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A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method

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Abstract

Salinity refers to the amount of salt in rivers, where the salt can be in many different forms. There are two main methods of defining the concentration of salt in water such as the total dissolved solid measurement (TDS) and the electrical conductivity measurement (EC). The salinity is measured by evaporating water to dryness and weighing the solid residue. The electrical conductivity measurement is measured by passing an electric current through the water and measuring how readily the current flows. The total amount of salt in the water can affect the taste of water. The World Health Organization's guideline on water palatability is that water with a salinity level of less than about 0.50–0.60 g/L is generally considered to be of a standard level. The drinking-water becomes significantly and increasingly unpalatable at salinity levels greater than about 1.0 g/L. In this research, a one-dimensional mathematical model of salinity measurement in a river is proposed. A modified model of salinity control in a river with a barrage dam is also introduced. An unconditionally stable explicit finite difference technique is used to approximate the salinity level under several conditions from the proposed model. The proposed computational technique gives good agreement results in realistic scenarios for water supply processes.

MSC: 65N06; 65M06; 92F99

Keywords: Salinity; Water quality; Barrage dam; River; Saulyev method

1 Introduction

Water production means the removal of surface water or raw water from natural water sources such as rivers, canals, reservoirs, and the sea into the production process for the quality and quantity as per requirement such as tap water and pure water for use in consumption, agriculture, and industry. Each type of production water can use different production technologies.

Water supply systems will use surface water or raw water to produce water, which will be used for consumption, agriculture, and certain industries that do not require high quality water. There are many factors that affect the quality of the water produced such as salinity of the water. It is a very important factor in the production because it cannot be treated in

Table 1 Water quality monitoring stations in river, Thailand where S_7 is the main water supply pumping station for Bangkok

Stations	Distance from the estuary
S_1	12
S_2	27
S_3	35
S_4	50
S_5	64
S_6	91
S_7	96
S_8	108

the normal way. So, for bringing the water to the water treatment process, it is necessary to have a salinity standard.

The Waterworks Authority of Thailand has eight water quality monitoring stations located throughout the river. Each station has a distance from the estuary as shown in Table 1. Currently, the station used to pump raw water for use in the water supply process for consumption in Bangkok has a problem of salinity of water over the standard. That makes an impact on the quality of water produced has a salinity up to standard.

In [1] and [2], the finite element method was used to solve the water pollution models. In [3], the finite difference method was used to solve the hydrodynamic model with the constant coefficients in the closed uniform reservoir. In [4], an analytical solution to the hydrodynamic model in a closed uniform reservoir was proposed. In [5], the Lax–Wendroff finite difference method was also proposed to approximate the water elevation and water flow velocity. In [6], the fourth-order method for a one-dimensional water quality model in a nonuniform flow stream was proposed. In [7], a nondimensional form of a two-dimensional hydrodynamic model with generalized boundary condition and initial conditions for describing the elevation of water wave in an open uniform reservoir was proposed.

Today, there are research studies on the effects of drinking water with salinity over standards, such as [8, 9], and [10]. We will see that the water is too salty to the standards that affect the body. Therefore, research has been presented on the increase of salt water, such as [11] and [12]. The well-known mathematical model uses the conservative property for defining the diffusion of salinity water in a one-dimensional equation [13]

$$A\frac{\partial S}{\partial t} + Q\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left[AD_x\frac{\partial S}{\partial x}\right], \tag{1}$$

where A is a cross-sectional area of the river (m^2), Q is flow rate (m^3/s), D_x is diffusion coefficient of water (m^2/s), S is salinity value (ppt), x is distance (m), and t is time (s).

In this research, a one-dimensional mathematical model of salinity measurement in a river is proposed. A modified model of salinity control in a river with a barrage dam is also introduced. An unconditionally stable explicit finite difference technique is used to approximate the salinity level under several conditions from the proposed model. The proposed computational technique can be applied in realistic scenarios for water supply processes.

2 Governing equations

2.1 Salinity water pollution measurement model

In a stream water quality model, the governing equation is the dynamic one-dimensional advection-dispersion equation. A simplified representation, averaging the equation over the depths, is shown in [6]:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2} \tag{2}$$

for all $(x,t) \in \Omega = [0,L] \times [0,T]$, u is the flow velocity and D is a given diffusion coefficient.

Assume that the salinity is diluted by the freshwater, then the salinity advection level is reduced by the freshwater velocity. The percentage ability of freshwater to dilute salinity is assumed to be $0 \leq k \leq 1$. The one-dimensional salinity water pollution measurement model in a river can be given as follows:

$$\frac{\partial c}{\partial t} + (u_s - ku_w)\frac{\partial c}{\partial x} = D_s\frac{\partial^2 c}{\partial x^2}, \tag{3}$$

where $c(x,t)$ is the salinity concentration (kg/m^3), u_s is advective velocity of salinity water (m/s), k is water salinity removal efficiency rate, u_w is the fresh water flow velocity.

2.2 Initial conditions

The initial condition is defined by an interpolation function of measured raw salinity data. It is aligned on the length of the river from the estuary to the end of the considered area. The initial condition is assumed to be

$$c(x,0) = f(x) \tag{4}$$

for all $x \in [0,L]$, where $f(x)$ is an interpolation function of measured salinity data.

2.3 Boundary condition

2.3.1 Left boundary condition

The left boundary condition is an interpolation function of measured raw data. It is based on the salinity of a river at the first station close to the estuary. The boundary condition is assumed to be

$$c(0,t) = g(t) \tag{5}$$

for all $t \in [0,T]$, where $g(t)$ is a given interpolation function by measured salinity data at the first monitoring station.

2.3.2 Right boundary condition

The right boundary condition is defined by the rate of change of salinity area of the water. The condition can be given as follows:

$$\frac{\partial c}{\partial x} = C_R \tag{6}$$

for all $t \in [0,T]$, where C_R is an approximated rate of change of salinity around the last monitoring station.

3 Explicit finite difference method for a one-dimensional salinity water pollution measurement model

We now discretize the domain by dividing the interval $[0, L]$ into M subintervals such that $M\Delta x = L$ and the time interval $[0, T]$ into N subintervals such that $N\Delta t = T$. The grid points (x_i, t_n) are defined by $x_i = i\Delta x$ for all $i = 1, 2, 3, \ldots, M$ and $t_n = n\Delta t$ for all $n = 1, 2, 3, \ldots, N$, in which M and N are positive integers. We can then approximate $c(x_i, t_n)$ by C_i^n, value of the difference approximation of $c(x, t)$ at point $x = i\Delta x$ and $t = n\Delta t$, where $0 \le i \le M$ and $0 \le n \le N$. We will employ the forward time central space finite difference scheme (FTCS) and the Saulyev method into Eq. (2).

3.1 Forward time central space finite difference scheme

Taking the forward time central space technique [4] into Eq. (2), we get the following discretization:

$$c(x_i, t_n) \cong C_i^n, \tag{7}$$

$$\frac{\partial c}{\partial t}\Big|_{(x_i,t_n)} \cong \frac{C_i^{n+1} - C_i^n}{\Delta t}, \tag{8}$$

$$\frac{\partial c}{\partial x}\Big|_{(x_i,t_n)} \cong \frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x}, \tag{9}$$

$$\frac{\partial^2 c}{\partial x^2}\Big|_{(x_i,t_n)} \cong \frac{C_{i+1}^n + C_{i-1}^n - 2C_i^n}{(\Delta x)^2}, \tag{10}$$

$$u_s(x_i, t_n) = u_{s_i}^n, \tag{11}$$

$$u_w(x_i, t_n) = u_{w_i}^n. \tag{12}$$

Substituting Eqs. (7–12) into Eq. (2), we get the finite difference equation:

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} + \left(u_{s_i}^n - ku_{w_i}^n\right)\left(\frac{C_{i+1}^n - C_{i-1}^n}{2\Delta x}\right) = D_s\left(\frac{C_{i+1}^n + C_{i-1}^n - 2C_i^n}{(\Delta x)^2}\right). \tag{13}$$

Then the explicit finite difference equation becomes

$$C_{i+1}^{n+1} = \left(\lambda + 0.5r_i^n\right)C_{i-1}^n + (1 - 2\lambda)C_i^n + \left(\lambda - 0.5r_i^n\right)C_{i+1}^n \tag{14}$$

for all $i = 1, 2, 3, \ldots, M - 1$, where $\lambda = \frac{D_s\Delta t}{(\Delta x)^2}$ and $r_i^n = \frac{(u_{s_i}^n - ku_{w_i}^n)\Delta t}{\Delta x}$. The forward time central space scheme is conditionally stable subject to constraints in Eq. (13). The stability requirements for the scheme are [6], $0 < \lambda < \frac{1}{2}$, and $0 < r_i^n < 1$.

3.1.1 Right boundary condition approximation

For the right boundary condition Eq. (6), the right boundary condition is defined by the rate of change of salinity area of the water. The right boundary condition is assumed to be

$$\frac{\partial c}{\partial x} \approx \frac{c(L_2, t) - c(L_1, t)}{L_2 - L_1} \tag{15}$$

for all $t \in [0, T]$, where L_1 and L_2 are the distance from the upstream to the point before and after the water supply source, respectively. If we substitute the approximate unknown value of the right boundary, we obtain

$$C^n_{M+1} = \left(\frac{C^n_{M_2} - C^n_{M_1}}{L_2 - L_1}\right)\Delta x + C^n_{M-1}. \tag{16}$$

The forward time central space scheme is conditionally stable subject to constraints in Eq. (13). The stability requirements for the scheme are [6]. It can be obtained that the strict stability requirements are the main disadvantage of this scheme.

3.2 Saulyev explicit finite difference scheme

The Saulyev scheme is unconditionally stable [3]. It is clear that the non-strict stability requirement of the Saulyev scheme is the main advantage and economical to use. Taking Saulyev technique [3] into Eq. (2), the following discretization can be obtained:

$$c(x_i, t_n) \cong C^n_i, \tag{17}$$

$$\frac{\partial c}{\partial t}\Big|_{(x_i,t_n)} \cong \frac{C^{n+1}_i - C^n_i}{\Delta t}, \tag{18}$$

$$\frac{\partial c}{\partial x}\Big|_{(x_i,t_n)} \cong \frac{C^n_{i+1} - C^{n+1}_{i-1}}{2\Delta x}, \tag{19}$$

$$\frac{\partial^2 c}{\partial x^2}\Big|_{(x_i,t_n)} \cong \frac{C^n_{i+1} - C^n_i - C^{n+1}_i + C^{n+1}_{i-1}}{(\Delta x)^2}. \tag{20}$$

Substituting Eqs. (17–20) into Eq. (2), we get the finite difference equation

$$\frac{C^{n+1}_i - C^n_i}{\Delta t} + \left(u^n_{s_i} - ku^n_{w_i}\right)\left(\frac{C^n_{i+1} - C^{n+1}_{i-1}}{2\Delta x}\right) = D_s\left(\frac{C^n_{i+1} - C^n_i - C^{n+1}_i + C^{n+1}_{i-1}}{(\Delta x)^2}\right). \tag{21}$$

Then the explicit finite difference equation becomes

$$C^{n+1}_{i+1} = \left(\frac{1}{1+\lambda}\right)\left[\left(\lambda + \frac{1}{2}r^n_i\right)C^{n+1}_{i-1} + (1-\lambda)C^n_i + \left(\lambda - \frac{1}{2}r^n_i\right)C^n_{i+1}\right] \tag{22}$$

for all $i = 1, 2, 3, \ldots, M-1$, where $\lambda = \frac{D_s\Delta t}{(\Delta x)^2}$ and $r^n_i = \frac{(u^n_{s_i} - ku^n_{w_i})\Delta t}{\Delta x}$. For $i = M$, the right boundary condition Eq. (5), if substituting the approximate unknown value of the right boundary, we obtain $C^n_{M+1} = (\frac{C^n_{M_2} - C^n_{M_1}}{L_2 - L_1})\Delta x + C^n_{M-1}$.

Using Taylor series expansions on the approximation, [14] has shown that the truncation error is $O\{(\Delta x)^2 + (\Delta t)^2 + (\Delta t/\Delta x)^2\}$.

The Saulyev method is an unconditionally stable method [15]. It follows that the application of the explicit Saulyev finite difference technique is economical in terms of computation implementation.

4 Numerical simulations

4.1 Simulation 1: salinity control in an ideal case

We consider a segment of a river with 108 km of length as shown in Table 1. Assume that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any

Table 2 Physical parameters of simulation 1

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (s)
0.1	0.065	0.25	0.3	108	100

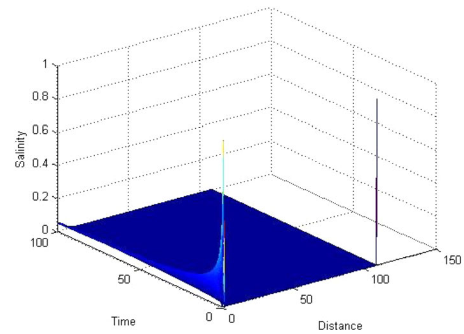


Figure 1 The exact solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$

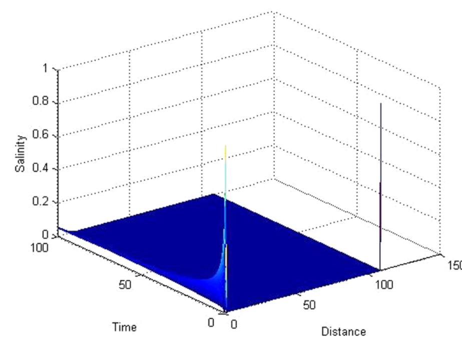


Figure 2 The FTCS solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$

Table 3 The maximum absolute error defined by $\mathbf{err}_{\max} = \max|\tilde{c}(x_i,T) - c(x_i,T)|$ for all $i = 0, 1, \ldots, N$, where $T = 10, 20, 30$, and 40

T	FTCS	Saulyev
	$\mathrm{err}_{\max}$	$\mathrm{err}_{\max}$
10	5.9442×10^{-4}	5.1141×10^{-4}
20	0.0044	0.0042
30	0.0083	0.0082
40	0.0107	0.0107

time is 100. Their physical parameters and given spacing are shown in Table 2. In [11], the theoretical solution is given by

$$c(x,t) = \frac{1}{\sqrt{4t+1}} \exp\left[-\frac{(x-1-(u_s-ku_w)t)^2}{D(4t+1)}\right]. \tag{23}$$

Actually, when using the FTCS scheme Eq. (14) and the Saulyev technique Eq. (23), when their physical parameters are as given in Table 2, we get the approximated solution $c(x,t)$. The theoretical solution is illustrated by a surface of solution in Fig. 1. The FTCS approximated solution is illustrated by Fig. 2. The Saulyev approximated solution is also illustrated by Fig. 3. The maximum absolute error of both finite difference approximations is compared in Table 3.

Figure 3 The Saulyev solution of simulation 1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$

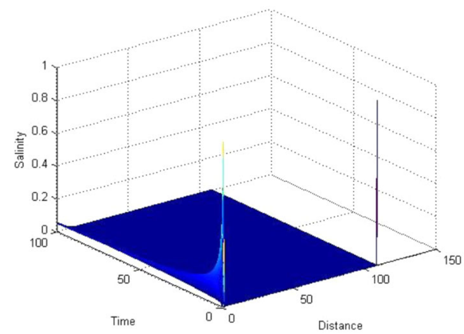


Table 4 Physical parameters of simulation 2

D_s (m^2/s)	u_s (m/s)	u_w (m/s)	K	L (km)	T (s)
0.1	0.065	0.3	0.3	108	100
0.1	0.065	0.25	0.3	108	100
0.1	0.065	0.2	0.3	108	100

Table 5 Convergence of FTCS method and Saulyev method for some grid spacing

T	Δx	Δt	FTCS	Saulyev
100	0.10	0.04	Stable	Stable
		0.05	Stable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable
100	0.05	0.04	Untable	Stable
		0.05	Untable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable
100	0.025	0.04	Untable	Stable
		0.05	Untable	Stable
		0.06	Untable	Stable
		0.07	Untable	Stable

4.2 Simulation 2: the salinity is diluted by releasing the fresh water from a barrage dam with different flow velocities.

We consider a segment of a river with 108 km of length as shown in Table 1. Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any time is 1000. Their physical parameters and given spacing are shown in Table 4. In this simulation, the Saulyev technique is used to approximate the solution since the technique will always give stable solutions as shown in Table 5. According to the good agreement of approximated solutions of the Saulyev method, the method in Eq. (23) is chosen to approximate the solution of the simulation. The several fresh water flow velocities $u_w = 0.20, 0.25, 0.30$ m/s from the barrage dam are simulated until the salinity level at the controlled monitoring station S_7 becomes standardized level as shown in Figs. 4–5.

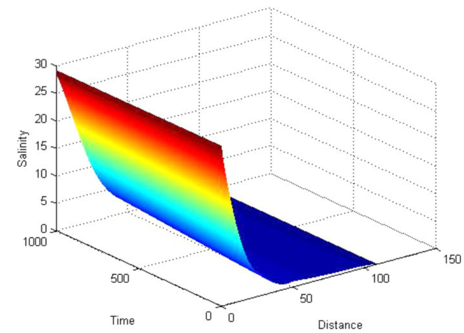


Figure 4 Approximated salinity along the river by using the Saulyev technique, where $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le x \le 108$ and $0 \le t \le 1000$

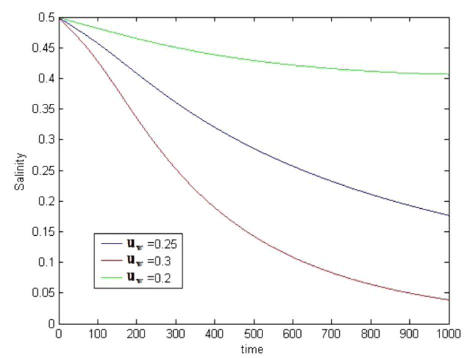


Figure 5 Approximated salinity at $c(96, t)$ by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \le t \le 1000$ when $u_W = 0.3, 0.25$ and 0.2

Table 6 Physical parameters of simulation 3

$c(x,t)$ at S_7	D (m^2/s)	u_s (m/s)	u_W (m/s)	K	T	L (km)	$c(0,t)$
$> C_{ST}$	0.1	0.065	0.25	0.3	1000	108	$g(t)$
$< C_{ST}$	0.1	0.065	0.205	0.3	1000	108	$g(t)$

4.3 Simulation 3: the salinity is diluted by releasing the fresh water from a barrage dam and changing flow velocities after the salinity comes to standard

We consider a segment of a river with 108 km of length as shown in Table 1. Assuming that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any time is 1000. Their physical parameters and given spacing are shown in Table 6. Assume that there are eight monitoring stations along a considered river segment as shown in Table 1. The controlled monitoring station is station S_7. We need to control the salinity level at station S_7 to be under the salinity standard level $C_{ST} = 0.3\ kg/m^3$. The salinity is controlled by a process as follows:

(1) If the salinity level at station S_7 $c(96, t) > C_{ST}$, then the fresh water will be released at a high speed from the barrage dam by controlled flow velocity.

(2) If the salinity level at station S_7 $c(96, t) < C_{ST}$, then the fresh water will be released at a low speed level from the barrage dam.

We can obtain the approximated salinity level along the considered river segment as shown in Fig. 6 and Table 7. The salinity level at several monitoring stations S_1, S_5, and S_7 is shown in Figs. 7, 8, and 9, respectively.

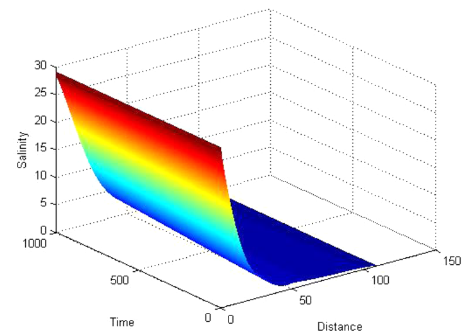


Figure 6 The numerical solution of simulation 2 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$

Table 7 Approximated salinity $c(x,t)$ of simulation 2 for all monitoring stations

t	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	12.1040	4.0187	2.0224	1.0978	0.7955	0.5316	0.4995	0.1444
5000	11.3456	3.6020	1.9844	1.0058	0.7667	0.4807	0.3844	0.0027
10,000	11.3391	3.6002	2.0397	1.0128	0.7476	0.4098	0.2993	0.0023
15,000	13.7382	4.4013	2.4788	1.1159	0.7704	0.3996	0.2987	0.0032
20,000	15.5617	5.3633	3.0060	1.2576	0.8025	0.3961	0.2983	0.0034

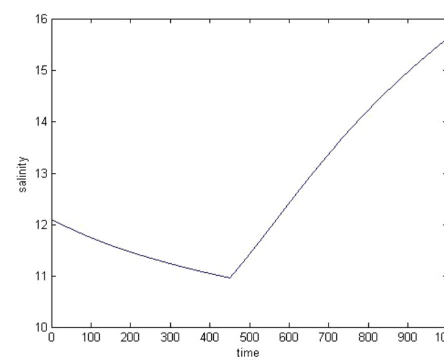


Figure 7 Approximated salinity of simulation 2 at station S_1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$

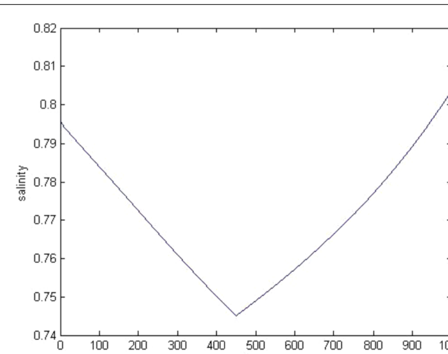


Figure 8 Approximated salinity of simulation at station S_5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$

4.4 Simulation 4: diluting the salinity of water by releasing fresh water before salinity water arrives at the pumping station

We consider a segment of a river with 108 km of length as shown in Table 1. Assume that the salinity diffusion coefficient is 0.1 m^2/s, the salinity flow velocity is 0.065 m/s, the ability percentage of fresh water dilution is 30%, and the given simulated station at any time is

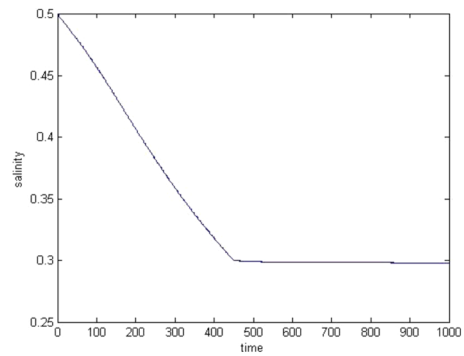


Figure 9 Approximated salinity of simulation at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$

Table 8 Physical parameters of simulation 2

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)	K	T	L (km)	$c(0,t)$
$< C_{ST}$	0.1	0.065	0	0.3	1000	108	$g(t)$
$> C_{ST}$	0.1	0.065	0.25	0.3	1000	108	$g(t)$

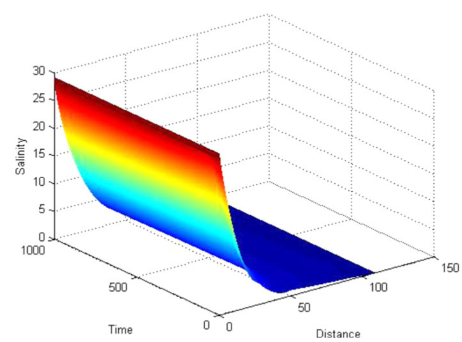


Figure 10 The numerical solution of simulation 3 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq x \leq 108$ and $0 \leq t \leq 1000$

1000. Their physical parameters and given spacing are shown in Table 8. Assume that there are eight monitoring stations along the considered river segment as shown in Table 1. The controlled monitoring station is station S_7. We need to control the salinity level at station S_7 before salinity level at station S_7 is over the salinity standard level $C_{ST} = 0.05$ kg/m^3 for about three days. In this simulation, the Saulyev technique is used to approximate the solution since the technique will always give stable solutions. The salinity is controlled by a process as follows:

(1) If the salinity level at station S_5 $c(91,t) < C_{ST}$, then the fresh water is released at a normal speed level from the barrage dam.

(2) If the salinity level a station S_5 $c(91,t) > C_{ST}$, then the fresh water will be released at a high speed from the barrage dam which is used to control the salinity.

We can obtain the approximated salinity level along the considered river segment as shown in Table 9 and Fig. 10. The salinity level at several monitoring stations S_1, S_5, and S_7 is shown in Figs. 11, 12, and 13, respectively.

5 Discussion

In simulation 1, we get good agreement between approximated solutions of the FTCS and the Saulyev finite difference techniques. The maximum error is less than 1%. In simulation 2, we can obtain that the Saulyev technique is better than the FTCS technique due to

Table 9 Approximated salinity $c(x,t)$ of simulation 3 for all monitoring stations

t	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	12.1040	4.0187	2.0224	1.0978	0.7957	0.4010	0.2544	0.1395
5000	20.3804	7.4884	4.0812	1.4095	0.9276	0.4791	0.3919	0.0040
10,000	17.8501	7.0540	3.9099	1.4447	0.8945	0.4144	0.2993	0.0020
15,000	16.1112	6.5658	3.7400	1.4458	0.8669	0.3456	0.2327	0.0015
20,000	14.8449	6.1007	3.5647	1.4298	0.8380	0.2888	0.1868	0.0011

Figure 11 Approximated salinity of simulation 3 at station S_1 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$

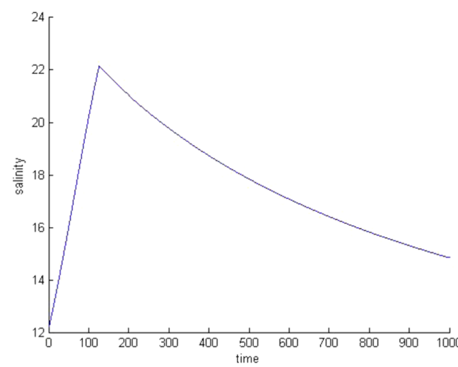


Figure 12 Approximated salinity of simulation 3 at station S_5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$

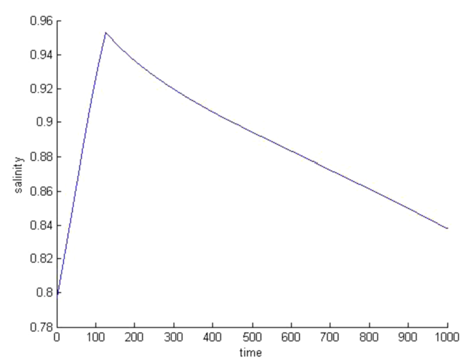
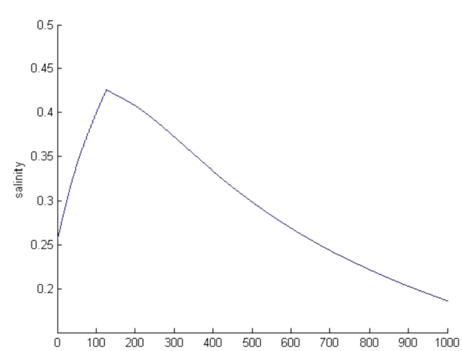


Figure 13 Approximated salinity of simulation 3 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $0 \leq t \leq 1000$



the limitation of stability conditions. The Saulyev technique gives a stable approximated solution. Otherwise, the FTCS is limited by its stability conditions. Thus the Saulyev technique is preferred in other realistic simulations. We can see that the salinity level will be reduced when the fresh water flow velocity is increasing as shown in Fig. 5. In simulation 3, a salinity control process is simulated. The salinity is reduced, the salinity level comes to standard, after that we can decrease the fresh water flow velocity to maintain the salinity level at the standard level as shown in Fig. 9. In simulation 4, a salinity control process is simulated. The salinity is reduced before the salinity level touches the standard salinity level. The proposed process can reduce the salinity level when the fresh water is released from the barrage dam at least amount as shown in Figs. 11–13.

6 Conclusion

We have proposed a one-dimensional mathematical model of salinity measurement in a river with a barrage dam. The proposed model deals with salinity advection to a river and the fresh water flow from the barrage dam effects. The traditional forward time central space finite difference method is compared with the proposed Saulyev technique. The proposed Saulyev technique gives a stable solution in any grid spacing. The technique also gives accurately approximated solutions. The realistic problem is also simulated. The proposed simulation can be used in several realistic salinity measurements. In the salinity control aspect, the proposed process can reduce the salinity level before the level is over the standard. The proposed numerical simulation can be applied in practical salinity control in a river with a barrage dam.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. The authors read and approved the final manuscript.

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