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A note on global properties for a stage structured predator–prey model with mutual interference

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Abstract

The global stability for a stage structured predator–prey model with mutual interference is investigated. By using the method of Lyapunov functionals, it is shown that the system has a unique interior equilibrium, which is always globally asymptotically stable without any additional assumptions. The results indicate that mutual interference helps the endangered predators survive under any maturation time delay of preys. This answers two open problems presented in (Discrete Contin. Dyn. Syst., Ser. B 19(1):173–187, 2014).

MSC: Primary 92D25; secondary 34D20

Keywords: Predator–prey model; Stage structure; Mutual interference; Global stability; Lyapunov functional

1 Introduction

To obtain useful predictions, mathematical models should be based on detailed observation data. One good example was the study by Hassell and Varely [9], who successfully fitted the data in [3] with the following model:

$$\lg_{10} a = \lg_{10} Q - m \lg_{10} p, \tag{1.1}$$

where a is the area of discovery and p is the density of searching parasites in a generation, Q indicates the level of efficiency of one parasite and m is the mutual interference constant. The concept of mutual interference was first introduced by Hassell [8] to capture the behavior between a host (a kind of bee) and parasite (a kind of butterfly). It is a measure of the degree of interference between parasites. Furthermore, mutual interference was considered by Freedman [5, 6] to describe the phenomenon that predators have the tendency to leave each other when they meet. Freedman [5] proposed a general Volterra model with mutual interference m $(0 < m \leq 1)$ as follows:

$$\begin{aligned} x'(t) &= xg(x) - y^m p(x), \\ y'(t) &= y\bigl(-s + cy^{m-1}p(x) - q(y)\bigr), \end{aligned} \tag{1.2}$$

where $g(0) > 0$, $g' \leq 0$, $g(K) = 0$ (for some $K > 0$), $p(0) = 0$, $p' > 0$, $q(0) = 0$, $q' \geq 0$. The author in [5] got the conditions for the existence of the interior equilibrium and analyzed its stability. The special case of $m = 1$ and $q(y) = 0$ was studied in [4].

Based on the fact that some individual members of the population may go through several stages in their whole life cycle [1], Barclay and Van den Driessche [2] exhibited two distinct stages of the populations, immature and mature ones, with delay τ representing the time from birth to maturity. The dynamics of stage structured predator–prey model is investigated in several studies (see, e.g., [12, 15, 19]). Recently, a predator–prey model with stage structure and mutual interference was proposed and investigated in [16]. The model is given as follows:

$$\begin{aligned}
x_1'(t) &= r_1x_2(t) - dx_1(t) - r_1e^{-d\tau}x_2(t-\tau), \\
x_2'(t) &= r_1e^{-d\tau}x_2(t-\tau) - b_1x_2^2(t) - c_1x_2(t)y^m(t), \\
y'(t) &= y(t)\bigl(-r_2 - b_2y(t) + c_2x_2(t)y^{m-1}(t)\bigr),
\end{aligned} \tag{1.3}$$

where $x_1(t)$, $x_2(t)$, and $y(t)$ denote the densities of immature prey, mature prey, and mature predator, respectively; m with $0 < m < 1$ is the mutual interference constant; τ is mature period of prey; r_1 is the birth rate of the mature prey; d and r_2 represent the death of immature prey and mature predator, respectively; b_1 and b_2 are the intra-specific competition among the mature prey and mature predator, respectively; c_1 describes the capturing rate of the mature predator; c_2 is the conversion rate for the predator; $e^{-d\tau}$ is the surviving rate of each immaturity to reach maturity in prey species. Given the assumption from biology, all of the coefficients presented in system (1.3) are positive constants.

Since the first equation of system (1.3) is uncoupled with the rest of the system, we investigate the global behavior for the subsystem of system (1.3) as follows:

$$\begin{aligned}
x_2'(t) &= r_1e^{-d\tau}x_2(t-\tau) - b_1x_2^2(t) - c_1x_2(t)y^m(t), \\
y'(t) &= y(t)\bigl(-r_2 - b_2y(t) + c_2x_2(t)y^{m-1}(t)\bigr).
\end{aligned} \tag{1.4}$$

From a biological point of view, it is reasonable to consider the following initial conditions for system (1.4):

$$x_2(\theta) = \phi(\theta) > 0, \qquad y(0) > 0, \qquad -\tau \leq \theta \leq 0, \tag{1.5}$$

where $\phi \in \mathcal{C}\{[-\tau, 0], \mathbb{R}_+\}$, the space of continuous functions mapping $[-\tau, 0]$ into $\mathbb{R}_+$.

The following results are taken from [16].

Proposition 1.1 *Solutions of system* (1.4) *with the initial conditions* (1.5) *are positive for all* $t > 0$.

Proposition 1.2 *System* (1.4) *has two boundary equilibria* $E_0(0, 0)$, $E_1(\frac{r_1e^{-d\tau}}{b_1}, 0)$ *and a unique interior equilibrium* $E^*(x_2^*, y^*)$ (*see Fig.* 1), *which satisfies*

$$\begin{aligned}
x_2 &= \frac{r_1e^{-d\tau}}{b_1} - \frac{c_1}{b_1}y^m \equiv f(y), \\
x_2 &= \frac{r_2}{c_2}y^{(1-m)} + \frac{b_2}{c_2}y^{(2-m)} \equiv g(y).
\end{aligned} \tag{1.6}$$

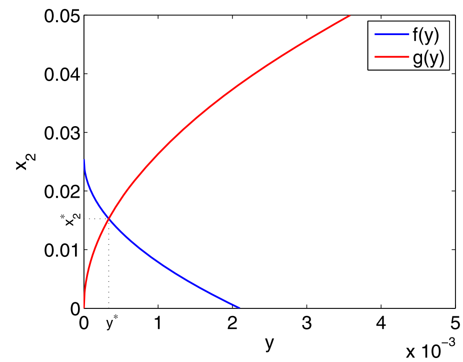


Figure 1 Existence of a unique interior equilibrium $E^*(x_2^*, y^*)$. Here we choose the parameters $d = 0.2$, $m = 0.5$, $r_1 = 2.5$, $b_1 = 1.8$, $c_1 = 1$, $r_2 = 1$, $b_2 = 0.5$, $c_2 = 1.2$, $\tau = 20$ from [16]

It is easy to see that the solutions $(x_2(t), y(t))$ of system (1.4) with the initial conditions (1.5) exist for all $t \geq 0$ and are unique.

By the analysis of the characteristic equations and the iterative schemes coupled with the comparison principle, the dynamical properties of system (1.4) were established in [16]. That is, when $(1-m)^2 r_2 > 2mr_1$, the interior equilibrium is locally asymptotically stable and globally attractive when the following inequality holds:

$$mr_1 e^{-d\tau}(r_2 + b_2 M) < r_2\bigl(r_1 e^{-d\tau} - c_1 M^m\bigr), \tag{1.7}$$

where $M = (\frac{c_2 r_1 e^{-d\tau}}{b_1 r_2})^{\frac{1}{1-m}}$.

However, numerical simulations suggest that the unique interior equilibrium E^* of system (1.4) is always globally stable when it exists. Hence the authors in [16] raise two worthy problems: (i) Under some weaker conditions, or even without preconditions, whether system (1.4) has a unique interior equilibrium, which is globally stable. (ii) In more general situations, whether the mutual interference $(0 < m < 1)$ can make the endangered species become globally stable.

The object of this study is to show that the interior equilibrium of system (1.4) is always globally asymptotically stable as long as it exists and to give straightforward positive answers to the questions above.

2 Main result

In this section, by constructing suitable Lyapunov functionals for delay differential equations system (1.4), we establish the global stability of the interior equilibrium when it exists for any $\tau \geq 0$.

Lemma 1 *For any positive constants w and m where* $0 < m \leq 1$, *the following two inequalities hold*:

$$\bigl(1 - w^{1-m}\bigr)\bigl(1 - w^m\bigr) \geq 0, \tag{2.1}$$

$$\bigl(1 - w^{2-m}\bigr)\bigl(1 - w^m\bigr) \geq 0. \tag{2.2}$$

Proof Let $f_1(w) = (1 - w^{1-m})(1 - w^m) = 1 - w^m - w^{1-m} + w$, then

$$f_1'(w) = -mw^{m-1} - (1-m)w^{-m} + 1 = -\frac{m}{w^{1-m}} - \frac{1-m}{w^m} + 1.$$

When $0 < w \leq 1$, we have $f_1'(w) < 0$. When $w > 1$, we have $f_1'(w) > 0$. Thus, it has

$$f_1(w) \geq f_1(1) = 0,$$

that is to say, $(1 - w^{1-m})(1 - w^m) \geq 0$ holds for any positive constants w and $0 < m \leq 1$.

Similarly, let $f_2(w) = (1 - w^{2-m})(1 - w^m) = 1 - w^m - w^{2-m} + w^2$, then

$$f_2'(w) = -mw^{m-1} - (2-m)w^{1-m} + 2w = m\left(w^{1-m} - \frac{1}{w^{1-m}}\right) + 2\left(w - w^{1-m}\right).$$

When $0 < w \leq 1$, we have $f_2'(w) < 0$. When $w > 1$, we have $f_2'(w) > 0$. One also has

$$f_2(w) \geq f_2(1) = 0.$$

It is easy to see that $(1 - w^{2-m})(1 - w^m) \geq 0$ holds for any positive constants w and $0 < m \leq 1$. □

Theorem 2.1 *The interior equilibrium $E^*(x_2^*, y^*)$ of system (1.4) is globally asymptotically stable for any delay $\tau \geq 0$.*

Proof Define the global Lyapunov functional for E^*,

$$U(t) = V_1(t) + r_1 e^{-d\tau} x_2^* \cdot V_+(t) + \frac{c_1}{c_2} \cdot V_2(t), \tag{2.3}$$

where

$$V_1(t) = x_2 - x_2^* - x_2^* \ln \frac{x_2}{x_2^*}, \tag{2.4}$$

$$V_2(t) = y - y^* - \int_{y^*}^{y} \frac{y^{*m}}{\sigma^m}\, d\sigma, \tag{2.5}$$

$$V_+(t) = \int_{t-\tau}^{t} \left\{ \frac{x_2(\sigma)}{x_2^*} - 1 - \ln \frac{x_2(\sigma)}{x_2^*} \right\} d\sigma. \tag{2.6}$$

Note that $h(z) = z - 1 - \ln z \geq 0$ for $z > 0$ and $h(z) = 0$ if and only if $z = 1$. So $V_1(t) = x_2^* h(\frac{x_2}{x_2^*}) \geq 0$ due to $\frac{x_2}{x_2^*} > 0$ and $V_1(t) = 0$ iff $x_2 = x_2^*$. Similarly, $V_+(t) = \int_{t-\tau}^{t} h(\frac{x_2(\sigma)}{x_2^*})\, d\sigma \geq 0$ due to $\frac{x_2}{x_2^*} > 0$ and $V_+(t) = 0$ iff $x_2 = x_2^*$. Let $g(y) = y - y^* - \int_{y^*}^{y} \frac{y^{*m}}{\sigma^m}\, d\sigma$. Then we have $g(y^*) = 0$, $g'(y) = 1 - (\frac{y^*}{y})^m \geq 0$ if $y \geq y^*$, and $g'(y) < 0$ if $y < y^*$. Besides, we have $\lim_{y \to 0} g(y) = \lim_{y \to +\infty} g(y) = +\infty$. So $g(y)$ has minimum value at $y = y^*$ in $[0, +\infty)$. Hence for any $y > 0$, $V_2(t)$ is nonnegative and $V_2(t) = 0$ iff $y = y^*$.

The Lyapunov functional $U(t)$ is nonnegative and defined with respect to the interior equilibrium $E^*(x_2^*, y^*)$, which is a global minimum. First, taking the derivative of V_+ with respect to time t, we get

$$\begin{aligned} \frac{dV_+}{dt} &= \frac{d}{dt} \int_{t-\tau}^{t} \left\{ \frac{x_2(\sigma)}{x_2^*} - 1 - \ln \frac{x_2(\sigma)}{x_2^*} \right\} d\sigma \\ &= \frac{x_2}{x_2^*} - 1 - \ln \frac{x_2}{x_2^*} - \frac{x_2(t-\tau)}{x_2^*} + 1 + \ln \frac{x_2(t-\tau)}{x_2^*} \end{aligned}$$

$$= -\frac{x_2(t-\tau)}{x_2^*} + \frac{x_2}{x_2^*} + \ln\frac{x_2(t-\tau)}{x_2}.$$

Let $U_1 = V_1(t) + r_1 e^{-d\tau_1} x_2^* \cdot V_+(t)$, we have

$$\begin{aligned}
\frac{dU_1}{dt} &= x_2'\left(1 - \frac{x_2^*}{x_2}\right) - r_1 e^{-d\tau}\left(x_2(t-\tau) - x_2 - x_2^* \ln\frac{x_2(t-\tau)}{x_2}\right) \\
&= \left(r_1 e^{-d\tau} x_2(t-\tau) - b_1 x_2^2 - c_1 x_2 y^m\right)\left(1 - \frac{x_2^*}{x_2}\right) - r_1 e^{-d\tau} x_2(t-\tau) \\
&\quad + r_1 e^{-d\tau} x_2 + r_1 e^{-d\tau} x_2^* \ln\frac{x_2(t-\tau)}{x_2} \\
&= r_1 e^{-d\tau}\left(x_2 - x_2^*\right) - b_1 x_2\left(x_2 - x_2^*\right) + c_1 y^m\left(x_2^* - x_2\right) \\
&\quad + r_1 e^{-d\tau} x_2^*\left(1 - \frac{x_2(t-\tau_1)}{x_2} + \ln\frac{x_2(t-\tau_1)}{x_2}\right) \\
&= \left(x_2 - x_2^*\right)\left[r_1 e^{-d\tau} - b_1 x_2 - c_1 y^m\right] + r_1 e^{-d\tau} x_2^*\left(1 - \frac{x_2(t-\tau)}{x_2} + \ln\frac{x_2(t-\tau)}{x_2}\right) \\
&= -b_1\left(x_2 - x_2^*\right)^2 + c_1\left(x_2 - x_2^*\right)\left(y^{*m} - y^m\right) \\
&\quad + r_1 e^{-d\tau} x_2^*\left(1 - \frac{x_2(t-\tau)}{x_2} + \ln\frac{x_2(t-\tau)}{x_2}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{c_1}{c_2}\frac{dV_2}{dt} &= \frac{c_1}{c_2}\left(1 - \frac{y^{*m}}{y^m}\right) y' \\
&= \left(1 - \frac{y^{*m}}{y^m}\right)\left(c_1 x_2 y^m - \frac{c_1 r_2}{c_2} y - \frac{c_1 b_2}{c_2} y^2\right) \\
&= c_1 x_2 y^m - \frac{c_1 r_2}{c_2} y - \frac{c_1 b_2}{c_2} y^2 - c_1 x_2 y^{*m} + \frac{c_1 r_2}{c_2} y \frac{y^{*m}}{y^m} + \frac{c_1 b_2}{c_2} y^2 \frac{y^{*m}}{y^m}.
\end{aligned}$$

Now, the time derivative of $U(t)$ computed along solutions of system (1.4) is

$$\begin{aligned}
\frac{dU}{dt} &= \frac{dU_1}{dt} + \frac{c_1}{c_2}\frac{dV_2}{dt} \\
&= -b_1\left(x_2 - x_2^*\right)^2 + r_1 e^{-d\tau} x_2^*\left(1 - \frac{x_2(t-\tau)}{x_2} + \ln\frac{x_2(t-\tau)}{x_2}\right) \\
&\quad - \frac{c_1 b_2}{c_2}\left(y^2 - y^2\frac{y^{*m}}{y^m} + y^{*2} - y^{*2}\frac{y^m}{y^{*m}}\right) - \frac{c_1 r_2}{c_2}\left(y - y\frac{y^{*m}}{y^m} + y^* - y^*\frac{y^m}{y^{*m}}\right),
\end{aligned}$$

and here we used $c_2 x_2^* y^{*m} = r_2 y^* + b_2 y^{*2}$.

By factoring the last two terms, we have

$$\begin{aligned}
\frac{dU}{dt} &= -b_1\left(x_2 - x_2^*\right)^2 - r_1 e^{-d\tau_1} x_2^*\left(\frac{x_2(t-\tau)}{x_2} - 1 - \ln\frac{x_2(t-\tau)}{x_2}\right) \\
&\quad - \frac{c_1 b_2}{c_2} y^2\left[1 - \left(\frac{y^*}{y}\right)^{2-m}\right]\left[1 - \left(\frac{y^*}{y}\right)^m\right] - \frac{c_1 r_2}{c_2} y\left[1 - \left(\frac{y^*}{y}\right)^{1-m}\right]\left[1 - \left(\frac{y^*}{y}\right)^m\right].
\end{aligned}$$

From Lemma 1, for $0 < m < 1$, we know that

$$\left[1-\left(\frac{y^*}{y}\right)^{2-m}\right]\left[1-\left(\frac{y^*}{y}\right)^{m}\right] \geq 0, \tag{2.7}$$

$$\left[1-\left(\frac{y^*}{y}\right)^{1-m}\right]\left[1-\left(\frac{y^*}{y}\right)^{m}\right] \geq 0. \tag{2.8}$$

Further, since the function $h(z) = z(t) - 1 - \ln z(t)$ is always nonnegative for any function $z(t) > 0$, and $h(z) = 0$ if and only if $z(t) = 1$, we know that

$$\frac{x_2(t-\tau)}{x_2} - 1 - \ln\frac{x_2(t-\tau)}{x_2} \geq 0. \tag{2.9}$$

It follows that the positive-definite functional $U(t)$ has non-positive derivative $\frac{d}{dt}U(t)$. Let $\mathcal{M}$ be the largest invariant subset of $\{(x_2(t), y(t)) \mid \frac{dU}{dt} = 0\}$. Since $\frac{dU}{dt}$ equals zero if and only if $x_2(t) = x_2^* = x_2(t-\tau), y(t) = y^*$, we see that $\mathcal{M}$ is the singleton $\{E^*\}$. By the LaSalle invariance principle [7], every solution of system (1.4) tends to the interior equilibrium E^*, which is globally asymptotically stable.

The proof is completed. □

Remark The type of Lyapunov function V_1 was first used for the Lotka–Volterra system, and then it was successfully applied to epidemiological models by Korobeinikov [13, 14]. Furthermore, McCluskey [17, 18] extended it as the Lyapunov function of form V_+ for some delay differential equations models.

3 Numerical simulations and conclusions

Here we perform numerical simulations to show that parameters m and τ have no effects on the stability of the interior equilibrium $E^*(x_2^*, y^*)$ of system (1.4). Parameters values are from [16] except for m and τ as follows:

$$\begin{aligned} &d = 0.2, \quad r_1 = 2.5, \quad b_1 = 1.8, \quad c_1 = 1, \\ &r_2 = 1, \quad b_2 = 0.5, \quad c_2 = 1.2. \end{aligned} \tag{3.1}$$

First, we fix $\tau = 2.5$ and vary $m = 0.1, 0.3, 0.7, 0.9$, which correspond to four values of $E^* = (0.346676, 0.320008), (0.444703, 0.328168), (0.62698, 0.258366), (0.741398, 0.150437)$. Figure 2 shows that all solutions tend to E^*. We can see that the value of x_2^* increases as the increase of m (see Fig. 3(a)), but the value of y^* increases first and then decreases as the increase of m (see Fig. 3(b)). Second, we fix $m = 0.5$ and vary $\tau = 1, 5, 10, 20$, which correspond to four values of $E^* = (0.742163, 0.505426)$, $(0.313992, 0.125679)$, $(0.113186, 0.0181181)$, $(0.0152641, 0.000335395)$. Figure 4 also reveals that all solutions tend to E^*. And we found that the values of both x_2^* and y^* decrease with the increase of τ (see Fig. 5). In conclusion, both m and τ can change the value of E^*, but they cannot affect the stability of E^*.

In this study, by constructing suitable Lyapunov functional, we establish the global asymptotic properties of the interior equilibrium of the stage structure predator–prey model with delay. Without any additional conditions, the interior equilibrium of system

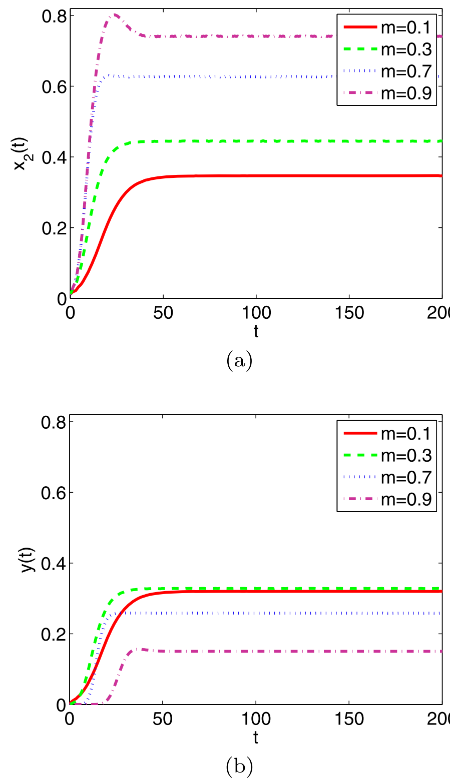


Figure 2 All solutions converge to the interior equilibrium E^* for different m. Here we fix $\tau = 2.5$ and choose the other parameters from (3.1) with the initial condition $(x_2(0), y(0)) = (0.01, 0.001)$

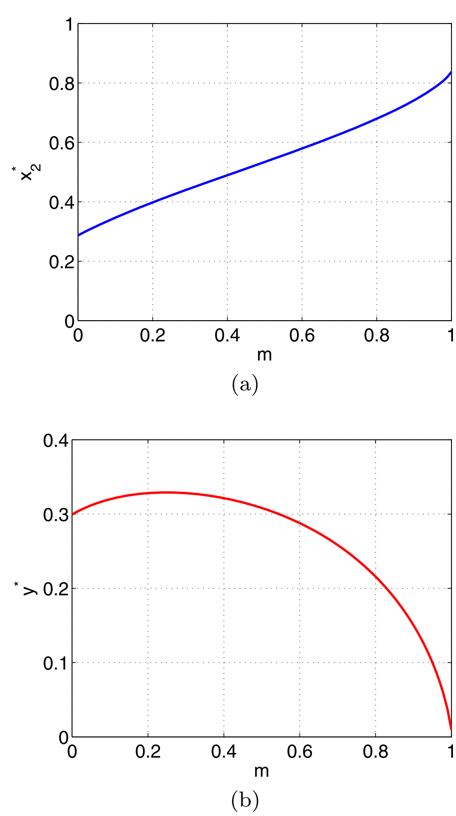


Figure 3 The relation curves of x_2^* and y^* versus m. Here we fix $\tau = 2.5$ and choose the other parameters from (3.1)

Figure 4 All solutions converge to the interior equilibrium E^* for different τ. Here we fix $m = 0.5$ and choose the other parameters from (3.1) with the initial condition $(x_2(0), y(0)) = (0.01, 0.001)$

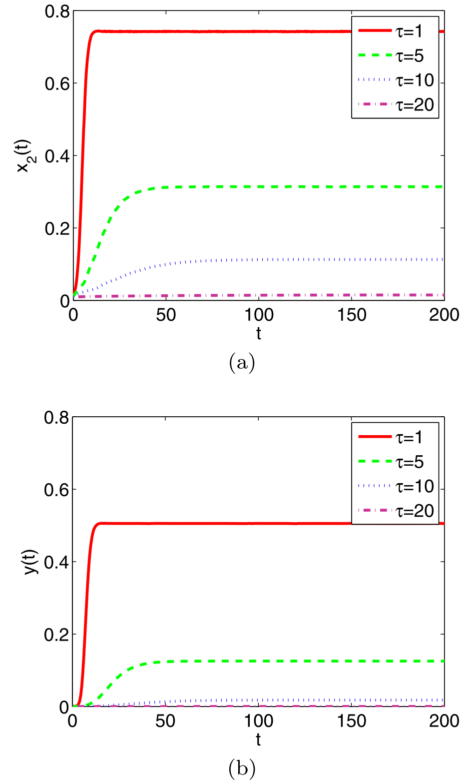
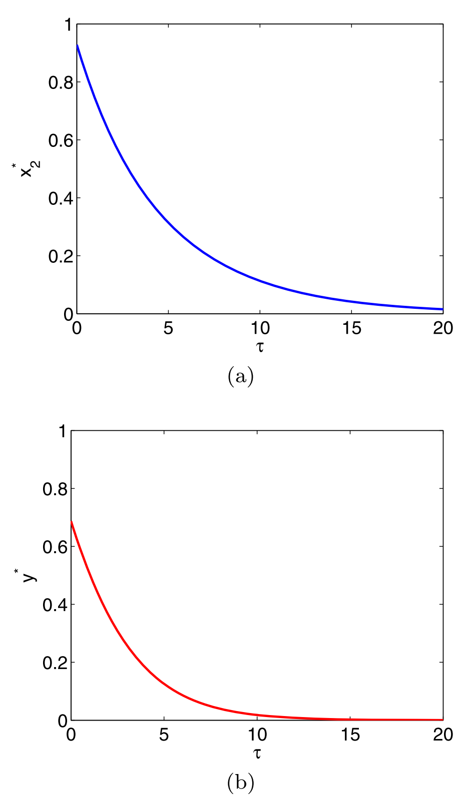


Figure 5 The relation curves of x_2^* and y^* versus τ. Here we fix $m = 0.5$ and choose the other parameters from (3.1)



(1.4) always exists and is globally asymptotically stable. This essentially improves the previous stability results in [16]. On the other hand, when $\tau = 0$, system (1.4) will be simplified to the ordinary differential system. Theorem 2.1 indicates that mature period delay of prey does not affect the global asymptotic properties of the model.

In the special case of $m = 1$ (that is, no mutual interference), system (1.4) has a positive equilibrium if and only if

$$\frac{r_2}{c_2} < \frac{r_1 e^{-d\tau}}{b_1}. \tag{3.2}$$

By using the same Lyapunov functional, it is easy to see that the positive equilibrium is globally asymptotically stable when it exists. When we introduce mutual interference ($0 < m < 1$), system (1.4) always has a positive equilibrium. That is, we do not need the condition (3.2) to ensure the existence of positive equilibrium for $0 < m < 1$. It means that the mutual interference ($0 < m < 1$) helps the endangered predators survive under any maturation time delay of preys.

We would like to point out that the Lyapunov approach in this study comes from the generalization of our previous work in Huang et al. [10, 11]. Here we applied the technology of constructing Lyapunov functionals to the delayed predator–prey model with mutual interference, and it can also be applied to some classes of systems similar to system (1.4).

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally to the writing of this paper. The authors read and approved the final manuscript.

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References

1. Aiello, W.G., Freedman, H.I.: A time-delay model of single-species growth with stage structure. Math. Biosci. **101**, 139–153 (1990)
2. Barclay, H.J., Van den Driessche, P.: A model for a species with two life history stages and added mortality. Ecol. Model. **11**, 157–166 (1980)
3. Burnett, T.: Effects of natural temperatures on oviposition of various numbers of an insect parasite (Hymenoptera, Chalcididae, Tenthredinidae). Ann. Entomol. Soc. Am. **49**, 55–59 (1956)
4. Freedman, H.I.: Graphical stability, enrichment, and pest control by a natural enemy. Math. Biosci. **31**, 207–225 (1976)
5. Freedman, H.I.: Stability analysis of a predator–prey system with mutual interference and density-dependent death rates. Bull. Math. Biol. **41**, 67–78 (1979)
6. Freedman, H.I., Rao, V.S.: The trade-off between mutual interference and time lags in predator–prey systems. Bull. Math. Biol. **45**, 991–1004 (1983)
7. Hale, J., Verduyn Lunel, S.M.: Introduction to Functional Differential Equations, vol. 9. Applied Mathematical Science, New York (1993)
8. Hassell, M.P.: Mutual interference between searching insect parasites. J. Anim. Ecol. **40**, 473–486 (1971)
9. Hassell, M.P., Varley, G.C.: New inductive population model for insect parasites and its bearing on biological control. Nature **223**, 1133–1137 (1969)
10. Huang, G., Liu, A., Foryś, U.: Global stability analysis of some nonlinear delay differential equations in population dynamics. J. Nonlinear Sci. **26**, 27–41 (2016)
11. Huang, G., Takeuchi, Y., Miyazaki, R.: Stability conditions for a class of delay differential equations in single species dynamics. Discrete Contin. Dyn. Syst., Ser. B **17**, 2451–2464 (2012)

12. Jiang, X., She, Z., Feng, Z., Zheng, X.: Bifurcation analysis of a predator–prey system with ratio-dependent functional response. Int. J. Bifurc. Chaos **27**, Article ID 1750222 (2017)
13. Korobeinikov, A., Maini, P.K.: A Lyapunov function and global properties for SIR and SEIR epidemiological models with nonlinear incidence. Math. Biosci. Eng. **1**, 57–60 (2004)
14. Korobeinikov, A., Wake, G.C.: Lyapunov functions and global stability for SIR, SIRS and SIS epidemiological models. Appl. Math. Lett. **15**, 955–961 (2002)
15. Li, H., She, Z.: Uniqueness of periodic solutions of a nonautonomous density-dependent predator–prey system. J. Math. Anal. Appl. **422**, 886–905 (2015)
16. Li, Z., Han, M., Chen, F.: Global stability of a predator–prey system with stage structure and mutual interference. Discrete Contin. Dyn. Syst., Ser. B **19**, 173–187 (2014)
17. McCluskey, C.C.: Global stability for an SEIR epidemiological model with varying infectivity and infinite delay. Math. Biosci. Eng. **6**, 603–610 (2009)
18. McCluskey, C.C.: Complete global stability for an SIR epidemic model with delay—distributed or discrete. Nonlinear Anal., Real World Appl. **11**, 55–59 (2010)
19. She, Z., Li, H.: Dynamics of a density-dependent stage-structured predator–prey system with Beddington–Deangelis functional response. J. Math. Anal. Appl. **406**, 188–202 (2013)