Suthar et al. *Advances in Difference Equations* (2019) 2019:213
https://doi.org/10.1186/s13662-019-2150-0

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Modified Saigo fractional integral operators involving multivariable H-function and general class of multivariable polynomials

D.L. Suthar[1]*, Belete Debalkie[1] and Mitku Andualem[1]

*Correspondence:
dlsuthar@gmail.com
[1]Department of Mathematics, College of Natural Sciences, Wollo University, Dessie, Ethiopia

Abstract

In this paper, we establish modified Saigo fractional integral operators involving the product of a general class of multivariable polynomials and the multivariable H-function. The results established here are of general nature and provide extension of some results obtained recently by Saxena et al.

MSC: Primary 26A33; secondary 26C05

Keywords: Multivariable H-function; General class of multivariable polynomial; Modified Saigo operators

1 Introduction and preliminaries

The multivariable H-function is defined and studied by Srivastava and Panda ([1], p. 271, Eq. (4.1)) in terms of Mellin–Barnes type contour integral as follows:

$$H[z_1,\ldots,z_r] = H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array}\middle| \begin{array}{l} (a_j;\alpha'_j,\ldots,\alpha^{(r)}_j)_{1,p}: \\ (b_j;\beta'_j,\ldots,\beta^{(r)}_j)_{1,q}: \end{array}\right.$$
$$\left.\begin{array}{l} (c'_j,\gamma'_j)_{1,p_1};\ldots;(c^{(r)}_j,\gamma^{(r)}_j)_{1,p_r} \\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d^{(r)}_j,\delta^{(r)}_j)_{1,q_r} \end{array}\right] \tag{1}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(\xi_1,\ldots,\xi_r)\left\{\prod_{i=1}^{r}\theta_i(\xi_i)z_i^{\xi_i}\right\}d\xi_1\cdots d\xi_r, \tag{2}$$

where $\omega=\sqrt{-1}$; and

$$\phi(\xi_1,\ldots,\xi_r) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\sum_{i=1}^{r}\alpha^{(i)}_j\xi_i)}{\prod_{j=n+1}^{p}\Gamma(a_j-\sum_{i=1}^{r}\alpha^{(i)}_j\xi_i)\prod_{j=1}^{n}\Gamma(1-b_j+\sum_{i=1}^{r}\beta^{(i)}_j\xi_i)}, \tag{3}$$

$$\theta_i(\xi_i) = \frac{\prod_{j=1}^{n_i}\Gamma(1-c^{(i)}_j+\gamma^{(i)}_j\xi_i+\sum_{i=1}^{r}\alpha^{(i)}_j\xi_i)\prod_{j=1}^{m_i}\Gamma(d^{(i)}_j-\delta^{(i)}_j\xi_i)}{\prod_{j=n_i+1}^{p_i}\Gamma(c^{(i)}_j-\gamma^{(i)}_j\xi_i)\prod_{j=m_i+1}^{q_i}\Gamma(1-d^{(i)}_j+\delta^{(i)}_j\xi_i)}, \tag{4}$$

and $L_j = L_{\omega\tau_j\infty}$ represents the contours which start at the point $\tau_j - \omega\infty$ and terminate at the points $\tau_j + \omega\infty$ with $\tau_j \in \Re = (-\infty, \infty)$ $(j = 1, \ldots, r)$.

In the case $r = 2$, (1) reduces to the H-function of two variables. For a detailed definition and convergence conditions of the multivariable H-function, the reader is referred to the original papers [2–9]. From Srivastava and Panda ([10], p. 131), we have

$$H[z_1, \ldots, z_r] = \mathrm{O}\bigl(|z_1|^{e_1} \cdots |z_r|^{e_r}\bigr)\Bigl(\max_{1\le j\le r} \|z_j\| \to 0\Bigr), \tag{5}$$

where

$$e_i = \min_{1\le j\le r}\left[\frac{\mathrm{Re}(d_j^{(i)})}{\delta_j^{(i)}}\right] \quad (i = 1, \ldots, r). \tag{6}$$

For $n = p = q = 0$, the multivariable H-function breaks up into product of 'r' H-function; consequently, there holds the following results:

$$\begin{aligned} &H^{0,0:m_1,n_1;\ldots;m_r,n_r}_{0,0:p_1,q_1;\ldots;p_r,q_r}\left[\begin{array}{c|c} z_1 & \\ \vdots & (c'_j, \gamma'_j)_{1,p_1}; \ldots; (c^{(r)}_j, \gamma^{(r)}_j)_{1,p_r} \\ & (d'_j, \delta'_j)_{1,q_1}; \ldots; (d^{(r)}_j, \delta^{(r)}_j)_{1,q_r} \\ z_r & \end{array}\right] \\ &\quad = \prod_{i=1}^{r} H^{m_i,n_i}_{p_i,q_i}\left[z \,\middle|\, \begin{array}{c} (c^{(i)}_j, \gamma^{(i)}_j)_{1,p_i} \\ (d^{(i)}_j, \delta^{(i)}_j)_{1,q_i} \end{array}\right], \end{aligned} \tag{7}$$

where $H^{m,n}_{p,q}(\cdot)$ is the familiar H-function.

In the sequel, Srivastava and Garg ([11], p. 686, Eq. (1.4)) gave the definition of multivariable generalization of the polynomials $S^m_n(x)$ as follows:

$$\begin{aligned} S^{h_1,\ldots,h_s}_L(x_1, \ldots, x_s) = {} & \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} (-L)_{h_1k_1+\cdots+h_sk_s} \\ & \times A(L; k_1, \ldots, k_s)\frac{x_1^{k_1}}{k_1!} \cdots \frac{x_s^{k_s}}{k_s!}, \end{aligned} \tag{8}$$

where the coefficients $A(L; h_1, \ldots, h_s)$ $(L, h_i \in N_0,\ i = 1, \ldots, s)$ are arbitrary. Choosing constants to be real or complex, as Srivastava [12] defined by $s = 1$ on the above polynomial, we obtain a polynomial of the form $S^m_n(x)$.

Let α', β', η' be complex numbers and $\theta > 0$. The modified Saigo integral operators are denoted by $I^{\alpha',\beta',\eta'}_{0,x,\theta}$ and $J^{\alpha',\beta',\eta'}_{x,\infty,\theta}$ respectively for $\Re(\alpha') > 0$:

$$\begin{aligned} I^{\alpha',\beta',\eta'}_{0,x,\theta} f = {} & \frac{\theta x^{-\theta(\alpha'+\beta')}}{\Gamma(\alpha')}\int_0^x \bigl(x^\theta - t^\theta\bigr)^{\alpha'-1} \\ & \times {}_2F_1\bigl(\alpha' + \beta', -\eta'; \alpha'; 1 - t^\theta/x^\theta\bigr)t^{\theta-1}f(t)\,dt \end{aligned} \tag{9}$$

$$= \frac{d^n}{d(x^\theta)^n} I^{\alpha'+n,\beta'-n,\eta'-n}_{0,x,\theta} f, \quad 0 < \Re\bigl(\alpha'\bigr) + n \leqq 1, \tag{10}$$

$$J^{\alpha',\beta',\eta'}_{x,\infty,\theta} f = \frac{\theta}{\Gamma(\alpha')}\int_x^\infty \bigl(t^\theta - x^\theta\bigr)^{\alpha'-1} t^{-\theta(\alpha'+\beta')}$$

$$\times {}_2F_1\left(\alpha' + \beta', -\eta'; \alpha'; 1 - x^\theta/t^\theta\right) t^{\theta-1} f(t)\, dt \tag{11}$$

$$= (-1)^n \frac{d^n}{d(x^\theta)^n} J_{x,\infty,\theta}^{\alpha'+n,\beta'-n,\eta'-n} f, \quad 0 < \Re\left(\alpha'\right) + n \leqq 1. \tag{12}$$

Sufficient conditions for the existence of (9) and (11) are

$$\theta > 0, \qquad \Re\left(\alpha'\right) > 1 - 1/2\theta; \qquad f(x) \in L_2(\Re_+) \tag{13}$$

and $\max[0, \Re(\beta' - \eta')] > 1 - 1/2\theta$; $\min[\Re(\beta'), \Re(\eta')] > -1/2\theta$. If these conditions are satisfied, then $I_{0,x,\theta}^{\alpha',\beta',\eta'} f(x)$, $J_{x,\infty,\theta}^{\alpha',\beta',\eta'} f(x)$ both exist and both $\in L_2(\Re_+)$.

The operators $I_{0,x,\theta}^{\alpha',\beta',\eta'}$ and $J_{x,\infty,\theta}^{\alpha',\beta',\eta'}$ include as their special case, $\beta' = -\alpha'$, the fractional calculus operators of Riemann–Liouville and Weyl types:

$$I_{0,x,\theta}^{\alpha',-\alpha',\eta'} f = R_{0,x,\theta}^{\alpha'} f, \qquad J_{x,\infty,\theta}^{\alpha',-\alpha',\eta'} f = W_{x,\infty,\theta}^{\alpha'} f. \tag{14}$$

Also, we obtain the following identities and inverses:

$$I_{0,x,\theta}^{0,0,\eta'} f = f(x); \qquad J_{x,\infty,\theta}^{0,0,\eta'} f = f(x). \tag{15}$$

$$\left[I_{0,x,\theta}^{\alpha',\beta',\eta'}\right]^{-1} = I_{0,x,\theta}^{-\alpha',-\beta',\alpha'+\eta'}; \qquad \left[J_{x,\infty,\theta}^{\alpha',\beta',\eta'}\right]^{-1} = J_{x,\infty,\theta}^{-\alpha',-\beta',\alpha'+\eta'}. \tag{16}$$

For the operators $I_{0,x,\theta}^{\alpha',\beta',\eta'}$ and $J_{x,\infty,\theta}^{\alpha',\beta',\eta'}$ there holds interesting results similar to the ones derived in a series of earlier papers [13–19].

In this paper, we shall study another generalization of (9) and (11) which is given in the following manner:

$$\begin{aligned} &I_{0,x,\theta;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}\left\{f(x)\right\} \\ &\quad = \frac{\theta x^{-\theta(\alpha'+\beta')}}{\Gamma(\alpha')} \int_0^x \left(x^\theta - t^\theta\right)^{\alpha'-1} {}_2F_1\left(\alpha' + \beta', -\eta'; \alpha'; 1 - t^\theta/x^\theta\right) t^{\theta-1} \\ &\qquad \times \Im_n^{\alpha,\beta,\tau}\left[zt^\rho; r, \varepsilon, q, C, D, m, k, l\right] f(t)\, dt, \end{aligned} \tag{17}$$

and

$$\begin{aligned} &J_{x,\infty,\theta;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}\left\{f(x)\right\} \\ &\quad = \frac{\theta}{\Gamma(\alpha')} \int_x^\infty \left(t^\theta - x^\theta\right)^{\alpha'-1} t^{-\theta(\alpha'+\beta')} {}_2F_1\left(\alpha' + \beta', -\eta'; \alpha'; 1 - x^\theta/t^\theta\right) t^{\theta-1} \\ &\qquad \times \Im_n^{\alpha,\beta,\tau}\left[zt^\rho; r, \varepsilon, q, C, D, m, k, l\right] f(t)\, dt, \end{aligned} \tag{18}$$

where $\Re(\alpha') > 0$, and $\Im_n^{\alpha,\beta,\tau}[z]$ stands for the generalized polynomial set defined by the following Rodrigues type formula ([20], p. 64, Eq. (2.18)):

$$\begin{aligned} &\Im_n^{\alpha,\beta,\tau}[x; r, \varepsilon, q, C, D, m, k, l] \\ &\quad = (Cx + D)^{-\alpha}\left(1 - \tau x^r\right)^{\frac{-\beta}{\tau}} T_{k,l}^{m+n}\left[(Cx + D)^{\alpha+qn}\left(1 - \tau x^r\right)^{\frac{\beta}{\tau+\varepsilon n}}\right], \end{aligned} \tag{19}$$

with the differential operator $T_{k,l}$ being defined as

$$T_{k,l} \equiv x^l\left(k + x\frac{d}{dx}\right). \tag{20}$$

An explicit form of this generalized polynomial set ([20], p. 71, Eq. (2.34)) is given by

$$\begin{aligned}
&\Im_n^{\alpha,\beta,\tau}[x;r,\varepsilon,q,C,D,m,k,l] \\
&\quad = D^{qn}x^{l(m+n)}\left(1-\tau x^r\right)^{\varepsilon n} l^{m+n} \\
&\qquad \times \sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\frac{(-1)^j(-j_i)(\alpha)_j(-\upsilon)_u(-\alpha-qn)_i}{i!j!u!\upsilon!(1-\alpha-j)_i} \\
&\qquad \times \left(-\frac{\beta}{\tau}-\varepsilon n\right)_\upsilon\left(\frac{i+k+ru}{l}\right)_{m+n}\left(\frac{-\tau x^r}{1-\tau x^r}\right)^\upsilon\left(\frac{Cx}{D}\right)^j.
\end{aligned} \tag{21}$$

It may be noted that the polynomial set defined by (19) is of general character and unifies and extends a number of classical polynomials introduced and studied by various authors (see [21–26]). Two special cases of (17) are given below ([20], p. 65).

1. If we set $C = 1$, $D = 0$ in (19), it gives

$$\begin{aligned}
&\Im_n^{\alpha,\beta,\tau}[x;r,\varepsilon,q,1,0,m,k,l] \\
&\quad = x^{qn+l(m+n)}\left(1-\tau x^r\right)^{\varepsilon n} l^{m+n} \\
&\qquad \times \sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\frac{(-\upsilon)_u}{u!\upsilon!}\left(-\frac{\beta}{\tau}-\varepsilon n\right)_\upsilon\left(\frac{\alpha+qn+k+ru}{l}\right)_{m+n}\left(\frac{-\tau x^r}{1-\tau x^r}\right)^\upsilon.
\end{aligned} \tag{22}$$

2. As $\tau \to 0$ in (21), by virtue of the well-known confluence principle $\lim_{|b|\to\infty}(b_n)(\frac{z}{b})^n = z^n$, it yields the following polynomial set:

$$\begin{aligned}
&\Im_n^{\alpha,\beta,0}[x;r,\varepsilon,q,1,0,m,k,l] \\
&\quad = x^{qn+l(m+n)} l^{m+n}\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\frac{(-\upsilon)_u}{u!\upsilon!}\left(\frac{\alpha+qn+k+ru}{l}\right)_{m+n}\left(\beta x^r\right)^\upsilon.
\end{aligned} \tag{23}$$

2 Main results

It will be shown here that

(I)

$$\begin{aligned}
&I_{0,x,\theta;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}\left\{t^\lambda S_L^{h_1,\ldots,h_s}\left(y_1t^{\eta_1},\ldots,y_st^{\eta_s}\right)H\left[a_1t^{\zeta_1},\ldots,a_rt^{\zeta_r}\right]\right\} \\
&\quad = \sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j} \\
&\qquad \times \sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L}\sum_{w=0}^{\infty}\frac{(-L)_{h_1k_1+\cdots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{k_1!\cdots k_s!w!}A(L;k_1,\ldots,k_s) \\
&\qquad \times y_1^{k_1}\cdots y_s^{k_s}x^{\lambda+G+\rho[l(m+n)+r\upsilon+rw+j]-\theta\beta'}H_{p+2,q+2:p_1,q_1;\ldots,p_r,q_r}^{0,n+2:m_1,n_1;\ldots,m_r,n_r}\left[\begin{array}{c} a_1x^{\zeta_1} \\ \vdots \\ a_rx^{\zeta_r} \end{array}\right|
\end{aligned}$$

$$\begin{aligned}&(1-\tfrac{\ell}{\theta},\tfrac{\zeta_1}{\theta},\ldots,\tfrac{\zeta_r}{\theta}),(1-\tfrac{\ell}{\theta}-\eta'+\beta',\tfrac{\zeta_1}{\theta},\ldots,\tfrac{\zeta_r}{\theta}),\\ &(1-\tfrac{\ell}{\theta}+\beta',\tfrac{\zeta_1}{\theta},\ldots,\tfrac{\zeta_r}{\theta}),(1-\tfrac{\ell}{\theta}-\eta'-\alpha',\tfrac{\zeta_1}{\theta},\ldots,\tfrac{\zeta_r}{\theta}),\\ &\left.\begin{matrix}(a_j;\alpha_j',\ldots,\alpha_j^{(r)})_{1,p}:(c_j',\gamma_j')_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j',\ldots,\beta_j^{(r)})_{1,q}:(d_j',\delta_j')_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right], \end{aligned}\tag{24}$$

where

$$\begin{aligned}\Omega(i,j,u,v)={}&D^{qn}l^{m+n}\frac{(-1)^j(-j_i)(\alpha)_j(-\upsilon)_u(-\alpha-qn)_i}{i!j!u!v!(1-\alpha-j)_i}\\ &\times\left(-\frac{\beta}{\tau}-\varepsilon n\right)_\upsilon\left(\frac{i+k+ru}{l}\right)_{m+n}\left(\frac{C}{D}\right)^j(-\tau)^\upsilon,\end{aligned}\tag{25}$$

$$\ell=\lambda+\theta+G+\rho l(m+n)+\rho r\upsilon+\rho rw+\rho j.\tag{26}$$

Proof In view of definition (17) and by using the general binomial theorem, we expand the term

$$(\alpha-\beta x)^{-\omega}=\alpha^{-\omega}\sum_{w=0}^{\infty}\frac{(\omega)_w}{w!}\left(\frac{\beta x}{\alpha}\right)^w$$

for ($|\frac{\beta x}{\alpha}|<1$) and the L.H.S. of (24)

$$\begin{aligned}&=\frac{\theta x^{-\theta(\alpha'+\beta')}}{\Gamma(\alpha')}\int_0^x\left(x^\theta-t^\theta\right)^{\alpha'-1}t^{\lambda+\theta-1}{}_2F_1\left(\alpha'+\beta',-\eta';\alpha';1-t^\theta/x^\theta\right)\\ &\quad\times\Im_n^{\alpha,\beta,\tau}\left[zt^\rho;r,\varepsilon,q,C,D,m,k,l\right]S_L^{h_1,\ldots,h_s}\left(y_1t^{\eta_1},\ldots,y_st^{\eta_s}\right)H\left[a_1t^{\zeta_1},\ldots,a_rt^{\zeta_r}\right]dt,\end{aligned}\tag{27}$$

using (21), (8), and (2), it is found that the L.H.S. of (24)

$$\begin{aligned}&=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j}\\ &\quad\times\sum_{k_1,\ldots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L}\sum_{w=0}^{\infty}\frac{(-L)_{h_1k_1+\cdots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^wz^{rw}}{k_1!\cdots k_s!w!}A(L;k_1,\ldots,k_s)\\ &\quad\times y_1^{k_1}\cdots y_s^{k_s}\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(\xi_1,\ldots,\xi_r)\left\{\prod_{i=1}^r\theta_i(\xi_i)a_i^{\xi_i}\right\}d\xi_1\cdots d\xi_r\\ &\quad\times\frac{\theta x^{-\theta(\alpha'+\beta')}}{\Gamma(\alpha')}\int_0^x\left(x^\theta-t^\theta\right)^{\alpha'-1}t^{\lambda+\theta+G+\rho l(m+n)+\rho rv+\rho rw+\rho j+L-1}\\ &\quad\times{}_2F_1\left(\alpha'+\beta',-\eta';\alpha';1-t^\theta/x^\theta\right)dt,\end{aligned}\tag{28}$$

where $G=\sum_{i=1}^s\eta_ik_i,L=\sum_{i=1}^r\zeta_i\xi_i$, $\Omega(i,j,u,v)$ and $\phi(\xi_1,\ldots,\xi_r)$ are defined by (25) and (3), respectively. □

Applying the following result given by Saigo and Saxena ([27], p. 57, Eq. (4.16))

$$\begin{aligned} &A\int_0^x u^{\rho-1}\left(x^A-u^A\right)^{\alpha-1}{}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{u^A}{x^A}\right)du \\ &\quad=\frac{\Gamma(\alpha)\Gamma(\frac{\rho}{A})\Gamma(\frac{\rho}{A}+\eta-\beta)}{\Gamma(\frac{\rho}{A}-\beta)\Gamma(\frac{\rho}{A}+\eta+\alpha)}x^{\alpha A+\rho-A}, \end{aligned} \tag{29}$$

where $\Re(\alpha)>0$, $\Re(\rho)>0$, $\Re((\rho/A)+\eta-\beta)>0$, $A>0$ in (28) and interchanging the order of integration and summation, we obtain (24).

Next, we prove that

(II)

$$\begin{aligned} &J^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}_{x,\infty,\theta;r,\varepsilon,q;m,k,l}\left\{t^{\lambda}S_L^{h_1,\dots,h_s}\left(y_1t^{\eta_1},\dots,y_st^{\eta_s}\right)H\left[a_1t^{\zeta_1},\dots,a_rt^{\zeta_r}\right]\right\} \\ &\quad=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j} \\ &\qquad\times\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}\sum_{w=0}^{\infty}\frac{(-L)_{h_1k_1+\dots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{k_1!\cdots k_s!w!}A(L;k_1,\dots,k_s) \\ &\qquad\times y_1^{k_1}\cdots y_s^{k_s}x^{\lambda+G+\rho[l(m+n)+r\upsilon+rw+j]-\theta\beta'}H^{0,n+2:m_1,n_1;\dots;m_r,n_r}_{p+2,q+2:p_1,q_1;\dots;p_r,q_r}\left[\begin{array}{c} a_1t^{\xi_1} \\ \vdots \\ a_rt^{\xi_r}\end{array}\right| \\ &\qquad\begin{array}{l}(1-\alpha'-\beta'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(1+\eta'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\\ (1-\alpha'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(1-\alpha'-\beta'+\eta'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\end{array} \\ &\qquad\left.\begin{array}{l}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{array}\right], \end{aligned} \tag{30}$$

where

$$\ell'=\lambda+\theta+G+\rho l(m+n)+\rho r v+\rho r w+\rho j-\theta\alpha'-\theta\beta'; \tag{31}$$

$\Omega(i,j,u,v)$ and $S_L^{h_1,\dots,h_s}(x)$ are defined in (25) and (8).

Proof In view of definition (18), the L.H.S. of (30)

$$\begin{aligned} &=\frac{\theta}{\Gamma(\alpha')}\int_x^{\infty}\left(t^{\theta}-x^{\theta}\right)^{\alpha'-1}t^{\lambda-\theta(\alpha'+\beta')+\theta-1}{}_2F_1\left(\alpha'+\beta',-\eta';\alpha';1-x^{\theta}/t^{\theta}\right) \\ &\quad\times\Im_n^{\alpha,\beta,\tau}\left[zt^{\rho};r,\varepsilon,q,C,D,m,k,l\right]S_L^{h_1,\dots,h_s}\left(y_1t^{\eta_1},\dots,y_st^{\eta_s}\right)H\left[a_1t^{\zeta_1},\dots,a_rt^{\zeta_r}\right]dt. \end{aligned} \tag{32}$$

If we apply (21), (8), and (2) in the above term, we get

$$
\begin{aligned}
&= \sum_{\upsilon=0}^{m+n} \sum_{u=0}^{\upsilon} \sum_{j=0}^{m+n} \sum_{i=0}^{j} \Omega(i,j,u,v) z^{l(m+n)+r\upsilon+j} \\
&\quad \times \sum_{k_1,\dots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} \sum_{w=0}^{\infty} \frac{(-L)_{h_1k_1+\cdots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{k_1!\cdots k_s! w!} A(L;k_1,\dots,k_s) y_1^{k_1}\cdots y_s^{k_s} \\
&\quad \times \frac{1}{(2\pi\omega)^r} \int_{L_1}\cdots\int_{L_r} \phi(\xi_1,\dots,\xi_r) \left\{ \prod_{i=1}^{r} \theta_i(\xi_i) a_i^{\xi_i} \right\} d\xi_1\dots d\xi_r \\
&\quad \times \frac{\theta}{\Gamma(\alpha')} \int_x^{\infty} \left(t^\theta - x^\theta\right)^{\alpha'-1} t^{\lambda+\theta+G+\rho l(m+n)+\rho r v+\rho j-\theta(\alpha'+\beta')+H-1} \\
&\quad \times {}_2F_1\left(\alpha'+\beta', -\eta'; \alpha'; 1-x^\theta/t^\theta\right) dt.
\end{aligned}
\tag{33}
$$

Now, by applying the integral given by Saigo and Saxena ([27], p. 57, Eq. (4.17))

$$
\begin{aligned}
&A \int_x^{\infty} u^{\rho-1}\left(u^A - x^A\right)^{\alpha-1} {}_2F_1\left(\alpha+\beta, -\eta; \alpha; 1-\frac{x^A}{u^A}\right) du \\
&= \frac{\Gamma(\alpha)\Gamma(1-\alpha-\frac{\rho}{A})\Gamma(1-\alpha-\beta+\eta-\frac{\rho}{A})}{\Gamma(1-\alpha-\beta-\frac{\rho}{A})\Gamma(1+\eta-\frac{\rho}{A})} x^{\alpha A+\rho-A},
\end{aligned}
\tag{34}
$$

where $\Re(\alpha) > 0$, $\Re(1-\alpha-\rho/A) > 0$, $\Re(1-\alpha-\beta+\eta-\rho/A) > 0$, $A > 0$ in (33) and interchanging the order of integration and summation, we arrive at the result (30). □

3 Special cases

(i) If we use the identity $I_{0,x,\theta}^{\alpha',-\alpha',\eta'} f = R_{0,x,\theta}^{\alpha'} f$ with $\theta = 1$ in (24), we find that

$$
\begin{aligned}
&R_{0,x,1;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha'} \left\{ t^\lambda S_L^{h_1,\dots,h_s}\left(y_1 t^{\eta_1},\dots,y_s t^{\eta_s}\right) H\left[a_1 t^{\zeta_1},\dots,a_r t^{\zeta_r}\right] \right\} \\
&\quad = \sum_{\upsilon=0}^{m+n} \sum_{u=0}^{\upsilon} \sum_{j=0}^{m+n} \sum_{i=0}^{j} \Omega(i,j,u,v) z^{l(m+n)+r\upsilon+j} \\
&\quad \times \sum_{k_1,\dots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} \sum_{w=0}^{\infty} \frac{(-L)_{h_1k_1+\cdots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{k_1!\cdots k_s! w!} \\
&\quad \times A(L;k_1,\dots,k_s) y_1^{k_1}\cdots y_s^{k_s} x^{\lambda+G+\rho[l(m+n)+r\upsilon+rw+j]-\alpha'} \\
&\quad \times H_{p+1,q+1:p_1,q_1;\dots;p_r,q_r}^{0,n+1:m_1,n_1;\dots;m_r,n_r} \left[\begin{matrix} a_1 x^{\zeta_1} \\ \vdots \\ a_r x^{\zeta_r} \end{matrix} \middle| \begin{matrix} (1-\Lambda,\zeta_1,\dots,\zeta_r), \\ (1-\Lambda-\alpha',\zeta_1,\dots,\zeta_r), \end{matrix} \right. \\
&\quad \left. \begin{matrix} (a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p} : (c_j',\gamma_j')_{1,q_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta_j',\dots,\beta_j^{(r)})_{1,p} : (d_j',\delta_j')_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r} \end{matrix} \right],
\end{aligned}
\tag{35}
$$

where $\Lambda = \lambda + G + \rho l(m+n) + \rho r\upsilon + \rho rw + \rho j + 1$.

(ii) The formula $J_{x,\infty,\theta}^{\alpha',-\alpha',\eta'}f = W_{x,\infty,\theta}^{\alpha'}f$ with $\theta = 1$, when used in (30), gives

$$\begin{aligned}
&W_{x,\infty,1;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha'}\{t^{\lambda}S_L^{h_1,\dots,h_s}(y_1t^{\eta_1},\dots,y_st^{\eta_s})H[a_1t^{\zeta_1},\dots,a_rt^{\zeta_r}]\}\\
&\quad=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j}\\
&\qquad\times\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}\sum_{w=0}^{\infty}\frac{(-L)_{h_1k_1+\dots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^wz^{rw}}{k_1!\cdots k_s!w!}\\
&\qquad\times A(L;k_1,\dots,k_s)y_1^{k_1}\cdots y_s^{k_s}x^{\lambda+G+\rho[l(m+n)+r\upsilon+rw+j]+\alpha'}\\
&\qquad\times H_{p+1,q+1:p_1,q_1;\dots;p_r,q_r}^{0,n+1:m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix}a_1t^{\xi_1}\\ \vdots\\ a_rt^{\xi_r}\end{matrix}\,\middle|\,\begin{matrix}(1-\Lambda',\zeta_1,\dots,\zeta_r),\\(1-\alpha'-\Lambda',\zeta_1,\dots,\zeta_r),\end{matrix}\right.\\
&\qquad\left.\begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j',\gamma_j')_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\(b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j',\delta_j')_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right],
\end{aligned}\tag{36}$$

where $\Lambda' = \lambda + G + \rho l(m+n) + \rho r\upsilon + \rho rw + \rho j + 1$.

(iii) If we take $n = p = q = 0$ in (24) and (30) with respect to H-function respectively, we obtain two fractional integral formulas involving product of the r, H-functions stated as follows:

$$\begin{aligned}
&I_{0,x,\theta;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}\left\{t^{\lambda}S_L^{h_1,\dots,h_s}(y_1t^{\eta_1},\dots,y_st^{\eta_s})\prod_{i=1}^{r}H_{p_i,q_i}^{m_i,n_i}\left[a_it^{\zeta_i}\,\middle|\,\begin{matrix}(c_j^{(i)},\gamma_j^{(i)})_{1,p_i}\\(d_j^{(i)},\delta_j^{(i)})_{1,q_i}\end{matrix}\right]\right\}\\
&\quad=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j}\\
&\qquad\times\sum_{k_1,\dots,k_s=0}^{h_1k_1+\dots+h_sk_s\le L}\sum_{w=0}^{\infty}\frac{(-L)_{h_1k_1+\dots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^wz^{rw}}{k_1!\cdots k_s!w!}\\
&\qquad\times A(L;k_1,\dots,k_s)y_1^{k_1}\cdots y_s^{k_s}x^{\lambda+G+\rho[l(m+n)+r\upsilon+rw+j]-\theta\beta'}\\
&\qquad\times H_{2,2:p_1,q_1;\dots;p_r,q_r}^{0,2:m_1,n_1;\dots;m_r,n_r}\left[a_ix^{\zeta_i}\,\middle|\,\begin{matrix}(1-\frac{\ell}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\\(1-\frac{\ell}{\theta}+\beta',\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\end{matrix}\right.\\
&\qquad\left.\begin{matrix}(1-\frac{\ell}{\theta}-\eta'+\beta',\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(c_j^{(1)},\gamma_j^{(1)})_{1,p_i}\\(1-\frac{\ell}{\theta}-\eta'-\alpha',\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(d_j^{(1)},\delta_j^{(1)})_{1,q_i}\end{matrix}\right];
\end{aligned}\tag{37}$$

and

$$\begin{aligned}
&J_{x,\infty,\theta;r,\varepsilon,q;m,k,l}^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}\left\{t^{\lambda}S_L^{h_1,\dots,h_s}(y_1t^{\eta_1},\dots,y_st^{\eta_s})\prod_{i=1}^{r}H_{p_i,q_i}^{m_i,n_i}\left[a_it^{\zeta_i}\,\middle|\,\begin{matrix}(c_j^{(i)},\gamma_j^{(i)})_{1,p_i}\\(d_j^{(i)},\delta_j^{(i)})_{1,q_i}\end{matrix}\right]\right\}\\
&\quad=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j}
\end{aligned}$$

$$\times \sum_{k_1,\dots,k_s=0}^{h_1k_1+\cdots+h_sk_s\le L} \sum_{w=0}^{\infty} \frac{(-L)_{h_1k_1+\cdots+h_sk_s}(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{k_1!\cdots k_s!w!}$$
$$\times A(L;k_1,\dots,k_s)y_1^{k_1}\cdots y_s^{k_s}x^{\lambda+G+\rho[l(m+n)+r\upsilon+rw+j]-\theta\beta'}$$
$$\times H^{0,2:m_1,n_1;\dots,m_r,n_r}_{2,2:p_1,q_1;\dots,p_r,q_r}\left[a_i x^{\zeta_i}\,\middle|\, \begin{matrix}(1-\alpha'-\beta'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\\ (1-\alpha'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\end{matrix}\right.$$
$$\left.\begin{matrix}(1+\eta'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(c_j^{(1)},\gamma_j^{(1)})_{1,p_r}\\ (1-\alpha'-\beta'+\eta'-\frac{\ell'}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(d_j^{(1)},\delta_j^{(1)})_{1,q_r}\end{matrix}\right]. \tag{38}$$

(iv) If we set $S_L^{h_j}(x)$ to reduce to unity, i.e., $S_0^{h_j}(x)\to 1$, in (24) and (30) respectively, then we arrive at the interesting results.

$$I^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}_{0,x,\theta;r,\varepsilon,q;m,k,l}\{t^\lambda H[a_1t^{\zeta_1},\dots,a_rt^{\zeta_r}]\}$$
$$=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j}\sum_{w=0}^{\infty}\frac{(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{w!}$$
$$\times x^{\lambda+\rho[l(m+n)+r\upsilon+rw+j]-\theta\beta'}H^{0,n+2:m_1,n_1;\dots,m_r,n_r}_{p+2,q+2:p_1,q_1;\dots,p_r,q_r}\left[\begin{matrix}a_1x^{\zeta_1}\\ \vdots\\ a_rx^{\zeta_r}\end{matrix}\,\middle|\right.$$
$$\begin{matrix}(1-\frac{\ell''}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(1-\frac{\ell''}{\theta}-\eta'+\beta',\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\\ (1-\frac{\ell''}{\theta}+\beta',\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(1-\frac{\ell''}{\theta}-\eta'-\alpha',\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\end{matrix}$$
$$\left.\begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j',\gamma_j')_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j',\delta_j')_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right], \tag{39}$$

where $\ell''=\lambda+\theta+\rho l(m+n)+\rho r\upsilon+\rho rw+\rho j$; and

$$J^{\rho;\alpha,\beta,\tau;C,D,\alpha',\beta',\eta'}_{x,\infty,\theta;r,\varepsilon,q;m,k,l}\{t^\lambda H[a_1t^{\zeta_1},\dots,a_rt^{\zeta_r}]\}$$
$$=\sum_{\upsilon=0}^{m+n}\sum_{u=0}^{\upsilon}\sum_{j=0}^{m+n}\sum_{i=0}^{j}\Omega(i,j,u,v)z^{l(m+n)+r\upsilon+j}\sum_{w=0}^{\infty}\frac{(\upsilon-\varepsilon n)_w(\tau)^w z^{rw}}{w!}$$
$$\times x^{\lambda+\rho[l(m+n)+r\upsilon+rw+j]-\theta\beta'}H^{0,n+2:m_1,n_1;\dots,m_r,n_r}_{p+2,q+2:p_1,q_1;\dots,p_r,q_r}\left[\begin{matrix}a_1t^{\xi_1}\\ \vdots\\ a_rt^{\xi_r}\end{matrix}\,\middle|\right.$$
$$\begin{matrix}(1-\alpha'-\beta'-\frac{\ell'''}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(1+\eta'-\frac{\ell'''}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\\ (1-\alpha'-\frac{\ell'''}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),(1-\alpha'-\beta'+\eta'-\frac{\ell'''}{\theta},\frac{\zeta_1}{\theta},\dots,\frac{\zeta_r}{\theta}),\end{matrix}$$
$$\left.\begin{matrix}(a_j;\alpha_j',\dots,\alpha_j^{(r)})_{1,p}:(c_j',\gamma_j')_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j',\dots,\beta_j^{(r)})_{1,q}:(d_j',\delta_j')_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right], \tag{40}$$

where $\ell'''=\lambda+\theta+\rho l(m+n)+\rho rv+\rho rw+\rho j-\theta\alpha'-\theta\beta'$.

4 Concluding remarks

The modified Saigo fractional integral operators have advantage that they generalize the Saigo, Erdélyi–Kober, Riemann–Liouville, and Weyl fractional integral operators. Therefore, several authors called them general operators. We also derived analogous results in the form of Riemann–Liouville and Weyl fractional integral operators, which have been depicted in corollaries. Now, we conclude this paper by interesting results that can be derived as the specific cases of our leading results I and II in the form of I-function and H-function. On the other hand, by putting the appropriate values to the arbitrary constant, the family of polynomials (defined by (8)) provide several well-known classical orthogonal polynomials as its special cases, which includes the Hermite, the Laguerre, the Jacobi, the Konhauser polynomials, and so on. Finally, it is interesting to observe that the results given earlier by Saxena et al. ([28], Eqs. (2.1), (2.11))) can be derived from the results (24) and (30) of this paper by virtue of the identity $r = 1$.

Acknowledgements
The authors are thankful to the referee for his/her valuable remarks and comments for the improvement of the paper.

Funding
Not available.

Availability of data and materials
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Publisher's Note

Received: 12 March 2019 Accepted: 21 May 2019 Published online: 30 May 2019

References

1. Srivastava, H.M., Panda, R.: Some bilateral generating functions for a class of generalized hypergeometric polynomials. J. Reine Angew. Math. **283/284**, 265–274 (1976)
2. Baleanu, D., Kumar, D., Purohit, S.D.: Generalized fractional integrals of product of two *H*-functions and a general class of polynomials. Int. J. Comput. Math. **93**(8), 1320–1329 (2016)
3. Kumar, D., Agarwal, P., Purohit, S.D.: Generalized fractional integration of the H-function involving general class of polynomials. Walailak J. Sci. Technol. **11**(12), 1019–1030 (2014)
4. Saigo, M., Saxena, R.K.: Unified fractional integral formulas for the multivariable H-function III. J. Fract. Calc. **20**, 45–68 (2001)
5. Saigo, M., Saxena, R.K., Ram, J.: Fractional integration of the product of F_3 and multivariable H-function. J. Fract. Calc. **27**, 31–42 (2005)
6. Saxena, R.K., Ram, J., Suthar, D.L.: Unified fractional derivative formulas for the multivariable H-function. Vijnana Parishad Anusandhan Patrika **49**(2), 159–175 (2006)
7. Srivastava, H.M., Gupta, K.C., Goyal, S.P.: The H-Functions of One and Two Variables with Applications. South Asian Publishers, New Delhi (1982)
8. Suthar, D.L., Habenom, H., Tadesse, H.: Generalized fractional calculus formulas for a product of Mittag-Leffler function and multivariable polynomials. Int. J. Appl. Comput. Math. **4**(1), 1–12 (2018)
9. Suthar, D.L., Agarwal, P.: Generalized Mittag-Leffler function and the multivariable H-function involving the generalized Mellin–Barnes contour integrals. Commun. Numer. Anal. **2017**(1), Article ID cna-00311, 25–33 (2017)
10. Srivastava, H.M., Panda, R.: Some expansion theorems and the generating relations for the H-function of several complex variables II. Comment. Math. Univ. St. Pauli **25**(2), 167–197 (1976)
11. Srivastava, H.M., Garg, M.: Some integrals involving a general class of polynomials and the multivariable H-function. Rev. Roum. Phys. **32**, 685–692 (1987)
12. Srivastava, H.M.: A contour integral involving Fox's H-function. Indian J. Math. **14**, 1–6 (1972)
13. Saigo, M.: A remark on integral operators involving the Gauss hypergeometric functions. Math. Rep. Coll. Gen. Educ. Kyushu Univ. **11**, 135–143 (1978)
14. Saigo, M.: A certain boundary value problem for the Euler–Darboux equation. Math. Jpn. **24**, 377–385 (1979)

15. Saigo, M.: A certain boundary value problem for the Euler–Darboux equation II. Math. Jpn. **24**, 211–220 (1979)
16. Saigo, M.: A certain boundary value problem for the Euler–Darboux equation III. Math. Jpn. **26**, 103–119 (1981)
17. Saigo, M.: A generalization of fractional calculus, fractional calculus. Res. Notes Math. **138**, 188–198 (1985)
18. Saigo, M., Saxena, R.K.: Application of generalized fractional calculus operators in the solution of an integral equation. J. Fract. Calc. **14**, 53–63 (1998)
19. Saigo, M., Saxena, R.K., Ram, J.: Certain properties of operators of fractional integrations associated with Mellin and Laplace transformations. In: Shrivastava, H.M., Owa, S. (eds.) Current Topics in Analytic Function Theory, pp. 291–304. World Scientific, Singapore (1992)
20. Raizada, S.K.: A study of unified representation of special function of mathematical physics and their use in statistics and boundary value problems. Ph.D. Thesis, Bundelkhand Univ., India (1991)
21. Chatterjea, S.K.: Quelques fonction génératrices des polynômes d'Hermite, du point de vue de l'algèbre de Lie. C. R. Acad. Sci. Paris, Ser. A–B **268**, A600–A602 (1969)
22. Dhillon, S.S.: A study of generalization of special functions of mathematical physics and applications. Ph.D. Thesis, Bundelkhand Univ., India (1989)
23. Gould, H.W., Hoppor, A.T.: Operational formulas connected with two generalization of Hermite polynomials. Duke Math. J. **29**, 51–63 (1962)
24. Krall, H.L., Frink, O.: A new class of orthogonal polynomials: the Bessel polynomials. Trans. Am. Math. Soc. **65**, 100–115 (1949)
25. Singh, A.: A study of special functions of mathematical physics and their applications in combinatorial analysis. Ph.D. Thesis, Bundelkhand Univ., India (1981)
26. Singh, R.P., Srivastava, K.N.: A note on generalization of Laguerre and Humbert polynomials. Ricerca (Napoli) (2) **14**, 11–21 (1963)
27. Saigo, M., Saxena, R.K.: Application of generalized fractional calculus operators in the solution of an integral equation. J. Fract. Calc. **14**, 53–63 (1998)
28. Saxena, R.K., Ram, J., Suthar, D.L.: Unified fractional integral formulas for the modified Saigo operator. Acta Cienc. Indica Math. **31**(2), 421–428 (2005)