. Figures 3, 4 show that as the time delay increases, the vibration amplitude of the uncontrolled system increases.

Case 2: The system under control.

Figure 7 Bifurcation diagram for θ versus τ of uncontrolled system (2) and controlled system (8), respectively

Figure 8 Bifurcation diagram for $\frac{d\theta}{dt}$ versus τ of uncontrolled system (2) and controlled system (8), respectively

Figure 9 The stability domain of system (8) varies with the increase of the control parameter b_1

Fix parameters $a_1 = 1, a_2 = a_3 = a_4 = 1, a_5 = -2, b_1 = 0.5, b_2 = b_3 = 0.01$. From Theorem 4, we can get $Q_1 = -0.75 < 0, Q_2 = -1.75 < 0, \omega_2^c = 1.323, \tau_{02} = 1.093$.

We choose $\tau = 1 < \tau_{01} = 1.093, \tau \in [0, \tau_{02})$ and note that the equilibrium of system (8) is stable, as shown in Fig. 5. As τ exceeds the critical value τ_{02} , system (8) generates a Hopf bifurcation at the expense of its stability. By choosing $\tau = 1.2 > \tau_{02} = 1.093$, we can get $c_1(0) = -1.2736 + 5.3826i, \mu = -0.7634, \beta_2 = -2.5472$, as depicted in Fig. 6. Figures 7, 8 show that the bifurcation point is delayed from 0.568 to 1.123 and demonstrate that the occurrence of the Hopf bifurcation can be effectively retarded.

Fix parameters $a_1 = 1, a_2 = a_3 = a_4 = 1, a_5 = -2$. By choosing b_1 as a parameter, we can obtain $\tau_{02} = \frac{1}{\omega_2^c} \arcsin\left(\frac{(a_1+b_1)\omega_2^c}{-a_5}\right)$. From Theorem 4: if $\tau < \tau_{02}$ holds, E_0 is locally stable; if $\tau > \tau_{02}$ holds, E_0 is unstable whenever $\tau \in (\tau_{02}, +\infty)$. Moreover, it suffers a Hopf bifurcation at E_0 when $\tau = \tau_{02}$. Stable domain and unstable domain of system (8) with the nonlinear

Figure 10 Waveforms and phase diagrams of uncontrolled system (2) and controlled system (8) with $\tau = 2.3$

Figure 11 Waveforms and phase diagrams of uncontrolled system (2) and controlled system (8) with $\tau = 2.7$

Figure 12 Waveforms and phase diagrams of uncontrolled system (2) and controlled system (8) with $\tau = 3.1$

state feedback control are shown in Fig. 9. From Fig. 9, we note that the threshold value of the bifurcation increases with the increase of the control parameter b_1 and reaches the maximum value around $b_1 = 0.6$, then the threshold value decreases with the increase of b_1 .

In Figs. 10, 11, 12, we show waveforms and phase diagrams of the uncontrolled system and controlled system (2) with the same time delay, respectively. The result indicates that the vibration amplitude of the system can be effectively reduced under nonlinear feedback control. Therefore, the proposed control strategy is feasible.

5 Conclusion

In this paper, the main drive delay system of the rolling mill is considered and a nonlinear controller is designed to control the Hopf bifurcation in the system. We give the conditions for the Hopf bifurcation to exist in the main drive delay system of rolling mill without control and under control, respectively. In the state under feedback control, the occurrence of Hopf bifurcation is effectively delayed, and the nature of bifurcating periodic solutions can be changed by selecting proper gain parameters of the nonlinear part of the controller. Besides, the relation graph between the stability of system (8) and control parameter is given. Some numerical results validate the above analysis.

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Availability of data and materials

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the final manuscript.

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