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Determining optimal policies for sugarcane harvesting in Thailand using bi-objective and quasi-Newton optimization methods

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Abstract

In Thailand, the harvesting season for sugarcane usually begins in November and ends the following May. At the beginning of each harvesting season, the Royal Thai government sets the price of two types of sugarcane, namely fresh and fired, based on sweetness (sugar content) and gross weight of sugarcane delivered to the sugar factories. The aim of the present research is to determine optimal harvesting policies for the two types of sugarcane in sugarcane producing regions of Thailand in order to maximize revenue and minimize harvesting cost. In this paper, a harvesting policy is defined as the amount of each type of sugarcane harvested and delivered to the sugar factories during each 15-day period of a harvesting season. Two optimization methods have been used to solve this optimization problem, namely the ε-constraints method and a quasi-Newton optimization method. In the ε-constraints method, the problem is considered as a bi-objective optimization problem with the main objective being to determine harvesting policies that maximize the total revenue subject to upper bounds on the harvesting cost. In the quasi-Newton method, the aim is to determine the harvesting policy which gives maximum profit to the farmers subject to constraints on the maximum amount that can be cut in a 15-day period. The methods are used to determine optimal harvesting policies for the north, central, east, and north-east regions of Thailand for harvesting seasons 2012/13, 2013/14, and 2014/15 based on the data obtained from the Ministry of Industry and the Ministry of Agriculture and Co-operatives of the Royal Thai government.

Keywords: Optimal harvesting policy; ε-constraints method; Bi-objective mathematical model; Quasi-Newton optimization

1 Introduction

The sugarcane industry is one of the major agro-industries in Thailand; it is important to the Thai economy because sugar is one of the top five agricultural products that the country exports. It helps create income of approximately 180 billion baht a year [1]. A survey in 2018 reported that Brazil is the largest sugarcane producing country followed by India and the EU [2]. Thailand is the world's 4th largest sugarcane producer and produced about 100, 103.5, and 106 million tonnes of sugarcane in 2012/13, 2013/14, and 2014/15, respectively [3]. Sugarcane is grown in the north, central, east, and north-east regions of Thailand. The north-east region has the largest growing area and production [4]. In 2013/14, the north-east region produced 45 million tonnes of the total of 103.5 million tonnes of sugarcane produced in the country, the north, central, and east regions producing 24, 30, and 4.5 million tonnes, respectively.

Since 1992/93 the Commercial Cane Sugar (CCS) System has been used as the main sugarcane trading system in Thailand [5]. In this system, the Royal Thai government determines the price of sugarcane for each of the four regions in the country based on two main factors of weight and quality. The quality is considered as a sweetness or CCS, where CCS means the percentage of sucrose produced from a tonne of sugarcane. The government determines the price of sugarcane in each region by combining sweetness and weight, where a "standard sweetness" is counted as 10 CCS. Generally, the level of sweetness and the price of the sugarcane increases during the harvesting season. Before the sugarcane is harvested by a farmer, a sugar factory will carry out a randomized check of CCS value. If the value lies above a standard value of 10 CCS, then that factory will accept the sugarcane from that farm immediately. However, a farmer might delay harvesting if they expect that they will obtain more revenue by waiting for their sugarcane to increase in value.

Many researchers have studied planning models for sugarcane farming using a variety of optimization methods. In this paper, we will compare results obtained from bi-objective and quasi-Newton optimization methods. Some examples of the use of the bi-objective optimization method are as follows. In 2012, Gomes [6] studied a bi-objective mathematical model for choosing sugarcane varieties which have a harvest biomass residual for use in electricity generation. The bi-objective optimization model was to maximize the revenue from sale of the generated electricity and to minimize the cost of harvesting the residual biomass. In 2016, Sungnul et al. [7] studied a multi-objective optimization model to find an optimal time of harvesting for sugarcane growers in the north-eastern region of Thailand. The aim of this work was to help farmers to find the optimal harvesting time in order to maximize the revenue and to minimize the cost. Sungnul et al. [8] used the ε-constraints method to solve the mathematical model by choosing the revenue as the objective function and the costs as constraints. In 2017, Sungnul et al. [9] extended the work in [7] to find the optimal harvesting times for all of the four regions of Thailand. Quasi-Newton optimization methods are a well-known method of optimization that have been used for many years to find optimal solutions for problems in many areas of science, finance, and industry (see, e.g., [10–12]). However, as far as the authors are aware, they have not previously been applied to the problem of optimizing sugarcane harvesting in Thailand.

In this paper, we will use the bi-objective and quasi-Newton optimization methods to determine optimal harvesting policies to maximize profit to the farmers in the four sugarcane producing regions (north, central, east, and north-east) of Thailand for harvesting seasons 2012/13, 2013/14, and 2014/15. We will consider a harvesting policy for a given type of sugarcane for a given region for a given harvesting season as the amounts of the given type harvested each 15-day period during the harvesting season. For this work, we will use the data obtained from the Ministry of Industry (Office of the Cane and Sugarcane Board: OCSB) and the Ministry of Agriculture and Co-operatives of the Royal Thai government. We will first develop mathematical models of sugarcane harvesting and then use the bi-objective and quasi-Newton optimization methods problem to maximize the revenue and minimize the harvesting cost.

2 Mathematical models

In general, farmers harvest sugarcane of two types, namely fresh or fired. The main difference is that fired sugarcane is burnt to remove leaves before it is cut so that it can be cut manually by workers. Fresh sugarcane is usually cut by machines which can remove the leaves as they cut.

In this paper, we consider harvesting of the two types of sugarcane (fresh and fired) in four regions of Thailand (north, central, east, north-east) for three crop years (2012/13, 2013/14, 2014/15). We will use the notation: A for fresh and B for fired sugarcane, $j = 1$ for north, $j = 2$ for central, $j = 3$ for east, $j = 4$ for north-east. The mathematical model for maximizing profit from sugarcane harvesting can be written as

$$\text{Maximize} \quad J[u] = \int_{t_0}^{t_f} P\bigl(t, x(t), u(t)\bigr)\, dt, \tag{1}$$

$$\text{subject to} \quad \frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - u(t), \tag{2}$$

$$0 \le u(t) \le u_{\max}, \qquad x(t_0) = a, \qquad x(t_f) = 0, \tag{3}$$

where $J[u]$ is the optimal profit functional to be maximized by selecting the control variable $u(t)$, which is the rate of cutting sugarcane at time t. Also, $P(t, x(t), u(t))$ is the profit for sugarcane cut at time t (baht/day), $x(t)$ is the total amount of sugarcane (tonnes) on the farms in a region at time t, $u_{\max}$ is the maximum rate of cutting sugarcane (tonnes/day), a is the total amount of sugarcane on farms at time t_0 (tonnes), t_0 is the initial time at the start of harvesting period (day), t_f is the final time at the end of harvesting period (day), r is the rate of change of weight at the initial time t_0 (tonnes/day), and K is a constant in the logistic growth function which represents the carrying capacity of the farms (tonnes) in the absence of cutting.

In order to solve the model in Eq. (1)–(3), it would be necessary to use optimal control (see, e.g., [13]). However, to gain some insight into the sugarcane harvesting problem, we have simplified the optimization problem by assuming that the harvesting time can be divided into twelve 15-day periods starting at the opening of the sugar mills on November 16 and ending at the closing of mills on May 15 the following year. Also, since the growth of sugarcane is slow over the cutting period, we have simplified the discretized model considered in this paper by setting $r = 0$, i.e., we have assumed that there is no growth during the cutting period. The optimization problem for sugarcane harvesting for a given type of sugarcane in a given region of Thailand for a given crop year can then be stated as follows:

$$\text{Maximize} \quad J[u] = \sum_{k=1}^{12} P\bigl(k, x(k), u(k)\bigr) \tag{4}$$

$$\text{subject to} \quad x(k) = x_k(t_k) = x_{k-1}(t_k) - u(k), \tag{5}$$

$$0 \le u(k) \le \min\bigl\{x(k), u_{\max}\bigr\}, \tag{6}$$

$$x(1) = a, \qquad x(12) = 0, \tag{7}$$

where $k = 1$ is the first 15-day harvesting period, $k = 12$ is final 15-day harvesting period, t_k is the time (days) at the start of period k, $x(k)$ is the weight (tonnes) of sugarcane remaining on the farms at the start of period k, $u(k)$ is the weight of sugarcane cut in period k, $u_{\max}$ is an upper bound on the amount of sugarcane that can be cut in a 15-day period, and a is the total amount of sugarcane on farms at the start of harvesting. Also, in calculating profits and costs for each 15-day period, we have assumed that the total cutting $u(k)$ in period k is carried out at an average daily rate of $u(k)/15$ tonnes per day.

The profit from sugarcane harvesting in a 15-day period is the difference between the revenue from the harvesting and the total costs of harvesting, i.e.,

$$P\bigl(k, x(k), u(k)\bigr) = \mathrm{RV}\bigl(k, x(k), u(k)\bigr) - C\bigl(k, x(k), u(k)\bigr), \tag{8}$$

where $\mathrm{RV}(k, x(k), u(k))$ is the revenue from the harvested sugarcane and $C(k, x(k), u(k))$ is the total cost of harvesting the sugarcane.

In this paper, we solve the optimization problem in (4)–(7) using the bi-objective optimization method and a quasi-Newton optimization method. In the bi-objective method, the two objectives are to maximize the revenue and minimize the cost. In the quasi-Newton method, the revenue and cost objectives are combined into the single objective of maximizing the profit.

3 Revenue from sugarcane selling [7]

There are three main factors determining sale price of sugarcane for a given region for a given crop year. These factors are type (fresh or fired), weight, and quality or CCS.

1. Revenue from weight of sugarcane: The basic price P_w of sugarcane (baht/tonne) is fixed by the Royal Thai government for each crop year. This basic price is the same for all regions. However, farmers who sell fired sugarcane will be deducted 20 baht/tonne from the basic sugarcane price determined by the government each year. Then, at the end of harvesting for the year, factories in each region will share the total amount of money deducted from fired sugarcane sales in that region to farmers who sold fresh sugarcane at a rate not exceeding 70 baht/tonne of fresh sugarcane delivered, thus increasing the price per tonne of fresh sugarcane above the basic price.

   The actual price based on weight received by farmers for fired sugarcane is given by

   $$\mathrm{PW}(B) = P_w - 20, \tag{9}$$

   where P_w is the basic price of sugarcane (baht/tonne) based on weight set by the government.

   The actual price per tonne received by farmers for fresh sugarcane in a region j is then

   $$\mathrm{PW}_j(A) = P_w + \frac{20a_j(B)}{a_j(A)} \le P_w + 70, \tag{10}$$

   where $a_j(A)$ is the total amount of fresh sugarcane (tonnes) from region j and $a_j(B)$ is the total amount of fired sugarcane (tonnes) from region j.
2. Revenue based on CCS: Each year, the Royal Thai government sets a basic price per tonne for sugarcane with 10 CCS. This price per tonne based on CCS is the same

Table 1 The price (baht/tonne) of fresh and fired sugarcane in each region for crop year 2012/13

Period	North		Central		East		North-east	
	Fresh	Fired	Fresh	Fired	Fresh	Fired	Fresh	Fired
(1) 16–30/11/12	1104.12	1045.66	1060.75	1002.28	1128.10	1069.64	1186.57	1128.10
(2) 1–15/12/12	1126.49	1068.02	1100.75	1042.28	1157.18	1098.71	1246.51	1188.04
(3) 16–31/12/12	1151.89	1093.43	1138.07	1079.60	1175.75	1117.29	1271.17	1212.71
(4) 1–15/01/13	1177.39	1118.92	1160.93	1102.46	1189.59	1131.12	1290.34	1231.88
(5) 16–31/01/13	1202.90	1144.44	1189.83	1131.37	1203.19	1144.73	1309.43	1250.97
(6) 1–14/02/13	1221.13	1162.66	1205.57	1147.10	1211.87	1153.41	1323.35	1264.88
(7) 15–28/02/13	1236.26	1177.79	1221.27	1162.81	1221.35	1162.88	1336.38	1277.91
(8) 1–15/03/13	1248.85	1190.38	1235.04	1176.58	1228.60	1170.14	1346.20	1287.73
(9) 16–31/03/13	1259.64	1201.17	1244.53	1186.07	1233.57	1175.10	1354.01	1295.55
(10) 1–15/04/13	1264.75	1206.29	1248.49	1190.02	1236.97	1178.51	1358.00	1299.53
(11) 16–30/04/13	1265.10	1206.63	–	–	1236.76	1178.29	1358.65	1300.19
(12) 1–15/05/13	–	–	–	–	1236.02	1177.56	1358.66	1300.20

Note: – means that no sugarcane was delivered to the mills in that period.

for fresh and fired sugarcane. The actual price per tonne received by farmers is then adjusted if the CCS is different from 10. For sugarcane from region j harvested in period k, the actual price per tonne is given by

$$\mathrm{PC}_j(k) = P_c\bigl(1 + 0.06y_j(k)\bigr), \tag{11}$$

where P_c is the basic price per tonne of sugarcane with 10 CCS set by the government, and $y_j(k) = \mathrm{CCS}_j(k) - 10$, where $\mathrm{CCS}_j(k)$ is the average CCS from sugarcane in region j harvested in period k and the factor 0.06 is the rate of change of price per 1 CCS from the base level of 10.

Therefore, the revenue $\mathrm{RV}_j(k, A)$ (baht/tonne) from selling fresh sugarcane from region j which is harvested in period k is determined by adding Eq. (10) and Eq. (11). We obtain

$$\mathrm{RV}_j(k, A) = \mathrm{PW}_j(A) + \mathrm{PC}_j(k). \tag{12}$$

Similarly, we obtain the revenue $\mathrm{RV}_j(k, B)$ (baht/tonne) from selling fired sugarcane by adding Eq. (9) and Eq. (11).

$$\mathrm{RV}_j(k, B) = \mathrm{PW}(B) + \mathrm{PC}_j(k). \tag{13}$$

The price and CCS data that we used in our optimization were obtained from the Office of the Cane and Sugar Board [14–16]. The real price data obtained from the OCSB for the four regions for the crop years 2012/13, 2013/14, and 2014/15 are shown in Tables 1–3, and the CCS data obtained from the OCSB for the four regions for the crop years 2012/13, 2013/14, and 2014/15 are shown in Tables 4–6.

4 Costs of production

The costs of production can be separated into (1) gathering cost and (2) maintenance cost.

4.1 Gathering cost of production

The gathering cost $\mathrm{GC}_j(k, i)$ (baht/tonne) of sugarcane of type $i = A$ or B from region j which is harvested in period k consists of a harvesting cost $\mathrm{CF}_j(k, i)$ (baht/tonne) of sug-

Table 2 The price (baht/tonne) of fresh and fired sugarcane in each region for crop year 2013/14

Period	North		Central		East		North-east	
	Fresh	Fired	Fresh	Fired	Fresh	Fired	Fresh	Fired
(1) 16–30/11/13	1125.13	1080.66	–	–	–	–	1136.35	1113.05
(2) 1–15/12/13	1133.91	1101.57	1143.24	1088.57	1148.33	1106.79	1207.06	1168.62
(3) 16–31/12/13	1153.82	1127.41	1158.55	1103.88	1179.29	1141.22	1236.04	1188.63
(4) 1–15/01/14	1180.59	1151.32	1183.10	1128.43	1204.55	1166.21	1255.78	1207.84
(5) 16–31/01/14	1206.55	1175.37	1208.38	1153.71	1225.70	1189.81	1276.81	1227.23
(6) 1–14/02/14	1228.21	1192.90	1228.41	1173.74	1245.09	1209.81	1292.84	1240.93
(7) 15–28/02/14	1244.67	1207.14	1244.40	1189.73	1259.62	1223.97	1306.00	1253.23
(8) 1–15/03/14	1258.73	1219.77	1257.03	1202.36	1270.83	1236.42	1316.52	1262.98
(9) 16–31/03/14	1268.07	1226.47	1263.99	1209.32	1280.35	1244.64	1325.57	1271.80
(10) 1–15/04/14	1270.65	1227.57	1265.45	1210.78	1283.22	1246.37	1330.67	1274.68
(11) 16–30/04/14	–	1227.57	–	–	1283.17	–	1331.37	1275.33
(12) 1–15/05/14	–	–	–	–	–	1245.87	1331.49	1275.43

Note: – means that no sugarcane was delivered to the mills in that period.

Table 3 The price (baht/tonne) of fresh and fired sugarcane in each region for crop year 2014/15

Period	North		Central		East		North-east	
	Fresh	Fired	Fresh	Fired	Fresh	Fired	Fresh	Fired
(1) 16–30/11/14	–	–	–	–	–	–	-	–
(2) 1–15/12/14	1071.89	1014.48	1049.40	991.99	1069.88	1012.47	1130.37	1072.96
(3) 16–31/12/14	1091.75	1034.34	1076.74	1019.33	1109.27	1051.86	1165.64	1108.23
(4) 1–15/01/15	1116.41	1059.00	1102.24	1044.83	1126.57	1069.16	1186.81	1129.40
(5) 16–31/01/15	1136.75	1079.34	1120.24	1062.83	1142.29	1084.88	1208.24	1150.83
(6) 1–14/02/15	1157.69	1100.28	1141.61	1084.20	1159.08	1101.67	1223.20	1165.79
(7) 15–28/02/15	1173.90	1116.49	1156.76	1099.35	1170.49	1113.08	1234.67	1177.26
(8) 1–15/03/15	1185.90	1128.49	1168.72	1111.31	1179.05	1121.64	1243.47	1186.06
(9) 16–31/03/15	1193.61	1136.20	1176.37	1118.96	1185.28	1127.87	1250.43	1193.02
(10) 1–15/04/15	1196.49	1139.08	1178.05	1120.64	1186.25	1128.84	1254.44	1197.03
(11) 16–30/04/15	1197.18	1139.77	–	–	–	–	1255.74	1198.33
(12) 1–15/05/15	–	–	–	–	–	–	1255.95	1198.54

Note: – means that no sugarcane was delivered to the mills in that period.

Table 4 The CCS value in each region for crop year 2012/13

Period	North	Central	East	North-east
(1) 16–30/11/12	8.440	7.716	8.840	9.815
(2) 1–15/12/12	8.813	8.383	9.325	10.815
(3) 16–31/12/12	9.236	9.006	9.635	11.226
(4) 1–15/01/13	9.662	9.387	9.865	11.546
(5) 16–31/01/13	10.087	9.869	10.092	11.864
(6) 1–14/02/13	10.391	10.132	10.237	12.096
(7) 15–28/02/13	10.644	10.394	10.395	12.314
(8) 1–15/03/13	10.854	10.623	10.516	12.477
(9) 16–31/03/13	11.034	10.782	10.599	12.608
(10) 1–15/04/13	11.119	10.848	10.656	12.674
(11) 16–30/04/13	11.125	–	10.652	12.685
(12) 1–15/05/13	–	–	10.640	12.685

Note: – means that no sugarcane was delivered to the mills in that period.

arcane and a transport cost $\mathrm{CT}_j(k,i)$ (baht/tonne) for delivering sugarcane to the mills.

$$\mathrm{GC}_j(k,i) = \mathrm{CF}_j(k,i) + \mathrm{CT}_j(k,i), \quad k = 1,2,\ldots,12, i = A,B. \tag{14}$$

Table 5 The CCS value in each region for crop year 2013/14

Period	North	Central	East	North-east
(1) 16–30/11/13	9.516	–	–	9.711
(2) 1–15/12/13	9.668	9.831	9.919	10.941
(3) 16–31/12/13	10.015	10.097	10.458	11.444
(4) 1–15/01/14	10.480	10.524	10.897	11.788
(5) 16–31/01/14	10.932	10.964	11.265	12.154
(6) 1–14/02/14	11.308	11.312	11.602	12.432
(7) 15–28/02/14	11.595	11.590	11.855	12.661
(8) 1–15/03/14	11.839	11.810	12.050	12.844
(9) 16–31/03/14	12.002	11.931	12.215	13.002
(10) 1–15/04/14	12.046	11.956	12.265	13.090
(11) 16–30/04/14	–	–	12.264	13.103
(12) 1–15/05/14	–	–	12.262	13.105

Note: – means that no sugarcane was delivered to the mills in that period.

Table 6 The CCS value in each region for crop year 2014/15

Period	North	Central	East	North-east
(1) 16–30/11/14	–	–	–	–
(2) 1–15/12/14	9.527	9.111	9.490	10.610
(3) 16–31/12/14	9.895	9.617	10.220	11.263
(4) 1–15/01/14	10.352	10.089	10.540	11.656
(5) 16–31/01/15	10.729	10.423	10.831	12.052
(6) 1–14/02/15	11.116	10.818	11.142	12.330
(7) 15–28/02/15	11.416	11.099	11.353	12.542
(8) 1–15/03/15	11.639	11.321	11.512	12.705
(9) 16–31/03/15	11.782	11.462	11.627	12.834
(10) 1–15/04/15	11.835	11.493	11.645	12.908
(11) 16–30/04/15	11.848	–	–	12.932
(12) 1–15/05/15	–	–	–	12.936

Note: – means that no sugarcane was delivered to the mills in that period.

4.2 Maintenance cost

We assume that the maintenance cost of sugarcane remaining on the farm is of the form $\mathrm{CM}_j(k,i)x_j(k,i)$, where $\mathrm{CM}_j(k,i)$ is a maintenance cost (baht/tonne) of maintaining a farm if an amount $x_j(k,i)$ (tonnes) of type i remains on the farm in period k in region j.

The total cost of production is then

$$C_j(k,i) = \mathrm{GC}_j(k,i) + \mathrm{CM}_j(k,i)x_j(k,i). \tag{15}$$

In general, the costs defined above will be a function of the type of sugarcane and the time of harvesting. However, in practice, it is difficult to obtain these detailed costs, and we have therefore carried out the numerical simulations assuming that the costs are constant in each region. Examples of the cost data obtained from the OCSB for the crop years 2012/13, 2013/14, and 2014/15 are shown in Table 7.

5 Bi-objective optimization

The ε-constraints method [8] was used to find the optimal patterns for fresh and fired sugarcane harvesting in the four regions of Thailand for the crop years 2012/13, 2013/14, and 2014/15. In this method, a multi-objective optimization problem is reformulated by choosing the most important objective to optimize while putting upper or lower bound

Table 7 The cost of fresh and fired sugarcane production (baht/tonne) in each region for each crop year

Year	Region	Fixed cost CF_j (baht/tonne)	Transport cost CT_j (baht/tonne)	Maintenance cost CM_j (baht/tonne)
2012/13	North	924.28	136.51	52.70
	Central	872.95	147.66	94.24
	East	836.25	148.89	92.51
	North-east	765.18	141.97	56.30
2013/14	North	815.89	149.46	66.07
	Central	781.41	147.04	86.59
	East	912.61	165.66	106.01
	North-east	875.84	151.64	59.55
2014/15	North	1061.42	182.36	79.38
	Central	954.16	155.30	82.32
	East	1024.71	194.89	94.30
	North-east	987.00	140.12	91.71

ε-constraints on the other objectives. In this paper, we choose maximizing the revenue as the most important objective and put upper bounds on the total cost which we are trying to minimize.

Using the ε-constraint method, we solve the following bi-objective optimization problem for the two types of sugarcane for the four regions of Thailand for the three crop years:

$$\text{Maximize} \quad J[X] = \sum_{k=1}^{12} \mathrm{RV}_j(k,i) a_j(i) X_j(k,i), \quad j = 1,2,3,4, i = A,B, \tag{16}$$

$$\text{subject to} \quad \sum_{k=1}^{12} C_j(k,i) a_j(i) X_j(k,i) \le \varepsilon_r, \quad k = 1,2,\ldots,12, \tag{17}$$

$$\sum_{k=1}^{12} X_j(k,i) = 1, \tag{18}$$

$$X_j(k,i) \in [0,1], \tag{19}$$

$$y_j(k) > -4, \tag{20}$$

where the aim is to select the values of the decision variables $X_j(k,i)$ to maximize the total revenue $J[X]$ from the harvesting of the sugarcane. The definitions of the parameters and variables in the model are as follows: $a_j(i)$ is the total amount of sugarcane of type i on the farms in region j at the beginning of the cutting season, and $X_j(k,i)$ is the fraction of the total amount of type i cut in period k in region j. The ε-constraints in (17) represent upper bounds on the second objective of the problem, which is to minimize the total cost of production of a given type in a given region in a given crop year. The condition equation (18) ensures that the total amount of sugarcane of type i harvested in region j in one year is equal to the amount of sugarcane of type i available. The condition equation (19) means that cutting in each period is non-negative and less than or equal to the total amount available. The constraint equation (20) means that the CCS of sugarcane that can be harvested must be greater than 6 ($y_j(k) = \mathrm{CCS}_j(k) - 10$).

Following the ε-constraints method [8], we solved the bi-objective optimization problem in Eq. (16)–(20) for p values of ε_r defined as follows:

$$\varepsilon_{r+1} = \varepsilon_r + \Delta\varepsilon; \quad e = 1, 2, \ldots, p-1, \tag{21}$$

where $\Delta\varepsilon = \frac{\mathrm{UB}-\mathrm{LB}}{p-1}$ and LB and UB are lower and upper limits on ε_r values defined as follows. LB is the minimum cost of cutting all sugarcane in one period using cost data for each period given in Tables 8–10, UB is the maximum cost of cutting all sugarcane in one period using cost data for each period given in Tables 8–10, p is the number of experiments.

Table 8 Optimum cutting of fresh and fired sugarcane in each region for year 2012/13 (bi-objective, mcf = 0). It is assumed that all sugarcane in each region can be cut in one 15-day period

Type	Region	Period	LB (10^9 baht)	UB (10^9 baht)	Revenue (10^9 baht)	Total cost (10^9 baht)	Profit (10^9 baht)
Fresh	North	16–30/04/13	7.464	7.835	8.902	7.464	1.438
	Central	1–15/04/13	9.501	10.379	11.623	9.501	2.122
	East	1–15/04/13	0.941	1.029	1.181	0.941	0.240
	North-east	1–15/05/13	15.339	16.291	22.973	15.339	7.634
Fired	North	16–30/04/13	18.629	19.555	21.190	18.629	2.561
	Central	1–15/04/13	21.626	23.623	25.215	21.626	3.589
	East	1–15/04/13	3.676	4.021	4.397	3.676	0.721
	North-east	1–15/05/13	21.147	22.459	30.309	21.147	9.162

Table 9 Optimum cutting of fresh and fired sugarcane in each region for year 2013/14 (bi-objective, mcf = 0). It is assumed that all sugarcane in each region can be cut in one 15-day period

Type	Region	Period	LB (10^9 baht)	UB (10^9 baht)	Revenue (10^9 baht)	Total cost (10^9 baht)	Profit (10^9 baht)
Fresh	North	1–15/04/14	6.831	7.298	8.991	6.831	2.160
	Central	1–15/04/14	8.913	9.744	12.148	8.913	3.235
	East	1–15/04/14	1.332	1.463	1.585	1.332	0.253
	North-east	1–15/05/14	20.563	21.755	26.647	20.563	6.084
Fired	North	1–15/04/14	16.540	17.672	21.033	16.540	4.493
	Central	1–15/04/14	19.014	20.787	24.796	19.014	5.782
	East	1–15/04/14	3.491	3.834	4.035	3.491	0.544
	North-east	1–15/05/14	25.576	27.058	31.748	25.576	6.172

Table 10 Optimum cutting of fresh and fired sugarcane in each region for year 2014/15 (bi-objective, mcf = 0). It is assumed that all sugarcane in each region can be cut in one 15-day period

Type	Region	Period	LB (10^9 baht)	UB (10^9 baht)	Revenue (10^9 baht)	Total cost (10^9 baht)	Profit (10^9 baht)
Fresh	North	–	9.184	9.77	–	–	–
	Central	1–15/05/14	10.246	11.006	10.879	10.246	0.633
	East	–	1.541	1.660	–	–	–
	North-east	1–15/05/14	21.447	23.192	23.898	21.447	2.451
Fired	North	–	22.387	23.815	–	–	–
	Central	1–15/04/14	20.820	22.364	21.029	20.820	0.210
	East	–	4.813	5.185	–	–	–
	North-east	1–15/05/14	31.940	34.539	33.964	31.940	2.024

Note: The – in column mean that no optimal solution could be found because the costs were greater than the revenue.

6 Quasi-Newton optimization

To obtain the results in this paper, we used the well-known quasi-Newton method (see, e.g., [10–12]) to find the optimal harvesting policies from the model given in Eq. (4)–(7) for the two types of sugarcane for the four regions of Thailand for the crop years 2012/13, 2013/14, and 2014/15. The numerical results were obtained using the constrained optimization function *fmincon* in Matlab with the "active-set" algorithm selected. For these solutions, we used the price and cost data supplied by the OCSB that has been discussed in the Mathematical Models section. Also, as stated earlier after Eq. (4)–(7), we assumed that $r = 0$, i.e., no growth of sugarcane during the cutting season.

For each type, region, and year, we have examined the effect of changing the values of the upper bound on the maximum cutting per period $u_{\max}$ (tonnes per 15 days) and the effect of reducing the maintenance costs by a factor mcf ($0 \leq \text{mcf} \leq 1$) of the actual maintenance cost.

7 Results

7.1 Bi-objective optimization

In the bi-objective optimization, we assumed that it was possible to cut all of the sugarcane in one 15-day period. With this assumption, it was found that with the given prices and costs, the optimal harvesting policy was to cut all of the sugarcane in one 15-day period. The results for the optimal harvesting period for fresh and fired sugarcane in the four regions of Thailand for the three crop years 2012/13, 2013/14, and 2014/15 are shown in Tables 8–10.

7.2 Quasi-Newton optimization

We first computed the optimal harvesting policy for the two types of sugarcane for the four regions for the three crop years assuming that it was possible to cut all of the sugarcane in one 15-day period. The results are shown in Figs. 1 and 2 for fresh and fired sugarcane for all four regions for the crop year 2013/14. It can be seen that, in agreement with the results from the bi-objective optimization, the optimal policy was to cut all sugarcane in one period.

We then computed the optimal harvesting policies if: (1) the maximum cutting in each period was bounded and (2) the maintenance costs could be changed, for example, by reducing the actual maintenance cost by a factor mcf ($0 \leq \text{mcf} \leq 1$) of the actual maintenance cost. Examples of the results are shown in Figs. 3 and 4 for the north-east region of Thailand for the crop year 2013/14. It can be seen that for the given price and cost data, the total profit to the farmers increases as the maximum possible cutting per period is increased. The reason for this is that the increase in maximum cutting per period means that more sugarcane can be cut in periods with the highest profits. The figures also show that the effect of reducing the maintenance cost of keeping uncut sugarcane on the farm by a factor mcf is to move the optimal cutting times to earlier periods. For example, if the maintenance cost is zero (mcf = 0), then with the given price and cost data, the optimal policy is to cut sugarcane as late as possible. If the maintenance cost is one quarter of the actual maintenance cost (mcf = 0.25), then the optimal policy is to cut in the middle of the cutting time. However, with the actual maintenance cost (mcf = 1), the optimal policy is

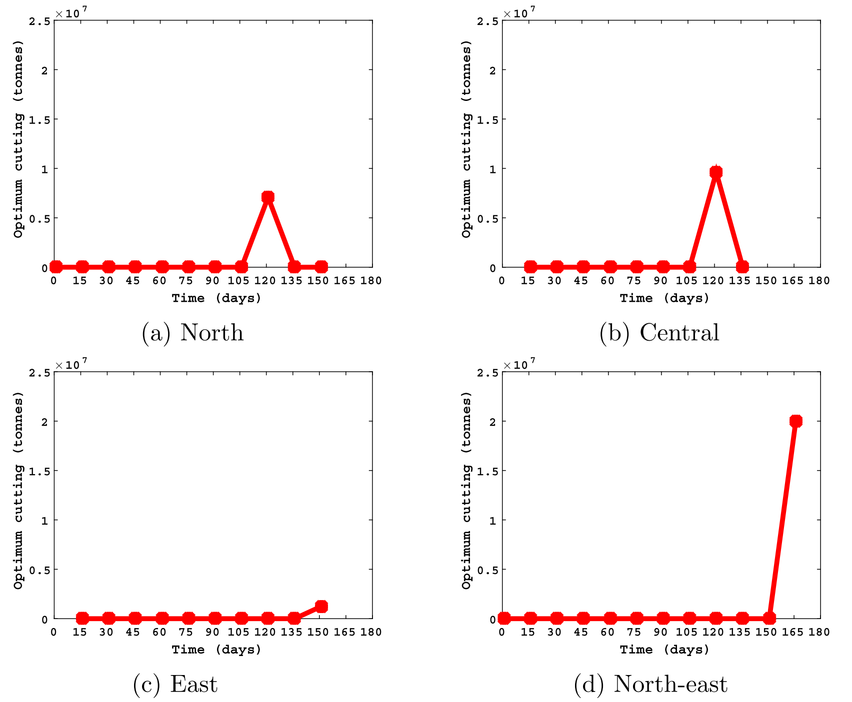

Figure 1 Optimum cutting of fresh sugarcane in each region for year 2013/14 (quasi-Newton, mcf = 0). It is assumed that all sugarcane in each region can be cut in one 15-day period

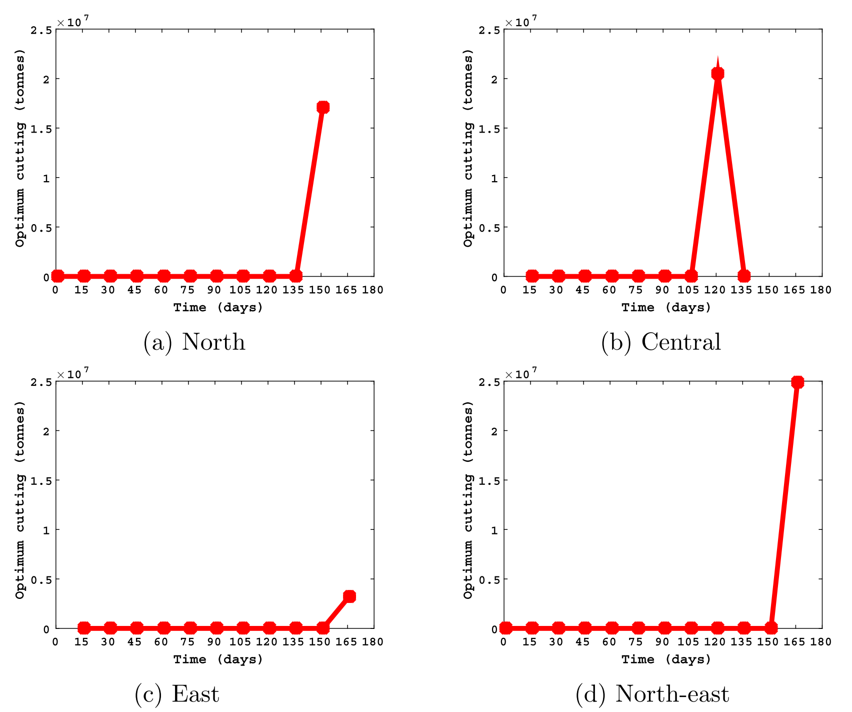

Figure 2 Optimum cutting of fired sugarcane in each region for year 2013/14 (quasi-Newton, mcf = 0). It is assumed that all sugarcane in each region can be cut in one 15-day period

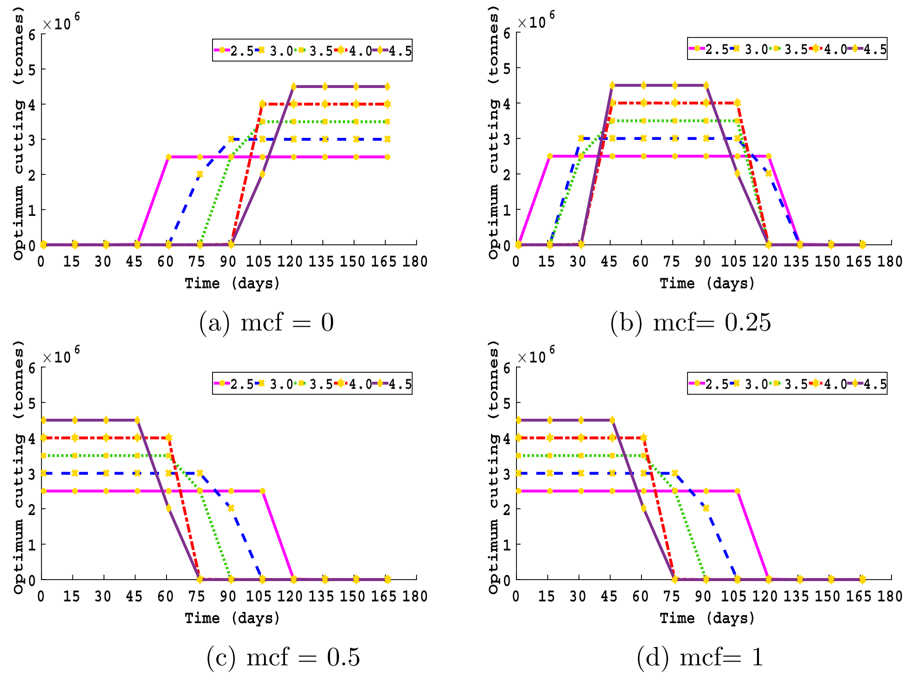

Figure 3 Optimum cutting of fresh sugarcane in the north-east region in year 2013/14 (quasi-Newton) showing effects of changing upper bound on cutting per period and of reducing the maintenance cost by fraction mcf of the actual cost

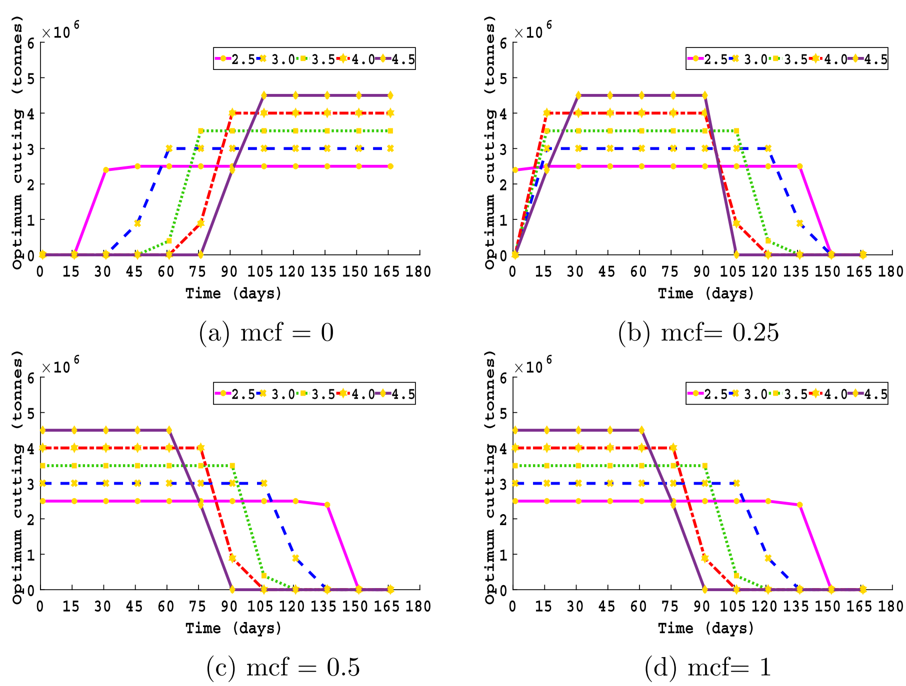

Figure 4 Optimum cutting of fired sugarcane in the north-east region in year 2013/14 (quasi-Newton) showing effects of changing upper bound on cutting per period and of reducing the maintenance cost by fraction mcf of the actual cost

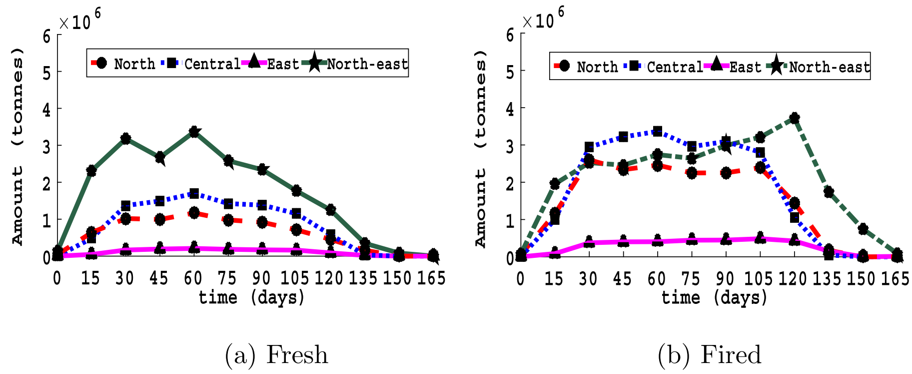

Figure 5 Actual amounts of fresh and fired sugarcane cut in each region for year 2013/14

to cut as early as possible. Similar results have been obtained for all regions for the three crop years.

7.3 Comparison of results from models with actual harvesting policies

The actual harvesting patterns for fresh and fired sugarcane are shown in Figs. 5(a) and (b) for the four regions of Thailand for the crop year 2013/14. A comparison of these actual cutting patterns with the results in Figs. 3(b) and 4(b) show that the actual cutting patterns are approximately the same for a maximum allowed cutting of 3.5 million tonnes per period and a maintenance cost of approximately one quarter (mcf = 0.25) of the cost estimated in the Mathematical Models section from the available OCSB data.

8 Conclusion

In this paper, optimization models have been developed to obtain optimal harvesting policies for fresh and fired sugarcane in the four main sugar-producing regions of Thailand (north, central, east, and north-east) for crop years 2012/13, 2013/14, and 2014/15. Tables 11–13 show a comparison between the actual profits reported by the Office of the Cane and Sugar Board [14–16] and the optimal profits computed from the optimization models.

As shown in Tables 11 and 12, the optimal profits computed from the bi-objective optimization model (fourth column) and quasi-Newton optimization models (fifth column) are greater than the actual profits for both fresh and fired sugarcane in all four regions in crop years 2012/13 and 2013/14. As shown in Table 13, there was an actual loss (negative profit) from sugarcane production in all regions for all regions in crop year 2014/15. For 2014/15, the bi-objective model had no feasible solutions for the north and east regions for any value of ε because the costs of cutting exceeded the revenue for all 15-day cutting periods. It can also be seen from Tables 11 and 12 that the optimal profit estimates from the quasi-Newton method and the bi-objective are approximately the same for fresh and fired sugarcane for all regions and years. These results suggest that optimization of a single objective, in this case the profit, is a more effective method of optimization for the sugar cane harvesting model than the bi-objective method of maximizing the revenue subject to upper bound constraints on the cost.

Table 14 shows the changes in optimal profit computed by the quasi-Newton method as the upper bound on the amount cut per period is varied with the maintenance cost fixed

Table 11 Comparison of actual profits, optimal total profits, and optimal selling profits ($\times 10^9$ baht) of sugarcane harvested in each region for year 2012/13 (mcf = 0)

Region	Type	Actual profit	Total profit (bi-objective)	Selling profit (quasi-Newton)	Total profit (quasi-Newton)
North	Fresh	0.571	1.438	7.940	1.437
	Fired	0.602	2.561	18.790	2.559
Central	Fresh	0.717	2.122	10.234	2.108
	Fired	0.451	3.589	22.054	3.558
East	Fresh	0.121	0.240	1.040	0.241
	Fired	0.262	0.721	3.845	0.725
North-east	Fresh	5.746	7.634	20.593	7.655
	Fired	6.851	9.162	27.027	9.190

Table 12 Comparison of actual profits, optimal total profits, and optimal selling profits ($\times 10^9$ baht) of sugarcane harvested in each region for year 2013/14 (mcf = 0)

Region	Type	Actual profit	Total profit (bi-objective)	Selling profit (quasi-Newton)	Total profit (quasi-Newton)
North	Fresh	1.241	2.160	7.924	2.151
	Fired	2.494	4.493	18.507	4.528
Central	Fresh	1.927	3.235	10.729	3.228
	Fired	3.213	5.782	21.769	5.768
East	Fresh	0.058	0.253	1.382	0.255
	Fired	0.062	0.544	3.509	0.555
North-east	Fresh	3.696	6.084	23.655	6.128
	Fired	3.738	6.172	27.972	6.172

Table 13 Comparison of actual profits, optimal total profits, and optimal selling profits ($\times 10^9$ baht) of sugarcane harvested in each region for year 2014/15 (mcf = 0)

Region	Type	Actual profit	Total profit (bi-objective)	Selling profit (quasi-Newton)	Total profit (quasi-Newton)
North	Fresh	–1.358	–	–	–
	Fired	–4.255	–	–	–
Central	Fresh	–0.583	0.633	9.437	0.626
	Fired	–2.281	0.210	18.099	0.195
East	Fresh	–0.219	–	–	–
	Fired	–0.889	–	–	–
North-east	Fresh	–0.355	2.451	21.241	2.461
	Fired	–1.651	2.024	30.007	2.039

Note: The – means that there is no optimal solution as profit is negative.

at one half of the actual maintenance cost (mcf = 0.5). It can be seen that the optimal profit increases as the upper bound on the amount cut per period is increased.

The results presented in this paper clearly need further development before the model can be useful to farmers in planning their harvesting policies. One important problem with the present model is that there are constraints on cutting that have not been included, for example, constraints imposed by the factories or by the availability of workers.

Table 14 Changes in optimal profit (quasi-Newton) as upper bound on cutting per period is changed for fresh and fired sugarcane in the north-east region in year 2013/14 (mcf = 0.5)

Maximum cutting (tonne per period)	Optimal total profit (10^9 baht)	
	Fresh	Fired
2,500,000	2.375	1.320
3,000,000	2.549	1.701
3,500,000	2.655	1.941
4,000,000	2.725	2.095
4,500,000	2.757	2.202

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