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Solvability of second order linear differential equations in the sequence space $n(\phi)$

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Abstract

We apply the concept of measure of noncompactness to study the existence of solution of second order differential equations with initial conditions in the sequence space $n(\phi)$.

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1 Introduction and preliminaries

In recent years, the notion of measure of noncompactness has been effectively utilized in sequence spaces for different classes of differential equations (see [4, 5, 8, 11–15]). By applying this notion, Aghajani and Pourhadi [2] investigated the infinite system of second-order differential equations in an ℓ_1-space. Since then Mohiuddine et al. [10] and Banaś et al. [6] focused on this system in the sequence space ℓ_p.

A measure of noncompactness is a nonnegative real-valued map defined on a collection of bounded subsets of a normed (metric) space which maps the class of relatively compact sets (known as kernel) to zero, while other sets are mapped to a positive value. There are several ways to define this notion on a given space. The widely used approach is the axiomatic one, introduced in [3], which is given below.

Let $\mathfrak{M}_E$ denote the family of all nonempty bounded subsets of a Banach space E and $\mathfrak{N}_E$ be its subfamily consisting of all relatively compact sets. Let $B(x,r)$ denote the closed ball centered at x with radius r and $B_r = B(\theta, r)$.

We recall the following definition given in [3].

Definition 1.1 ([3, Definition 3.1.3]) A mapping $\mu: \mathfrak{M}_E \longrightarrow \mathbb{R}^+$ is called a *measure of noncompactness* (MNC for short) if

(i) $\ker \mu$ is nonempty and a subset of $\mathfrak{N}_E$.
(ii) $\mu(X) \leq \mu(Y)$ for $X \subset Y$.
(iii) $\mu(\overline{X}) = \mu(X)$.
(iv) $\mu(\operatorname{Conv} X) = \mu(X)$.

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(v) For all $\lambda \in [0,1]$,

$$\mu\big(\lambda X + (1-\lambda)Y\big) \leq \lambda\mu(X) + (1-\lambda)\mu(Y).$$

(vi) If $(X_n)_{n\in\mathbb{N}}$ is a sequence of closed sets from $\mathfrak{M}_E$ satisfying

$$X_{n+1} \subset X_n \quad \text{for all } n \in \mathbb{N} \quad \text{and} \quad \mu(X_n) \to 0 \quad \text{as } n \to \infty,$$

then

$$X_\infty = \bigcap_{n=1}^{\infty} X_n \neq \varnothing.$$

Definition 1.2 ([5, Definition 3.1.3]) For a measure of noncompactness μ in E, the mapping $G\colon B \subseteq E \longrightarrow E$ is said to be a μ_E-*contraction* if there exists a constant $0 < k < 1$ such that

$$\mu\big(G(Y)\big) \leq k\mu(Y) \tag{1.1}$$

for any bounded closed subset $Y \subseteq B$.

Darbo [7] used the idea of measure of noncompactness to obtain a new fixed point theorem which generalizes the Banach contraction principle and assures the existence of a fixed point concerning the so-called condensing operators.

Theorem 1.1 ([7]) *Let $\complement$ be a nonempty, closed, bounded, and convex subset of a Banach space E, and let $\mathcal{G} : \complement \mapsto \complement$ be a continuous mapping such that there exists a constant $\theta \in [0,1)$ with the property $\mu(\mathcal{G}(\complement)) \leq \theta\mu(\complement)$. Then $\mathcal{G}$ has a fixed point in $\complement$.*

The following definition was given in [1] which is a generalization of Meir–Keeler contraction (MKC) given in [9].

Definition 1.3 ([1]) For an arbitrary measure of noncompactness μ on a Banach space X, we say that an operator $\mathfrak{T} : B \mapsto B$ is a Meir–Keeler condensing operator if for any $\epsilon > 0$ there exists $\delta > 0$ such that

$$\epsilon \leq \mu(E) < \epsilon + \delta \quad \Longrightarrow \quad \mu\big(\mathfrak{T}(E)\big) < \epsilon \tag{1.2}$$

for any bounded subset E of B; where B is a nonempty subset of X.

Now we state the following theorem for Meir–Keeler condensing operators which will be applied in our main results.

Theorem 1.2 ([1]) *Let μ be an arbitrary measure of noncompactness on a Banach space X. If $\mathfrak{T} : B \mapsto B$ is a continuous and Meir–Keeler condensing operator, then $\mathfrak{T}$ has at least one fixed point and the set of all fixed points of $\mathfrak{T}$ in B is compact, where B is a nonempty, bounded, closed, and convex subset of X.*

2 The sequence space $n(\phi)$

We denote by $\mathcal{C}$ the space of finite sets of distinct positive integers. For any $\sigma \in \mathcal{C}$, we define $\alpha(\sigma) = \{\alpha_n(\sigma)\}$ such that $\alpha_n(\sigma)$ is 1 if n is in σ; and 0 elsewhere. Write

$$\mathcal{C}_r = \left\{\sigma \in \mathcal{C} : \sum_{n=1}^{\infty} \alpha_n(\sigma) \le r\right\},$$

and define

$$\Phi = \{\phi = (\phi_k) : 0 < \phi_1 \le \phi_n \le \phi_{n+1} \text{ and } (n+1)\phi_n \ge n\phi_{n+1}\}.$$

Sargent [16] defined the following sequence spaces which were further studied in [11]. Write $S(x)$ for the set of all sequences that are rearrangements of x. For $\phi \in \Phi$,

$$m(\phi) = \left\{x = (x_k) : \|x\|_{m(\phi)} = \sup_{r \ge 1} \sup_{\sigma \in \mathcal{C}_r} \left(\frac{1}{\phi_r} \sum_{k \in \sigma} |x_k|\right) < \infty\right\},$$

$$n(\phi) = \left\{x = (x_k) : \|x\|_{n(\phi)} = \sup_{u \in S(x)} \left(\sum_{k=1}^{\infty} |u_k| \Delta\phi_k\right) < \infty\right\},$$

where $\Delta\phi_k = \phi_k - \phi_{k-1}$. Note that, for all $n \in \mathbb{N} = \{1, 2, 3, \ldots\}$, $m(\phi) = \ell_1$, $n(\phi) = \ell_\infty$ if $\phi_n = 1$; and $m(\phi) = \ell_\infty$, $n(\phi) = \ell_1$ if $\phi_n = n$.

We have the following important result.

Theorem 2.1 ([12]) *For any bounded subset $\mathcal{Q}$ of $n(\phi)$, we have*

$$\chi(\mathcal{Q}) = \lim_{k \to \infty} \sup_{x \in Q} \left(\sup_{u \in S(x)} \left(\sum_{n=k}^{\infty} |u_n| \Delta\phi_n\right)\right),$$

where $\chi(Q)$ denotes the *Hausdorff measure of noncompactness* of the set Q which is defined by

$$\chi(Q) := \inf\left\{\epsilon > 0 : Q \subset \bigcup_{i=1}^{n} B(x_i, r_i), x_i \in X, r_i < \epsilon (i = 1, 2, \ldots)\right\}.$$

3 Infinite system of second order differential equations in $n(\phi)$

We study the following infinite system:

$$\frac{d^2 u_i}{dt^2} = -f_i(t, u_1(t), u_2(t), u_3(t), \ldots); \quad u_i(0) = u_i(T) = 0, t \in [0, T], i = 1, 2, 3 \ldots \tag{3.1}$$

Let $C(I, \mathbb{R})$ be the space of all continuous real functions on the interval $I = [a, b]$ and $C^2(I, \mathbb{R})$ be the class of functions with the second continuous derivative on I. A function $u = (u_i) \in C^2(I, \mathbb{R})$ is a solution of (3.1) if and only if $u \in C(I, \mathbb{R})$ is a solution of the system of integral equations

$$u_i(t) = \int_0^T \mathfrak{G}(t, s) f_i(s, u(s))\, ds \quad \text{for } t \in I, \tag{3.2}$$

where $f_i(t,u) \in C(I \times \mathbb{R}^\infty, \mathbb{R})$, $i = 1,2,3,\ldots$; and the Green's function associated with (3.1) is given by

$$\mathfrak{G}(t,s) = \begin{cases} \frac{t}{T}(T-s), & 0 \le t \le s \le T, \\ \frac{s}{T}(T-t), & 0 \le s \le t \le T. \end{cases} \tag{3.3}$$

From (3.2) and (3.3)

$$u_i(t) = \int_0^t \frac{s}{T}(T-t) f_i\big(s,u(s)\big)\,ds + \int_t^T \frac{t}{T}(T-s) f_i\big(s,u(s)\big)\,ds.$$

Now compute

$$\frac{d}{dt}u_i(t) = -\frac{1}{T}\int_0^t s f_i\big(s,u(s)\big)\,ds + \frac{1}{T}\int_t^T (T-s) f_i\big(s,u(s)\big)\,ds.$$

Again differentiating we get

$$\frac{d^2u_i(t)}{dt^2} = -\frac{1}{T}\big(t f_i\big(t,u(t)\big)\big) + \frac{1}{T}(t-T) f_i\big(t,u(t)\big)\big) = -f_i\big(t,u(t)\big)\big).$$

The solution of the infinite system (3.1) in the sequence space ℓ_1 was discussed by Aghajani and Pourhadi [2] by establishing a generalization of Darbo type fixed point theorem using the concept of α-admissibility function and Schauder's fixed point theorem. Here, we determine the solvability of system (3.1) in Banach sequence spaces $n(\phi)$. Our result is more general than that of [2].

Assume that

(i) The functions f_i are defined on the set $I \times \mathbb{R}^\infty$ and take real values. The operator f defined on the space $I \times n(\phi)$ into $n(\phi)$ as

$$(t,u) \to (fu)(t) = \big(f_1\big(t,u(t)\big), f_2\big(t,u(t)\big), f_3\big(t,u(t)\big),\ldots\big)$$

is such that the class of all functions $((fu)(t))_{t\in I}$ is equicontinuous at every point of the space $n(\phi)$.

(ii) The following inequality holds:

$$\big|f_n\big(t,u_1(t),u_2(t),u_3(t),\ldots\big)\big| \le g_n(t) + h_n(t)\big|u_n(t)\big|,$$

where $g_n(t)$ and $h_n(t)$ are real functions defined and continuous on I such that $\sum_{k=1}^\infty g_k(t)\Delta\phi_k$ converges uniformly on I and the sequence $(h_n(t))$ is equibounded on I.

Write

$$G = \sup_{t\in I}\sum_{k=1}^\infty g_k(t)\Delta\phi_k$$

and

$$H = \sup_{n\in\mathbb{N},t\in I} h_n(t).$$

Theorem 3.1 *Let conditions* (i)–(ii) *hold. Then system* (3.1) *has at least one solution* $u(t) = (u_i(t)) \in n(\phi)$ *for all* $t \in [0, T]$.

Proof Let $S(u(t))$ denote the set of all sequences that are rearrangements of $u(t)$. If $v(t) \in S(u(t))$, then $\sum_{k=1}^{\infty} |v_k(t)|\Delta\phi_k \le M$, where M is a finite positive real number for all $u(t) = (u_i(t)) \in n(\phi)$ for all $t \in I$. Using (3.2) and (ii), we have, for all $t \in I$,

$$\begin{aligned}
&\|u(t)\|_{n(\phi)} \\
&\quad = \sup_{v\in S(u(t))}\left(\sum_{k=1}^{\infty}\left|\int_0^T \mathfrak{G}(t,s)f_k\bigl(s,u(s)\bigr)\,ds\right|\Delta\phi_k\right) \\
&\quad \le \sup_{v\in S(u(t))}\left(\sum_{k=1}^{\infty}\int_0^T \bigl|\mathfrak{G}(t,s)f_k\bigl(s,u(s)\bigr)\bigr|\,ds\Delta\phi_k\right) \\
&\quad \le \sup_{v\in S(u(t))}\left(\sum_{k=1}^{\infty}\int_0^T \bigl|\mathfrak{G}(t,s)\bigr|\bigl(g_k(t)+h_k(t)\bigl|v_k(t)\bigr|\bigr)\,ds\Delta\phi_k\right) \\
&\quad = \sup_{v\in S(u(t))}\left(\sum_{k=1}^{\infty}\int_0^T \mathfrak{G}(t,s)g_k(t)\Delta\phi_k\,ds + \sum_{k=1}^{\infty}\int_0^T \mathfrak{G}(t,s)\bigl|v_k(t)\bigr|\Delta\phi_k\,ds\right) \\
&\quad \le \sup_{v\in S(u(t))}\left(\int_0^T \mathfrak{G}(t,s)\left\{\sum_{k=1}^{\infty}g_k(t)\Delta\phi_k\right\}ds + H\int_0^T \mathfrak{G}(t,s)\left\{\sum_{k=1}^{\infty}\bigl|u_k(t)\bigr|\Delta\phi_k\right\}ds\right) \\
&\quad \le G\sup_{v\in S(u(t))}\int_0^T \mathfrak{G}(t,s)\,ds + H\sup_{v\in S(u(t))}\int_0^T \mathfrak{G}(t,s)M\,ds \\
&\quad \le \frac{GT^2}{8} + \frac{HMT^2}{8} = R,
\end{aligned}$$

say.

Let $u^0(t) = (u_i^0(t))$ where $u_i^0(t) = 0$ for all $t \in I$.

Consider the closed ball $\bar{B} = \bar{B}(u^0, r_1)$ centered at u^0 and of radius $r_1 \le r$ which is of course a nonempty, bounded, closed, and convex subset of $n(\phi)$. Consider the operator $\mathcal{F} = (\mathcal{F}_i)$ on $C(I, \bar{B})$ defined as follows. For $t \in I$,

$$(\mathcal{F}u)(t) = \bigl\{(\mathcal{F}_i u)(t)\bigr\} = \left\{\int_0^T \mathfrak{G}(t,s)f_i\bigl(s,u(s)\bigr)\,ds\right\},$$

where $u(t) = (u_i(t))$ and $u_i(t) \in C(I, \mathbb{R})$.

We have $(\mathcal{F}u)(t) = \{(\mathcal{F}_i u)(t)\} \in n(\phi)$ for each $t \in I$. Since $(f_i(t, u(t))) \in n(\phi)$ for each $t \in I$, we have

$$\sup_{v\in S(u(t))}\left(\sum_{k=1}^{\infty}\bigl|(\mathcal{F}_k u)(t)\bigr|\Delta\phi_k\}\,ds\right) \le R < \infty.$$

Also since $(\mathcal{F}_i u)(t)$ satisfies the boundary conditions, we have

$$(\mathcal{F}_i u)(0) = \int_0^T \mathfrak{G}(0,s)f_i\bigl(s,u(s)\bigr)\,ds = 0$$

and

$$(\mathcal{F}_i u)(T) = \int_0^T \mathfrak{G}(T,s) f_i\big(s,u(s)\big)\,ds = 0.$$

Since $\|(\mathcal{F}u)(t) - u^0(t)\|_{n(\phi)} \leq R$, $\mathcal{F}$ is an operator on $\bar{B}$.

The operator $\mathcal{F}$ is continuous on $C(I,\bar{B})$ by assumption (i). Now, we shall show that $\mathcal{F}$ is a Meir–Keeler condensing operator. For $\varepsilon > 0$, we have to find $\delta > 0$ such that $\varepsilon \leq \chi(\bar{B}) < \varepsilon + \delta \Rightarrow \chi(\mathcal{F}\bar{B}) < \varepsilon$. Now

$$\begin{aligned}
&\lim_{k\to\infty}\left\{\sup_{u(t)\in\bar{B}}\left(\sup_{v\in S(u(t))}\left(\sum_{n=k}^{\infty}\left|\int_0^T \mathfrak{G}(t,s)f_n\big(s,v(s)\big)\,ds\right|\Delta\phi_n\right)\right)\right\}\\
&\quad\leq \lim_{k\to\infty}\left\{\sup_{u(t)\in\bar{B}}\left(\sup_{v\in S(u(t))}\left(\sum_{n=k}^{\infty}\int_0^T \left|\mathfrak{G}(t,s)f_n\big(s,v(s)\big)\right|\Delta\phi_n\,ds\right)\right)\right\}\\
&\quad\leq \lim_{k\to\infty}\left\{\sup_{u(t)\in\bar{B}}\left(\sup_{v\in S(u(t))}\left(\sum_{n=k}^{\infty}\int_0^T \mathfrak{G}(t,s)g_n(s)\Delta\phi_n\,ds\right)\right)\right\}\\
&\qquad+\lim_{k\to\infty}\left\{\sup_{u(t)\in\bar{B}}\left(\sup_{v\in S(u(t))}\left(\sum_{n=k}^{\infty}\int_0^T \mathfrak{G}(t,s)h_n(s)\left|v_n(s)\right|\Delta\phi_n\,ds\right)\right)\right\}\\
&\quad\leq \lim_{k\to\infty}\left\{\sup_{u(t)\in\bar{B}}\left(\sup_{v\in S(u(t))}\left(\int_0^T \mathfrak{G}(t,s)\left(\sum_{n=k}^{\infty}g_n(s)\Delta\phi_n\right)ds\right)\right)\right\}\\
&\qquad+H\lim_{k\to\infty}\left\{\sup_{u(t)\in\bar{B}}\left(\sup_{v\in S(u(t))}\int_0^T \mathfrak{G}(t,s)\left(\sum_{n=k}^{\infty}\left|v_n(s)\right|\Delta\phi_n\right)ds\right)\right\}\\
&\quad\leq H\chi(\bar{B})\int_0^T \mathfrak{G}(t,s)\,ds \leq \frac{HT^2}{8}\chi(\bar{B}).
\end{aligned}$$

Hence $\chi(\mathcal{F}\bar{B}) < \frac{HT^2}{8}\chi(\bar{B}) < \varepsilon \Rightarrow \chi(\bar{B}) < \frac{8\varepsilon}{HT^2}$.

Taking $\delta = \frac{\varepsilon}{HT^2}(8 - HT^2)$, we get $\varepsilon \leq \chi(\bar{B}) < \varepsilon + \delta$. Therefore, $\mathcal{F}$ is a Meir–Keeler condensing operator defined on the set $\bar{B} \subset n(\phi)$. So $\mathcal{F}$ satisfies all the conditions of Theorem 1.2 which implies that $\mathcal{F}$ has a fixed point in $\bar{B}$, which is a required solution of system (3.1). □

Remark 3.1 For $\phi_n = n$, for all $n \in \mathbb{N}$, the above result is reduced to that of Aghajani and Pourhadi [2] but our proof is quite different.

4 Example

In order to illustrate the above result, we provide the following example.

Example 4.1 Let us consider the system of second order differential equations

$$-\frac{d^2u_j(t)}{dt^2} = \frac{\sqrt[j]{t}}{j^4} + \sum_{i=j}^{\infty}\frac{t\cos(t)u_i(t)}{i^4}, \quad j \in \mathbb{N}, t \in I = [0,T]. \tag{4.1}$$

Here $f_i(t,u_1(t),u_2(t),u_3(t),\dots) = \frac{\sqrt[j]{t}}{j^4} + \sum_{i=j}^{\infty}\frac{t\cos(t)u_i(t)}{i^4}$, and so (4.1) is a special case of the considered system (3.1). Clearly $\frac{\sqrt[j]{t}}{j^4}$ and $\sum_{i=j}^{\infty}\frac{t\cos(t)u_i(t)}{i4}$ are continuous on I for each $n \in \mathbb{N}$.

Notice that, for any $t \in I = [0, T]$, $(f_k(t, u(t))) \in n(\phi)$ if $(u_k(t)) \in n(\phi)$. Moreover, we have

$$\begin{aligned}
\sum_{k=1}^{\infty} \left|f_k\bigl(t, u(t)\bigr)\right| &= \sum_{k=1}^{\infty} \left| \frac{\sqrt[k]{t}}{k^4} + \sum_{i=k}^{\infty} \frac{t\cos(t) u_i(t)}{i^4} \right| \\
&\leq \sum_{k=1}^{\infty} \frac{\sqrt[k]{t}}{k^4} + \sum_{k=1}^{\infty} \sum_{i=k}^{\infty} \left| \frac{t\cos(t) u_i(t)}{i^4} \right| \\
&\leq \frac{T\pi^4}{90} + \sum_{k=1}^{\infty} \sum_{i=k}^{\infty} \frac{t}{i^4} \left|u_i(t)\right| \\
&\leq \frac{T\pi^4}{90} + T\left\|u(t)\right\|_{n(\phi)} < \infty.
\end{aligned}$$

We will show that assumption (i) is satisfied. Let us fix $\epsilon > 0$ arbitrarily and $u(t) = (u_k(t)) \in n(\phi)$. Then, taking $v(t) = (v_k(t)) \in n(\phi)$ with $\|u(t) - v(t)\| \leq \delta(\epsilon) := \frac{\epsilon}{T}$, we have

$$\begin{aligned}
\left|f\bigl(t, u(t)\bigr) - f\bigl(t, v(t)\bigr)\right| &= \sum_{i=j}^{\infty} \frac{t(u_i(t) - v_i(t))}{i^4} \\
&\leq T\left\|u(t) - v(t)\right\|_{n(\phi)} \\
&\leq T\delta < \epsilon,
\end{aligned}$$

which implies continuity as in assumption (i). Now, we show that assumption (ii) is satisfied.

$$\begin{aligned}
\left|f_j\bigl(t, u(t)\bigr)\right| &= \left| \frac{\sqrt[j]{t}}{j^4} + \sum_{i=j}^{\infty} \frac{t\cos(t) u_i(t)}{i^4} \right| \\
&\leq \frac{\sqrt{t}}{j^4} + \sum_{i=j}^{\infty} \frac{t}{i^4} \left|u_i(t)\right| \\
&\leq g_j(t) + h_j(t)\left|u_j(t)\right|.
\end{aligned}$$

The function $g_j(t) = \frac{\sqrt{t}}{j^4}$ is continuous and $\sum_{j\geq 1} g_j(t)$ converges uniformly to $\frac{\sqrt{t}\pi^4}{90}$, also $h_j(t) = \frac{t\pi^4}{90}$ is continuous and the sequence $(h_j(t))$ is equibounded on I by $H = \frac{T\pi^4}{80}$. Also $\frac{HT^2}{8} < 1$ is satisfied by taking $T = 1.2$, which gives $H \approx 1.9739$ and $G \approx 1.9739$.

Thus, from Theorem 3.1, for a suitable value of r_1 (as discussed in Theorem 3.1) the operator $\mathcal{F}$ as defined in Theorem 3.1 on $\bar{B}(u^0, r_1)$ has a fixed point $u(t) = ((u_i(t)) \in n(\phi)$, which is a solution of system (4.1).

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Competing interests
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Authors' contributions
The authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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References

1. Aghajani, A., Mursaleen, M., Haghighi, A.S.: A fixed point theorem for Meir–Keeler condensing operator via measure of noncompactness. Acta Math. Sci. **35B**(3), 552–556 (2015)
2. Aghajani, A., Pourhadi, E.: Application of measure of noncompactness to ℓ_1-solvability of infinite systems of second order differential equations. Bull. Belg. Math. Soc. Simon Stevin **22**(1), 105–118 (2015)
3. Banaś, J., Goebel, K.: Measures of Noncompactness in Banach Spaces. Lecture Notes in Pure and Applied Mathematics, vol. 60. Dekker, New York (1980)
4. Banaś, J., Lecko, M.: Solvability of infinite systems of differential equations in Banach sequence spaces. J. Comput. Appl. Math. **137**, 363–375 (2001)
5. Banaś, J., Mursaleen, M.: Sequence Spaces and Measure of Noncompactness with Applications to Differential and Integral Equation. Springer, India (2014)
6. Banaś, J., Mursaleen, M., Rizvi, S.M.H.: Existence of solutions to a boundary-value problem for an infinite system of differential equations. Electron. J. Differ. Equ. **2017**, 262 (2017)
7. Darbo, G.: Punti uniti in transformazioni a condominio non compatto. Rend. Semin. Mat. Univ. Padova **24**, 84–92 (1955)
8. Das, A., Hazarika, B., Arab, R., Mursaleen, M.: Solvability of the infinite system of integral equations in two variables in the sequence spaces c_0 and ℓ_1. J. Comput. Appl. Math. **326**, 183–192 (2017)
9. Meir, A., Keeler, E.: A theorem on contraction mappings. J. Math. Anal. Appl. **28**, 326–329 (1969)
10. Mohiuddine, S.A., Srivastava, H.M., Alotaibi, A.: Application of measures of noncompactness to the infinite system of second-order differential equations in ℓ_p spaces. Adv. Differ. Equ. **2016**, 317 (2016)
11. Mursaleen, M.: On some geometric properties of a sequence space related to l_p. Bull. Aust. Math. Soc. **67**, 343–347 (2003)
12. Mursaleen, M.: Application of measure of noncompactness to infinite system of differential equations. Can. Math. Bull. **56**, 388–394 (2013)
13. Mursaleen, M., Bilalov, B., Rizvi, S.M.H.: Applications of measures of noncompactness to infinite system of fractional differential equations. Filomat **31**(11), 3421–3432 (2017)
14. Mursaleen, M., Mohiuddine, S.A.: Applications of measure of noncompactness to the infinite system of differential equations in ℓ_p spaces. Nonlinear Anal. **75**, 2111–2115 (2012)
15. Mursaleen, M., Rizvi, S.M.H.: Solvability of infinite system of second order differential equations in c_0 and ℓ_1 by Meir–Keeler condensing operator. Proc. Am. Math. Soc. **144**(10), 4279–4289 (2016)
16. Sargent, W.L.C.: Some sequence spaces related to the l_p spaces. J. Lond. Math. Soc. **35**, 161–171 (1960)