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Stationary distribution of a stochastic SIRD epidemic model of Ebola with double saturated incidence rates and vaccination

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Abstract

In this paper, a stochastic SIRD model of Ebola with double saturated incidence rates and vaccination is considered. Firstly, the existence and uniqueness of a global positive solution are obtained. Secondly, by constructing suitable Lyapunov functions and using Khasminskii's theory, we show that the stochastic model has a unique stationary distribution. Moreover, the extinction of the disease is also analyzed. Finally, numerical simulations are carried out to portray the analytical results.

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1 Introduction

Ebola virus is a potent virus which causes Ebola hemorrhagic fever in humans and primates. It is a very rare virus. After discovering its existence in southern Sudan and the Ebola region of Congo in 1976, it attracted widespread attention in the medical community. Ebola virus has a high mortality rate between 50 percent and 90 percent. It is mainly transmitted by contacting with the blood, body fluids, and infected corpses of animals or human. For the spread of Ebola virus, on the basis of models (see e.g. Refs. [1–6]), we divide the total group into four categories: susceptible group S , infected group I , recovered group R , and infected corpse group D . The infected corpse can spread disease through contacting with the susceptible, and its infection rate is higher than that of an infected individual.

Considering the above discussion, the deterministic SIRD epidemic model is formulated as follows:

$$\begin{cases} \frac{dS}{dt} = A - \frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)} - \frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)} - \mu S(t) - qS(t), \\ \frac{dI}{dt} = \frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)} - \frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)} - (\rho + \mu + \delta)I(t), \\ \frac{dR}{dt} = \rho I(t) + qS(t) - \mu R(t), \\ \frac{dD}{dt} = (\mu + \delta)I(t) - \gamma D(t), \end{cases} \quad (1.1)$$

where A is a constant input rate, β_1 and β_2 represent the transmission coefficient. Functions $\frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)}$ and $\frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)}$ represent two different types of saturated incidence rates. μ is

the natural mortality rate of $S, I,$ and R compartments, δ is mortality due to diseases, $\frac{1}{\gamma}$ represents the average period of infectiousness after death in human corpses, and ρ is the recovery rate. On the basis of biological significance, all parameter values are assumed to be positive, and the quantity of infected corpse is less than the quantity of infected individuals, that is, $I(t) > D(t)$, see [7].

Noticing that R has no influence on the transmission dynamics, we can exclude the third equation. Therefore, we only need to study the following subsystem:

$$\begin{cases} \frac{dS}{dt} = A - \frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)} - \frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)} - \mu S(t) - qS(t), \\ \frac{dI}{dt} = \frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)} - \frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)} - (\rho + \mu + \delta)I(t), \\ \frac{dD}{dt} = (\mu + \delta)I(t) - \gamma D(t). \end{cases} \tag{1.2}$$

Obviously, the feasible region of system (1.2) is \mathbf{R}_+^3 . Referring to [8], we have the basic production number of system (1.2), its expression is

$$\mathfrak{R}_0 = \frac{\beta_1 A \gamma + \beta_2 A (\mu + \delta)}{(\mu + q) \gamma (\rho + \mu + \delta)}.$$

It is easy to draw two conclusions about system (1.2).

Lemma 1.1 *If $\mathfrak{R}_0 < 1$, the disease-free equilibrium $E_0 = (\frac{A}{\mu+q}, 0, 0)$ exists, and the E_0 is globally asymptotically stable.*

Lemma 1.2 *If $\mathfrak{R}_0 > 1$, the endemic equilibrium $E_* = (S_*, I_*, D_*)$ exists, and the E_* is globally asymptotically stable.*

As mentioned above, we show that the epidemic model is depicted by deterministic model. However, epidemic models are inevitably influenced by multifarious unpredictable environmental noise types because of the actual situations. Owing to this reason, many authors have studied epidemic models with stochastic perturbations (see e.g. Refs. [9–18]). But as far as we know, the studies on the dynamics of the stochastic SIRD model of Ebola seem to be rare. In this paper, we assume that stochastic perturbations are of the white noise type which are directly proportional to $S(t), I(t), D(t)$, influenced on the $\dot{S}(t), \dot{I}(t), \dot{D}(t)$ in system (1.2) respectively, then we build the following stochastic model:

$$\begin{cases} dS(t) = (A - \frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)} - \frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)} - \mu S(t) - qS(t)) dt + \sigma_1 S(t) dB_1(t), \\ dI(t) = (\frac{\beta_1 S(t)I(t)}{1+\alpha_1 I(t)} - \frac{\beta_2 S(t)D(t)}{1+\alpha_2 D(t)} - (\rho + \mu + \delta)I(t)) dt + \sigma_2 I(t) dB_2(t), \\ dD(t) = ((\mu + \delta)I(t) - \gamma D(t)) dt + \sigma_3 D(t) dB_3(t), \end{cases} \tag{1.3}$$

where σ_i ($i = 1, 2, 3$) denotes the intensities of the white noises which satisfy nonnegativity. $B_i(t)$ ($i = 1, 2, 3$) is a standard Brownian motion that is defined on a complete probability space $(\Omega, \mathbf{F}, \mathbf{P})$ with a filtration $\{\mathbf{F}_t\}_{t \in \mathbf{R}_+}$ satisfying the usual conditions, that is, $\{\mathbf{F}_t\}_{t \in \mathbf{R}_+}$ is right continuous and \mathbf{F}_0 contains all P-null sets. Other parameters are the same as for system (1.2).

Compared with model (1.2), model (1.3) has the advantage of introducing stochastic perturbations, which makes the model closer to reality. Of course, the results of model (1.3) are more refined.

This paper is arranged as follows. In Sect. 2, we study the existence of an ergodic stationary distribution of stochastic system (1.3). In Sect. 3, we deduce the conditions for extinction of the disease. Numerical simulations are given to support our theoretical results in Sect. 4. Finally, Sect. 5 presents some conclusions.

2 Stationary distribution of system (1.3)

In this section, we construct a suitable Lyapunov function to obtain the conditions of the existence of a unique ergodic stationary distribution to system (1.3). Before this, we introduce the following lemma.

Lemma 2.1 ([19]) *The Markov process $X(t)$ has a unique ergodic stationary distribution $\pi(\cdot)$ if a bounded domain $D_\varepsilon \subset \mathbf{R}^d$ with regular boundary Γ exists and*

(H1) *there is a constant $M > 0$ satisfying $\sum_{i,j=1}^d a_{ij}(x)\xi_i\xi_j \geq M|\xi|^2, x \in D_\varepsilon, \xi \in \mathbf{R}^d$;*

(H2) *there exists a nonnegative C^2 -function V such that LV is negative for any $\mathbf{R}^d \setminus D_\varepsilon$.*

Then

$$\mathbf{P} \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T f(X(t)) dt = \int_D f(x)\pi(dx) \right\} = 1$$

for all $x \in D$, where $f(\cdot)$ is an integrable function with respect to the measure π .

Theorem 2.1 *For any initial value $(S(0), I(0), D(0)) \in \mathfrak{R}_+^3$, there exists a unique positive solution $(S(t), I(t), D(t))$ of system (1.3) for $t \geq 0$, and the solution will remain in \mathfrak{R}_+^3 with probability one.*

The proof of this theorem is standard and we omit it.

Theorem 2.2 *If*

$$\mathfrak{R}_0^s := \frac{\beta_1}{(\mu + q + \frac{\sigma_1^2}{2})(\rho + \mu + \delta + \frac{\sigma_2^2}{2})} > 1,$$

then system (1.3) has a unique stationary distribution $\pi(\cdot)$, and the solution $(S(t), I(t), D(t))$ of system (1.3) is ergodic.

Proof The diffusion matrix of system (1.3) is

$$A = \begin{pmatrix} \sigma_1^2 S^2 & 0 & 0 \\ 0 & \sigma_2^2 I^2 & 0 \\ 0 & 0 & \sigma_3^2 D^2 \end{pmatrix}.$$

Let $M = \min_{(S,I,D) \in \bar{D}_\sigma} \{\sigma_1^2 S^2, \sigma_2^2 I^2, \sigma_3^2 D^2\}$, then we can get

$$\sum_{i,j=1}^3 a_{ij}(x)\xi_i\xi_j = \sigma_1^2 S^2 \xi_1^2 + \sigma_2^2 I^2 \xi_2^2 + \sigma_3^2 D^2 \xi_3^2 \geq M|\xi|^2, \quad (S, I, D) \in \bar{D}_\sigma, \xi \in \mathbf{R}_+^3.$$

Therefore, condition (H1) in Lemma 2.1 is proven.

Next, the validity of condition (H2) in Lemma 2.1 will be verified.

We define a C^2 -function $\bar{V}(S, I, D): \mathbf{R}^3 \rightarrow \mathbf{R}$ as follows:

$$\begin{aligned} \bar{V}(S, I, D) &= M \left(S + \left(1 + \frac{\alpha_1}{\rho + \mu + \delta} I \right) + 2D - c_1 \ln S - c_2 \ln I \right) \\ &\quad - \ln S - \ln D + \frac{1}{m+1} (S + I + D)^{m+1} \\ &:= MV_1 + V_2 + V_3 + V_4, \end{aligned} \tag{2.1}$$

where M, c_1, c_2 will be determined later. m and δ are positive constants and satisfy the following inequality:

$$\rho := \delta - \frac{m}{2} (\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2) > 0, \quad \delta = \min\{\mu + q, \rho, \gamma\}.$$

Obviously,

$$\lim_{k \rightarrow \infty, (S, I, D) \in \mathbf{R}^3 \setminus U_k} \bar{V}(S, I, D) = +\infty,$$

herein, $U_k = (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k)$. Then, $\bar{V}(S, I, D)$ is a continuous function and has a minimum point $(\bar{S}_0, \bar{I}_0, \bar{D}_0)$ in the interior of \mathbf{R}_+^3 . A nonnegative C^2 -function $V(S, I, D): \mathbf{R}^3 \rightarrow \mathbf{R}$ can be defined as

$$V(S, I, D) = \bar{V}(S, I, D) - \bar{V}(\bar{S}_0, \bar{I}_0, \bar{D}_0).$$

Making use of Itô's formula, we get

$$\begin{aligned} LV_1 &= A - \frac{\beta_1 SI}{1 + \alpha_1 I} - \frac{\beta_2 SD}{1 + \alpha_2 D} - \mu S - qS \\ &\quad + \frac{\alpha_1 \beta_1 SI}{(\rho + \mu + \delta)(1 + \alpha_1 I)} + \frac{\alpha_1 \beta_2 SD}{(\rho + \mu + \delta)(1 + \alpha_2 D)} - \alpha_1 I + 2(\mu + \delta)I - 2\gamma D \\ &\quad - c_1 \frac{A}{S} + c_1 \frac{\beta_1 I}{1 + \alpha_1 I} + c_1 \frac{\beta_2 D}{1 + \alpha_2 D} + c_1(\mu + q) + \frac{c_1}{2} \sigma_1^2 \\ &\quad - c_2 \frac{\beta_1 S}{1 + \alpha_1 I} - c_2 \frac{\beta_2 SD}{I(1 + \alpha_2 D)} + c_2(\rho + \mu + \delta) + \frac{c_2}{2} \sigma_2^2 \\ &\leq -\alpha_1 I - 1 - c_1 \frac{A}{S} - c_2 \frac{\beta_1 S}{1 + \alpha_1 I} + 1 + A + \frac{\alpha_1 S}{\rho + \mu + \delta} \left[\frac{\beta_1 I}{1 + \alpha_1 I} + \frac{\beta_2 D}{1 + \alpha_2 D} \right] \\ &\quad + 2(\mu + \delta)I + c_1 \left[\frac{\beta_1 I}{1 + \alpha_1 I} + \frac{\beta_2 D}{1 + \alpha_2 D} \right] + c_1(\mu + q) + \frac{c_1}{2} \sigma_1^2 \\ &\quad + c_2(\rho + \mu + \delta) + \frac{c_2}{2} \sigma_2^2 \\ &\leq -3(A\beta_1 c_1 c_2)^{\frac{1}{3}} + c_1 \left(\mu + q + \frac{\sigma_1^2}{2} \right) + c_2 \left(\rho + \mu + \delta + \frac{\sigma_2^2}{2} \right) + 1 + A \\ &\quad + \frac{\alpha_1 S}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2(\mu + \delta)I + c_1 \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right). \end{aligned} \tag{2.2}$$

Choosing

$$c_1 = \frac{A}{\mu + q + \frac{\sigma_1^2}{2}}, \quad c_2 = \frac{A}{\rho + \mu + \delta + \frac{\sigma_2^2}{2}},$$

we have

$$\begin{aligned} LV_1 &\leq -3 \left[\frac{A^3 \beta_1}{(\mu + q + \frac{\sigma_1^2}{2})(\rho + \mu + \delta + \frac{\sigma_2^2}{2})} \right]^{\frac{1}{3}} + 3A + 1 + \frac{\alpha_1 S}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\ &\quad + 2(\mu + \delta)I + \frac{A}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\ &= -3A \left((\mathfrak{N}_0^S)^{\frac{1}{3}} - 1 \right) + 1 + \frac{\alpha_1 S}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2(\mu + \delta)I \\ &\quad + \frac{A}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\ &:= -\lambda + 1 + \frac{\alpha_1 S}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2(\mu + \delta)I + \frac{A}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right), \end{aligned}$$

where $\lambda = 3A \left((\mathfrak{N}_0^S)^{\frac{1}{3}} - 1 \right) > 0$.

Applying Itô's formula, we have

$$\begin{aligned} LV_2 &= -\frac{A}{S} + \frac{\beta_1 I}{1 + \alpha_1 I} + \frac{\beta_2 D}{1 + \alpha_2 D} + (\mu + q) + \frac{\sigma_1^2}{2} \\ &\leq -\frac{A}{S} + \frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} + (\mu + q) + \frac{\sigma_1^2}{2}, \\ LV_3 &= -(\mu + \delta) \frac{I}{D} + \gamma + \frac{\sigma_3^2}{2}, \end{aligned}$$

and

$$\begin{aligned} LV_4 &= (S + I + D)^m (A - \mu S - qS - \rho I - \gamma D) \\ &\quad + \frac{m}{2} (S + I + D)^{m-1} (\sigma_1^2 S^2 + \sigma_2^2 I^2 + \sigma_3^2 D^2) \\ &\leq (S + I + D)^m [A - \delta(S + I + D)] + \frac{m}{2} (\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2) (S + I + D)^{m+1} \\ &= A(S + I + D)^m - \left[\delta - \frac{m}{2} (\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2) \right] (S + I + D)^{m+1} \\ &= F - \frac{\rho}{2} (S + I + D)^{m+1} \\ &\leq F - \frac{\rho}{2} (S^{m+1} + I^{m+1} + D^{m+1}), \end{aligned}$$

where

$$F = \sup_{(S,I,D) \in \mathbb{R}_+^3} \left\{ A(S + I + D)^m - \frac{\rho}{2} (S + I + D)^{m+1} \right\} < \infty.$$

Therefore,

$$\begin{aligned}
 LV \leq & -M\lambda + M + \frac{\alpha_1 MS}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\
 & - \frac{A}{S} + \frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} + (\mu + q + \gamma) - (\mu + \delta) \frac{I}{D} + \frac{\sigma_1^2 + \sigma_3^2}{2} \\
 & + F - \frac{\rho}{2} (S^{m+1} + I^{m+1} + D^{m+1}).
 \end{aligned} \tag{2.3}$$

We can choose a positive constant M such that

$$-M\lambda + E \leq -2,$$

herein,

$$\begin{aligned}
 E = \sup_{(S,I,D) \in \mathbf{R}_+^3} & \left\{ M + \frac{\alpha_1 MS}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \right. \\
 & \left. + \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F - \frac{\rho}{2} (S^{m+1} + I^{m+1} + D^{m+1}) \right\}.
 \end{aligned}$$

Now, we construct the following bounded closed set:

$$D_\varepsilon = \left\{ (S, I, D) \in \mathbf{R}_+^3 : \varepsilon \leq S \leq \frac{1}{\varepsilon}, \varepsilon \leq I \leq \frac{1}{\varepsilon}, \varepsilon^2 \leq D \leq \frac{1}{\varepsilon^2} \right\},$$

where $\varepsilon > 0$ is a sufficiently small constant. In the set $\mathbf{R}_+^3 \setminus D_\varepsilon$, we can choose ε sufficiently small and the following conditions hold:

$$-\frac{A}{\varepsilon} + G \leq -1, \tag{2.4}$$

$$-M\lambda + 2M(\mu + \delta)\varepsilon + E \leq -1, \tag{2.5}$$

$$-\frac{\mu + \delta}{\varepsilon} + G \leq -1, \tag{2.6}$$

$$-\frac{\rho}{4} \frac{1}{\varepsilon^{m+1}} + H \leq -1, \tag{2.7}$$

$$-\frac{\rho}{4} \frac{1}{\varepsilon^{m+1}} + J \leq -1, \tag{2.8}$$

$$-\frac{\rho}{4} \frac{1}{\varepsilon^{2m+2}} + N \leq -1, \tag{2.9}$$

where G, H, J, N will be determined later. Hence,

$$\mathbf{R}_+^3 \setminus D_\varepsilon = D_1^c \cup D_2^c \cup D_3^c \cup D_4^c \cup D_5^c \cup D_6^c,$$

with

$$D_1^c = \{(S, I, D) \in \Omega, 0 < S < \varepsilon\}, \quad D_2^c = \{(S, I, D) \in \Omega, 0 < I < \varepsilon\},$$

$$D_3^c = \{(S, I, D) \in \Omega, 0 < D < \varepsilon, I \geq \varepsilon\}, \quad D_4^c = \left\{ (S, I, D) \in \Omega, S > \frac{1}{\varepsilon} \right\},$$

$$D_5^c = \left\{ (S, I, D) \in \Omega, I \geq \frac{1}{\varepsilon} \right\}, \quad D_6^c = \left\{ (S, I, D) \in \Omega, D > \frac{1}{\varepsilon^2} \right\},$$

then we will prove that $LV(S, I, D) \leq -1$ on $\mathbf{R}_+^3 \setminus D_\varepsilon$.

Case 1. If $(S, I, D) \in D_1^c$, we achieve that

$$\begin{aligned} LV &\leq -M\lambda + M + \frac{\alpha_1 MS}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\ &\quad - \frac{A}{S} + \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F \\ &\quad - \frac{\rho}{2}(S^{m+1} + I^{m+1} + D^{m+1}) \\ &\leq -\frac{A}{S} + G \\ &\leq -\frac{A}{\varepsilon} + G, \end{aligned} \tag{2.10}$$

where

$$\begin{aligned} G = \sup_{(S, I, D) \in \mathbf{R}_+^3} &\left\{ M + \frac{\alpha_1 MS}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \right. \\ &\left. + \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F - \frac{\rho}{2}(S^{m+1} + I^{m+1} + D^{m+1}) \right\}. \end{aligned}$$

Making use of (2.4), we have that $LV \leq -1$ for all $(S, I, D) \in D_1^c$.

Case 2. If $(S, I, D) \in D_2^c$, we obtain that

$$\begin{aligned} LV &\leq -M\lambda + M + \frac{\alpha_1 MS}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\ &\quad - \frac{A}{S} + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2} \right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F \\ &\quad - \frac{\rho}{2}(S^{m+1} + I^{m+1} + D^{m+1}) \\ &\leq -M\lambda + 2M(\mu + \delta)I + E \\ &\leq -M\lambda + 2M(\mu + \delta)\varepsilon + E. \end{aligned} \tag{2.11}$$

By inequality (2.5), we can achieve that $LV \leq -1$ for all $(S, I, D) \in D_2^c$.

Case 3. If $(S, I, D) \in D_3^c$, we get that

$$\begin{aligned} LV &\leq -(\mu + \delta)\frac{I}{D} + M + \frac{\alpha_1 MS}{\rho + \mu + \delta} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}} \left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) \\ &\quad + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2} \right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F - \frac{\rho}{2}(S^{m+1} + I^{m+1} + D^{m+1}) \\ &\leq -\frac{\mu + \delta}{\varepsilon} + G. \end{aligned} \tag{2.12}$$

Applying (2.6), we have that $LV \leq -1$ for all $(S, I, D) \in D_3^c$.

Case 4. If $(S, I, D) \in D_4^c$, we have that

$$\begin{aligned}
 LV &\leq -\frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]S^{m+1} - \frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]S^{m+1} \\
 &\quad - \frac{1}{2}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](I^{m+1} + D^{m+1}) + M + \frac{\alpha_1 MS}{\rho + \mu + \delta}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) \\
 &\quad + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2}\right) + (\mu + q + \gamma) \\
 &\quad + \frac{\sigma_1^2 + \sigma_3^2}{2} + F \\
 &\leq -\frac{\rho}{4}S^{m+1} + H \\
 &\leq -\frac{\rho}{4}\frac{1}{\varepsilon^{m+1}} + H,
 \end{aligned} \tag{2.13}$$

where

$$\begin{aligned}
 H &= \sup_{(S, I, D) \in \mathbb{R}_+^3} \left\{ -\frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]S^{m+1} \right. \\
 &\quad - \frac{1}{2}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](I^{m+1} + D^{m+1}) \\
 &\quad + M + \frac{\alpha_1 MS}{\rho + \mu + \delta}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) \\
 &\quad \left. + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2}\right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F \right\}.
 \end{aligned}$$

Combining with (2.7), it can be obtained that $LV \leq -1$ for all $(S, I, D) \in D_4^c$.

Case 5. If $(S, I, D) \in D_5^c$, we can see that

$$\begin{aligned}
 LV &\leq -\frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]I^{m+1} - \frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]I^{m+1} \\
 &\quad - \frac{1}{2}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{m+1} + D^{m+1}) + M + \frac{\alpha_1 MS}{\rho + \mu + \delta}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) \\
 &\quad + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2}\right) + (\mu + q + \gamma) \\
 &\quad + \frac{\sigma_1^2 + \sigma_3^2}{2} + F \\
 &\leq -\frac{\rho}{4}I^{m+1} + J \\
 &\leq -\frac{\rho}{4}\frac{1}{\varepsilon^{m+1}} + J,
 \end{aligned} \tag{2.14}$$

where

$$J = \sup_{(S, I, D) \in \mathbb{R}_+^3} \left\{ -\frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]I^{m+1} \right\}$$

$$\begin{aligned}
 & -\frac{1}{2}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{m+1} + D^{m+1}) \\
 & + M + \frac{\alpha_1 MS}{\rho + \mu + \delta}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) \\
 & + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2}\right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F\}.
 \end{aligned}$$

According to inequality (2.8), it can be achieved that $LV \leq -1$ for all $(S, I, D) \in D_5^c$.

Case 6. If $(S, I, D) \in D_6^c$, we can see that

$$\begin{aligned}
 LV & \leq -\frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]D^{m+1} - \frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]D^{m+1} \\
 & - \frac{1}{2}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{m+1} + I^{m+1}) + M + \frac{\alpha_1 MS}{\rho + \mu + \delta}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) \\
 & + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2}\right) + (\mu + q + \gamma) \\
 & + \frac{\sigma_1^2 + \sigma_3^2}{2} + F \\
 & \leq -\frac{\rho}{4}D^{m+1} + N \\
 & \leq -\frac{\rho}{4}\frac{1}{\varepsilon^{2m+2}} + N,
 \end{aligned} \tag{2.15}$$

where

$$\begin{aligned}
 N & = \sup_{(S, I, D) \in \mathbb{R}_+^3} \left\{ -\frac{1}{4}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right]D^{m+1} \right. \\
 & - \frac{1}{2}\left[\delta - \frac{m}{2}(\sigma_1^2 \vee \sigma_2^2 \vee \sigma_3^2)\right](S^{m+1} + I^{m+1}) \\
 & + M + \frac{\alpha_1 MS}{\rho + \mu + \delta}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) + 2M(\mu + \delta)I + \frac{MA}{\mu + q + \frac{\sigma_1^2}{2}}\left(\frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2}\right) \\
 & \left. + \left(\frac{\beta_1}{\alpha_1 I} + \frac{\beta_2}{\alpha_2}\right) + (\mu + q + \gamma) + \frac{\sigma_1^2 + \sigma_3^2}{2} + F\right\}.
 \end{aligned}$$

By inequality (2.9), it can be seen that $LV \leq -1$ for all $(S, I, D) \in D_6^c$.

Thus condition (H2) of Lemma 2.1 holds. By Lemma 2.1, we know that the solution of system (1.3) is ergodic and system (1.3) has a unique stationary distribution $\pi(\cdot)$.

This completes the proof. □

Remark 1 Here, we construct a C^2 -function, and then we are going to do a step by step calculation of the function, to make the calculation process more logical. Then, the discussion is divided into six cases. Finally, the ideal result is obtained.

3 Extinction

In this section, we give the sufficient conditions that lead to the extinction of the disease. For convenience, we introduce the following notation and lemma.

If $f(t)$ is an integral function on $[0, \infty)$, define $\langle f(t) \rangle = \frac{1}{t} \int_0^t f(\tau) \, d\tau$.

Lemma 3.1 *Let $(S(t), I(t), D(t))$ be the solution of system (1.3) with the initial value $(S(0), I(0), D(0)) \in \mathbf{R}_+^3$. Then $S(t) + I(t) \leq \frac{A}{\mu}$, $D(t) \leq \frac{A(\mu + \delta)}{\gamma\mu}$.*

Proof Summing the first two equations of system (1.3), we can get

$$\frac{d(S(t) + I(t))}{dt} \leq A - \mu(S(t) + I(t)),$$

so we have

$$\limsup_{t \rightarrow \infty} (S(t) + I(t)) \leq \frac{A}{\mu}.$$

In the same way, we have

$$\limsup_{t \rightarrow \infty} D(t) \leq \frac{A(\mu + \delta)}{\gamma\mu}.$$

Therefore, the positive invariant set of system (1.3) is

$$\Omega = \left\{ (S, I, D) \in \mathbf{R}_+^3 : 0 \leq S, I \leq \frac{A}{\mu}, \quad 0 \leq D \leq \frac{A(\mu + \delta)}{\gamma\mu} \right\}.$$

This completes the proof of Lemma 3.1. □

Lemma 3.2 *Let $(S(t), I(t), D(t))$ be the solution of system (1.3) with the initial value $(S(0), I(0), D(0)) \in \Omega$. Then*

$$\lim_{t \rightarrow \infty} \frac{\int_0^t \sigma_1 S(\tau) \, dB_1(\tau)}{t} = 0, \quad \lim_{t \rightarrow \infty} \frac{\int_0^t \sigma_2 I(\tau) \, dB_2(\tau)}{t} = 0.$$

Proof Let $M_1(t) = \int_0^t \sigma_1 S(\tau) \, dB_1(\tau)$, $M_1(t)$ is an integral and corresponds to the Brownian motion and it is local continuous martingale. Also, if we replace the upper bound with $t = 0$ in $M_1(t)$, then we have $M(0) = 0$. Next, we can discover the quadratic variation and get the following limits:

$$\limsup_{t \rightarrow \infty} \frac{\langle M_1, M_1 \rangle_t}{t} \leq \frac{\sigma_1^2 A^2}{\mu^2} < \infty.$$

Applying the strong law of large numbers [9], we can deduce that

$$\lim_{t \rightarrow \infty} \frac{M_1(t)}{t} = 0 \quad a.s.$$

Let $M_2(t) = \int_0^t \sigma_2 I(\tau) \, dB_2(\tau)$. Similarly, we can get

$$\limsup_{t \rightarrow \infty} \frac{\langle M_2, M_2 \rangle_t}{t} \leq \frac{\sigma_2^2 A^2}{\mu^2} < \infty,$$

then

$$\lim_{t \rightarrow \infty} \frac{M_2(t)}{t} = 0 \quad a.s.$$

This completes the proof of Lemma 3.2. □

Theorem 3.1 *Let $(S(t), I(t), D(t))$ be the solution of system (1.3) with the initial value $(S(0), I(0), D(0)) \in \mathfrak{N}_+^3$. If*

$$\tilde{\mathfrak{N}}_0^s := \frac{(\beta_1 + \beta_2)A}{(\mu + q)(\rho + \mu + \delta + \frac{\sigma_2^2}{2})} < 1,$$

then

$$\limsup_{t \rightarrow \infty} \frac{\ln I(t)}{t} \leq \left(\rho + \mu + \delta + \frac{\sigma_2^2}{2} \right) (\tilde{\mathfrak{N}}_0^s - 1) < 0 \quad a.s.,$$

that is,

$$\lim_{t \rightarrow \infty} I(t) = 0 \quad a.s.,$$

which means the disease dies out with probability one.

Moreover,

$$\lim_{t \rightarrow \infty} S(t) = \frac{A}{\mu + q} \quad a.s., \quad \lim_{t \rightarrow \infty} D(t) = 0 \quad a.s.$$

Proof Integrating system (1.3), we get

$$\begin{aligned} \frac{S(t) - S(0)}{t} + \frac{I(t) - I(0)}{t} &= A - (\mu + q)\langle S(t) \rangle - (\rho + \mu + \delta)\langle I(t) \rangle \\ &\quad + \frac{\sigma_1}{t} \int_0^t \sigma_1 S(\tau) \, dB_1(\tau) + \frac{\sigma_2}{t} \int_0^t \sigma_2 I(\tau) \, dB_2(\tau), \end{aligned} \tag{3.1}$$

which means

$$\langle S(t) \rangle = \frac{A}{\mu + q} - \frac{\rho + \mu + \delta}{\mu + q} \langle I(t) \rangle + \Phi(t), \tag{3.2}$$

where

$$\Phi(t) = \frac{1}{\mu + q} \left[\frac{\sigma_1}{t} \int_0^t S(\tau) \, dB_1(\tau) + \frac{\sigma_2}{t} \int_0^t I(\tau) \, dB_2(\tau) - \frac{S(t) - S(0)}{t} - \frac{I(t) - I(0)}{t} \right].$$

By Lemma 3.1 and Lemma 3.2 we can obtain

$$\lim_{t \rightarrow \infty} \Phi(t) = 0 \quad a.s. \tag{3.3}$$

Applying Itô's formula to system (1.3), we get that

$$d \ln I(t) = \frac{\beta_1 S(t)}{1 + \alpha_1 I(t)} + \frac{\beta_2 S(t) D(t)}{I(t)(1 + \alpha_2 D(t))} - (\rho + \mu + \delta) - \frac{\sigma_2^2}{2} + \sigma_2 dB_2(t). \tag{3.4}$$

Integrating both sides of (3.4) from 0 to t and division by t lead to the following equation:

$$\begin{aligned}
 \frac{\ln I(t)}{t} &= \beta_1 \left\langle \frac{S(t)}{1 + \alpha_1 I(t)} \right\rangle + \beta_2 \left\langle \frac{S(t)D(t)}{I(t)(1 + \alpha_2 D(t))} \right\rangle - (\rho + \mu + \delta) - \frac{\sigma_2^2}{2} + \frac{\sigma_2 B_2(t)}{t} \\
 &\leq \beta_1 \langle S(t) \rangle + \beta_2 \langle S(t) \rangle - (\rho + \mu + \delta) - \frac{\sigma_2^2}{2} + \frac{\sigma_2 B_2(t)}{t} \\
 &= (\beta_1 + \beta_2) \left[\frac{A}{\mu + q} - \frac{\rho + \mu + \delta}{\mu + q} \langle I(t) \rangle + \Phi(t) \right] - (\rho + \mu + \delta) - \frac{\sigma_2^2}{2} + \frac{\sigma_2 B_2(t)}{t} \\
 &= \frac{(\beta_1 + \beta_2)A}{\mu + q} - \frac{(\beta_1 + \beta_2)(\rho + \mu + \delta)}{\mu + q} \langle I(t) \rangle + (\beta_1 + \beta_2)\Phi(t) \\
 &\quad - (\rho + \mu + \delta) - \frac{\sigma_2^2}{2} + \frac{\sigma_2 B_2(t)}{t} \\
 &= \left(\rho + \mu + \delta + \frac{\sigma_2^2}{2} \right) (\tilde{\mathfrak{H}}_0^s - 1) - \frac{(\beta_1 + \beta_2)(\rho + \mu + \delta)}{\mu + q} \langle I(t) \rangle \\
 &\quad + (\beta_1 + \beta_2)\Phi(t) + \frac{\sigma_2 B_2(t)}{t} \\
 &\leq \left(\rho + \mu + \delta + \frac{\sigma_2^2}{2} \right) (\tilde{\mathfrak{H}}_0^s - 1) + (\beta_1 + \beta_2)\Phi(t) + \frac{\sigma_2 B_2(t)}{t}. \tag{3.5}
 \end{aligned}$$

From strong law of large numbers [9], we have

$$\lim_{t \rightarrow \infty} \frac{\sigma_2 B_2(t)}{t} = 0 \quad a.s. \tag{3.6}$$

Combining (3.3) and (3.6), then taking the limit superior of both sides of (3.5), we have

$$\limsup_{t \rightarrow \infty} \frac{\ln I(t)}{t} \leq \left(\rho + \mu + \delta + \frac{\sigma_2^2}{2} \right) (\tilde{\mathfrak{H}}_0^s - 1) \quad a.s.$$

This implies that if $\tilde{\mathfrak{H}}_0^s < 1$, then

$$\lim_{t \rightarrow \infty} I(t) = 0 \quad a.s.$$

According to system (1.3), the third equation with limiting system becomes

$$dD(t) = -\gamma D(t) dt,$$

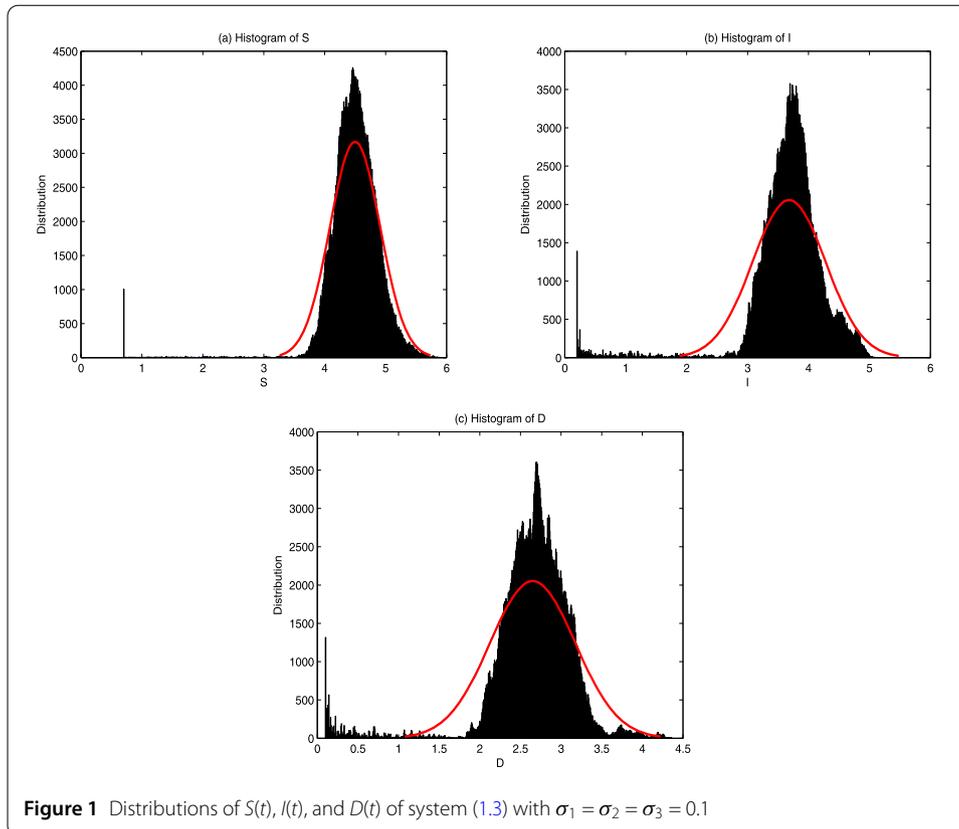
thus, we obtain

$$\lim_{t \rightarrow \infty} D(t) = 0 \quad a.s.$$

Similarly, according to the first equation of (1.3), we have

$$\lim_{t \rightarrow \infty} S(t) = \frac{A}{\mu + q} \quad a.s.$$

This completes the proof of Theorem 3.1. □



Remark 2 Theorem 3.1 shows that the disease will become extinct if $\tilde{\mathfrak{R}}_0^s < 1$. Note that $\tilde{\mathfrak{R}}_0^s < 1$ independent of σ_1 and σ_3 , that is to say, when $\tilde{\mathfrak{R}}_0^s < 1$, even if the white noises σ_1 and σ_3 are zero, the disease is still extinct.

4 Numerical simulations

In this section, we use the Milstein method [20] and numerical simulations to verify the conclusions in this paper. The discrimination equations of system (1.3) are presented as follows:

$$\begin{aligned}
 S_{i+1} &= S_i + \left(A - S_i \frac{\beta_1 I_i}{1 + \alpha_1 I_i} - S_i \frac{\beta_2 D_i}{1 + \alpha_2 D_i} - \mu S_i - q S_i \right) \Delta t \\
 &\quad + S_i \left(\sigma_1 \xi_i \sqrt{\Delta t} + \frac{\sigma_1^2}{2} (\xi_i^2 - 1) \Delta t \right), \\
 I_{i+1} &= I_i + \left(S_i \frac{\beta_1 I_i}{1 + \alpha_1 I_i} + S_i \frac{\beta_2 D_i}{1 + \alpha_2 D_i} - (\rho + \mu + \delta) I_i \right) \Delta t \\
 &\quad + I_i \left(\sigma_2 \eta_i \sqrt{\Delta t} + \frac{\sigma_2^2}{2} (\eta_i^2 - 1) \Delta t \right), \\
 D_{i+1} &= D_i + ((\mu + \delta) I_i - \gamma D_i) \Delta t + D_i \left(\sigma_3 \zeta_i \sqrt{\Delta t} + \frac{\sigma_3^2}{2} (\zeta_i^2 - 1) \Delta t \right),
 \end{aligned}$$

where ξ_i, η_i, ζ_i ($i = 1, 2, \dots$) are independent N-distributed Gaussian random variables.

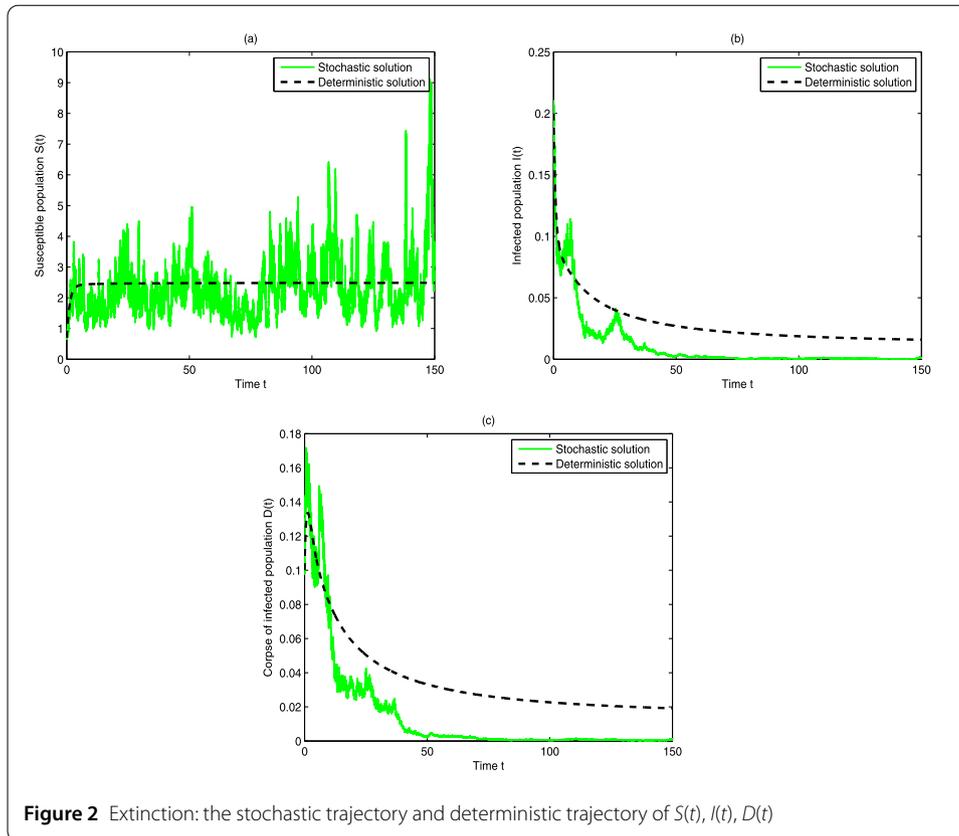


Figure 2 Extinction: the stochastic trajectory and deterministic trajectory of $S(t)$, $I(t)$, $D(t)$

Example 4.1 We take the parameters in system (1.3) as follows: $A = 5$, $\alpha_1 = 2$, $\alpha_2 = 2$, $\beta_1 = 0.35$, $\beta_2 = 0.45$, $\mu = 0.06$, $\delta = 0.3$, $\rho = 0.05$, $q = 0.7$, $\gamma = 0.5$, $\sigma_1 = 0.1$, $\sigma_2 = 0.1$, $\sigma_3 = 0.1$, and the initial value is $(S(0), I(0), D(0)) = (0.7, 0.2, 0.1)$. We can obtain that

$$\mathfrak{R}_0^s := \frac{\beta_1}{(\mu + q + \frac{\sigma_1^2}{2})(\rho + \mu + \delta + \frac{\sigma_2^2}{2})} = 1.1024 > 1.$$

Theorem 2.2 implies that system (1.3) has a unique stationary distribution(Fig. 1). From Fig. 1, we can see that the numbers of susceptible group, infected group, and infected corpse group are normally distributed.

Example 4.2 We take the parameters as follows: $A = 2$, $\alpha_1 = 2$, $\alpha_2 = 2$, $\beta_1 = 0.05$, $\beta_2 = 0.2$, $\mu = 0.1$, $\delta = 0.5$, $\rho = 0.1$, $q = 0.7$, $\gamma = 0.5$, $\sigma_1 = 0.5$, $\sigma_2 = 0.2$, $\sigma_3 = 0.2$, and the initial value is $(S(0), I(0), D(0)) = (0.7, 0.2, 0.1)$. We can easily calculate the basic reproduction number $\mathfrak{R}_0 = \frac{\beta_1 A \gamma + \beta_2 A (\mu + \delta)}{(\mu + q) \gamma (\rho + \mu + \delta)} = 1.036 > 1$.

$$\tilde{\mathfrak{R}}_0^s = \frac{(\beta_1 + \beta_2) A}{(\mu + q) (\rho + \mu + \delta + \frac{\sigma_2^2}{2})} = 0.7576 < 1,$$

which satisfies the condition of Theorem 3.1. So, the disease will go to extinction, that is to show that the white noise may lead to the extinction of disease. However, for deterministic models (1.2), the disease persists since $\mathfrak{R}_0 > 1$. Thus, the numerical simulations validate our theoretical results (Fig. 2). Figure 2 shows the change curves of the numbers of susceptible group, infected group, and infected corpse group over time. We can also see that

the comparison of curves of the same variable makes the numerical simulation of model (1.3) closer to the reality.

5 Conclusions

In this paper, a new SIRD epidemic model of Ebola with double saturated incidence rates and vaccination is proposed and studied. Firstly, we construct suitable Lyapunov functions, we get that if $\mathfrak{R}_0^s = \frac{\beta_1}{(\mu+q+\frac{\sigma_1^2}{2})(\rho+\mu+\delta+\frac{\sigma_2^2}{2})} > 1$, then system (1.3) has a unique stationary distribution. Next, we find that if $\tilde{\mathfrak{R}}_0^s = \frac{(\beta_1+\beta_2)A}{(\mu+q)(\rho+\mu+\delta+\frac{\sigma_2^2}{2})} < 1$, then the disease dies out with probability one. Finally, numerical simulations are shown to verify our results.

Some interesting topics deserve further consideration. On the one hand, one may propose some more realistic but complex models, such as considering the effects of impulsive perturbations on system (1.3). On the other hand, in our model (1.2), we only introduce the white noise into it, one can also introduce the colored noise into model (1.2). Moreover, our model is autonomous, it is interesting to investigate the nonautonomous system. We will leave these problems as our future work.

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Authors' contributions

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