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Solvability of functional-integral equations (fractional order) using measure of noncompactness

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Abstract

We investigate the solutions of functional-integral equation of fractional order in the setting of a measure of noncompactness on real-valued bounded and continuous Banach space. We introduce a new μ-set contraction operator and derive generalized Darbo fixed point results using an arbitrary measure of noncompactness in Banach spaces. An illustration is given in support of the solution of a functional-integral equation of fractional order.

MSC: 35K90; 47H10

Keywords: Fixed point; Measure of noncompactness; Functional-integral equation

1 Introduction

We will discuss the solutions $u \in C(I, X)$ of functional-integral equation of fractional order

$$u(t) = f\big(t, u(t)\big) + \frac{Hu(t)}{\Gamma(\gamma)} \int_0^t \frac{g'(s)}{(g(t) - g(s))^{1-\gamma}} k\big(t, s, u(s)\big)\, ds,$$
$$t \in I = [0,1], 0 < \gamma < 1,$$

in the setting of measure of noncompactness (MNC) on real-valued bounded and continuous Banach space. In particular, we also discuss

$$u(t) = \frac{2t^2 e^{-\lambda(t+2)}}{t^4 + 1} \cos\big(|u(t)|\big) + \frac{\sqrt[3]{|u(t)|}}{8(1 + |u(t)|^2)\Gamma(\frac{1}{2})} \int_0^t \frac{2s}{\sqrt{t^2 - s^2}} \frac{t}{(1 + s^2)(1 + u^2(s))}\, ds, \quad \lambda > 0,$$

and its solution in $C(I, \mathbb{R})$ (the space of all continuous mappings $u : I = [0,1] \to \mathbb{R}$).

Denote $\mathbb{R}$ and $\mathbb{N}$ as the set of real numbers, the set of natural numbers, respectively, and $\mathbb{R}^+ = [0, +\infty)$ and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$. Let $(E, \|\cdot\|)$ be a real Banach space with zero element θ. Let $\mathcal{B}(x, r)$ denote the closed ball centered at x with radius r. The symbol $\mathcal{B}_r$ stands for the ball $\mathcal{B}(\theta, r)$. For X, a nonempty subset of E, we denote by $\overline{X}$ and $\operatorname{Conv} X$ the closure and the convex closure of X, respectively. Moreover, let us denote by $\mathfrak{M}_E$ the family of all nonempty bounded subsets of E and by $\mathfrak{N}_E$ its subfamily consisting of all relatively compact sets.

Definition 1.1 ([9]) A mapping $\mu : \mathfrak{M}_E \to \mathbb{R}^+$ is said to be a MNC in E if

- (1^0) the family $\ker \mu = \{X \in \mathfrak{M}_E : \mu(X) = 0\}$ is nonempty and $\ker \mu \subset \mathfrak{N}_E$,
- (2^0) $X \subset Y \Rightarrow \mu(X) \leq \mu(Y)$,
- (3^0) $\mu(\overline{X}) = \mu(X)$,
- (4^0) $\mu(\operatorname{Conv} X) = \mu(X)$,
- (5^0) $\mu(\lambda X + (1-\lambda)Y) \leq \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [0,1]$,
- (6^0) if (X_n) is a decreasing sequence of nonempty, closed sets in $\mathfrak{M}_E$ such that $X_{n+1} \subset X_n$ $(n = 1,2,\ldots)$ and if $\lim_{n\to\infty} \mu(X_n) = 0$, then the set $X_\infty = \bigcap_{n=1}^{\infty} X_n$ is nonempty and compact.

The family $\ker \mu$ defined in axiom (1^0) is called the kernel of the MNC μ.

One of the properties of the MNC is $X_\infty \in \ker \mu$. Indeed, from the inequality $\mu(X_\infty) \leq \mu(X_n)$ for $n = 1,2,3,\ldots$, we infer that $\mu(X_\infty) = 0$.

The Kuratowski MNC is the map $\alpha : \mathfrak{M}_E \to \mathbb{R}^+$ with

$$\alpha(\mathcal{Q}) = \inf\left\{\epsilon > 0 : \mathcal{Q} \subset \bigcup_{k=1}^{n} S_k, S_k \subset E, \operatorname{diam}(S_k) < \epsilon \ (k \in \mathbb{N})\right\}. \tag{1.1}$$

We denote $\operatorname{fix}(T)$ as set of fixed points of T.

In 1955, Darbo [10] used the notion of Kuratowski MNC, α, to prove fixed point theorem (FPT) and generalized topological Schauder FPT [9] and classical Banach FPT [8].

Theorem 1.2 ([9]) *Let X be a closed, convex subset of a Banach space E. Then every compact, continuous map $T : X \to X$ has at least one fixed point.*

We denote by Ω a nonempty, bounded, closed and convex subset of a Banach space E.

Theorem 1.3 ([10]) *Let $T : \Omega \to \Omega$ be a continuous and μ-set contraction operator, that is, there exists a constant $k \in [0,1)$ with*

$$\mu(TM) \leq k\mu(M)$$

for any $\phi \neq M \subset \Omega$; let μ be the Kuratowski MNC on E, then $\operatorname{fix}(T) \neq \phi$.

Various Darbo-type FPT and coupled theorems by using different types of control functions arise (for instant, see [1–7, 10–12, 14–21, 23]). In this paper, we introduce a μ-set contraction operator using new control functions and establish some new fixed point result, a Krasnoselskii fixed point result, that generalizes the results in [1–3, 10, 12, 13].

2 Generalized Darbo-type fixed point theorems

We introduce the following notion as a generalization of a concept given in [22].

Definition 2.1 Let Θ_F be a family of all functions $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ such that:

(Θ_1) F is continuous and strictly increasing;
(Θ_2) for each sequences $\{t_n\}, \{s_n\} \subseteq \mathbb{R}^+$, $\lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = 0 \Leftrightarrow \lim_{n\to\infty} F(t_n, s_n) = -\infty$.

$\Pi_{G,\beta}$ denotes the set of pairs (G, β), where $G : \mathbb{R}^+ \to \mathbb{R}$ and $\beta : \mathbb{R}^+ \times \mathbb{R}^+ \to [0, 1)$, such that:
(Π_1) for each sequence $\{t_n\} \subseteq \mathbb{R}^+$, $\limsup_{n\to\infty} G(t_n) \geq 0 \Leftrightarrow \limsup_{n\to\infty} t_n \geq 1$;
(Π_2) for the sequences $\{t_n\}, \{s_n\} \subseteq \mathbb{R}^+$, $\limsup_{n\to\infty} \beta(t_n, s_n) = 1 \Rightarrow \lim_{n\to\infty} t_n = \lim_{n\to\infty} s_n = 0$;
(Π_3) for the sequences $\{t_n\}, \{s_n\} \subseteq \mathbb{R}^+$, $\sum_{n=1}^{\infty} G(\beta(t_n, s_n)) = -\infty$.

Theorem 2.2 *Let $T : \Omega \to \Omega$ is continuous operator. If there exist $F \in \Theta_F$, $(G, \beta) \in \Pi_{G,\beta}$ and a continuous and strictly increasing mapping $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\mu(TM) > 0$ implies*

$$F\bigl(\mu(TM), \varphi\bigl(\mu(TM)\bigr)\bigr) \leq F\bigl(\mu(M), \varphi\bigl(\mu(M)\bigr)\bigr) + G\bigl(\beta\bigl(\mu(M), \varphi\bigl(\mu(M)\bigr)\bigr)\bigr), \tag{2.1}$$

for all $\emptyset \neq M \subset \Omega$, where μ is an arbitrary MNC, then $\operatorname{fix}(T) \neq \emptyset$.

Proof We start with the assumption $\Omega_0 = \Omega$ and define a sequence $\{\Omega_n\}$ by $\Omega_{n+1} = \operatorname{Conv}(T\Omega_n)$, for $n \in \mathbb{N}^*$. If $\mu(\Omega_{n_0}) = 0$ for some natural number $n_0 \in \mathbb{N}$, then Ω_{n_0} is compact. We have $T(\Omega_{n_0}) \subseteq \operatorname{Conv}(T\Omega_{n_0}) = \Omega_{n_0+1} \subseteq \Omega_{n_0}$. In Theorem 1.2 we have $\mu(\Omega_n) > 0$, for all $n \in \mathbb{N}^*$. From (2.1) and (4^0) of Definition 1.1,

$$\begin{aligned}
&F\bigl(\mu(\Omega_{n+1}), \varphi\bigl(\mu(\Omega_{n+1})\bigr)\bigr) \\
&\quad= F\bigl(\mu\bigl(\operatorname{Conv}(T\Omega_n)\bigr), \varphi\bigl(\mu\bigl(\operatorname{Conv}(T\Omega_n)\bigr)\bigr)\bigr) \\
&\quad= F\bigl(\mu(T\Omega_n), \varphi\bigl(\mu(T\Omega_n)\bigr)\bigr) \\
&\quad\leq F\bigl(\mu(\Omega_n), \varphi\bigl(\mu(\Omega_n)\bigr)\bigr) + G\bigl(\beta\bigl(\mu(\Omega_n), \varphi\bigl(\mu(\Omega_n)\bigr)\bigr)\bigr) \\
&\quad\leq F\bigl(\mu(\Omega_{n-1}), \varphi\bigl(\mu(\Omega_{n-1})\bigr)\bigr) + G\bigl(\beta\bigl(\mu(\Omega_n), \varphi\bigl(\mu(\Omega_n)\bigr)\bigr)\bigr) \\
&\qquad+ G\bigl(\beta\bigl(\mu(\Omega_{n-1}), \varphi\bigl(\mu(\Omega_{n-1})\bigr)\bigr)\bigr) \\
&\quad\vdots \\
&\quad\leq F\bigl(\mu(\Omega_0), \varphi\bigl(\mu(\Omega_0)\bigr)\bigr) + \sum_{i=0}^{n} G\bigl(\beta\bigl(\mu(\Omega_i), \varphi\bigl(\mu(\Omega_i)\bigr)\bigr)\bigr),
\end{aligned}$$

that is,

$$F\bigl(\mu(\Omega_{n+1}), \varphi\bigl(\mu(\Omega_{n+1})\bigr)\bigr) \leq F\bigl(\mu(\Omega_0), \varphi\bigl(\mu(\Omega_0)\bigr)\bigr) + \sum_{i=0}^{n} G\bigl(\beta\bigl(\mu(\Omega_i), \varphi\bigl(\mu(\Omega_i)\bigr)\bigr)\bigr), \tag{2.2}$$

for all $n \in \mathbb{N}$.

By the properties of $(G, \beta) \in \Pi_{G,\beta}$, $F(\mu(\Omega_{n+1}), \varphi(\mu(\Omega_{n+1}))) \to -\infty$ as $n \to \infty$ and by (Θ_2), we have

$$\lim_{n\to\infty} \mu(\Omega_n) = \lim_{n\to\infty} \varphi\bigl(\mu(\Omega_n)\bigr) = 0.$$

From (6^0) of Definition 1.1, $\Omega_\infty = \bigcap_{n=1}^{\infty} \Omega_n$ is a nonempty, closed, convex set and $\Omega_\infty \subseteq \Omega_n$ for all $n \in \mathbb{N}$. Also $T(\Omega_\infty) \subset \Omega_\infty$ and $\Omega_\infty \in \ker \mu$. Therefore, by Theorem 1.2, $\operatorname{fix}(T) \neq \emptyset$. □

Corollary 2.3 *Let $T:\Omega\to\Omega$ is continuous operator. If there exist $\tau>0$, $F\in\Theta_F$, and a continuous and strictly increasing mapping $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ such that*

$$\mu(TM)>0\quad\Rightarrow\quad\tau+F(\mu(TM),\varphi(\mu(TM))\le F(\mu(M),\varphi(\mu(M))) \tag{2.3}$$

for all $\emptyset\neq M\subset\Omega$, where μ is an arbitrary MNC, then $\operatorname{fix}(T)\neq\emptyset$.

Proof If we consider $G(t)=\ln t$ $(t>0)$, $\beta(t,s)=\lambda\in(0,1)$ and $\tau=-\ln\lambda>0$ in (2.1) of Theorems 2.2, we have (2.3), and the result follows from Theorem 2.2. □

Corollary 2.4 *Let $T:\Omega\to\Omega$ is continuous operator. If there exists a continuous and strictly increasing mapping $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ such that for $\lambda\in(0,1)$*

$$\mu(TM)>0\quad\Rightarrow\quad\mu(TM)+\varphi(\mu(TM))\le\lambda\big[\mu(M)+\varphi(\mu(M))\big] \tag{2.4}$$

for all $\emptyset\neq M\subset\Omega$, where μ is an arbitrary MNC. Then $\operatorname{fix}(T)\neq\emptyset$.

Proof If we consider $F(t,s)=\ln(t+s)$ $(t,s>0)$ and $\tau=\ln(\frac{1}{\lambda})$ $(\lambda\in(0,1))$ in (2.3) of Corollary 2.3, we have condition (2.4). □

Proposition 2.5 *Let $T:\Omega\to\Omega$ is continuous operator. If there exist $F\in\Theta_F$, $(G,\beta)\in\Pi_{G,\beta}$ and a continuous mapping $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ such that* $\operatorname{diam}(TM)>0$ *implies*

$$\begin{aligned}F\big(\operatorname{diam}(TM),\varphi(\operatorname{diam}(TM))\big)&\le F\big(\operatorname{diam}(M),\varphi(\operatorname{diam}(M))\big)\\&\quad+G\big(\beta\big(\operatorname{diam}(M),\varphi(\operatorname{diam}(M))\big)\big)\end{aligned} \tag{2.5}$$

for all $\emptyset\neq M\subset\Omega$, then $\operatorname{fix}(T)\neq\emptyset$.

Proof Following the argument of Proposition 3.2 [12], Theorem 2.2 guarantees the existence of a T-invariant nonempty closed convex subset M with $\operatorname{diam}(M_\infty)=0$, which means that M_∞ is a singleton and therefore $\operatorname{fix}(T)\neq\emptyset$.

Uniqueness. In order to get a contradiction we may suppose that there exist two different fixed points $\zeta\neq\xi\in\Omega$, then we may define the set $M:=\{\zeta,\xi\}$. In this case $\operatorname{diam}(M)=\operatorname{diam}(T(M))=\|\xi-\zeta\|>0$. Then using (2.5)

$$\begin{aligned}F\big(\operatorname{diam}\big(T(M)\big),\varphi\big(\operatorname{diam}\big(T(M)\big)\big)\big)&\le F\big(\operatorname{diam}(M),\varphi\big(\operatorname{diam}(M)\big)\big)\\&\quad+G\big(\beta\big(\operatorname{diam}(M),\varphi\big(\operatorname{diam}(M)\big)\big)\big).\end{aligned}$$

Therefore, $G(\beta(\operatorname{diam}(M),\varphi(\operatorname{diam}(M))))\ge0$ and hence $\beta(\operatorname{diam}(M),\varphi(\operatorname{diam}(M)))\ge1$, which is a contradiction, and hence $\xi=\zeta$. □

A generalized classical fixed point result derived from Proposition 2.5 follows.

Corollary 2.6 *Let $T:\Omega\to\Omega$ be an operator. It there exist $F\in\Theta_F$, $(G,\beta)\in\Pi_{G,\beta}$ and a continuous and strictly increasing mapping $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ such that $\|Tu-Tv\|>0$ im-*

plies

$$F\big(\|Tu-Tv\|,\varphi\big(\|Tu-Tv\|\big)\big)\leq F\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)+G\big(\beta\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)\big) \tag{2.6}$$

for all $u,v\in\Omega$, *then* $\operatorname{fix}(T)\neq\emptyset$.

Proof Let $\mu:\mathfrak{M}_E\to\mathbb{R}^+$ be a set quantity defined by $\mu(\Omega)=\operatorname{diam}\Omega$, where $\operatorname{diam}\Omega=\sup\{\|u-v\|:u,v\in\Omega\}$, the diameter of Ω. Therefore μ is a MNC in a space E in the sense of Definition 1.1, and from (2.6)

$$\begin{aligned}
&\sup_{u,v\in\Omega}\|Tu-Tv\|>0\\
&\quad\Rightarrow\quad F\Big(\sup_{u,v\in\Omega}\|Tu-Tv\|,\varphi\Big(\sup_{u,v\in\Omega}\|Tu-Tv\|\Big)\Big)\\
&\qquad\qquad=\sup_{u,v\in\Omega}F\big(\|Tu-Tv\|,\varphi\big(\|Tu-Tv\|\big)\big)\\
&\qquad\qquad\leq\sup_{u,v\in\Omega}\big[F\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)+G\big(\beta\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)\big)\big]\\
&\qquad\qquad\leq F\Big(\sup_{u,v\in\Omega}\|u-v\|,\varphi\Big(\sup_{u,v\in\Omega}\|u-v\|\Big)\Big)\\
&\qquad\qquad\quad+G\Big(\beta\Big(\sup_{u,v\in\Omega}\|u-v\|,\varphi\Big(\sup_{u,v\in\Omega}\|u-v\|\Big)\Big)\Big),
\end{aligned}$$

that is, $\operatorname{diam}(T(\Omega))>0$, which implies

$$\begin{aligned}
F\big(\operatorname{diam}\big(T(\Omega)\big),\varphi\big(\operatorname{diam}\big(T(\Omega)\big)\big)\big)&\leq F(\operatorname{diam}(\Omega),\varphi\big(\operatorname{diam}(\Omega)\big)\\
&\quad+G\big(\beta\big(\operatorname{diam}(\Omega),\varphi\big(\operatorname{diam}(\Omega)\big)\big)\big).
\end{aligned}$$

Thus following Proposition 2.5, $\operatorname{fix}(T)\neq\emptyset$. □

Corollary 2.7 *Let* $(E,\|\cdot\|)$ *be a Banach space and let* Ω *be a closed convex subset of* E. *Let* $T_1,T_2:\Omega\to\Omega$ *be two operators satisfying the following conditions*:

(I) $(T_1+T_2)(X)\subseteq\Omega$, *for* $X\in\Omega$;

(II) *there exist* $F\in\Theta_F$ *and* $(G,\beta)\in\Pi_{G,\beta}$ *and a continuous and increasing mapping* $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ *such that* $\|T_1u-T_1v\|>0$ *implies*

$$\begin{aligned}
&F\big(\|T_1u-T_1v\|,\varphi\big(\|T_1u-T_1v\|\big)\big)\\
&\quad\leq F\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)+G\big(\beta\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)\big);
\end{aligned}\tag{2.7}$$

(III) T_2 *is a continuous and compact operator.*

Then $\mathcal{J}:=T_1+T_2:\Omega\to\Omega$ *has a fixed point* $u\in\Omega$.

Proof Suppose $M\subset\Omega$ with $\alpha(M)>0$. Invoking the notion of a Kuratowski MNC, for each $n\in\mathbb{N}$, there exist $\mathcal{C}_1,\dots,\mathcal{C}_{m(n)}$ bounded subsets such that $M\subseteq\bigcup_{i=1}^{m(n)}\mathcal{C}_i$ and $\operatorname{diam}(\mathcal{C}_i)\leq\alpha(M)+\frac{1}{n}$. Suppose that $\alpha(T_1(M))>0$. Since $T_1(M)\subseteq\bigcup_{i=1}^{m(n)}T_1(\mathcal{C}_i)$, there exists $i_0\in\{1,2,\dots,m(n)\}$ such that $\alpha(T_1(M))\leq\operatorname{diam}(T_1(\mathcal{C}_{i_0}))$. Using (2.7) the condition of

T_1 with the discussed arguments, we have

$$\begin{aligned}
&F\big(\alpha\big(T_1(M)\big),\varphi\big(\alpha\big(T_1(M)\big)\big)\big)\\
&\quad\le F\big(\operatorname{diam}\big(T_1(\mathcal{C}_{i_0})\big),\varphi\big(\operatorname{diam}\big(T_1(\mathcal{C}_{i_0})\big)\big)\big)\\
&\quad\le F(\operatorname{diam}(\mathcal{C}_{i_0}),\varphi\big(\operatorname{diam}(\mathcal{C}_{i_0})\big)+G(\beta\big(\operatorname{diam}(\mathcal{C}_{i_0}),\varphi\big(\operatorname{diam}(\mathcal{C}_{i_0})\big)\big)\\
&\quad\le F\left(\alpha(M)+\frac{1}{n},\varphi\left(\alpha(M)+\frac{1}{n}\right)\right)+G\left(\beta\left(\alpha(M)+\frac{1}{n},\varphi\left(\alpha(M)+\frac{1}{n}\right)\right)\right).
\end{aligned}\tag{2.8}$$

Passing to the limit in (2.8) as $n\to\infty$, we get

$$F\big(\alpha\big(T_1(M)\big),\varphi\big(\alpha\big(T_1(M)\big)\big)\big)\le F\big(\alpha(M),\varphi\big(\alpha(M)\big)\big)+G\big(\beta\big(\alpha(M),\varphi\big(\alpha(M)\big)\big)\big).$$

Using hypothesis (III), we have, invoking the notion of α,

$$\begin{aligned}
&F\big(\alpha\big(\mathcal{J}(M)\big),\varphi\big(\alpha\big(\mathcal{J}(M)\big)\big)\big)\\
&\quad= F\big(\alpha\big(T_1(M)+T_2(M)\big),\varphi\big(\alpha\big(T_1(M)+T_2(M)\big)\big)\big)\\
&\quad\le F\big(\alpha\big(T_1(M)\big)+\alpha\big(T_2(M)\big),\varphi\big(\alpha\big(T_1(M)\big)+\alpha\big(T_2(M)\big)\big)\big)\\
&\quad= F\big(\alpha\big(T_1(M)\big),\varphi\big(\alpha\big(T_1(M)\big)\big)\big)\\
&\quad\le F\big(\alpha(M),\varphi\big(\alpha(M)\big)\big)+G\big(\beta\big(\alpha(M),\varphi\big(\alpha(M)\big)\big)\big).
\end{aligned}$$

Thus by Theorem 2.2, $\operatorname{fix}(\mathcal{J})\neq\emptyset$. □

3 Application

Let $(X,\|\cdot\|)$ be a real Banach algebra and let the symbol $C(I,X)$ stand for the space consisting of all continuous mappings $u:I=[0,1]\to X$ and $C_+(I)$ for the space of positive real-valued continuous function defined on I and $C_+^1(I)$ for the space of positive real-valued continuous differential function defined on I. We will consider the existence of a solution $u\in C(I,X)$ to the integral equation

$$\begin{aligned}
&u(t)=f\big(t,u(t)\big)+\frac{Hu(t)}{\Gamma(\gamma)}\int_0^t\frac{g'(s)}{(g(t)-g(s))^{1-\gamma}}k\big(t,s,u(s)\big)\,ds,\\
&\quad t\in I=[0,1],0<\gamma<1.
\end{aligned}\tag{3.1}$$

Assume:

(A_1) $f:I\times X\to X$ is a continuous mapping such that there exist $F\in\Theta_F$, $(G,\beta)\in\Pi_{G,\beta}$ and a nondecreasing function $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ such that

$$\begin{aligned}
&\big\|f(t,u)-f(t,v)\big\|>0\\
&\quad\Rightarrow\quad F\big(\big\|f(t,u)-f(t,v)\big\|,\varphi\big(\big\|f(t,u)-f(t,v)\big\|\big)\big)\\
&\qquad\le F\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)+G\big(\beta\big(\big(\|u-v\|,\varphi\big(\|u-v\|\big)\big)\big)\big).
\end{aligned}\tag{3.2}$$

Also, there exist a function $\phi_1:\mathbb{R}^+\to\mathbb{R}^+$ such that

$$\big\|f(t,u)\big\|\le\phi_1\big(\|u\|\big)$$

and

$$M_0 = \sup\{|\phi_1(t)| : t \in \mathbb{R}^+\} < \infty.$$

(A_2) H is some operator acting continuously from the space $C(I,X)$ into itself and there is an increasing function $\psi_1 : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$\|H(u)\| \leq \psi_1(\|u\|).$$

(A_3) The function $k : I \times I \times \mathbb{R} \to \mathbb{R}$ is continuous such that $k(I \times I \times \mathbb{R}_+) \subseteq \mathbb{R}_+$ and

$$K_0 = \sup\{|k(t,s,u(s))| : t,s \in I, u \in C_+(I)\} < \infty.$$

(A_4) The function $g : I \to \mathbb{R}_+$ is C^1_+ and nondecreasing.
(A_5) $\liminf_{\zeta\to\infty} \frac{\psi_1(\zeta)K_0(g(1)-g(0))^\gamma}{\zeta\Gamma(\gamma+1)} < 1$.

Theorem 3.1 *Under assumptions* (A_1)–(A_6), *Eq.* (3.1) *has at least one solution in the space* $u \in C(I,X)$.

Proof Define an integral operator T: $C(I,X) \to C(I,X)$ by

$$Tu(t) = f(t,u(t)) + Hu(t)\mathcal{F}u(t),$$

where

$$\mathcal{F}u(t) = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{g'(s)}{((g(t)-g(s))^{1-\gamma}} k(t,s,u(s))\,ds.$$

We prove $\operatorname{fix}(T) \neq \emptyset$.

Consider the two mappings $T_1, T_2 : C(I,X) \to C(I,X)$,

$$T_1u(t) = f(t,u(t)),$$
$$T_2u(t) = Hu(t)\mathcal{F}u(t),$$

where $T = T_1 + T_2$. It is easy to see that T_1 is well defined. Now we show that T_2 is well defined. Let $\varepsilon > 0$ be arbitrary and let $u \in C(I,X)$ be given and fixed and let $\eta_1, \eta_2 \in I$ (without loss of generality assume that $\eta_2 \geq \eta_1$) and $|\eta_2 - \eta_1| \leq \epsilon$ and $r_0 = \|u\|$. Then we get

$$\begin{aligned}
&\Gamma(\gamma)\big|(\mathcal{F}u)(\eta_2) - (\mathcal{F}u)(\eta_1)\big| \\
&\quad= \left|\int_0^{\eta_2} \frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}} k(\eta_2,s,u(s))\,ds - \int_0^{t_1} \frac{g'(s)}{(g(t_1)-g(s))^{1-\gamma}} k(t_1,s,u(s))\,ds\right| \\
&\quad\leq \left|\int_0^{\eta_2} \frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}} k(\eta_2,s,u(s))\,ds - \int_0^{\eta_2} \frac{g'(s)}{(g(\eta_2)-f(s))^{1-\gamma}} k(\eta_1,s,u(s))\,ds\right| \\
&\qquad+ \left|\int_0^{\eta_2} \frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}} k(\eta_1,s,u(s))\,ds - \int_0^{t_1} \frac{g'(s)}{(fg(t_2)-g(s))^{1-\gamma}} k(t_1,s,u(s))\,ds\right|
\end{aligned}$$

$$\begin{aligned}
&+\left|\int_0^{\eta_1}\frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}}k\big(\eta_1,s,u(s)\big)\,ds-\int_0^{t_1}\frac{g'(s)}{(g(t_1)-g(s))^{1-\gamma}}k\big(t_1,s,u(s)\big)\,ds\right| \\
\leq{}&\int_0^{\eta_2}\frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}}\big|k\big(\eta_2,s,u(s)\big)-k\big(\eta_1,s,u(s)\big)\big|\,ds \\
&+\int_{\eta_1}^{\eta_2}\frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}}\big|k\big(\eta_1,s,u(s)\big)\big|\,ds \\
&+\int_0^{\eta_1}\left|\frac{g'(s)}{(g(\eta_2)-g(s))^{1-\gamma}}-\frac{g'(s)}{(g(t_1)-g(s))^{1-\gamma}}\right|\big|k\big(\eta_1,s,u(s)\big)\big|\,ds.
\end{aligned}$$

Denote

$$\omega(k,\epsilon)=\sup\bigl\{\big|k(t,s,u)-k\big(t',s,u\big)\big|:t,t',s\in I,\big|t-t'\big|\leq\epsilon,u\in[-r_0,r_0]\bigr\}.$$

Then

$$\begin{aligned}
&\Gamma(\gamma)\big|(\mathcal{F}u)(\eta_2)-(\mathcal{F}u)(\eta_1)\big| \\
&\quad\leq\frac{\omega(k,\epsilon)}{\gamma}\big(g(\eta_2)-g(0)\big)^{\gamma}+\frac{K_0}{\gamma}\big(g(\eta_2)-g(\eta_1)\big)^{\gamma} \\
&\qquad+\frac{K_0}{\gamma}\bigl[\big(g(\eta_2)-g(t_0)\big)^{\gamma}-\big(g(\eta_2)-g(\eta_1)\big)^{\gamma}-\big(g(\eta_1)-g(t_0)\big)^{\gamma}\bigr] \\
&\quad\leq\frac{\omega(k,\epsilon)}{\gamma}\big(g(\eta_2)-g(0)\big)^{\gamma}+\frac{2K_0}{\gamma}\big(g(\eta_2)-g(\eta_1)\big)^{\gamma} \\
&\quad\leq\frac{\omega(k,\epsilon)}{\gamma}\big(g(1)-g(0)\big)^{\gamma}+\frac{2K_0}{\gamma}\omega(g,\epsilon)^{\gamma},
\end{aligned}$$

that is,

$$\big\|(\mathcal{F}u)(\eta_2)-(\mathcal{F}u)(\eta_1)\big\|\leq\frac{\omega(k,\epsilon)}{\Gamma(\gamma+1)}\big(g(1)-g(0)\big)^{\gamma}+\frac{2K_0}{\Gamma(\gamma+1)}\omega(g,\epsilon)^{\gamma}.$$

Using the notion of uniform continuity of the function k on the set $I^2\times[-r_0,r_0]$ and g on the set I, we have $\omega(k,\epsilon)\to0$ and $\omega(g,\epsilon)\to0$ as $\epsilon\to0$, consequently $\mathcal{F}u\in C(I,X)$, and thus $T_2u\in C(I,X)$.

We prove that T_2 is a continuous operator. Fix $v\in C(I,X)$ and let $\varepsilon>0$ be given. Since H is some operator acting continuously from the space $C(I,X)$ into itself, there exists $\delta_1>0$, such that

$$\forall u\in C(I,X),\quad\big(\|u-v\|<\delta_1\Rightarrow\|Hu-Hv\|<\varepsilon_1(\varepsilon)\big),$$

for each $t\in I$, we have

$$\begin{aligned}
&\Gamma(\gamma)\big|(\mathcal{F}u)(t)-(\mathcal{F}v)(t)\big| \\
&\quad=\left|\int_0^t\frac{g'(s)}{(g(t)-g(s))^{1-\gamma}}k\big(t,s,u(s)\big)\,ds-\int_0^t\frac{g'(s)}{(g(t)-g(s))^{1-\gamma}}k\big(t,s,v(s)\big)\,ds\right| \\
&\quad\leq\int_0^t\frac{g'(s)}{(g(t)-g(s))^{1-\gamma}}\big|k\big(t,s,u(s)\big)-k\big(t,s,v(s)\big)\big|\,ds \\
&\quad\leq\frac{(g(1)-g(0))^{\gamma}}{\gamma}K_{\epsilon},
\end{aligned}$$

where

$$K_{\delta_2} = \sup\{|k(t,s,u) - k(t,s,v)| : t,s \in I, \|u - v\| \leq \delta_2\}.$$

Thus

$$\|(\mathcal{F}u) - (\mathcal{F}v)\| \leq \frac{(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} K_{\delta_2}.$$

Also, we have

$$\begin{aligned} |(\mathcal{F}u)(t)| &\leq \frac{1}{\Gamma(\gamma)} \int_0^t \frac{g'(s)}{(g(t) - g(s))^{1-\gamma}} |k(t,s,u(s))| \, ds, \\ &\leq \frac{K_0}{\Gamma(\gamma)} \int_0^t \frac{g'(s)}{(g(t) - g(s))^{1-\gamma}} \, ds \leq \frac{K_0(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)}, \end{aligned} \tag{3.3}$$

for all $t \in I$. Now if we put $\delta = \min\{\delta_1, \delta_2\}$, then for any $u \in C(I,X)$ such that $\|u - v\| < \delta$, by the triangle inequality we obtain

$$\begin{aligned} \|T_2u(t) - T_2v(t)\| &= \|Hu(t)\mathcal{F}u(t) - Hv(t)\mathcal{F}v(t)\| \\ &\leq \|Hu(t) - Hv(t)\| \|\mathcal{F}u(t)\| + \|Hv(t)\| \|\mathcal{F}u(t) - \mathcal{F}v(t)\| \\ &\leq \varepsilon_1 \frac{K_0(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} + \|Hy\| \frac{(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} K_{\delta_2} \\ &\leq \varepsilon_1 \frac{K_0(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} + \psi_1(\|y\|) \frac{(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} K_{\delta_2} \\ &\leq \varepsilon_1 \frac{K_0(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} + \psi_1(\|y\|) \frac{(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} \varepsilon_2 \\ &\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon, \end{aligned}$$

where

$$\begin{aligned} \varepsilon_1 &= \frac{\Gamma(\gamma + 1)\varepsilon}{2[1 + K_0(g(1) - g(0))^\gamma]}, \\ \varepsilon_2 &= \frac{\Gamma(\gamma + 1)\varepsilon}{2[1 + \psi_1(\|y\|)(g(1) - g(0))^\gamma]}. \end{aligned}$$

To prove T_2 is a compact operator. If $B = \{u \in C(I,X) : \|u\| < 1\}$ is the open unit ball of $C(I,X)$, then we claim that $\overline{T_2(B)}$ is a compact subset of $C(I,X)$. To see this, by the Arzelà–Ascoli theorem, we need only to show that $T_2(B)$ is an uniformly bounded and equi-continuous subset of $C(I,X)$. First we show that $T_2(B) = \{T_2u : u \in B\}$ is uniformly bounded. By the conditions (A_2) for any $u \in B$,

$$\begin{aligned} \|T_2u(t)\| &= \|Hu(t)\mathcal{F}u(t)\| \leq \|Hu(t)\| \|\mathcal{F}u(t)\| \\ &\leq \|Hu\| \|\mathcal{F}u\| \leq \psi_1(\|u\|) \frac{K_0(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)} \\ &\leq \psi_1(1) \frac{K_0(g(1) - g(0))^\gamma}{\Gamma(\gamma + 1)}. \end{aligned}$$

Hence, putting $M := \psi_1(1)\frac{K_0(g(1)-g(0))^\gamma}{\Gamma(\gamma+1)}$, we conclude that $T_2(B)$ is uniformly bounded. Now we show that $T_2(B)$ is an uniformly equi-continuous subset of $C(I,X)$. To see this, let $u \in B$ be arbitrary, and let $\varepsilon > 0$. Since Hu and $\mathcal{F}u$ are uniformly continuous, there exist some $\delta_1(\varepsilon), \delta_2(\varepsilon) > 0$ such that

$$\begin{aligned}
&\forall \eta_1, \eta_2 \in I, \quad \bigl(|\eta_2 - \eta_1| < \delta_1(\varepsilon) \Rightarrow \bigl\|Hu(\eta_2) - Hu(\eta_1)\bigr\| < \varepsilon_1\bigr), \\
&\forall \eta_1, \eta_2 \in I, \quad \bigl(|\eta_2 - \eta_1| < \delta_2(\varepsilon) \Rightarrow \bigl\|\mathcal{F}u(\eta_2) - \mathcal{F}u(\eta_1)\bigr\| < \varepsilon_2\bigr).
\end{aligned}$$

Let $\delta(\varepsilon) = \min\{\delta_1(\varepsilon), \delta_2(\varepsilon), \varepsilon_1, \varepsilon_2\}$, where the given ε_1 and ε_2 depend on ε. Therefore, if $\eta_1, \eta_2 \in I$ satisfies $0 < \eta_2 - \eta_1 < \delta(\varepsilon)$ and $x \in B$,

$$\begin{aligned}
\bigl\|T_2u(\eta_2) - T_2u(\eta_1)\bigr\| &= \bigl\|Hu(\eta_2)\mathcal{F}u(\eta_2) - Hu(\eta_1)\mathcal{F}u(\eta_1)\bigr\| \\
&\leq \bigl\|Hu(\eta_2) - Hu(\eta_1)\bigr\| \bigl\|\mathcal{F}u(\eta_2)\bigr\| \\
&\quad + \bigl\|Hu(\eta_1)\bigr\| \bigl\|\mathcal{F}x(\eta_2) - \mathcal{F}x(\eta_1)\bigr\| \\
&\leq \varepsilon_1 \frac{K_0(g(1)-g(0))^\gamma}{\Gamma(\gamma+1)} + \psi_2\bigl(\|u\|\bigr)\varepsilon_2 \\
&\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon,
\end{aligned}$$

where

$$\begin{aligned}
\varepsilon_1 &= \frac{\Gamma(\gamma+1)\varepsilon}{2(1 + K_0(g(1)-g(0))^\gamma)}, \\
\varepsilon_2 &= \frac{\varepsilon}{2(1+\psi_1(1))}.
\end{aligned}$$

Therefore T_2 is a compact operator. Next, we show that T_1 satisfies (3.2). Let $u, v \in C(I,X)$, and $\|T_1u - T_1y\| > 0$. By applying the fact that every continuous function attains its maximum on a compact set, there exists $t \in I$ such that $0 < \|T_1u - T_1v\| = \|f(t,u(t)) - f(t,v(t))\|$. By (A_1) and using the fact that F and φ are strictly increasing functions we obtain

$$\begin{aligned}
&F\bigl(\|T_1u - T_1v\|, \varphi\bigl(\|T_1u - T_1v\|\bigr)\bigr) \\
&\quad = F\bigl(\bigl\|f\bigl(t,u(t)\bigr) - f\bigl(t,v(t)\bigr)\bigr\|, \varphi\bigl(\bigl\|f\bigl(t,u(t)\bigr) - f\bigl(t,v(t)\bigr)\bigr\|\bigr)\bigr) \\
&\quad \leq F\bigl(\|u-v\|, \varphi\bigl(\|u-v\|\bigr)\bigr) + G\bigl(\beta\bigl(\bigl(\|u-v\|, \varphi\bigl(\|u-v\|\bigr)\bigr)\bigr)\bigr).
\end{aligned}$$

Hence T_1 satisfies (3.2). Now we show that there exists some $M_1 > 0$ such that $\|T_1u\| \leq M_1$ holds for each $u \in C(I,X)$. By (A_1)

$$\bigl\|T_1u(t)\bigr\| = \bigl\|f(t,u)\bigr\| \leq \phi_1\bigl(\|u\|\bigr) \leq M_0,$$

Therefore

$$\exists M_0 > 0, \forall u, \quad \bigl(u \in C(I,X) \Rightarrow \|T_1u\| \leq M_0\bigr).$$

Finally, we claim that there exists some $r > 0$, such that $T(B_r(\theta)) \subseteq B_r(\theta)$ with $B_r(\theta) = \{u \in C(I,X) : \|u\| \leq r\}$. On the contrary, for any $\zeta > 0$ there exists some $u_\zeta \in B_r(\theta)$ such that $\|T(u_\zeta)\| > \zeta$. This implies that $\liminf_{\zeta\to\infty} \frac{1}{\zeta}\|T(u_\zeta)\| \geq 1$. On the other hand, we have

$$\begin{aligned}
\|Tu_\zeta(t)\| &\leq \|f(t,u_\zeta(t))\| + \|Hu_\zeta(t)Fu_\zeta(t)\| \\
&\leq \|T_1u_\zeta\| + \|Hu_\zeta(t)\|\|Fu_\zeta(t)\| \\
&\leq M_0 + \|Hu_\zeta\|.\|Fu_\zeta\| \\
&\leq M_0 + \psi_1(\|u_\zeta\|).\frac{K_0(g(1)-g(0))^\gamma}{\Gamma(\gamma+1)} \\
&\leq M_0 + \psi_1(\zeta).\frac{K_0(g(1)-g(0))^\gamma}{\Gamma(\gamma+1)}.
\end{aligned}$$

Hence, by the above estimate and condition (A_5) we get

$$\liminf_{\zeta\to\infty}\frac{1}{\zeta}\|T(u_\zeta)\| \leq \liminf_{\zeta\to\infty}\frac{\psi_1(\zeta)K_0(g(1)-g(0))^\gamma}{\zeta\Gamma(\gamma+1)} < 1$$

which is a contradiction. Thus in view of the above discussions and Corollary 2.7 we conclude that Eq. (3.1) has at least one solution in $B_r(\theta) \subseteq C(I,X)$. □

Example Consider the functional-integral equation of fractional order

$$\begin{aligned}
u(t) &= \frac{2t^2e^{-\lambda(t+2)}}{t^4+1}\cos(|u(t)|) \\
&\quad + \frac{\sqrt[3]{|u(t)|}}{8(1+|u(t)|^2)\Gamma(\frac{1}{2})}\int_0^t \frac{2s}{\sqrt{t^2-s^2}}\frac{t}{(1+s^2)(1+u^2(s))}\,ds, \quad \lambda > 0.
\end{aligned} \tag{3.4}$$

Define the continuous operator $H : C(I,\mathbb{R}) \to C(I,\mathbb{R})$ given by

$$Hu = \frac{\sqrt[3]{|u|}}{2(1+|u|^2)}.$$

Define the functions $f : [0,1]\times\mathbb{R} \to \mathbb{R}$ given by $f(t,u(t)) = \frac{2t^2e^{-\lambda(t+2)}}{t^4+1}\cos(u(t))$, f is continuous and

$$|f(t,u(t)) - f(t,v(t))| \leq \frac{2t^2e^{-\lambda(t+2)}}{t^4+1}|\cos(u)-\cos(v)| \leq e^{-2\lambda}|u-v|. \tag{3.5}$$

Also, $\phi_1 : \mathbb{R}^+ \to \mathbb{R}^+$ by $\phi_1(t) = \cos(t)$ with $M_0 = 1$ such that

$$|f(t,u(t))| = \frac{2t^2e^{-\lambda(t+2)}}{t^4+1}|\cos(|u(t)|)| \leq |\cos(|u(t)|)| = \phi_1(|u(t)|).$$

Now, by choosing the function $F : \mathbb{R}^+\times\mathbb{R}^+ \to \mathbb{R}$ given by $F(t,s) = \ln(t+s)$, $G : \mathbb{R}^+ \to \mathbb{R}$ by $G(t) = \ln(t)$, $\beta : \mathbb{R}^+\times\mathbb{R}^+ \to [0,1)$ by $\beta(t,s) = e^{-2\lambda}$ and the function $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ given by $\varphi(t) = t^2$, it is easy to see that the inequality (3.5) implies that the condition (3.2) holds.

Indeed if $|f(t,u(t))-f(t,v(t))|>0$, then we have

$$\begin{aligned}
&F\big[\big|f\big(t,u(t)\big)-f\big(t,v(t)\big)\big|,\varphi\big(\big|f\big(t,u(t)\big)-f\big(t,v(t)\big)\big|\big)\big] \\
&\quad= F\big[\big|f\big(t,u(t)\big)-f\big(t,v(t)\big)\big|,\big|f\big(t,u(t)\big)-f\big(t,v(t)\big)\big|^2\big] \\
&\quad= \ln\big[\big|f\big(t,u(t)\big)-f\big(t,v(t)\big)\big| + \big|f\big(t,u(t)\big)-f\big(t,v(t)\big)\big|^2\big] \\
&\quad\le \ln\big[e^{-2\lambda}\big(|u-v|+|u-v|^2\big)\big] \\
&\quad= \ln\big(|u-v|+|u-v|^2\big)+\ln\big(e^{-2\lambda}\big) \\
&\quad= F\big(|u-v|,\varphi\big(|u-v|\big)\big)+G\big(\beta\big(|u-v|,\varphi\big(|u-v|\big)\big)\big).
\end{aligned}$$

Here $g(t)=t^2$, $k(t,s,u)=\frac{t}{4(1+s^2)(1+u^2)}$, with $K_0=\frac{1}{4}$. By choosing the strictly continuous function $\psi_1:\mathbb{R}^+\to\mathbb{R}^+$ given by $\psi_1(t)=\frac{\sqrt[3]{t}}{2}$, we have

$$\begin{aligned}
&\big\|H(u)\big\| \le \psi_1\big(\|u\|\big), \\
&\liminf_{\zeta\to\infty}\frac{\psi_1(\zeta)K_0(g(1)-g(0))^{\gamma}}{\zeta\Gamma(\gamma+1)} = \liminf_{\zeta\to\infty}\frac{\sqrt[3]{\zeta}}{4\Gamma(\frac{1}{2})\zeta}=0<1,
\end{aligned}$$

and this satisfies assumption (A_5). Thus from all above results, it is clear that Eq. (3.4) satisfies all the requirements of Theorem 3.1 and, hence, the functional-integral equation (3.1) has a solution in $C(I,\mathbb{R})$.

4 Conclusions

In this work, some new generalized Darbo-type fixed point results have been discussed for the notion of a μ-set contraction operator using some control functions, on an arbitrary measure of noncompactness in Banach spaces. The obtained results include related existing results mentioned in the references. Finally, to justify our work, we have given an application for the solution of a functional-integral equations of fractional order, followed by a suitable example.

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