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Parameter-range-dependent robust stability conditions for quaternion-valued neural networks with time delays

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Abstract

In this paper, a model of discrete delayed quaternion-valued neural networks with interval parameter uncertainties is considered. Some sufficient conditions for the existence, uniqueness and globally robust stability of the equilibrium point are obtained. The approach is on the basis of the homeomorphic mapping method, the Lyapunov stability theorem and inequality techniques. Finally, a numerical example is given to demonstrate the application of the obtained results.

Keywords: Quaternion-valued; Neural networks; Discrete delays; Robust stability; Interval parameter uncertainty

1 Introduction

Since neural networks had been proposed in the 1940s, they have been widely studied by many researchers, due to the many applications in various areas, such as associative memory, optimization, pattern recognition, fault diagnosis and signal processing. A lot of excellent work as regards the real-valued neural networks (RVNNs) and complex-valued neural networks (CVNNs) has appeared in the study of their dynamics [1–6]. However, there are some problems that the RVNNs and the CVNNs cannot deal with straightforwardly, such as 4-D signals, body images which are four or more dimensional [7–9], new methods or theories have to be put forward to, the theory of quaternion-valued neural networks (QVNNs) is one of those approaches, since it can handle not only real-valued and complex-valued cases but also the multidimensional data. For details, see Refs. [10–17].

In the practical system, due to the internal and external disturbances, the stability may be destroyed and this may bring about many faults and problems. On the other hand, the inaccurate measurement, owing to technical reasons, will also result in the instability of the systems. The general stability cannot describe the complete inner characters of the systems, so it is quite necessary to refer to robust stability, while discussing the neural networks with interval parameter uncertainties. So far, a large number of authors in the literature have considered the important and interesting issue, some sufficient conditions have been established to guarantee existence, uniqueness and robust stability of the equilibrium point for RVNNs and CVNNs with interval parameter uncertainties [18–25].

Moreover, between neurons, there exists a limited transmission speed, it is inevitable to see the delayed phenomenon in the neural networks, thus delays become one of the essential parts in analyzing its dynamics. Compared to the general systems, the dynamics of neural networks with delays becomes more complicated and it may lead to volatility, instability and even chaos. In fact, some authors also have considered the existence, uniqueness and global stability of the equilibrium point for delayed quaternion-valued neural networks [14, 26–34].

Although the work above is very excellent, little research considered robust stability of quaternion-valued neural networks with delay and interval parameter uncertainties. Recently, Chen in [35, 36] considered the robust stability with different kinds of delays for quaternion-valued neural networks, some sufficient conditions have also been obtained about the existence, uniqueness and global asymptotic robust stability. But in his judging criteria, the negative definiteness of two matrices is needed, moreover, the entries in these matrices are all the maximal values, which are decided by the absolution of the upper and lower bounds of elements for the connection weight matrix, the sign of connection weight matrix is ignored. In this article, inspired by the methods of [20], we proposed a new approach to overcome this defects, in our unique criteria matrix, the given elements depend on not only the lower bounds but also the upper bounds of the interval parameters, which is different from previous contributions and extends the relevant work in Refs. [19, 20, 22, 27, 35].

The rest of this paper is arranged as follows. In Sect. 2, the discrete delayed QVNN model is proposed, some basic knowledge, preliminaries and lemmas are also presented. Our main results are given in Sect. 3, in which the sufficient condition for the existence, uniqueness and global robust stability of the equilibrium point are obtained relying on The homeomorphic mapping method, the Lyapunov stability theorem and inequality techniques. In order to illustrate the effectiveness of our main results, the numerical simulation is addressed in Sect. 4, and then the relevant conclusions at the end.

2 Problem formulation and preliminaries

For convenience, we give some notations used throughout this paper before introducing the model.

$\mathbb{R}$, $\mathbb{C}$ and $\mathbb{H}$ mean the real field, complex field and the skew field of quaternions, respectively. $\mathbb{R}^{n\times m}$, $\mathbb{C}^{n\times m}$ and $\mathbb{H}^{n\times m}$ denote $n \times m$ real-valued matrices, complex-valued matrices, and quaternion-valued matrices, respectively. $\bar{A}$, A^T and A^* denote the conjugate, the transpose and the conjugate transpose of matrix A, respectively. Let $\|z\|$ denote the norm of $z \in \mathbb{C}^n$, where $\|z\| = \sqrt{z^* z}$, and $\|A\|$ means the norm of $A \in \mathbb{C}^n$. The symbol I denotes the identity matrix with appropriate dimensions. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ stand for the largest and the smallest eigenvalue of the Hermitian matrix P. The notation $X \geq Y$ (respectively, $X > Y$) means that $X - Y$ is a positive semi-definite (respectively, positive definite). Moreover, the notation $\star$ means the conjugate transpose of a suitable block in a Hermitian matrix.

As for quaternion-valued operation and related issues, we can refer to [35]. Note that, if $p, q \in \mathbb{H}$ with $p = p_0 + p_1\imath + p_2\jmath + p_3\kappa$, $q = q_0 + q_1\imath + q_2\jmath + q_3\kappa$, then $p \preceq q$ denotes $p_i \leq q_i$ $(i = 0, 1, 2, 3)$. If $A, B \in \mathbb{H}^{n\times n}$ with $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$, then $A \preceq B$ denotes $a_{ij} \preceq b_{ij}$ $(i = 1, 2, \ldots, n)$. For $A \in H^{n\times n}$, let A^R, A^I, A^J and A^K be the real part and three imaginary components.

2.1 Model description

In this paper, we consider the following discrete delayed QVNNs with interval parameter uncertainties:

$$\dot{q}(t) = -Cq(t-\delta) + Af\bigl(q(t)\bigr) + Bf\bigl(q(t-\tau)\bigr) + J, \tag{1}$$

where $q(t) = (q_1(t), q_2(t), \dots, q_n(t))^T \in \mathbb{H}^n$, $q_i(t)$ is the state of the ith neural neuron of the neural network at time t, $\delta \geq 0$ and $\tau \geq 0$ represent the leakage delay and transmission delay, respectively, $f(q(t)) = (f_1(q_1(t)), f_2(q_2(t)), \dots, f_n(q_n(t)))^T \in \mathbb{H}^n$ refers to the neuron activation function, $C = \operatorname{diag}\{c_1, c_2, \dots, c_n\} \in \mathbb{R}_d^{n\times n}$ with $c_i > 0$ is the self-feedback connection weight matrix, $A \in \mathbb{H}^{n\times n}$ is the connection weight matrix, $B \in \mathbb{H}^{n\times n}$ is the delayed connection weight matrix, $J = (J_1, J_2, \dots, J_n)^T \in \mathbb{H}^n$ refers to the input vector.

Next, we will consider the dynamical behaviors of system (1).

Some assumptions as regards Eq. (1) are given first:

($\mathbf{A}_1$) For $i \in \{1, 2, \dots, n\}$, the neuron activation function f_i is continuous and satisfies

$$\bigl|f_i(\varphi_1) - f_i(\varphi_2)\bigr| \leq k_i|\varphi_1 - \varphi_2|, \quad \forall \varphi_1, \varphi_2 \in \mathbb{H},$$

where k_i is a real-valued positive constant; in the next proof, we define $K = \operatorname{diag}\{k_1, k_2, \dots, k_n\}$.

($\mathbf{A}_2$) The matrices C, A, B and J in model (1) are included in the following sets, respectively:

$$\begin{aligned}
C_I &= \bigl\{C \in \mathbb{R}_d^{n\times n} : 0 \prec \check{C} \preceq C \preceq \hat{C}, \check{C}, \hat{C} \in \mathbb{R}_d^{n\times n}\bigr\}, \\
A_I &= \bigl\{A \in \mathbb{H}^{n\times n} : \check{A} \preceq A \preceq \hat{A}, \check{A}, \hat{A} \in \mathbb{H}^{n\times n}\bigr\}, \\
B_I &= \bigl\{B \in \mathbb{H}^{n\times n} : \check{B} \preceq B \preceq \hat{B}, \check{B}, \hat{B} \in \mathbb{H}^{n\times n}\bigr\}, \\
J &= \bigl\{J \in \mathbb{H}^{n} : \check{J} \preceq J \preceq \hat{J}, \check{J}, \hat{J} \in \mathbb{H}^{n}\bigr\},
\end{aligned}$$

where $\check{C} = \operatorname{diag}\{\check{c}_1, \check{c}_2, \dots, \check{c}_n\}$, $\hat{C} = \operatorname{diag}\{\hat{c}_1, \hat{c}_2, \dots, \hat{c}_n\}$, $\check{A} = (\check{a}_{ij})_{n\times n}$, $\hat{A} = (\hat{a}_{ij})_{n\times n}$, $\check{B} = (\check{b}_{ij})_{n\times n}$, $\hat{B} = (\hat{b}_{ij})_{n\times n}$.

Before analyzing the existence, uniqueness and robust stability of the equilibrium point in system (1), we need clear some definitions and notation.

For every $A_i = A_i^R + \imath A_i^I + \jmath A_i^J + \kappa A_i^K$, $B_i = B_i^R + \imath B_i^I + \jmath B_i^J + \kappa B_i^K \in \mathbb{H}^{n\times n}$, $i = 0, 1$, let

$$\begin{aligned}
A_0^R &= \frac{1}{2}\bigl(\hat{A}^R + \check{A}^R\bigr), & A_1^R &= \frac{1}{2}\bigl(\hat{A}^R - \check{A}^R\bigr) = \bigl(\mu_{ij}^R\bigr)_{n\times n}, \\
A_0^I &= \frac{1}{2}\bigl(\hat{A}^I + \check{A}^I\bigr), & A_1^I &= \frac{1}{2}\bigl(\hat{A}^I - \check{A}^I\bigr) = \bigl(\mu_{ij}^I\bigr)_{n\times n}, \\
A_0^J &= \frac{1}{2}\bigl(\hat{A}^J + \check{A}^J\bigr), & A_1^J &= \frac{1}{2}\bigl(\hat{A}^J - \check{A}^J\bigr) = \bigl(\mu_{ij}^J\bigr)_{n\times n}, \\
A_0^K &= \frac{1}{2}\bigl(\hat{A}^K + \check{A}^K\bigr), & A_1^K &= \frac{1}{2}\bigl(\hat{A}^K - \check{A}^K\bigr) = \bigl(\mu_{ij}^K\bigr)_{n\times n}, \\
B_0^R &= \frac{1}{2}\bigl(\hat{B}^R + \check{B}^R\bigr), & B_1^R &= \frac{1}{2}\bigl(\hat{B}^R - \check{B}^R\bigr) = \bigl(\nu_{ij}^R\bigr)_{n\times n}, \\
B_0^I &= \frac{1}{2}\bigl(\hat{B}^I + \check{B}^I\bigr), & B_1^I &= \frac{1}{2}\bigl(\hat{B}^I - \check{B}^I\bigr) = \bigl(\nu_{ij}^I\bigr)_{n\times n},
\end{aligned}$$

$$
\begin{aligned}
B_0^J &= \frac{1}{2}\left(\hat{B}^J + \check{B}^J\right), \qquad B_1^J = \frac{1}{2}\left(\hat{B}^J - \check{B}^J\right) = \left(\nu_{ij}^J\right)_{n\times n}, \\
B_0^K &= \frac{1}{2}\left(\hat{B}^K + \check{B}^K\right), \qquad B_1^K = \frac{1}{2}\left(\hat{B}^K - \check{B}^K\right) = \left(\nu_{ij}^K\right)_{n\times n}, \\
C_0 &= \frac{1}{2}(\hat{C} + \check{C}), \\
C_1 &= \frac{1}{2}(\hat{C} - \check{C}) = (\omega_{ij})_{n\times n} = \frac{1}{2}\operatorname{diag}\{\hat{c}_1 - \check{c}_1, \hat{c}_1 - \check{c}_2, \ldots, \hat{c}_n - \check{c}_n\}.
\end{aligned}
$$

Obviously, $A_1^X \succeq 0$, $B_1^X \succeq 0$, here X expresses R, I, J, K, respectively, and $C_1 \succeq 0$, let $e_i = (0, 0, \ldots, 1, \ldots, 0)_{n\times 1}^T$ where 1 belongs to the ith value in the unit row vector, define

$$
\begin{aligned}
M_A^R &= \left(\sqrt{\mu_{11}^R}e_1, \ldots, \sqrt{\mu_{1n}^R}e_1, \sqrt{\mu_{21}^R}e_2, \ldots, \sqrt{\mu_{2n}^R}e_2, \ldots, \sqrt{\mu_{n1}^R}e_n, \ldots, \sqrt{\mu_{nn}^R}e_n\right)_{n\times n^2}, \\
M_A^I &= \left(\sqrt{\mu_{11}^I}e_1, \ldots, \sqrt{\mu_{1n}^I}e_1, \sqrt{\mu_{21}^I}e_2, \ldots, \sqrt{\mu_{2n}^I}e_2, \ldots, \sqrt{\mu_{n1}^I}e_n, \ldots, \sqrt{\mu_{nn}^I}e_n\right)_{n\times n^2}, \\
M_A^J &= \left(\sqrt{\mu_{11}^J}e_1, \ldots, \sqrt{\mu_{1n}^J}e_1, \sqrt{\mu_{21}^J}e_2, \ldots, \sqrt{\mu_{2n}^J}e_2, \ldots, \sqrt{\mu_{n1}^J}e_n, \ldots, \sqrt{\mu_{nn}^J}e_n\right)_{n\times n^2}, \\
M_A^K &= \left(\sqrt{\mu_{11}^K}e_1, \ldots, \sqrt{\mu_{1n}^K}e_1, \sqrt{\mu_{21}^K}e_2, \ldots, \sqrt{\mu_{2n}^K}e_2, \ldots, \sqrt{\mu_{n1}^K}e_n, \ldots, \sqrt{\mu_{nn}^K}e_n\right)_{n\times n^2}, \\
M_B^R &= \left(\sqrt{\nu_{11}^R}e_1, \ldots, \sqrt{\nu_{1n}^R}e_1, \sqrt{\nu_{21}^R}e_2, \ldots, \sqrt{\nu_{2n}^R}e_2, \ldots, \sqrt{\nu_{n1}^R}e_n, \ldots, \sqrt{\nu_{nn}^R}e_n\right)_{n\times n^2}, \\
M_B^I &= \left(\sqrt{\nu_{11}^I}e_1, \ldots, \sqrt{\nu_{1n}^I}e_1, \sqrt{\nu_{21}^I}e_2, \ldots, \sqrt{\nu_{2n}^I}e_2, \ldots, \sqrt{\nu_{n1}^I}e_n, \ldots, \sqrt{\nu_{nn}^I}e_n\right)_{n\times n^2}, \\
M_B^J &= \left(\sqrt{\nu_{11}^J}e_1, \ldots, \sqrt{\nu_{1n}^J}e_1, \sqrt{\nu_{21}^J}e_2, \ldots, \sqrt{\nu_{2n}^J}e_2, \ldots, \sqrt{\nu_{n1}^J}e_n, \ldots, \sqrt{\nu_{nn}^J}e_n\right)_{n\times n^2}, \\
M_B^K &= \left(\sqrt{\nu_{11}^K}e_1, \ldots, \sqrt{\nu_{1n}^K}e_1, \sqrt{\nu_{21}^K}e_2, \ldots, \sqrt{\nu_{2n}^K}e_2, \ldots, \sqrt{\nu_{n1}^K}e_n, \ldots, \sqrt{\nu_{nn}^K}e_n\right)_{n\times n^2}, \\
M_C &= (\sqrt{\omega_{11}}e_1, \ldots, \sqrt{\omega_{1n}}e_1, \sqrt{\omega_{21}}e_2, \ldots, \sqrt{\omega_{2n}}e_2, \ldots, \sqrt{\omega_{n1}}e_n, \ldots, \sqrt{\omega_{nn}}e_n)_{n\times n^2}, \\
N_A^R &= \left(\sqrt{\mu_{11}^R}e_1, \ldots, \sqrt{\mu_{1n}^R}e_n, \sqrt{\mu_{21}^R}e_1, \ldots, \sqrt{\mu_{2n}^R}e_n, \ldots, \sqrt{\mu_{n1}^R}e_1, \ldots, \sqrt{\mu_{nn}^R}e_n\right)_{n^2\times n}^T, \\
N_A^I &= \left(\sqrt{\mu_{11}^I}e_1, \ldots, \sqrt{\mu_{1n}^I}e_n, \sqrt{\mu_{21}^I}e_1, \ldots, \sqrt{\mu_{2n}^I}e_n, \ldots, \sqrt{\mu_{n1}^I}e_1, \ldots, \sqrt{\mu_{nn}^I}e_n\right)_{n^2\times n}^T, \\
N_A^J &= \left(\sqrt{\mu_{11}^J}e_1, \ldots, \sqrt{\mu_{1n}^J}e_n, \sqrt{\mu_{21}^J}e_1, \ldots, \sqrt{\mu_{2n}^J}e_n, \ldots, \sqrt{\mu_{n1}^J}e_1, \ldots, \sqrt{\mu_{nn}^J}e_n\right)_{n^2\times n}^T, \\
N_A^K &= \left(\sqrt{\mu_{11}^K}e_1, \ldots, \sqrt{\mu_{1n}^K}e_n, \sqrt{\mu_{21}^K}e_1, \ldots, \sqrt{\mu_{2n}^K}e_n, \ldots, \sqrt{\mu_{n1}^K}e_1, \ldots, \sqrt{\mu_{nn}^K}e_n\right)_{n^2\times n}^T, \\
N_B^R &= \left(\sqrt{\nu_{11}^R}e_1, \ldots, \sqrt{\nu_{1n}^R}e_n, \sqrt{\nu_{21}^R}e_1, \ldots, \sqrt{\nu_{2n}^R}e_n, \ldots, \sqrt{\nu_{n1}^R}e_1, \ldots, \sqrt{\nu_{nn}^R}e_n\right)_{n^2\times n}^T, \\
N_B^I &= \left(\sqrt{\nu_{11}^I}e_1, \ldots, \sqrt{\nu_{1n}^I}e_n, \sqrt{\nu_{21}^I}e_1, \ldots, \sqrt{\nu_{2n}^I}e_n, \ldots, \sqrt{\nu_{n1}^I}e_1, \ldots, \sqrt{\nu_{nn}^I}e_n\right)_{n^2\times n}^T, \\
N_B^J &= \left(\sqrt{\nu_{11}^J}e_1, \ldots, \sqrt{\nu_{1n}^J}e_n, \sqrt{\nu_{21}^J}e_1, \ldots, \sqrt{\nu_{2n}^J}e_n, \ldots, \sqrt{\nu_{n1}^J}e_1, \ldots, \sqrt{\nu_{nn}^J}e_n\right)_{n^2\times n}^T, \\
N_B^K &= \left(\sqrt{\nu_{11}^K}e_1, \ldots, \sqrt{\nu_{1n}^K}e_n, \sqrt{\nu_{21}^K}e_1, \ldots, \sqrt{\nu_{2n}^K}e_n, \ldots, \sqrt{\nu_{n1}^K}e_1, \ldots, \sqrt{\nu_{nn}^K}e_n\right)_{n^2\times n}^T, \\
N_C &= (\sqrt{\omega_{11}}e_1, \ldots, \sqrt{\omega_{1n}}e_n, \sqrt{\omega_{21}}e_1, \ldots, \sqrt{\omega_{2n}}e_n, \ldots, \sqrt{\omega_{n1}}e_1, \ldots, \sqrt{\omega_{nn}}e_n)_{n^2\times n}^T.
\end{aligned}
$$

It is obvious that

$$M_A^R(M_A^R)^T = \operatorname{diag}\left(\sum_{j=1}^n \mu_{1j}^R, \dots, \sum_{j=1}^n \mu_{nj}^R\right), \qquad M_A^I(M_A^I)^T = \operatorname{diag}\left(\sum_{j=1}^n \mu_{1j}^I, \dots, \sum_{j=1}^n \mu_{nj}^I\right),$$

$$M_A^J(M_A^J)^T = \operatorname{diag}\left(\sum_{j=1}^n \mu_{1j}^J, \dots, \sum_{j=1}^n \mu_{nj}^J\right), \qquad M_A^K(M_A^K)^T = \operatorname{diag}\left(\sum_{j=1}^n \mu_{1j}^K, \dots, \sum_{j=1}^n \mu_{nj}^K\right),$$

$$M_B^R(M_B^R)^T = \operatorname{diag}\left(\sum_{j=1}^n \nu_{1j}^R, \dots, \sum_{j=1}^n \nu_{nj}^R\right), \qquad M_B^I(M_B^I)^T = \operatorname{diag}\left(\sum_{j=1}^n \nu_{1j}^I, \dots, \sum_{j=1}^n \nu_{nj}^I\right),$$

$$M_B^J(M_B^J)^T = \operatorname{diag}\left(\sum_{j=1}^n \nu_{1j}^J, \dots, \sum_{j=1}^n \nu_{nj}^J\right), \qquad M_B^K(M_B^K)^T = \operatorname{diag}\left(\sum_{j=1}^n \nu_{1j}^K, \dots, \sum_{j=1}^n \nu_{nj}^K\right),$$

$$M_C(M_C)^T = \operatorname{diag}\left(\sum_{j=1}^n \omega_{1j}, \dots, \sum_{j=1}^n \omega_{nj}\right), \qquad (N_A^R)^T N_A^R = \operatorname{diag}\left(\sum_{j=1}^n \mu_{j1}^R, \dots, \sum_{j=1}^n \mu_{jn}^R\right),$$

$$(N_A^I)^T N_A^I = \operatorname{diag}\left(\sum_{j=1}^n \mu_{j1}^I, \dots, \sum_{j=1}^n \mu_{jn}^I\right), \qquad (N_A^J)^T N_A^J = \operatorname{diag}\left(\sum_{j=1}^n \mu_{j1}^J, \dots, \sum_{j=1}^n \mu_{jn}^J\right),$$

$$(N_A^K)^T N_A^K = \operatorname{diag}\left(\sum_{j=1}^n \mu_{j1}^K, \dots, \sum_{j=1}^n \mu_{jn}^K\right), \qquad (N_B^R)^T N_B^R = \operatorname{diag}\left(\sum_{j=1}^n \nu_{j1}^R, \dots, \sum_{j=1}^n \nu_{jn}^R\right),$$

$$(N_B^I)^T N_B^I = \operatorname{diag}\left(\sum_{j=1}^n \nu_{j1}^I, \dots, \sum_{j=1}^n \nu_{jn}^I\right), \qquad (N_B^J)^T N_B^J = \operatorname{diag}\left(\sum_{j=1}^n \nu_{j1}^J, \dots, \sum_{j=1}^n \nu_{jn}^J\right),$$

$$(N_B^K)^T N_B^K = \operatorname{diag}\left(\sum_{j=1}^n \nu_{j1}^K, \dots, \sum_{j=1}^n \nu_{jn}^K\right), \qquad (N_C)^T N_C = \operatorname{diag}\left(\sum_{j=1}^n \omega_{j1}, \dots, \sum_{j=1}^n \omega_{jn}\right).$$

Moreover, we define

$$L_A^R = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \mu_{1j}^R}, \dots, \sqrt{\sum_{j=1}^n \mu_{nj}^R}\right), \qquad L_A^I = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \mu_{1j}^I}, \dots, \sqrt{\sum_{j=1}^n \mu_{nj}^I}\right),$$

$$L_A^J = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \mu_{1j}^J}, \dots, \sqrt{\sum_{j=1}^n \mu_{nj}^J}\right), \qquad L_A^K = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \mu_{1j}^K}, \dots, \sqrt{\sum_{j=1}^n \mu_{nj}^K}\right),$$

$$L_B^R = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \nu_{1j}^R}, \dots, \sqrt{\sum_{j=1}^n \nu_{nj}^R}\right), \qquad L_B^I = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \nu_{1j}^I}, \dots, \sqrt{\sum_{j=1}^n \nu_{nj}^I}\right),$$

$$L_B^J = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \nu_{1j}^J}, \dots, \sqrt{\sum_{j=1}^n \nu_{nj}^J}\right), \qquad L_B^K = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \nu_{1j}^K}, \dots, \sqrt{\sum_{j=1}^n \nu_{nj}^K}\right),$$

$$L_C = \operatorname{diag}\left(\sqrt{\sum_{j=1}^n \omega_{1j}}, \dots, \sqrt{\sum_{j=1}^n \omega_{nj}}\right).$$

2.2 Basic lemmas

The following lemmas will play important roles in the proof of our main results.

Lemma 1 *Let* $\Sigma^* = \{\Sigma \in \mathbb{R}^{n^2\times n^2} : \Sigma = \operatorname{diag}(\varepsilon_{11},\dots,\varepsilon_{1n},\dots,\varepsilon_{n1},\dots,\varepsilon_{nn})$, *where* $|\varepsilon_{ij}| \le 1$, $i,j = 1,2,\dots,n\}$, *then* $\Sigma^T\Sigma \le I$. *Furthermore, let*

$$\begin{aligned}
\widetilde{C} &= \{C = C_0 + M_C\Sigma_C N_C\},\\
\widetilde{A} &= \{A = A_0 + M_A^R\Sigma_A^R N_A^R + \imath M_A^I\Sigma_A^I N_A^I + \jmath M_A^J\Sigma_A^J N_A^J + \kappa M_A^K\Sigma_A^K N_A^K\},\\
\widetilde{B} &= \{B = B_0 + M_B^R\Sigma_B^R N_B^R + \imath M_B^I\Sigma_B^I N_B^I + \jmath M_B^J\Sigma_B^J N_B^J + \kappa M_B^K\Sigma_B^K N_B^K\},
\end{aligned}$$

then for $\Sigma_C, \Sigma_A^R, \Sigma_A^I, \Sigma_A^J, \Sigma_A^K, \Sigma_B^R, \Sigma_B^I, \Sigma_B^J, \Sigma_B^K \in \Sigma^*$, $C_I = \widetilde{C}$, $A_I = \widetilde{A}$, $B_I = \widetilde{B}$.

Proof The result can easily proved through calculation directly, which is similar to the approach in [20]. So the details are omitted. □

Lemma 2 ([20]) *If* U_i, V_i *and* W_i $(i = 1,2,\dots,m)$ *are complex-valued matrices of appropriate dimension with* M *satisfying* $M^* = M$, *then*

$$M + \sum_{i=1}^{m}\bigl(U_iV_iW_i + W_i^*V_i^*U_i^*\bigr) < 0$$

for all $V_i^*V_i \le I$ $(i = 1,2,\dots,m)$, *if and only if there exist positive constants* ε_i $(i = 1,2,\dots,m)$ *such that*

$$M + \sum_{i=1}^{m}\bigl(\varepsilon_i^{-1}U_iU_i^* + \varepsilon_iW_i^*W_i\bigr) < 0.$$

Lemma 3 ([19]) *For a given Hermitian matrix, if* $S_{11}^* = S_{11}$, $S_{12}^* = S_{21}$ *and* $S_{22}^* = S_{22}$, *then*

$$S = \begin{pmatrix} S_{11} & S_{12}\\ S_{21} & S_{22}\end{pmatrix} < 0$$

is equivalent to the following conditions:

(i) $S_{22} < 0$ *and* $S_{11} - S_{12}S_{22}^{-1}S_{21} < 0$,

(ii) $S_{11} < 0$ *and* $S_{22} - S_{21}S_{11}^{-1}S_{12} < 0$.

Lemma 4 ([35]) *For any* $a,b \in \mathbb{H}^n$, *if* $P \in \mathbb{H}^{n\times n}$ *is a positive definite Hermitian matrix, then*

$$a^*b + b^*a \le a^*Pa + b^*P^{-1}b.$$

Lemma 5 ([35]) *If* $H(z)$: $\mathbb{H}^n \to \mathbb{H}^n$ *is a continuous map and satisfies the following conditions*:

(i) $H(z)$ *is injective on* $\mathbb{H}^n$,

(ii) $\lim_{\|z\|\to\infty}\|H(z)\| = \infty$,

then $H(z)$ *is a homeomorphism of* $\mathbb{H}^n$ *onto itself.*

Lemma 6 ([35]) *For any positive definite constant Hermitian matrix* $W \in \mathbb{H}^{n\times n}$ *and any scalar function* $\omega(s)$: $[a,b] \to \mathbb{H}^n$, *if the integrations concerned are well defined, then*

$$\left(\int_a^b \omega(s)\,ds\right)^* W\left(\int_a^b \omega(s)\,ds\right) \le (b-a)\int_a^b \omega^*(s)W\omega(s)\,ds.$$

Lemma 7 ([10]) *Let $A = A_1 + A_{2J}$ and $B = B_1 + B_{2J}$, where $A_1, A_2, B_1, B_2 \in \mathbb{C}^{n\times n}$ and $A, B \in \mathbb{H}^{n\times n}$, then*

(i) $A^* = A_1^* - A_{2J}^T$;

(ii) $AB = (A_1B_1 - A_2\bar{B}_2) + (A_1B_2 + A_2\bar{B}_1)_J$,

where $\bar{B}_1$ and $\bar{B}_2$ denote the conjugate matrices of B_1 and B_2, respectively.

Lemma 8 ([10]) *Let $P \in \mathbb{H}^{n\times n}$ be a Hermite matrix, $P = P_1 + P_{2j}$, then $P < 0$ is equivalent to*

$$\begin{pmatrix} P_1 & -P_2 \\ \bar{P}_2 & \bar{P}_1 \end{pmatrix} < 0,$$

where $P_1, P_2 \in \mathbb{C}^{n\times n}$ and $\bar{P}_1$, $\bar{P}_2$ denote the conjugate matrices of P_1, P_2, respectively.

3 Global robust stability results

In this section, we will present the existence and uniqueness of the equilibrium point of the delayed QVNNs on the basis of Assumptions ($\mathbf{A}_1$) and ($\mathbf{A}_2$), then we investigate the global robust stability of the equilibrium point of the delayed QVNNs.

Theorem 1 *Assume assumptions ($\mathbf{A}_1$) and ($\mathbf{A}_2$) are satisfied, then QVNNs (1) has a unique equilibrium point which is globally robust stable, if there exist a positive definite Hermitian matrix Q and four positive diagonal matrices R_i ($i = 1, 2, 3, 4$), and 27 positive constants λ_i ($i = 1, 2, \ldots, 27$) such that the following linear matrix inequality holds*:

$$\Omega = (\Omega_{ij})_{34\times 34} < 0, \tag{2}$$

where

$$\begin{aligned}
\Omega_{11} &= -QC_0 - C_0^TQ + R_1 + \delta^2R_2 + R_3 + KR_1K + \lambda_1(N_C)^TN_C \\
&\quad + \lambda_{11}(N_C)^TN_C, \qquad \Omega_{12} = QA_0, \qquad \Omega_{13} = QB_0, \qquad \Omega_{14} = C_0^TQC_0,
\end{aligned}$$

$$\Omega_{18} = QL_C, \qquad \Omega_{19} = QL_A^R, \qquad \Omega_{1.10} = QL_A^I, \qquad \Omega_{1.11} = QL_A^J, \qquad \Omega_{1.12} = QL_A^K,$$

$$\Omega_{1.13} = QL_B^R, \qquad \Omega_{1.14} = QL_B^I, \qquad \Omega_{1.15} = QL_B^J, \qquad \Omega_{1.16} = QL_B^K,$$

$$\Omega_{1.17} = C_0^TQL_C, \qquad \Omega_{1.19} = (M_C\Sigma_CN_C)^TQL_C, \qquad \Omega_{21} = (A_0)^*Q,$$

$$\begin{aligned}
\Omega_{22} &= R_4 - R_1 + \lambda_2\big(N_A^R\big)^TN_A^R + \lambda_3\big(N_A^I\big)^TN_A^I + \lambda_4\big(N_A^J\big)^TN_A^J + \lambda_5\big(N_A^K\big)^TN_A^K \\
&\quad + \lambda_{19}\big(N_A^R\big)^TN_A^R + \lambda_{20}\big(N_A^I\big)^TN_A^I + \lambda_{21}\big(N_A^J\big)^TN_A^J + \lambda_{22}\big(N_A^K\big)^TN_A^K,
\end{aligned}$$

$$\Omega_{27} = (A_0)^*Q, \qquad \Omega_{31} = (B_0)^*Q,$$

$$\begin{aligned}
\Omega_{33} &= -R_4 - R_2 + \lambda_6\big(N_B^R\big)^TN_B^R + \lambda_7\big(N_B^I\big)^TN_B^I + \lambda_8\big(N_B^J\big)^TN_B^J + \lambda_9\big(N_B^K\big)^TN_B^K \\
&\quad + \lambda_{23}\big(N_B^R\big)^TN_B^R + \lambda_{24}\big(N_B^I\big)^TN_B^I + \lambda_{25}\big(N_B^J\big)^TN_B^J + \lambda_{26}\big(N_B^K\big)^TN_B^K,
\end{aligned}$$

$$\Omega_{37} = (B_0)^*Q, \qquad \Omega_{41} = C_0^TQC_0,$$

$$\begin{aligned}
\Omega_{44} &= -R_2 + C_0^TQC_0 + \lambda_{10}(N_C)^TN_C + \lambda_{12}(N_C)^TN_C + \lambda_{13}(N_C)^TN_C \\
&\quad + \lambda_{14}(N_C)^TN_C + \lambda_{15}(N_C)^TN_C + \lambda_{17}(N_C)^TN_C,
\end{aligned}$$

$$\Omega_{45} = C_0^T QC_0, \quad \Omega_{4.18} = C_0^T QL_C, \quad \Omega_{4.20} = C_0^T QL_C, \quad \Omega_{4.21} = C_0^T QL_C,$$
$$\Omega_{4.22} = (M_C \Sigma_C N_C)^T QL_C, \quad \Omega_{4.23} = C_0^T QL_C, \quad \Omega_{4.25} = (M_C \Sigma_C N_C)^T QL_C,$$
$$\Omega_{54} = C_0^T QC_0, \quad \Omega_{55} = -R_1 + \lambda_{16}(N_C)^T N_C + \lambda_{18}(N_C)^T N_C + \lambda_{27}(N_C)^T N_C,$$
$$\Omega_{57} = C_0^T Q, \quad \Omega_{5.24} = C_0^T QL_C, \quad \Omega_{66} = -R_3 + KR_2K, \quad \Omega_{72} = QA_0,$$
$$\Omega_{73} = QB_0, \quad \Omega_{75} = QC_0, \quad \Omega_{77} = -Q^*, \quad \Omega_{7.26} = QL_A^R, \quad \Omega_{7.27} = QL_A^I,$$
$$\Omega_{7.28} = QL_A^J, \quad \Omega_{7.29} = QL_A^K, \quad \Omega_{7.30} = QL_B^R, \quad \Omega_{7.31} = QL_B^I, \quad \Omega_{7.32} = QL_B^J,$$
$$\Omega_{7.33} = QL_B^K, \quad \Omega_{7.34} = QL_C, \quad \Omega_{81} = (L_C)^T Q, \quad \Omega_{88} = -\lambda_1 I,$$
$$\Omega_{91} = \left(L_A^R\right)^T Q, \quad \Omega_{99} = -\lambda_2 I, \quad \Omega_{10.1} = \left(L_A^I\right)^T Q, \quad \Omega_{10.10} = -\lambda_3 I,$$
$$\Omega_{11.1} = \left(L_A^J\right)^T Q, \quad \Omega_{11.11} = -\lambda_4 I, \quad \Omega_{12.1} = \left(L_A^K\right)^T Q, \quad \Omega_{12.12} = -\lambda_5 I,$$
$$\Omega_{13.1} = \left(L_B^R\right)^T Q, \quad \Omega_{13.13} = -\lambda_6 I, \quad \Omega_{14.1} = \left(L_B^I\right)^T Q, \quad \Omega_{14.14} = -\lambda_7 I,$$
$$\Omega_{15.1} = \left(L_B^J\right)^T Q, \quad \Omega_{15.15} = -\lambda_8 I, \quad \Omega_{16.1} = \left(L_B^K\right)^T Q, \quad \Omega_{16.16} = -\lambda_9 I,$$
$$\Omega_{17.1} = (L_C)^T QC_0, \quad \Omega_{17.17} = -\lambda_{10} I, \quad \Omega_{18.4} = (L_C)^T QC_0, \quad \Omega_{18.18} = -\lambda_{11} I,$$
$$\Omega_{19.1} = (L_C)^T QM_C \Sigma_C N_C, \quad \Omega_{19.19} = -\lambda_{12} I, \quad \Omega_{20.4} = (L_C)^T QC_0,$$
$$\Omega_{20.20} = -\lambda_{13} I, \quad \Omega_{21.4} = (L_C)^T QC_0, \quad \Omega_{21.21} = -\lambda_{14} I,$$
$$\Omega_{22.4} = (L_C)^T QM_C \Sigma_C N_C, \quad \Omega_{22.22} = -\lambda_{15} I,$$
$$\Omega_{23.4} = (L_C)^T QC_0, \quad \Omega_{23.23} = -\lambda_{16} I, \quad \Omega_{24.5} = (L_C)^T QC_0, \quad \Omega_{24.24} = -\lambda_{17} I,$$
$$\Omega_{25.4} = (L_C)^T QM_C \Sigma_C N_C, \quad \Omega_{25.25} = -\lambda_{18} I, \quad \Omega_{26.7} = \left(L_A^R\right)^T Q,$$
$$\Omega_{26.26} = -\lambda_{19} I, \quad \Omega_{27.7} = \left(L_A^I\right)^T Q, \quad \Omega_{27.27} = -\lambda_{20} I, \quad \Omega_{28.7} = \left(L_A^J\right)^T Q,$$
$$\Omega_{28.28} = -\lambda_{21} I, \quad \Omega_{29.7} = \left(L_A^K\right)^T Q, \quad \Omega_{29.29} = -\lambda_{22} I, \quad \Omega_{30.7} = \left(L_B^R\right)^T Q,$$
$$\Omega_{30.30} = -\lambda_{23} I, \quad \Omega_{31.7} = \left(L_B^I\right)^T Q, \quad \Omega_{31.31} = -\lambda_{24} I, \quad \Omega_{32.7} = \left(L_B^J\right)^T Q,$$
$$\Omega_{32.32} = -\lambda_{25} I, \quad \Omega_{33.7} = \left(L_B^K\right)^T Q, \quad \Omega_{33.33} = -\lambda_{26} I, \quad \Omega_{34.7} = (L_C)^T Q,$$
$$\Omega_{34.34} = -\lambda_{27} I,$$

the other entries of Ω are zeros.

Proof We will prove the theorem in three steps.

Step 1: The negative definiteness of the following matrix Δ is equivalent to the negative definiteness of matrix Ω,

$$\Delta = \begin{pmatrix} \Delta_{11} & QA & QB & C^T QC & 0 & 0 & 0 \\ \star & R_4 - R_1 & 0 & 0 & 0 & 0 & A^* Q \\ \star & \star & -R_4 - R_2 & 0 & 0 & 0 & B^* Q \\ \star & \star & \star & \Delta_{44} & C^T QC & 0 & 0 \\ \star & \star & \star & \star & -R_1 & 0 & C^T Q \\ \star & \star & \star & \star & \star & \Delta_{66} & 0 \\ \star & \star & \star & \star & \star & \star & -Q^* \end{pmatrix}, \quad (3)$$

where

$$\Delta_{11} = -QC - C^TQ + R_1 + \delta^2 R_2 + R_3 + KR_1K,$$
$$\Delta_{44} = -R_2 + C^TQC, \qquad \Delta_{66} = -R_3 + KR_2K.$$

From Lemma 1, for any $C \in C_I$, $A \in A_I$, $B \in B_I$, we obtain

$$C = C_0 + M_C\Sigma_C N_C, \tag{4}$$
$$A = A_0 + M_A^R\Sigma_A^R N_A^R + \imath M_A^I\Sigma_A^I N_A^I + \jmath M_A^J\Sigma_A^J N_A^J + \kappa M_A^K\Sigma_A^K N_A^K, \tag{5}$$
$$B = B_0 + M_B^R\Sigma_B^R N_B^R + \imath M_B^I\Sigma_B^I N_B^I + \jmath M_B^J\Sigma_B^J N_B^J + \kappa M_B^K\Sigma_B^K N_B^K. \tag{6}$$

Note that, for $(\Sigma_C), (\Sigma_A^X), (\Sigma_B^X) \in \Sigma^*$, here X expresses R, I, J, K, respectively, it is to get $(\Sigma_A^X)^T\Sigma_A^X \le I$, $(\Sigma_B^X)^T\Sigma_B^X \le I$, $(\Sigma_C)^T\Sigma_C \le I$.

Taking (4), (5), (6) into (3), we obtain

$$\begin{aligned}
\Delta = {}& \begin{pmatrix}
\Delta_{11} & QA_0 & QB_0 & C_0^TQC_0 & 0 & 0 & 0 \\
\star & R_4 - R_1 & 0 & 0 & 0 & 0 & (A_0)^*Q \\
\star & \star & -R_4 - R_2 & 0 & 0 & 0 & (B_0)^*Q \\
\star & \star & \star & \Delta_{44} & C_0^TQC_0 & 0 & 0 \\
\star & \star & \star & \star & -R_1 & 0 & C_0^TQ \\
\star & \star & \star & \star & \star & \Delta_{66} & 0 \\
\star & \star & \star & \star & \star & \star & -Q^*
\end{pmatrix} \\
&+ \big((-M_C^TQ)\varepsilon_1\big)^T\Sigma_C(N_C\varepsilon_1) + (N_C\varepsilon_1)^T(\Sigma_C)^T\big((-M_C^TQ)\varepsilon_1\big) \\
&+ \big(\big((M_A^R)^TQ\big)\varepsilon_1\big)^T\Sigma_A^R\big(N_A^R\varepsilon_2\big) + \big(N_A^R\varepsilon_2\big)^T\big(\Sigma_A^R\big)^T\big(\big((M_A^R)^TQ\big)\varepsilon_1\big) \\
&+ \big(\big((\imath M_A^I)^*Q\big)\varepsilon_1\big)^*\Sigma_A^I\big(N_A^I\varepsilon_2\big) + \big(N_A^I\varepsilon_2\big)^T\big(\Sigma_A^I\big)^T\big(\big((\imath M_A^I)^*Q\big)\varepsilon_1\big) \\
&+ \big(\big((\jmath M_A^J)^*Q\big)\varepsilon_1\big)^*\Sigma_A^J\big(N_A^J\varepsilon_2\big) + \big(N_A^J\varepsilon_2\big)^T\big(\Sigma_A^J\big)^T\big(\big((\jmath M_A^J)^*Q\big)\varepsilon_1\big) \\
&+ \big(\big((\kappa M_A^K)^*Q\big)\varepsilon_1\big)^*\Sigma_A^K\big(N_A^K\varepsilon_2\big) + \big(N_A^K\varepsilon_2\big)^T\big(\Sigma_A^K\big)^T\big(\big((\kappa M_A^K)^*Q\big)\varepsilon_1\big) \\
&+ \big(\big((M_B^R)^TQ\big)\varepsilon_1\big)^T\Sigma_B^R\big(N_B^R\varepsilon_3\big) + \big(N_B^R\varepsilon_3\big)^T\big(\Sigma_B^R\big)^T\big(\big((M_B^R)^TQ\big)\varepsilon_1\big) \\
&+ \big(\big((\imath M_B^I)^*Q\big)\varepsilon_1\big)^*\Sigma_B^I\big(N_B^I\varepsilon_3\big) + \big(N_B^I\varepsilon_3\big)^T\big(\Sigma_B^I\big)^T\big(\big((\imath M_B^I)^*Q\big)\varepsilon_1\big) \\
&+ \big(\big((\jmath M_B^J)^*Q\big)\varepsilon_1\big)^*\Sigma_B^J\big(N_B^J\varepsilon_3\big) + \big(N_B^J\varepsilon_3\big)^T\big(\Sigma_B^J\big)^T\big(\big((\jmath M_B^J)^*Q\big)\varepsilon_1\big) \\
&+ \big(\big((\kappa M_B^K)^*Q\big)\varepsilon_1\big)^*\Sigma_B^K\big(N_B^K\varepsilon_3\big) + \big(N_B^K\varepsilon_3\big)^T\big(\Sigma_B^K\big)^T\big(\big((\kappa M_B^K)^*Q\big)\varepsilon_1\big) \\
&+ \big(\big((M_C)^TQC_0\big)\varepsilon_1\big)^T\Sigma_C(N_C\varepsilon_4) + (N_C\varepsilon_4)^T(\Sigma_C)^T\big(\big((M_C)^TQC_0\big)\varepsilon_1\big) \\
&+ (N_C\varepsilon_1)^T(\Sigma_C)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big) + \big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\Sigma_C(N_C\varepsilon_1) \\
&+ \big(\big((M_C)^TQM_C\Sigma_C N_C\big)\varepsilon_1\big)^T\Sigma_C(N_C\varepsilon_4) + (N_C\varepsilon_4)^T(\Sigma_C)^T\big(\big((M_C)^TQM_C\Sigma_C N_C\big)\varepsilon_1\big) \\
&+ \big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\Sigma_C(N_C\varepsilon_4) + (N_C\varepsilon_4)^T(\Sigma_C)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big) \\
&+ (N_C\varepsilon_4)^T(\Sigma_C)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big) + \big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\Sigma_C(N_C\varepsilon_4) \\
&+ \big(\big((M_C)^TQM_C\Sigma_C N_C\big)\varepsilon_4\big)^T\Sigma_C(N_C\varepsilon_4) + (N_C\varepsilon_4)^T(\Sigma_C)^T\big(\big((M_C)^TQM_C\Sigma_C N_C\big)\varepsilon_4\big)
\end{aligned}$$

$$
\begin{aligned}
&+\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\Sigma_C(N_C\varepsilon_5)+(N_C\varepsilon_5)^T(\Sigma_C)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)\\
&+(N_C\varepsilon_4)^T(\Sigma_C)^T\big(\big((M_C)^TQC_0\big)\varepsilon_5\big)+\big(\big((M_C)^TQC_0\big)\varepsilon_5\big)^T\Sigma_C(N_C\varepsilon_4)\\
&+\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_4\big)^T\Sigma_C(N_C\varepsilon_5)+(N_C\varepsilon_5)^T(\Sigma_C)^T\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_4\big)\\
&+\big(N_A^R\varepsilon_2\big)^T\big(\Sigma_A^R\big)^T\big(\big(\big(M_A^R\big)^TQ\big)\varepsilon_7\big)+\big(\big(\big(M_A^R\big)^TQ\big)\varepsilon_7\big)^T\Sigma_A^R\big(N_A^R\varepsilon_2\big)\\
&+\big(N_A^I\varepsilon_2\big)^T\big(\Sigma_A^I\big)^T\big(\big(\big(\imath M_A^I\big)^*Q\big)\varepsilon_7\big)+\big(\big(\big(\imath M_A^I\big)^*Q\big)\varepsilon_7\big)^*\Sigma_A^I\big(N_A^I\varepsilon_2\big)\\
&+\big(N_A^J\varepsilon_2\big)^T\big(\Sigma_A^J\big)^T\big(\big(\big(\jmath M_A^J\big)^*Q\big)\varepsilon_7\big)+\big(\big(\big(\jmath M_A^J\big)^*Q\big)\varepsilon_7\big)^*\Sigma_A^J\big(N_A^J\varepsilon_2\big)\\
&+\big(N_A^K\varepsilon_2\big)^T\big(\Sigma_A^K\big)^T\big(\big(\big(\kappa M_A^K\big)^*Q\big)\varepsilon_7\big)+\big(\big(\big(\kappa M_A^K\big)^*Q\big)\varepsilon_7\big)^*\Sigma_A^K\big(N_A^K\varepsilon_2\big)\\
&+\big(N_B^R\varepsilon_3\big)^T\big(\Sigma_B^R\big)^T\big(\big(\big(M_B^R\big)^TQ\big)\varepsilon_7\big)+\big(\big(\big(M_B^R\big)^TQ\big)\varepsilon_7\big)^T\Sigma_B^R\big(N_B^R\varepsilon_3\big)\\
&+\big(N_B^I\varepsilon_3\big)^T\big(\Sigma_B^I\big)^T\big(\big(\big(\imath M_B^I\big)^*Q\big)\varepsilon_7\big)+\big(\big(\big(\imath M_B^I\big)^*Q\big)\varepsilon_7\big)^*\Sigma_B^I\big(N_B^I\varepsilon_3\big)\\
&+\big(N_B^J\varepsilon_3\big)^T\big(\Sigma_B^J\big)^T\big(\big(\big(\jmath M_B^J\big)^*Q\big)\varepsilon_7\big)+\big(\big(\big(\jmath M_B^J\big)^*Q\big)\varepsilon_7\big)^*\Sigma_B^J\big(N_B^J\varepsilon_3\big)\\
&+\big(N_B^K\varepsilon_3\big)^T\big(\Sigma_B^K\big)^T\big(\big(\big(\kappa M_B^K\big)^*Q\big)\varepsilon_7\big)+\big(\big(\big(\kappa M_B^K\big)^*Q\big)\varepsilon_7\big)^*\Sigma_B^K\big(N_B^K\varepsilon_3\big)\\
&+(N_C\varepsilon_5)^T(\Sigma_C)^T\big(\big((M_C)^TQ\big)\varepsilon_7\big)+\big(\big((M_C)^TQ\big)\varepsilon_7\big)^T\Sigma_C(N_C\varepsilon_5).
\end{aligned}
$$

Here, $(X\varepsilon_i)_{n^2\times 7n}$ denotes a block matrix with the matrix $X_{n^2\times n}$ lies in the ith column, here $i=1,2,\ldots,7$,

$$
\begin{aligned}
&\Delta_{11}=-QC_0-C_0^TQ+R_1+\delta^2R_2+R_3+KR_1K,\\
&\Delta_{44}=-R_2+C_0^TQC_0,\qquad \Delta_{66}=-R_3+KR_2K.
\end{aligned}
$$

From Lemma 2 we can see that $\Delta<0$ if and only if there exist positive constants λ_i ($i=1,2,\ldots,27$) such that

$$
\begin{aligned}
\bar{\Delta}=&\begin{pmatrix}
\Delta_{11} & QA_0 & QB_0 & C_0^TQC_0 & 0 & 0 & 0\\
\star & R_4-R_1 & 0 & 0 & 0 & 0 & (A_0)^*Q\\
\star & \star & -R_4-R_2 & 0 & 0 & 0 & (B_0)^*Q\\
\star & \star & \star & -R_2+C_0^TQC_0 & C_0^TQC_0 & 0 & 0\\
\star & \star & \star & \star & -R_1 & 0 & C_0^TQ\\
\star & \star & \star & \star & \star & \Delta_{66} & 0\\
\star & \star & \star & \star & \star & \star & -Q^*
\end{pmatrix}\\
&+\lambda_1^{-1}\big(\big(-M_C^TQ\big)\varepsilon_1\big)^T\big(\big(-M_C^TQ\big)\varepsilon_1\big)+\lambda_1(N_C\varepsilon_1)^T(N_C\varepsilon_1)\\
&+\lambda_2^{-1}\big(\big(\big(M_A^R\big)^TQ\big)\varepsilon_1\big)^T\big(\big(\big(M_A^R\big)^TQ\big)\varepsilon_1\big)+\lambda_2\big(N_A^R\varepsilon_2\big)^T\big(N_A^R\varepsilon_2\big)\\
&+\lambda_3^{-1}\big(\big(\big(\imath M_A^I\big)^*Q\big)\varepsilon_1\big)^*\big(\big(\big(\imath M_A^I\big)^*Q\big)\varepsilon_1\big)+\lambda_3\big(N_A^I\varepsilon_2\big)^T\big(N_A^I\varepsilon_2\big)\\
&+\lambda_4^{-1}\big(\big(\big(\jmath M_A^J\big)^*Q\big)\varepsilon_1\big)^*\big(\big(\big(\jmath M_A^J\big)^*Q\big)\varepsilon_1\big)+\lambda_4\big(N_A^J\varepsilon_2\big)^T\big(N_A^J\varepsilon_2\big)\\
&+\lambda_5^{-1}\big(\big(\big(\kappa M_A^K\big)^*Q\big)\varepsilon_1\big)^*\big(\big(\big(\kappa M_A^K\big)^*Q\big)\varepsilon_1\big)+\lambda_5\big(N_A^K\varepsilon_2\big)^T\big(N_A^K\varepsilon_2\big)\\
&+\lambda_6^{-1}\big(\big(\big(M_B^R\big)^TQ\big)\varepsilon_1\big)^T\big(\big(\big(M_B^R\big)^TQ\big)\varepsilon_1\big)+\lambda_6\big(N_B^R\varepsilon_3\big)^T\big(N_B^R\varepsilon_3\big)\\
&+\lambda_7^{-1}\big(\big(\big(\imath M_B^I\big)^*Q\big)\varepsilon_1\big)^*\big(\big(\big(\imath M_B^I\big)^*Q\big)\varepsilon_1\big)+\lambda_7\big(N_B^I\varepsilon_3\big)^T\big(N_B^I\varepsilon_3\big)
\end{aligned}
$$

$$
\begin{aligned}
&+\lambda_8^{-1}\big(\big((\jmath M_B^J)^*Q\big)\varepsilon_1\big)^*\big(\big((\jmath M_B^J)^*Q\big)\varepsilon_1\big)+\lambda_8\big(N_B^J\varepsilon_3\big)^T\big(N_B^J\varepsilon_3\big)\\
&+\lambda_9^{-1}\big(\big((\kappa M_B^K)^*Q\big)\varepsilon_1\big)^*\big(\big((\kappa M_B^K)^*Q\big)\varepsilon_1\big)+\lambda_9\big(N_B^K\varepsilon_3\big)^T\big(N_B^K\varepsilon_3\big)\\
&+\lambda_{10}^{-1}\big(\big((M_C)^TQC_0\big)\varepsilon_1\big)^T\big(\big((M_C)^TQC_0\big)\varepsilon_1\big)+\lambda_{10}(N_C\varepsilon_4)^T(N_C\varepsilon_4)\\
&+\lambda_{11}^{-1}\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)+\lambda_{11}(N_C\varepsilon_1)^T(N_C\varepsilon_1)\\
&+\lambda_{12}^{-1}\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_1\big)^T\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_1\big)+\lambda_{12}(N_C\varepsilon_4)^T(N_C\varepsilon_4)\\
&+\lambda_{13}^{-1}\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)+\lambda_{13}(N_C\varepsilon_4)^T(N_C\varepsilon_4)\\
&+\lambda_{14}^{-1}\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)+\lambda_{14}(N_C\varepsilon_4)^T(N_C\varepsilon_4)\\
&+\lambda_{15}^{-1}\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_4\big)^T\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_4\big)+\lambda_{15}(N_C\varepsilon_4)^T(N_C\varepsilon_4)\\
&+\lambda_{16}^{-1}\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)^T\big(\big((M_C)^TQC_0\big)\varepsilon_4\big)+\lambda_{16}(N_C\varepsilon_5)^T(N_C\varepsilon_5)\\
&+\lambda_{17}^{-1}\big(\big((M_C)^TQC_0\big)\varepsilon_5\big)^T\big(\big((M_C)^TQC_0\big)\varepsilon_5\big)+\lambda_{17}(N_C\varepsilon_4)^T(N_C\varepsilon_4)\\
&+\lambda_{18}^{-1}\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_4\big)^T\big(\big((M_C)^TQM_C\Sigma_CN_C\big)\varepsilon_4\big)+\lambda_{18}(N_C\varepsilon_5)^T(N_C\varepsilon_5)\\
&+\lambda_{19}^{-1}\big(\big((M_A^R)^TQ\big)\varepsilon_7\big)^T\big(\big((M_A^R)^TQ\big)\varepsilon_7\big)+\lambda_{19}\big(N_A^R\varepsilon_2\big)^T\big(N_A^R\varepsilon_2\big)\\
&+\lambda_{20}^{-1}\big(\big((\imath M_A^I)^*Q\big)\varepsilon_7\big)^*\big(\big((\imath M_A^I)^*Q\big)\varepsilon_7\big)+\lambda_{20}\big(N_A^I\varepsilon_2\big)^T\big(N_A^I\varepsilon_2\big)\\
&+\lambda_{21}^{-1}\big(\big((\jmath M_A^J)^*Q\big)\varepsilon_7\big)^*\big(\big((\jmath M_A^J)^*Q\big)\varepsilon_7\big)+\lambda_{21}\big(N_A^J\varepsilon_2\big)^T\big(N_A^J\varepsilon_2\big)\\
&+\lambda_{22}^{-1}\big(\big((\kappa M_A^K)^*Q\big)\varepsilon_7\big)^*\big(\big((\kappa M_A^K)^*Q\big)\varepsilon_7\big)+\lambda_{22}\big(N_A^K\varepsilon_2\big)^T\big(N_A^K\varepsilon_2\big)\\
&+\lambda_{23}^{-1}\big(\big((M_B^R)^TQ\big)\varepsilon_7\big)^T\big(\big((M_B^R)^TQ\big)\varepsilon_7\big)+\lambda_{23}\big(N_B^R\varepsilon_3\big)^T\big(N_B^R\varepsilon_3\big)\\
&+\lambda_{24}^{-1}\big(\big((\imath M_B^I)^*Q\big)\varepsilon_7\big)^*\big(\big((\imath M_B^I)^*Q\big)\varepsilon_7\big)+\lambda_{24}\big(N_B^I\varepsilon_3\big)^T\big(N_B^I\varepsilon_3\big)\\
&+\lambda_{25}^{-1}\big(\big((\jmath M_B^J)^*Q\big)\varepsilon_7\big)^*\big(\big((\jmath M_B^J)^*Q\big)\varepsilon_7\big)+\lambda_{25}\big(N_B^J\varepsilon_3\big)^T\big(N_B^J\varepsilon_3\big)\\
&+\lambda_{26}^{-1}\big(\big((\kappa M_B^K)^*Q\big)\varepsilon_7\big)^*\big(\big((\kappa M_B^K)^*Q\big)\varepsilon_7\big)+\lambda_{26}\big(N_B^K\varepsilon_3\big)^T\big(N_B^K\varepsilon_3\big)\\
&+\lambda_{27}^{-1}\big(\big((M_C)^TQ\big)\varepsilon_7\big)^T\big(\big((M_C)^TQ\big)\varepsilon_7\big)+\lambda_{27}(N_C\varepsilon_5)^T(N_C\varepsilon_5)\\
&<0. \qquad (7)
\end{aligned}
$$

Rewrite (7) in the form of Lemma 3 and by direct calculation, we have $\bar{\Delta} = S_{11} - S_{12}S_{22}^{-1}S_{21}$, on the other hand $\Omega = \left(\begin{smallmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{smallmatrix}\right)$. It is obvious that $S_{22} < 0$. Here, $(S_{11})_{7n\times 7n}$, $(S_{12})_{7n\times 27n}$, $(S_{21})_{27n\times 7n}$, $(S_{22})_{27n\times 27n}$ are the sub-block matrices of the judging matrix Ω. We find that (7) is equivalent to (2) by Lemma 3. Because $\bar{\Delta} < 0$ is equivalent to $\Omega < 0$, thus, $\Delta < 0$ and $\Omega < 0$ are equivalent.

Step 2: We will show system (1) has a unique equilibrium point under the condition $\Delta < 0$. Let $\tilde{q}$ be an equilibrium point of the system, then $\tilde{q}$ satisfies

$$-C\tilde{q} + Af(\tilde{q}) + Bf(\tilde{q}) + J = 0.$$

Let $\mathcal{F}(q) = -Cq + Af(q) + Bf(q) + J$. In the following, we prove the map $\mathcal{F} : \mathbb{H}^n \to \mathbb{H}^n$ is a homeomorphism map.

Firstly, we prove that $\mathcal{F}(q)$ is an injective map on $\mathbb{H}^n$.

Supposing there exist $q_1, q_2 \in \mathbb{H}^n$ with $q_1 \neq q_2$ such that $\mathcal{F}(q_1) = \mathcal{F}(q_2)$, then we can get

$$-C(q_1 - q_2) + A\bigl(f(q_1) - f(q_2)\bigr) + B\bigl(f(q_1) - f(q_2)\bigr) = 0. \tag{8}$$

Left-multiplying both sides of (8) by $(q_1 - q_2)^* Q$ leads to

$$\begin{aligned} &-(q_1 - q_2)^* QC(q_1 - q_2) + (q_1 - q_2)^* QA\bigl(f(q_1) - f(q_2)\bigr) \\ &\quad + (q_1 - q_2)^* QB\bigl(f(q_1) - f(q_2)\bigr) = 0. \end{aligned} \tag{9}$$

Taking the conjugate transpose on both sides of (9), we have

$$\begin{aligned} &-(q_1 - q_2)^* C^* Q(q_1 - q_2) + \bigl(f(q_1) - f(q_2)\bigr)^* A^* Q(q_1 - q_2) \\ &\quad + \bigl(f(q_1) - f(q_2)\bigr)^* B^* Q(q_1 - q_2) = 0. \end{aligned} \tag{10}$$

Summing (9) and (10) results in

$$\begin{aligned} 0 &= (q_1 - q_2)^* \bigl(-QC - C^* Q\bigr)(q_1 - q_2) + (q_1 - q_2)^* QA\bigl(f(q_1) - f(q_2)\bigr) \\ &\quad + \bigl(f(q_1) - f(q_2)\bigr)^* A^* Q(q_1 - q_2) + (q_1 - q_2)^* QB\bigl(f(q_1) - f(q_2)\bigr) \\ &\quad + \bigl(f(q_1) - f(q_2)\bigr)^* B^* Q(q_1 - q_2). \end{aligned} \tag{11}$$

From $\Delta < 0$, we see

$$R_4 - R_1 < 0, \qquad -R_1 < 0, \qquad -\delta^2 R_2 < 0, \qquad -R_3 + KR_2K < 0, \tag{12}$$

$$\begin{pmatrix} -QC - C^T Q + R_1 + \delta^2 R_2 + R_3 + KR_1K & QA & QB \\ \star & R_4 - R_1 & 0 \\ \star & 0 & -R_4 - R_2 \end{pmatrix} < 0. \tag{13}$$

Hence, from (12) (13) we can get

$$\begin{pmatrix} -QC - C^T Q + K(R_1 + R_2)K & QA & QB \\ \star & R_4 - R_1 & 0 \\ \star & 0 & -R_4 - R_2 \end{pmatrix} < 0.$$

By Lemma 3, we obtain

$$-QC - C^* Q + QA(R_1 - R_4)^{-1} A^* Q + QB(R_2 + R_4)^{-1} B^* Q + K(R_1 + R_2)K < 0. \tag{14}$$

Considering equality (11) and the negative definite quality of $R_4 - R_1$ and $R_2 + R_4$, it is easy to obtain the following inequality by Lemma 4:

$$\begin{aligned} 0 &\leq (q_1 - q_2)^* \bigl[-QC - C^* Q + QA(R_1 - R_4)^{-1} A^* Q + QB(R_2 + R_4)^{-1} B^* Q\bigr] \\ &\quad \times (q_1 - q_2) + \bigl(f(q_1) - f(q_2)\bigr)^* (R_1 + R_2)\bigl(f(q_1) - f(q_2)\bigr). \end{aligned} \tag{15}$$

Since R_1 *and* R_2 is real-valued positive diagonal matrix, we can see from Assumptions $(\mathbf{A}_1)$ that

$$\big(f(q_1)-f(q_2)\big)^*(R_1+R_2)\big(f(q_1)-f(q_2)\big) \le (q_1-q_2)^*K(R_1+R_2)K(q_1-q_2). \tag{16}$$

Then, by (15) and (16), we get

$$\begin{aligned} 0 \le{}& (q_1-q_2)^*\big[-QC-C^*Q+QA(R_1-R_4)^{-1}A^*Q+QB(R_2+R_4)^{-1}B^*Q \\ &+K(R_1+R_2)K\big](q_1-q_2). \end{aligned} \tag{17}$$

From inequalities (14) and (17), we can get $q_1=q_2$, which contradicts the supposed condition. Hence, $\mathcal{F}(q)$ is an injective map on $\mathbb{H}^n$.

Secondly, we prove that $\|\mathcal{F}(q)\| \to +\infty$ as $\|q\| \to +\infty$.

From the definition of $\mathcal{F}(q)$, we can get

$$\mathcal{F}(q)-\mathcal{F}(0) = -Cq + A\big(f(q)-f(0)\big) + B\big(f(q)-f(0)\big). \tag{18}$$

Left-multiplying both sides of (18) by q^*Q brings about

$$q^*Q\big(\mathcal{F}(q)-\mathcal{F}(0)\big) = -q^*QCq + q^*QA\big(f(q)-f(0)\big) + q^*QB\big(f(q)-f(0)\big). \tag{19}$$

Taking the conjugate transpose on equality (19), we have

$$\big(\mathcal{F}(q)-\mathcal{F}(0)\big)^*Qq = -q^*C^*Qq + \big(f(q)-f(0)\big)^*A^*Qq + \big(f(q)-f(0)\big)^*B^*Qq. \tag{20}$$

Summing (19) and (20) one derives that

$$\begin{aligned} q^*Q\big(\mathcal{F}(q)-\mathcal{F}(0)\big) + \big(\mathcal{F}(q)-\mathcal{F}(0)\big)^*Qq ={}& q^*\big(-QC-C^*Q\big)q \\ &+ q^*QA\big(f(q)-f(0)\big) + \big(f(q)-f(0)\big)^*A^*Qq \\ &+ q^*QB\big(f(q)-f(0)\big) + \big(f(q)-f(0)\big)^*B^*Qq. \end{aligned}$$

Similar to the proof of the injective map, we obtain

$$\begin{aligned} &q^*Q\big(\mathcal{F}(q)-\mathcal{F}(0)\big) + \big(\mathcal{F}(q)-\mathcal{F}(0)\big)^*Qq \\ &\quad \le q^*\big[-QC-C^*Q+QA(R_1-R_4)^{-1} \\ &\qquad \times A^*Q+QB(R_2+R_4)^{-1}B^*Q+K(R_1+R_2)K\big]q \\ &\quad \le -\lambda_{\min}(-\Lambda)\|q\|^2, \end{aligned}$$

where

$$\Lambda = -QC-C^*Q+QA(R_1-R_4)^{-1}A^*Q+QB(R_2+R_4)^{-1}B^*Q+K(R_1+R_2) < 0.$$

Applying the Cauchy–Schwarz inequality brings about

$$\lambda_{\min}(-\Lambda)\|q\|^2 \le -q^*Q\big(\mathcal{F}(q)-\mathcal{F}(0)\big) + \big(\mathcal{F}(q)-\mathcal{F}(0)\big)^*Qq$$

$$
\begin{aligned}
&= -2\,\mathrm{Re}\big(q^* Q\big(\mathcal{F}(q) - \mathcal{F}(0)\big)\big) \\
&\le 2\big|q^* Q\big(\mathcal{F}(q) - \mathcal{F}(0)\big)\big| \\
&\le 2\|q\| \cdot \|Q\| \cdot \big\|\mathcal{F}(q) - \mathcal{F}(0)\big\| \\
&\le 2\|q\| \cdot \|Q\| \cdot \big(\big\|\mathcal{F}(q)\big\| + \big\|\mathcal{F}(0)\big\|\big).
\end{aligned} \tag{21}
$$

So, $\|\mathcal{F}(q)\| \to +\infty$ as $\|q\| \to +\infty$, by Lemma 5, we know that $\mathcal{F}(q)$ is a homeomorphism of $\mathbb{H}^n$. Therefore, system (1) has a unique equilibrium point.

Step 3: We will analyze the globally asymptotic robust stability of the equilibrium point.

Let $z(t) = q(t) - \tilde{q}$, then system (1) can be rewritten as

$$
\dot{z}(t) = -Cz(t-\delta) + Ah\big(z(t)\big) + Bh\big(z(t-\tau)\big), \tag{22}
$$

where $h(z(t)) = f(q(t)) - f(\tilde{q})$ *and* $h(z(t-\tau)) = f(q(t-\tau)) - f(\tilde{q})$.

Considering the following Lyapunov function:

$$
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t),
$$

where

$$
V_1(t) = \left(z(t) - C\int_{t-\delta}^{t} z(s)\,ds\right)^* Q\left(z(t) - C\int_{t-\delta}^{t} z(s)\,ds\right), \tag{23}
$$

$$
V_2(t) = \int_{t-\delta}^{t} z^*(s)R_1 z(s)\,ds, \tag{24}
$$

$$
V_3(t) = \delta\int_0^{\delta}\int_{t-u}^{t} z^*(s)R_2 z(s)\,ds\,du, \tag{25}
$$

$$
V_4(t) = \int_{t-\tau}^{t} z^*(s)R_3 z(s)\,ds, \tag{26}
$$

$$
V_5(t) = \int_{t-\tau}^{t} h^*\big(z(s)\big)R_4 h\big(z(s)\big)\,ds. \tag{27}
$$

Calculating the time-derivative of $V_1(t)$, $V_2(t)$, $V_3(t)$, $V_4(t)$ and $V_5(t)$, we can get

$$
\begin{aligned}
\dot{V}_1(t) = {} & \left(z(t) - C\int_{t-\delta}^{t} z(s)\,ds\right)^* Q\big(\dot{z}(t) - Cz(t) + Cz(t-\delta)\big) \\
& + \big(\dot{z}(t) - Cz(t) + Cz(t-\delta)\big)^* Q\left(z(t) - C\int_{t-\delta}^{t} z(s)\,ds\right) \\
= {} & -z^*(t)\big(QC + C^T Q\big)z(t) + z^*(t)QAh\big(z(t)\big) + h^*\big(z(t)\big)A^*Qz(t) \\
& + z^*(t)QBh\big(z(t-\tau)\big) + h^*\big(z(t-\tau)\big)B^*Qz(t) + \left(\int_{t-\delta}^{t} z(s)\,ds\right)^* C^T QCz(t) \\
& + z^*(t)C^T QC\left(\int_{t-\delta}^{t} z(s)\,ds\right) - \left(\int_{t-\delta}^{t} z(s)\,ds\right)^* C^T QAh\big(z(t)\big) \\
& - h^*\big(z(t)\big)A^*QC\left(\int_{t-\delta}^{t} z(s)\,ds\right) - \left(\int_{t-\delta}^{t} z(s)\,ds\right)^* C^T QBh\big(z(t-\tau)\big) \\
& - h^*\big(z(t-\tau)\big)B^*QC\left(\int_{t-\delta}^{t} z(s)\,ds\right),
\end{aligned} \tag{28}
$$

$$\begin{aligned}
\dot{V}_2(t) &= z^*(t)R_1 z(t) - z^*(t-\delta)R_1 z(t-\delta), \qquad (29)\\
\dot{V}_3(t) &= \delta^2 z^*(t)R_2 z(t) - \delta \int_0^\delta z^*(t-u)R_2 z(t-u)\,du \\
&= \delta^2 z^*(t)R_2 z(t) - \delta \int_{t-\delta}^t z^*(s)R_2 z(s)\,ds.
\end{aligned}$$

By Lemma 6, enlarging the equation $\dot{V}_3(t)$

$$\dot{V}_3(t) \le \delta^2 z^*(t)R_2 z(t) - \left(\int_{t-\delta}^t z(s)\,ds\right)^* R_2 \left(\int_{t-\delta}^t z(s)\,ds\right), \tag{30}$$

$$\dot{V}_4(t) = z^*(t)R_3 z(t) - z^*(t-\tau)R_3 z(t-\tau), \tag{31}$$

$$\dot{V}_5(t) = h^*\big(z(t)\big)R_4 h\big(z(t)\big) - h^*\big(z(t-\tau)\big)R_4 h\big(z(t-\tau)\big). \tag{32}$$

Further, for real-valued positive diagonal matrices R_1 *and* R_2, we can get from Assumption $(\mathbf{A}_1)$

$$0 \le z^*(t)KR_1Kz(t) - h^*\big(z(t)\big)R_1 h\big(z(t)\big), \tag{33}$$

$$0 \le z^*(t-\tau)KR_2Kz(t-\tau) - h^*\big(z(t-\tau)\big)R_2 h\big(z(t-\tau)\big). \tag{34}$$

From system (22), we obtain

$$\begin{aligned}
0 &= \left[Q\dot{z}(t) + QC\int_{t-\delta}^t z(s)\,ds\right]^* \left[-\dot{z}(t) - Cz(t-\delta) + Ah\big(z(t)\big) + Bh\big(z(t-\tau)\big)\right] \\
&\quad + \left[-\dot{z}(t) - Cz(t-\delta) + Ah\big(z(t)\big) + Bh\big(z(t-\tau)\big)\right]^* \left[Q\dot{z}(t) + QC\int_{t-\delta}^t z(s)\,ds\right] \\
&= \dot{z}^*(t)\big(-Q^*\big)\dot{z}(t) - \dot{z}^*(t)Q^*Cz(t-\delta) + \dot{z}^*(t)Q^*Ah\big(z(t)\big) + \dot{z}^*(t)Q^*Bh\big(z(t-\tau)\big) \\
&\quad - \left(\int_{t-\delta}^t z(s)\,ds\right)^* C^T Q\dot{z}(t) - \left(\int_{t-\delta}^t z(s)\,ds\right)^* C^T QCz(t-\delta) \\
&\quad + \left(\int_{t-\delta}^t z(s)\,ds\right)^* C^T QAh\big(z(t)\big) \\
&\quad + \left(\int_{t-\delta}^t z(s)\,ds\right)^* C^T QBh\big(z(t-\tau)\big) + \dot{z}^*(t)(-Q)\dot{z}(t) - z^*(t-\delta)C^T Q\dot{z}(t) \\
&\quad + h^*\big(z(t)\big)A^*Q\dot{z}(t) + h^*\big(z(t-\tau)\big)B^*Q\dot{z}(t) - \dot{z}^*(t)QC\left(\int_{t-\delta}^t z(s)\,ds\right) \\
&\quad - z^*(t-\delta)C^T QC\left(\int_{t-\delta}^t z(s)\,ds\right) + h^*\big(z(t)\big)A^*QC\left(\int_{t-\delta}^t z(s)\,ds\right) \\
&\quad + h^*\big(z(t-\tau)\big)B^*QC\left(\int_{t-\delta}^t z(s)\,ds\right).
\end{aligned} \tag{35}$$

It follows from (28) to (35) that

$$\begin{aligned}
\dot{V}(t) &\le -z^*(t)\big(QC + C^T Q\big)z(t) + z^*(t)QAh\big(z(t)\big) + h^*\big(z(t)\big)A^*Qz(t) \\
&\quad + z^*(t)QBh\big(z(t-\tau)\big) + h^*\big(z(t-\tau)\big)B^*Qz(t)
\end{aligned}$$

$$
\begin{aligned}
&+ \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q C z(t) + z^{*}(t) C^{T} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
&- \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q A h\bigl(z(t)\bigr) - h^{*}\bigl(z(t)\bigr) A^{*} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
&- \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q B h\bigl(z(t-\tau)\bigr) - h^{*}\bigl(z(t-\tau)\bigr) B^{*} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
&+ z^{*}(t) R_1 z(t) - z^{*}(t-\delta) R_1 z(t-\delta) \\
&+ \delta^2 z^{*}(t) R_2 z(t) - \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} R_2 \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
&+ z^{*}(t) R_3 z(t) - z^{*}(t-\tau) R_3 z(t-\tau) \\
&+ h^{*}\bigl(z(t)\bigr) R_4 h\bigl(z(t)\bigr) - h^{*}\bigl(z(t-\tau)\bigr) R_4 h\bigl(z(t-\tau)\bigr) \\
&+ z^{*}(t) K R_1 K z(t) - h^{*}\bigl(z(t)\bigr) R_1 h\bigl(z(t)\bigr) \\
&+ z^{*}(t-\tau) K R_2 K z(t-\tau) - h^{*}\bigl(z(t-\tau)\bigr) R_2 h\bigl(z(t-\tau)\bigr) \\
&+ \dot{z}^{*}(t)\bigl(-Q^{*}\bigr)\dot{z}(t) - \dot{z}^{*}(t) Q C z(t-\delta) + \dot{z}^{*}(t) Q^{*} A h\bigl(z(t)\bigr) \\
&+ \dot{z}^{*}(t) Q^{*} B h\bigl(z(t-\tau)\bigr) - \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q \dot{z}(t) \\
&- \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q C z(t-\delta) + \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q A h\bigl(z(t)\bigr) \\
&+ \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q B h\bigl(z(t-\tau)\bigr) + \dot{z}^{*}(t)(-Q)\dot{z}(t) - z^{*}(t-\delta) C^{T} Q \dot{z}(t) \\
&+ h^{*}\bigl(z(t)\bigr) A^{*} Q \dot{z}(t) + h^{*}\bigl(z(t-\tau)\bigr) B^{*} Q \dot{z}(t) - \dot{z}^{*}(t) Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
&- z^{*}(t-\delta) C^{T} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) + h^{*}\bigl(z(t)\bigr) A^{*} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
&+ h^{*}\bigl(z(t-\tau)\bigr) B^{*} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) \\
={}& z^{*}(t)\bigl(-QC - C^{T}Q + R_1 + \delta^2 R_2 + R_3 + K R_1 K\bigr) z(t) \\
&+ z^{*}(t) Q A h\bigl(z(t)\bigr) + h^{*}\bigl(z(t)\bigr) A^{*} Q z(t) \\
&+ z^{*}(t) Q B h\bigl(z(t-\tau)\bigr) + h^{*}\bigl(z(t-\tau)\bigr) B^{*} Q z(t) \\
&+ z^{*}(t) C^{T} Q C \left(\int_{t-\delta}^{t} z(s)\,ds\right) + \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} C^{T} Q C z(t) \\
&+ h^{*}\bigl(z(t)\bigr)(R_4 - R_1) h\bigl(z(t)\bigr) \\
&+ h^{*}\bigl(z(t)\bigr) A^{*} Q \dot{z}(t) + \dot{z}^{*}(t) Q A h\bigl(z(t)\bigr) \\
&+ h^{*}\bigl(z(t-\tau)\bigr)(-R_4 - R_2) h\bigl(z(t-\tau)\bigr) \\
&+ h^{*}\bigl(z(t-\tau)\bigr) B^{*} Q \dot{z}(t) + \dot{z}^{*}(t) Q B h\bigl(z(t-\tau)\bigr) \\
&+ \left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*} (-R_2) \left(\int_{t-\delta}^{t} z(s)\,ds\right)
\end{aligned}
$$

$$\begin{aligned}
&+\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}\left(C^{T}QC\right)\left(-z(t-\delta)\right)+\left(-z^{*}(t-\delta)\right)\left(C^{T}QC\right)\left(\int_{t-\delta}^{t} z(s)\,ds\right)\\
&+\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}\left(-C^{T}Q\right)\left(\dot{z}(t)\right)+\left(\dot{z}^{*}(t)\right)(-QC)\left(\int_{t-\delta}^{t} z(s)\,ds\right)\\
&+\left(-z^{*}(t-\delta)\right)(-R_1)\left(-z(t-\delta)\right)\\
&+\left(-z^{*}(t-\delta)\right)\left(C^{T}Q\right)\dot{z}(t)+\dot{z}^{*}(t)(QC)\left(-z(t-\delta)\right)\\
&+z^{*}(t-\tau)(-R_3+KR_2K)z(t-\tau)\\
&+\dot{z}^{*}(t)\left(-Q^{*}-Q\right)\dot{z}(t).
\end{aligned}\tag{36}$$

By Lemma 4, we can deduce that

$$\begin{aligned}
&\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}\left(-C^{T}Q\right)\left(\dot{z}(t)\right)+\left(\dot{z}^{*}(t)\right)(-QC)\left(\int_{t-\delta}^{t} z(s)\,ds\right)\\
&\quad\le\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}\left(C^{T}QC\right)\left(\int_{t-\delta}^{t} z(s)\,ds\right)+\dot{z}^{*}(t)Q\dot{z}(t).
\end{aligned}\tag{37}$$

Combining (37) with (36), we have

$$\begin{aligned}
\dot{V}(t)\le{}& z^{*}(t)\left(-QC-C^{T}Q+R_1+\delta^{2}R_2+R_3+KR_1K\right)z(t)\\
&+z^{*}(t)QAh\left(z(t)\right)+h^{*}\left(z(t)\right)A^{*}Qz(t)\\
&+z^{*}(t)QBh\left(z(t-\tau)\right)+h^{*}\left(z(t-\tau)\right)B^{*}Qz(t)\\
&+z^{*}(t)C^{T}QC\left(\int_{t-\delta}^{t} z(s)\,ds\right)+\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}C^{T}QCz(t)\\
&+h^{*}\left(z(t)\right)(R_4-R_1)h\left(z(t)\right)\\
&+h^{*}\left(z(t)\right)A^{*}Q\dot{z}(t)+\dot{z}^{*}(t)QAh\left(z(t)\right)\\
&+h^{*}\left(z(t-\tau)\right)(-R_4-R_2)h\left(z(t-\tau)\right)\\
&+h^{*}\left(z(t-\tau)\right)B^{*}Q\dot{z}(t)+\dot{z}^{*}(t)QBh\left(z(t-\tau)\right)\\
&+\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}(-R_2)\left(\int_{t-\delta}^{t} z(s)\,ds\right)+\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}\left(C^{T}QC\right)\left(\int_{t-\delta}^{t} z(s)\,ds\right)\\
&+\left(\int_{t-\delta}^{t} z(s)\,ds\right)^{*}\left(C^{T}QC\right)\left(-z(t-\delta)\right)+\left(-z^{*}(t-\delta)\right)\left(C^{T}QC\right)\left(\int_{t-\delta}^{t} z(s)\,ds\right)\\
&+\left(-z^{*}(t-\delta)\right)(-R_1)\left(-z(t-\delta)\right)\\
&+\left(-z^{*}(t-\delta)\right)\left(C^{T}Q\right)\dot{z}(t)+\dot{z}^{*}(t)(QC)\left(-z(t-\delta)\right)\\
&+z^{*}(t-\tau)(-R_3+KR_2K)z(t-\tau)\\
&+\dot{z}^{*}(t)\left(-Q^{*}\right)\dot{z}(t)\\
={}&\xi^{*}(t)\Delta\xi(t),
\end{aligned}\tag{38}$$

where $\xi^{*}(t)=[z^{*}(t),h^{*}(z(t)),h^{*}(z(t-\tau)),(\int_{t-\delta}^{t} z(s)\,ds)^{*},(-z^{*}(t-\delta)),z^{*}(t-\tau),\dot{z}^{*}(t)]$.

Obviously, $\dot{V}(t)\le 0$ since $\Delta<0$.

By making use of Lyapunov theory, we derive that the unique equilibrium point $\tilde{q}$ in system (1) is globally robust stable.

The proof is completed. □

Remark 1 In [20], the authors discussed the robust stability of complex-valued neural networks with interval parameter uncertainties and obtained the existence, uniqueness and global robust stability of the equilibrium, we add the leak delay, extend the model to quaternion-valued neural networks and obtain the corresponding results under the relatively weak conditions. Namely, in system (1), when $\delta = 0$, $q(t) \in C^n$ and $C, A, B, J \in C^{n\times n}$, it is easy to get the relevant conclusions of the literature [20] by our Theorem 1.

Remark 2 In the literature [35], the negative definiteness of two judging matrices is needed, moreover, the entries in these matrices are all the maximal values, which rely on resolving of the upper and lower bounds of elements for the connection weight matrix, the sign of connection weight matrix is ignored, but in our article, the given elements depend on not only the lower bounds but also the upper bounds of the interval parameters, which is different from previous contributions and extends the relevant work in Refs. [19, 20, 22, 35].

Since LMI (2) is defined in $\mathbb{H}^{n\times n}$, we cannot handle it directly using the Matlab LMI toolbox, in order to simulate our results, it is an effective way to turn the quaternion LMI into a complex one. In the following, the following corollary is required theoretically, which can easily be obtained by Theorem 1 of our article.

Expressing the parameters as pairs of complex parts, $A_0^R + \imath A_0^R + \jmath A_0^J + \kappa A_0^K = A_0^R + \imath A_0^R + (A_0^J + \imath A_0^K)\jmath = A_1 + A_{2\jmath}$ and $B_0^R + \imath B_0^R + \jmath B_0^J + \kappa B_0^K = B_0^R + \imath B_0^R + (B_0^J + \imath B_0^K)\jmath = B_1 + B_{2\jmath}$, where $A_1, A_2, B_1, B_2 \in \mathbb{C}^{n\times n}$, then we have the following result.

Corollary 1 *Supposing assumptions* $(\mathbf{A}_1)$ *and* $(\mathbf{A}_2)$ *valid, then system* (1) *is globally robust stable, if there exist a positive definite Hermitian matrix* $Q_1 \in \mathbb{C}^{n\times n}$, *a skew-symmetric matrix* $Q_2 \in \mathbb{C}^{n\times n}$, *four real positive diagonal matrices* $R_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2, 3, 4)$, 27 *positive constants* λ_i $(i = 1, 2, \ldots, 27)$, *such that the following CVLMIs hold*:

$$\begin{pmatrix} Q_1 & -Q_2 \\ \bar{Q}_2 & \bar{Q}_1 \end{pmatrix} > 0, \qquad \begin{pmatrix} \Omega_1 & -\Omega_2 \\ \bar{\Omega}_2 & \bar{\Omega}_1 \end{pmatrix} < 0, \tag{39}$$

where Ω_1 *and* Ω_2 *are defined in* (40) *and* (41),

$$\Omega_1 = (\Omega_{ij})_{34\times 34}, \tag{40}$$

where

$$\Omega_{11} = -Q_1 C_0 - C_0^T Q_1 + R_1 + \delta^2 R_2 + R_3 + K R_1 K + \lambda_1 (N_C)^T N_C + \lambda_{11} (N_C)^T N_C,$$

$$\Omega_{12} = Q_1 A_1 - Q_2 \bar{A}_2, \qquad \Omega_{13} = Q_1 B_1 - Q_2 \bar{B}_2, \qquad \Omega_{14} = C_0^T Q_1 C_0, \qquad \Omega_{18} = Q_1 L_C,$$

$$\Omega_{19} = Q_1 L_A^R, \qquad \Omega_{1.10} = Q_1 L_A^I, \qquad \Omega_{1.11} = Q_1 L_A^J, \qquad \Omega_{1.12} = Q_1 L_A^K,$$

$$\Omega_{1.13} = Q_1 L_B^R, \qquad \Omega_{1.14} = Q_1 L_B^I, \qquad \Omega_{1.15} = Q_1 L_B^J, \qquad \Omega_{1.16} = Q_1 L_B^K,$$

$$\Omega_{1.17} = C_0^T Q_1 L_C, \qquad \Omega_{1.19} = (M_C \Sigma_C N_C)^T Q_1 L_C, \qquad \Omega_{21} = A_1^* Q_1^* - A_2^T Q_2^*,$$

$$\begin{aligned}\Omega_{22} = {}& R_4 - R_1 + \lambda_2 \left(N_A^R\right)^T N_A^R + \lambda_3 \left(N_A^I\right)^T N_A^I + \lambda_4 \left(N_A^J\right)^T N_A^J + \lambda_5 \left(N_A^K\right)^T N_A^K \\ & + \lambda_{19} \left(N_A^R\right)^T N_A^R + \lambda_{20} \left(N_A^I\right)^T N_A^I + \lambda_{21} \left(N_A^J\right)^T N_A^J + \lambda_{22} \left(N_A^K\right)^T N_A^K,\end{aligned}$$

$$\Omega_{27} = A_1^* Q_1 + A_2^T \bar{Q}_2, \qquad \Omega_{31} = B_1^* Q_1^* - B_2^T Q_2^*,$$

$$\begin{aligned}\Omega_{33} = {}& -R_4 - R_2 + \lambda_6 \left(N_B^R\right)^T N_B^R + \lambda_7 \left(N_B^I\right)^T N_B^I + \lambda_8 \left(N_B^J\right)^T N_B^J + \lambda_9 \left(N_B^K\right)^T N_B^K \\ & + \lambda_{23} \left(N_B^R\right)^T N_B^R + \lambda_{24} \left(N_B^I\right)^T N_B^I + \lambda_{25} \left(N_B^J\right)^T N_B^J + \lambda_{26} \left(N_B^K\right)^T N_B^K,\end{aligned}$$

$$\Omega_{37} = B_1^* Q_1 + B_2^T \bar{Q}_2, \qquad \Omega_{41} = C_0^T Q_1^* C_0, \qquad \Omega_{44} = -R_2 + C_0^T Q_1 C_0,$$

$$\Omega_{45} = C_0^T Q_1 C_0, \qquad \Omega_{4.18} = C_0^T Q_1 L_C, \qquad \Omega_{4.20} = C_0^T Q_1 L_C, \qquad \Omega_{4.21} = C_0^T Q_1 L_C,$$

$$\Omega_{4.22} = (M_C \Sigma_C N_C)^T Q_1 L_C, \qquad \Omega_{4.23} = C_0^T Q_1 L_C, \qquad \Omega_{4.25} = (M_C \Sigma_C N_C)^T Q_1 L_C,$$

$$\Omega_{54} = C_0^T Q_1^* C_0, \qquad \Omega_{55} = -R_1 + \lambda_{16} (N_C)^T N_C + \lambda_{18} (N_C)^T N_C + \lambda_{27} (N_C)^T N_C,$$

$$\Omega_{57} = C_0^T Q_1, \qquad \Omega_{5.24} = C_0^T Q_1 L_C, \qquad \Omega_{66} = -R_3 + K R_2 K, \qquad \Omega_{72} = Q_1^* A_1 + Q_2^T \bar{A}_2,$$

$$\Omega_{73} = Q_1^* B_1 + Q_2^T \bar{B}_2, \qquad \Omega_{75} = Q_1^* C_0, \qquad \Omega_{77} = -Q_1^*, \qquad \Omega_{7.26} = Q_1 L_A^R,$$

$$\Omega_{7.27} = Q_1 L_A^I, \qquad \Omega_{7.28} = Q_1 L_A^J, \qquad \Omega_{7.29} = Q_1 L_A^K, \qquad \Omega_{7.30} = Q_1 L_B^R,$$

$$\Omega_{7.31} = Q_1 L_B^I, \qquad \Omega_{7.32} = Q_1 L_B^J, \qquad \Omega_{7.33} = Q_1 L_B^K, \qquad \Omega_{7.34} = Q_1 L_C,$$

$$\Omega_{81} = (L_C)^T Q_1^*, \qquad \Omega_{88} = -\lambda_1 I, \qquad \Omega_{91} = \left(L_A^R\right)^T Q_1^*, \qquad \Omega_{99} = -\lambda_2 I,$$

$$\Omega_{10.1} = \left(L_A^I\right)^T Q_1^*, \qquad \Omega_{10.10} = -\lambda_3 I, \qquad \Omega_{11.1} = \left(L_A^J\right)^T Q_1^*, \qquad \Omega_{11.11} = -\lambda_4 I,$$

$$\Omega_{12.1} = \left(L_A^K\right)^T Q_1^*, \qquad \Omega_{12.12} = -\lambda_5 I, \qquad \Omega_{13.1} = \left(L_B^R\right)^T Q_1^*, \qquad \Omega_{13.13} = -\lambda_6 I,$$

$$\Omega_{14.1} = \left(L_B^I\right)^T Q_1^*, \qquad \Omega_{14.14} = -\lambda_7 I, \qquad \Omega_{15.1} = \left(L_B^J\right)^T Q_1^*, \qquad \Omega_{15.15} = -\lambda_8 I,$$

$$\Omega_{16.1} = \left(L_B^K\right)^T Q_1^*, \qquad \Omega_{16.16} = -\lambda_9 I, \qquad \Omega_{17.1} = (L_C)^T Q_1^* C_0, \qquad \Omega_{17.17} = -\lambda_{10} I,$$

$$\Omega_{18.4} = (L_C)^T Q_1^* C_0, \qquad \Omega_{18.18} = -\lambda_{11} I, \qquad \Omega_{19.1} = (L_C)^T Q_1^* M_C \Sigma_C N_C,$$

$$\Omega_{19.19} = -\lambda_{12} I, \qquad \Omega_{20.4} = (L_C)^T Q_1^* C_0, \qquad \Omega_{20.20} = -\lambda_{13} I, \qquad \Omega_{21.4} = (L_C)^T Q_1^* C_0,$$

$$\Omega_{21.21} = -\lambda_{14} I, \qquad \Omega_{22.4} = (L_C)^T Q_1^* M_C \Sigma_C N_C, \qquad \Omega_{22.22} = -\lambda_{15} I,$$

$$\Omega_{23.4} = (L_C)^T Q_1^* C_0, \qquad \Omega_{23.23} = -\lambda_{16} I, \qquad \Omega_{24.5} = (L_C)^T Q_1^* C_0, \qquad \Omega_{24.24} = -\lambda_{17} I,$$

$$\Omega_{25.4} = (L_C)^T Q_1^* M_C \Sigma_C N_C, \qquad \Omega_{25.25} = -\lambda_{18} I, \qquad \Omega_{26.7} = \left(L_A^R\right)^T Q_1^*,$$

$$\Omega_{26.26} = -\lambda_{19} I, \qquad \Omega_{27.7} = \left(L_A^I\right)^T Q_1^*, \qquad \Omega_{27.27} = -\lambda_{20} I, \qquad \Omega_{28.7} = \left(L_A^J\right)^T Q_1^*,$$

$$\Omega_{28.28} = -\lambda_{21} I, \qquad \Omega_{29.7} = \left(L_A^K\right)^T Q_1^*, \qquad \Omega_{29.29} = -\lambda_{22} I, \qquad \Omega_{30.7} = \left(L_B^R\right)^T Q_1^*,$$

$$\Omega_{30.30} = -\lambda_{23} I, \qquad \Omega_{31.7} = \left(L_B^I\right)^T Q_1^*, \qquad \Omega_{31.31} = -\lambda_{24} I, \qquad \Omega_{32.7} = \left(L_B^J\right)^T Q_1^*,$$

$$\Omega_{32.32} = -\lambda_{25} I, \qquad \Omega_{33.7} = \left(L_B^K\right)^T Q_1^*, \qquad \Omega_{33.33} = -\lambda_{26} I, \qquad \Omega_{34.7} = L_C^T Q_1^*,$$

$$\Omega_{34.34} = -\lambda_{27} I, \quad \textit{and}$$

$$\Omega_2 = (\Omega_{ij})_{34 \times 34}, \tag{41}$$

where

$$\begin{aligned}
&\Omega_{11} = -Q_2C_0 - C_0^TQ_2, \quad \Omega_{12} = Q_1A_2 + Q_2\bar{A}_1, \quad \Omega_{13} = Q_1B_2 + Q_2\bar{B}_1,\\
&\Omega_{14} = C_0^TQ_2C_0, \quad \Omega_{18} = Q_2L_C, \quad \Omega_{19} = Q_2L_A^R, \quad \Omega_{1.10} = Q_2L_A^I,\\
&\Omega_{1.11} = Q_2L_A^J, \quad \Omega_{1.12} = Q_2L_A^K, \quad \Omega_{1.13} = Q_2L_B^R, \quad \Omega_{1.14} = Q_2L_B^I,\\
&\Omega_{1.15} = Q_2L_B^J, \quad \Omega_{1.16} = Q_2L_B^K, \quad \Omega_{1.17} = C_0^TQ_2L_C,\\
&\Omega_{1.19} = (M_C\Sigma_CN_C)^TQ_2L_C, \quad \Omega_{21} = -A_2^TQ_1^T - A_1^*Q_2^T, \quad \Omega_{27} = A_1^*Q_2 - A_2^T\bar{Q}_1,\\
&\Omega_{31} = -B_2^TQ_1^T - B_1^*Q_2^T, \quad \Omega_{37} = B_1^*Q_2 - B_2^T\bar{Q}_1, \quad \Omega_{41} = -C_0^TQ_2^TC_0,\\
&\Omega_{44} = C_0^TQ_2C_0, \quad \Omega_{45} = C_0^TQ_2C_0, \quad \Omega_{4.18} = C_0^TQ_2L_C, \quad \Omega_{4.20} = C_0^TQ_2L_C,\\
&\Omega_{4.21} = C_0^TQ_2L_C, \quad \Omega_{4.22} = (M_C\Sigma_CN_C)^TQ_2L_C, \quad \Omega_{4.23} = C_0^TQ_2L_C,\\
&\Omega_{4.25} = (M_C\Sigma_CN_C)^TQ_2L_C, \quad \Omega_{54} = -C_0^TQ_2^TC_0, \quad \Omega_{57} = C_0^TQ_2,\\
&\Omega_{5.24} = C_0^TQ_2L_C, \quad \Omega_{72} = -Q_2^T\bar{A}_1 + Q_1^*A_2, \quad \Omega_{73} = -Q_2^T\bar{B}_1 + Q_1^*B_2,\\
&\Omega_{75} = -Q_2^TC_0, \quad \Omega_{77} = -Q_2^*, \quad \Omega_{7.26} = Q_2L_A^R, \quad \Omega_{7.27} = Q_2L_A^I,\\
&\Omega_{7.28} = Q_2L_A^J, \quad \Omega_{7.29} = Q_2L_A^K, \quad \Omega_{7.30} = Q_2L_B^R, \quad \Omega_{7.31} = Q_2L_B^I,\\
&\Omega_{7.32} = Q_2L_B^J, \quad \Omega_{7.33} = Q_2L_B^K, \quad \Omega_{7.34} = Q_2L_C, \quad \Omega_{81} = -(L_C)^TQ_2^T,\\
&\Omega_{91} = -\left(L_A^R\right)^TQ_2^T, \quad \Omega_{10.1} = -\left(L_A^I\right)^TQ_2^T,\\
&\Omega_{11.1} = -\left(L_A^J\right)^TQ_2^T, \quad \Omega_{12.1} = -\left(L_A^K\right)^TQ_2^T, \quad \Omega_{13.1} = -\left(L_B^R\right)^TQ_2^T,\\
&\Omega_{14.1} = -\left(L_B^I\right)^TQ_2^T, \quad \Omega_{15.1} = -\left(L_B^J\right)^TQ_2^T, \quad \Omega_{16.1} = -\left(L_B^K\right)^TQ_2^T,\\
&\Omega_{17.1} = -(L_C)^TQ_2^TC_0, \quad \Omega_{18.4} = -(L_C)^TQ_2^TC_0, \quad \Omega_{19.1} = -(L_C)^TQ_2^TM_C\Sigma_CN_C,\\
&\Omega_{20.4} = -(L_C)^TQ_2^TC_0, \quad \Omega_{21.4} = -(L_C)^TQ_2^TC_0, \quad \Omega_{22.4} = -(L_C)^TQ_2^TM_C\Sigma_CN_C,\\
&\Omega_{23.4} = -(L_C)^TQ_2^TC_0, \quad \Omega_{24.5} = -(L_C)^TQ_2^TC_0, \quad \Omega_{25.4} = -(L_C)^TQ_2^TM_C\Sigma_CN_C,\\
&\Omega_{26.7} = -\left(L_A^R\right)^TQ_2^T, \quad \Omega_{27.7} = -\left(L_A^I\right)^TQ_2^T, \quad \Omega_{28.7} = -\left(L_A^J\right)^TQ_2^T,\\
&\Omega_{29.7} = -\left(L_A^K\right)^TQ_2^T, \quad \Omega_{30.7} = -\left(L_B^R\right)^TQ_2^T, \quad \Omega_{31.7} = -\left(L_B^I\right)^TQ_2^T,\\
&\Omega_{32.7} = -\left(L_B^J\right)^TQ_2^T, \quad \Omega_{33.7} = -\left(L_B^K\right)^TQ_2^T, \quad \Omega_{34.7} = -L_C^TQ_2^T,
\end{aligned}$$

and the other ones in Ω_1 and Ω_2 are zeros.

Proof Based on Lemmas 7 and 8 and Theorem 1, the result easily can be checked. □

4 Numerical example

In this section, we will demonstrate the effectiveness of our obtained results by the following example.

Example 1 Assume that the parameters of QVNNs (1) are given as follows:

$$\check{C} = \begin{pmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}, \qquad \hat{C} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.6 \end{pmatrix},$$

$$K = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \qquad \delta = 0.1, \qquad \tau = 0.2,$$

$$\check{A} = (\check{a}_{ij})_{3\times 3}, \qquad \hat{A} = (\hat{a}_{ij})_{3\times 3}, \qquad \check{B} = (\check{b}_{ij})_{3\times 3}, \qquad \hat{B} = (\hat{b}_{ij})_{3\times 3},$$

where

$$\check{a}_{11} = -0.075 + 0.02\imath + 0.025\jmath + 0.02\kappa, \qquad \check{a}_{12} = -0.02 - 0.025\imath - 0.025\jmath - 0.02\kappa,$$
$$\check{a}_{13} = -0.06 - 0.033\imath - 0.033\jmath - 0.03\kappa, \qquad \check{a}_{21} = -0.06 + 0\imath + 0\jmath + 0\kappa,$$
$$\check{a}_{22} = -0.025 + 0\imath + 0\jmath + 0\kappa, \qquad \check{a}_{23} = -0.02 - 0.014\imath - 0.015\jmath - 0.014\kappa,$$
$$\check{a}_{31} = -0.04 + 0\imath + 0\jmath + 0\kappa, \qquad \check{a}_{32} = -0.045 - 0.004\imath - 0.006\jmath - 0.004\kappa,$$
$$\check{a}_{33} = -0.07 - 0.006\imath - 0.006\jmath - 0.008\kappa, \qquad \hat{a}_{11} = 0.05 + 0.025\imath + 0.0255\jmath + 0.0256\kappa,$$
$$\hat{a}_{12} = 0.075 + 0.015\imath + 0.015\jmath + 0.016\kappa, \qquad \hat{a}_{13} = 0.02 + 0.0124\imath + 0.0125\jmath + 0.012\kappa,$$
$$\hat{a}_{21} = 0.025 + 0.004\imath + 0.007\jmath + 0.006\kappa,$$
$$\hat{a}_{22} = 0.035 + 0.0006\imath + 0.0005\jmath + 0.0005\kappa,$$
$$\hat{a}_{23} = 0.01 + 0.0125\imath + 0.0125\jmath + 0.012\kappa, \qquad \hat{a}_{31} = 0.04 + 0.003\imath + 0.003\jmath + 0.003\kappa,$$
$$\hat{a}_{32} = 0.005 + 0.006\imath + 0.005\jmath + 0.005\kappa, \qquad \hat{a}_{33} = 0.095 + 0.008\imath + 0.008\jmath + 0.006\kappa,$$
$$\check{b}_{11} = -0.04 - 0.001\imath - 0.002\jmath - 0.003\kappa, \qquad \check{b}_{12} = -0.055 + 0\imath + 0\jmath + 0\kappa,$$
$$\check{b}_{13} = -0.02 - 0.003\imath - 0.004\jmath - 0.006\kappa, \qquad \check{b}_{21} = -0.01 + 0\imath + 0\jmath + 0\kappa,$$
$$\check{b}_{22} = -0.15 - 0.016\imath - 0.015\jmath - 0.014\kappa, \qquad \check{b}_{23} = -0.085 - 0.031\imath - 0.031\jmath - 0.05\kappa,$$
$$\check{b}_{31} = -0.03 - 0.001\imath - 0.002\jmath - 0.003\kappa, \qquad \check{b}_{32} = -0.015 - 0.015\imath - 0.016\jmath - 0.016\kappa,$$
$$\check{b}_{33} = -0.01 - 0.003\imath - 0.003\jmath - 0.005\kappa, \qquad \hat{b}_{11} = 0.032 + 0\imath + 0\jmath + 0\kappa,$$
$$\hat{b}_{12} = 0.008 + 0.03\imath + 0.02\jmath + 0.02\kappa, \qquad \hat{b}_{13} = 0.16 + 0.004\imath + 0.003\jmath + 0.004\kappa,$$
$$\hat{b}_{21} = 0.08 + 0.012\imath + 0.012\jmath + 0.012\kappa, \qquad \hat{b}_{22} = 0.12 + 0.014\imath + 0.015\jmath + 0.016\kappa,$$
$$\hat{b}_{23} = 0.096 + 0.05\imath + 0.04\jmath + 0.08\kappa, \qquad \hat{b}_{31} = 0.032 + 0\imath + 0\jmath + 0\kappa,$$
$$\hat{b}_{32} = 0.016 + 0.03\imath + 0.05\jmath + 0.04\kappa, \qquad \hat{b}_{33} = 0.04 + 0.019\imath + 0.016\jmath + 0.019\kappa.$$

Then we choose the following activation function of the QVNNs (1):

$$f_1(q) = f_2(q) = f_3(q) = \bigl(|q+1| - |q-1|\bigr) \times 0.05,$$
$$\forall q = q_0 + iq_1 + jq_2 + kq_3 \in \mathbb{H}.$$

It is obvious that the function satisfies Assumption ($\mathbf{A}_1$) and Assumption ($\mathbf{A}_2$). Applying YALMIP with the solver of SDPT3 in MATLAB, we obtain the following feasible solution to the LMI (2) in Theorem 1:

$$Q = \begin{pmatrix} 23.8873 & -0.0148 + 0.0017\imath & 0.0833 - 0.0013\imath \\ -0.0148 - 0.0017\imath & 22.2007 & -0.0262 - 0.0007\imath \\ 0.0833 + 0.0013\imath & -0.0262 + 0.0007\imath & 45.3548 + 0.0000\imath \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 19.1380 & 0 & 0 \\ 0 & 19.6444 & 0 \\ 0 & 0 & 28.8901 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 166.0871 & 0 & 0 \\ 0 & 151.3635 & 0 \\ 0 & 0 & 168.7452 \end{pmatrix},$$

$$R_3 = \begin{pmatrix} 1.4655 & 0 & 0 \\ 0 & 0.4237 & 0 \\ 0 & 0 & 0.8634 \end{pmatrix},$$

$$R_4 = \begin{pmatrix} 3.1066 & 0 & 0 \\ 0 & 2.5973 & 0 \\ 0 & 0 & 4.3724 \end{pmatrix},$$

$$\begin{aligned}
&\lambda_1 = 28.5537, \quad \lambda_2 = 53.6157, \quad \lambda_3 = 49.4532, \quad \lambda_4 = 49.5432, \\
&\lambda_5 = 49.8468, \quad \lambda_6 = 120.9964, \quad \lambda_7 = 80.8760, \quad \lambda_8 = 82.2778, \\
&\lambda_9 = 85.6456, \quad \lambda_{10} = 74.4314, \quad \lambda_{11} = 8.8261, \quad \lambda_{12} = 57.8808, \\
&\lambda_{13} = 58.2264, \quad \lambda_{14} = 58.2264, \quad \lambda_{15} = 57.8808, \quad \lambda_{16} = 13.2412, \\
&\lambda_{17} = 69.3583, \quad \lambda_{18} = 12.7879, \quad \lambda_{19} = 37.2494, \quad \lambda_{20} = 43.9406, \\
&\lambda_{21} = 43.9092, \quad \lambda_{22} = 44.4620, \quad \lambda_{23} = 101.3891, \quad \lambda_{24} = 71.3626, \\
&\lambda_{25} = 71.7247, \quad \lambda_{26} = 74.5952, \quad \lambda_{27} = 26.2634.
\end{aligned}$$

Thus, the condition of Theorem 1 is satisfied, then system (1) has a unique equilibrium point and the equilibrium point is globally robust stable.

In the following, we choose the following fixed network parameters:

$$\begin{aligned}
&C = \begin{pmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.55 \end{pmatrix}, \qquad A = (a_{ij})_{3\times 3}, \qquad B = (b_{ij})_{3\times 3}, \\
&J = \begin{pmatrix} 0.1 - 0.1\imath - 0.2\jmath + 0.05\kappa \\ -0.2 + 0.1\imath + 0.05\jmath - 0.1\kappa \\ 0.1 + 0.1\imath + 0.2\jmath + 0.5\kappa \end{pmatrix},
\end{aligned} \tag{42}$$

where

$$\begin{aligned}
&a_{11} = 0.02 + 0.021\imath + 0.0252\jmath + 0.025\kappa, \qquad a_{12} = 0.02 + 0.01\imath + 0.01\jmath + 0.012\kappa, \\
&a_{13} = 0.01 + 0.012\imath + 0.012\jmath + 0.01\kappa, \qquad a_{21} = 0.015 + 0.001\imath + 0.005\jmath + 0.004\kappa, \\
&a_{22} = 0.02 + 0.0004\imath + 0.0001\jmath + 0.0002\kappa, \qquad a_{23} = -0.01 + 0.012\imath + 0.012\jmath + 0.01\kappa, \\
&a_{31} = 0.02 + 0.0015\imath + 0.0015\jmath + 0.0015\kappa, \qquad a_{32} = 0.01 + 0.003\imath + 0.003\jmath + 0.001\kappa, \\
&a_{33} = 0.05 + 0.005\imath + 0.005\jmath + 0.005\kappa, \qquad b_{11} = 0.03 - 0.001\imath - 0.001\jmath - 0.002\kappa, \\
&b_{12} = 0.005 + 0.02\imath + 0.01\jmath + 0.01\kappa, \qquad b_{13} = 0.15 + 0.003\imath + 0.002\jmath + 0.003\kappa,
\end{aligned}$$

Figure 1 The first part of the state trajectories for system (1) with parameters

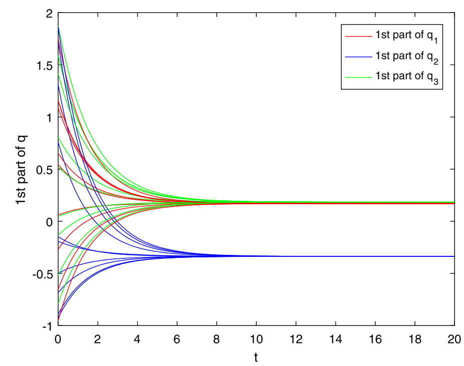


Figure 2 The second part of the state trajectories for system (1) with parameters

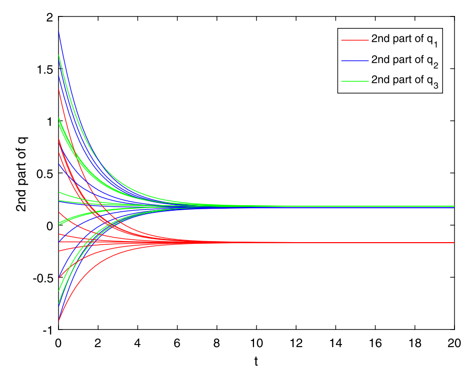
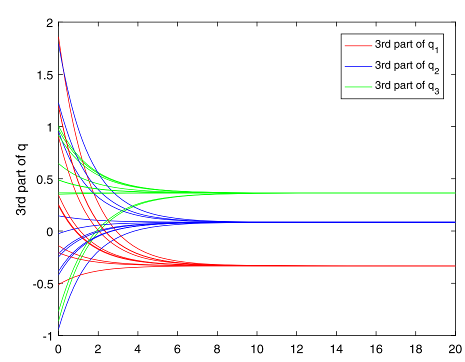


Figure 3 The third part of the state trajectories for system (1) with parameters



$$
\begin{aligned}
b_{21} &= 0.05 + 0.01\imath + 0.01\jmath + 0.01\kappa, & b_{22} &= 0.1 + 0.01\imath + 0.012\jmath + 0.014\kappa, \\
b_{23} &= 0.08 + 0.03\imath + 0.02\jmath + 0.03\kappa, & b_{31} &= 0.02 - 0.001\imath - 0.002\jmath - 0.0015\kappa, \\
b_{32} &= 0.012 + 0.015\imath + 0.01\jmath + 0.02\kappa, & b_{33} &= 0.03 + 0.016\imath + 0.015\jmath + 0.014\kappa.
\end{aligned}
$$

Figures 1, 2, 3 and 4 depict the four parts of the states of the considered quaternion-valued neural network system. It can be seen from these figures that each neuron state converges to the stable equilibrium point, which is $(0.1701 - 0.1674\imath - 0.3335\jmath + 0.0830\kappa, -0.3366 + 0.1673\imath + 0.0838\jmath - 0.1662\kappa, 0.1836 + 0.1812\imath + 0.3633\jmath + 0.9083\kappa)^T$.

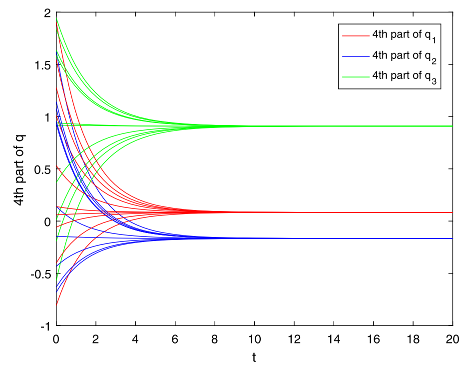


Figure 4 The fourth part of the state trajectories for system (1) with parameters

5 Conclusion

In this paper, the issue of globally robust stability of delayed quaternion-valued neural networks with interval parameter uncertainties has been investigated by using a quaternion-valued inequality, a Lyapunov function and a homeomorphic mapping, some new sufficient conditions to the existence, uniqueness and global robust stability of the equilibrium point for delayed quaternion-valued neural networks with interval parameter uncertainties have been derived. To the study of [20] we add the leak delay, extend the model to quaternion-valued neural networks and obtain the corresponding results under the relatively weak conditions. In addition, the elements of a given unique criteria matrix in our proposed results depend on not only the lower bounds but also the upper bounds of the interval parameters, which is less conservative than some previous contributions [19, 22, 27, 35].

We would like to point out that more quaternion-valued neural networks can be generalized based on our main results such as discrete-time neural networks [26], stochastic perturbations [37], and Markovian jumping parameters [38]. As future work, we plan to focus on discrete-time quaternion-valued neural networks with linear threshold function, the relevant results will be presented in the near future.

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Competing interests

The authors declare that they have no conflict of interest.

Authors' contributions

All authors conceived of the study, participated in its design and coordination, and read and approved the final manuscript.

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