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Dynamic output feedback control based on virtual feedback control law for planar switched nonlinear systems with time-varying delays and multiple subsystems

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Abstract

This paper addresses the dynamic output feedback control problem for a class of discrete-time planar switched nonlinear systems with time-varying delays and multiple subsystems. First, the virtual state feedback control law is designed based on adding a power integrator approach. Secondly, the nonlinear reduced-order compensator is designed for the subsystems of the planar switched nonlinear systems under arbitrary switchings. Then, the dynamic output feedback controller is constructed based on the virtual state feedback control law and the nonlinear reduced-order compensator. By introducing the new discrete Lyapunov–Krasovskii functional, it can be seen that the solutions of the closed-loop system converge to an adjustable bounded region. Compared with the previous works, the subsystems of this planar switched system may contain the uncontrollable/unobservable Jacobian linearizations. In addition, the obtained results are further extended to the general nonlinear case and one-link manipulator case with the motor dynamics. Finally, two simulation examples are performed to show the effectiveness of the proposed method.

Keywords: Planar switched systems; Time-varying delays; Adding a power integrator approach; Dynamic output feedback; Lyapunov–Krasovskii functional

1 Introduction

The planar switched nonlinear system is a challenging topic of control technology with a number of traditional and potential applications. The planar switched systems consist of some subsystems described by the difference or differential equations [1]. In addition, the switching control laws are often employed to orchestrate the switching between these subsystems [2]. Many applications of planar switched nonlinear systems have advantages in the practice industry. The different characteristics of switched systems, especially the system feedback stabilization and asymptotic stability, have attracted much more attention [3]. The Lyapunov–Krasovskii functional method is a very important technique for the controller design of a planar switched nonlinear system, and the obtained results are often given in the form of linear matrix inequalities (LMIs) [4]. Stability analysis of the switched systems is often implemented with a common Lyapunov–Krasovskii functional and multiple Lyapunov–Krasovskii functionals [5]. A common Lyapunov–Krasovskii functional is used to discuss the stability of switched systems under arbitrary switching; however, the construction of the common Lyapunov–Krasovskii functional is a difficult task and there are only few results for some special systems [5]. Multiple Lyapunov–Krasovskii functionals method is often used in the stability analysis of slowly switched systems [6]. With the help of the proper switching signals, such as the dwell time and average dwell time switching signals, the stability analysis can be approached by the virtue of multiple Lyapunov–Krasovskii functionals. However, how to design a proper Lyapunov–Krasovskii functional for the switched nonlinear systems is generally a difficult task [7]. By employing the Lyapunov–Krasovskii method, some new results were obtained based on the sleeping strategy optimization strategy in [8].

It is well known that the time-delay issues are encountered frequently in various engineering systems and can be a cause of instabilities [9]. The information transmission among the subsystems often induces the time-delays in the planar switched nonlinear systems [10]. On the other hand, with the rapid development of an industrial nonlinear system, planar switched systems play a more and more important role in the nonlinear system, the stability analysis and controller design problem is increasingly concerned and studied. To deal with this problem, many important techniques are presented to design the intelligent controllers. An efficient method to tackle this problem is to use the adaptive control scheme such as the adaptive generalized predictive and the model predictive methods in a force control scheme proposed in [11]. In [9, 10], the adaptive control theory was employed for a class of nonlinear systems with multiple dead-zone inputs and nonlinear uncertainties, respectively. In order to avoid the problem of "explosion of complexity", Han and Lee [12] proposed a dynamic surface control method for a class of planar switched nonlinear strict-feedback systems. With the help of the neural network approximation theory, a variable neural adaptive robust controller was constructed for the planar switched nonlinear systems to deal with the dead-zone case in [13]. Then, a low-complexity global approximation-free control scheme with prescribed performance was proposed for a class of SISO pure-feedback nonlinear systems in [14]. Compared with the controller design for the SISO system, the control issues for the MIMO system are more difficult and challenging. For the MIMO nonlinear dynamic systems with time-delay and dead-zone inputs, the delay-dependent exponential stabilization criteria were proposed for a class of nonlinear systems with mixed time-varying delays in [15]. In [16], the MIMO underactuated nonlinear system was in the form of block-triangular structure, and the novel controller was designed based on the back-stepping method. Moreover, the state feedback stabilization of switched nonlinear systems, which have special structures, especially in lower triangular forms [17], has drawn much attention, but the output feedback stabilization of switched nonlinear systems under arbitrary switchings is still a challenging task [18]. There was a switch Lyapunov–Krasovskii functional proposed for every subsystem, and some observable conditions about output feedback controllers and switching logic were presented in [18]. However, the proposed control strategy in [18] did not mention how to construct the Lyapunov–Krasovskii functional. Moreover, if the subsystem is not observable any more, the above method may be unavailable. Recently, for the switched nonlinear systems, output feedback stabilization has been achieved successfully based on the reduced-order observers, and it was designed in a constructive manner [19]. Even though many effective control algorithms have been proposed for the nonlinear dynamic system, only the steady-state performance is considered. Few control schemes consider both the transient-state performance and the steady-state performance for the planar switched nonlinear system with multiple time-varying delays.

The output feedback stabilization is one of the most important problems in the area of nonlinear robust control. On the other hand, it should be pointed out that the separation principle does not hold normally for the nonlinear time-delay system anymore [20]. Hence, compared with the global stabilization via state feedback, the output feedback stabilization for the planar switched nonlinear system is much more important and challenging [20]. In addition, the output feedback control approach has been investigated extensively and used widely in the control and applications problems, see [18, 21, 22] and the references therein. In [21], the feedback linearization for planar switched nonlinear systems with time-varying input and output delays was investigated. Then, in order to obtain the better transient-state behavior of the closed-loop system, the adaptive output feedback control was investigated in [22]. The output feedback control means that the solutions of the resulting closed-loop system are uniformly ultimately bounded and convergent towards a ball with adjustable radius, exhibiting a maximum overshoot less than a sufficiently small constant [22]. Recently, some intelligent control strategies were proposed for switched nonlinear systems via the adaptive fuzzy and output feedback design techniques [23–25]. But the planar switched nonlinear systems are all in pure-feedback or strict-feedback forms, rather than in non-strict-feedback forms [25]. More recently, Tong et al. [26] proposed an adaptive fuzzy output feedback tracking control method for the switched nonlinear systems without considering the time-delay issue. In addition, since Tong et al. [26] employed the average dwell-time method, the switching signals in [26] are not arbitrary and need to satisfy some restrictive conditions. Since the first approximation of the nonlinear system is neither observable nor controllable, adding a power integrator approach has been employed for the output feedback stabilization of the systems. Such as the global output feedback stabilization of nonlinear systems via sampled-data control [27], non-smooth output feedback stabilization of nonlinear system [28], and smooth output feedback stabilization of planar system [29]. In general, a popular assumption in the above-mentioned results is that the switch behaviour is required to occur at the same time for the controllers and system modes, i.e., they switch synchronously all the time. However, this assumption rarely holds. In practical systems, some time is always spent on the mode identification process before the application of the corresponding controller [30]. With the above observations, we consider the dynamic output feedback control problem for a class of planar switched nonlinear systems with multiple time-varying delays under arbitrary switchings and aim to design the smooth dynamic output feedback controller.

In this paper, the smooth dynamic output feedback control approach is proposed for a class of planar switched nonlinear systems in the presence of multiple time-varying delays. Different from the previous work for a planar switched nonlinear system, both transient-state and steady-state performances are guaranteed under arbitrary switchings. And the control design conditions are relaxed because of the developed nonlinear reduced-order compensator. The contributions of this paper can be summarized as follows.

(1) The virtual state feedback control laws and Lyapunov–Krasovskii functional are presented based on adding a power integrator approach. And the nonlinear reduced-order compensator is designed for the subsystems to deal with the nonlinear uncertainties under arbitrary switchings.

(2) The developed dynamic output feedback controller in this paper is memoryless and smooth, which only uses the system output. The control design conditions are relaxed because of the developed dynamic controller.
(3) By employing the discrete Lyapunov–Krasovskii functional, it can be seen that the solutions of the closed-loop system converge to an adjustable bounded region. The results are further extended to the general nonlinear case and one-link manipulator case with the motor dynamics.

This paper is organized as follows. In Sect. 2, some preliminary knowledge for the planar switched nonlinear system with multiple time-varying delays is presented. In Sect. 3, the smooth dynamic output-feedback controller is designed and the main results are presented. In Sect. 4, two simulation examples are provided to show the effectiveness of the proposed method. Finally, Sect. 5 concludes with a summary of the obtained results.

Notations $\mathbb{R}^n$ denotes the real n-dimensional space; q_i denotes the power integrator parameter; $\max\{\ldots\}$ denotes the maximum value of the parameters in $\{\ldots\}$; $G > 0$ denotes the positive term; ΔV_1 denotes the forward difference of V_1; $|x|$ denotes the standard Euclidean norm for vector x; the superscript "-1" denotes the inverse of the term.

2 Problem formulation

Consider a class of discrete-time planar switched nonlinear systems with time-varying delays and multiple subsystems as follows:

$$\begin{cases} x_1(k+1) = x_2^{q_i}(k-\tau_2(k)) + \varphi_{i,1}(x_1(k-\tau_1(k))), \\ x_2(k+1) = u_i(k) + \varphi_{i,2}(x_1(k-\tau_1(k)), x_2(k-\tau_2(k))), \\ y(k) = x(k), \quad i \in N_S, N_S = \{1,2,\ldots,N\}, \end{cases} \tag{1}$$

where $x(k) = [x_1(k-\tau_1), x_2(k-\tau_2)]^T \in \mathbb{R}^2$, $u_i(k) \in \mathbb{R}^2$, and $y(k) \in \mathbb{R}^2$ are the state vector, control input, and output of the system, respectively. $q_i \in [1,3,\ldots,2n+1]$ is the power integrator parameter with n being a nonnegative integer. $x_2^{q_i}$ is the q_i square of the state variable x_2. $\varphi_{i,1}(x_1(k-\tau_1))$ and $\varphi_{i,2}(x_1(k-\tau_1), x_2(k-\tau_2))$ are the nonlinear functions with $\varphi_{i,1}(0) = 0$, $\varphi_{i,2}(0,0) = 0$. N_S is the number of the subsystems for system (1).

For the planar switched nonlinear time-delay system (1), there exists Assumption 1 as follows.

Assumption 1 ([28, 29]) For the state variables x_1 and x_2, there exist the positive scalars $a_{i,1}$ and $a_{i,2}$ such that

$$\begin{cases} a_{i,1}|x_1(k-\tau_1)|^{q_i} \geq |\varphi_{i,1}(x_1(k-\tau_1))|, \\ a_{i,2}(|x_1(k-\tau_1)|^{q_i} + |x_2(k-\tau_2)|^{q_i}) \geq |\varphi_{i,2}(x_1(k-\tau_1), x_2(k-\tau_2))|. \end{cases} \tag{2}$$

Remark 1 The dynamic output feedback technique is more flexible and the required conditions on the considered systems are less conservative. In addition, the proposed method is very efficient for the control design of the time-delay systems with nonlinear uncertainties, and the precise time-delays are not required for the control implementation [31]. Thus, with Assumption 1, the dynamic output feedback controller will be constructed for system (1). First, the virtual state feedback control law is designed based on adding a power integrator technique. Then, the nonlinear reduced-order compensators are designed to estimate the unmeasurable state of system (1). Moreover, based on the developed virtual state feedback control law and nonlinear reduced-order compensators, the dynamic output feedback controller is constructed to stabilize system (1) under arbitrary switchings. The detailed design process of the controller will be presented in Sect. 3.

3 Controller design

In this section, the virtual state feedback control law is designed for the nonlinear system (Sect. 3.1). The nonlinear reduced-order compensator is designed for the subsystems of the planar switched nonlinear systems (Sect. 3.2). Then the dynamic output feedback controller is constructed based on the virtual state feedback control law and the nonlinear reduced-order compensator (Sect. 3.3).

The dynamic output feedback controller is designed for system (1) with Assumption 1. The graphical abstract of the proposed methodology is shown in Fig. 1. For system (1), the smooth dynamic output feedback controller is designed as follows:

$$\begin{cases} z(k+1) = -G(\varphi_{i,1}(x_1(k-\tau_1)) + (z(k) + Gx_1(k-\tau_1))^{q_i}) + u_i(k), \\ u_i(k) = -(1+\sigma_i)(z(k) + (G-M)x_1(k-\tau_1))^{q_i}, \end{cases} \tag{3}$$

where $G > 0$, $M > 0$, and $\sigma_i \geq 0$ $(i \in N_S)$ are the positive scalars such that the solutions of the closed-loop system converge to an adjustable bounded region under arbitrary switchings.

The design scheme of controller (3) is divided into three steps (Sects. 3.1, 3.2, and 3.3).

Remark 2 It is worth pointing out that Assumption 1 is a standard assumption for the output tracking control of nonlinear systems, and the similar assumption can be found in other literature sources see [32, 33] and the references therein. For some other control

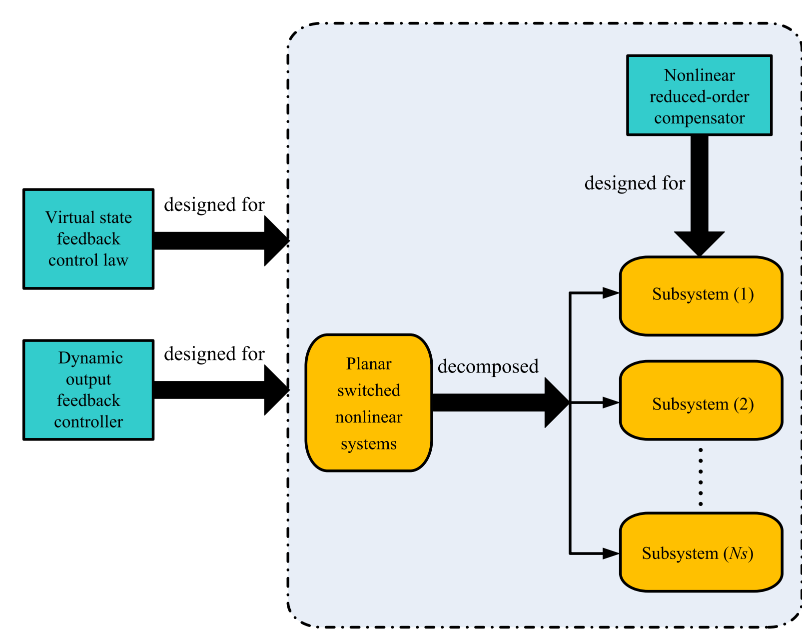

Figure 1 The graphical abstract of the proposed methodology

schemes of the nonlinear systems with unmeasured states, a restrictive assumption on the Lyapunov–Krasovskii functional stability condition is often needed [34, 35]. Therefore, we will introduce some technical Lemmas (Lemmas 1, 2, and 3) and LMIs related to the Lyapunov–Krasovskii functional in this paper, which will play the important roles in the subsequent developments and will be used frequently in Sect. 3.1.

3.1 Virtual state feedback control law design

In this section, the virtual state feedback control law is designed for the planar switched nonlinear system (1).

For system (1), choose a discrete Lyapunov–Krasovskii functional as follows:

$$V_1(x_1) = \frac{x_1^2}{2}. \tag{4}$$

Now, taking the forward difference of V_1 along system (1) yields

$$\Delta V_1(x_1) \le x_1(k-\tau_1)\bigl(x_2^{q_i}(k-\tau_2) - x_2^{*q_i}(k)\bigr) + x_1^{1+q_i}(k-\tau_1)a_{i,1} + x_1(k-\tau_1)x_2^{*q_i}(k), \tag{5}$$

where x_2^* is the virtual state feedback control law.

In this paper, the virtual state feedback control law is designed as follows:

$$x_2^*(k) = -Mx_1(k-\tau_1), \tag{6}$$

where $M > 0$ is the virtual state feedback gain.

With (5), the following inequality holds:

$$\Delta V_1(x_1) \le x_1(k-\tau_1)\bigl(x_2^{q_i}(k-\tau_2) - x_2^{*q_i}(k)\bigr) + \bigl(a_{i,1} - M^{q_i}\bigr)x_1^{1+q_i}(k-\tau_1). \tag{7}$$

Let $M \ge \max_{i\in N_S}\{(2+a_{i,1})^{q_i^{-1}}\}$, one has

$$\Delta V_1(x_1) \le x_1(k-\tau_1)\bigl(x_2^{q_i}(k-\tau_2) - x_2^{*q_i}(k)\bigr) - 2x_1^{1+q_i}(k-\tau_1). \tag{8}$$

For system (1), choose a new discrete Lyapunov–Krasovskii functional as follows:

$$V_2(x_1,x_2) = \frac{\zeta_2^2}{2} + V_1(x_1), \tag{9}$$

where $\zeta_2 = x_2(k-\tau_2) - x_2^*(k)$.

Then, taking the forward difference of V_2 yields

$$\begin{aligned}\Delta V_2(x_1,x_2) \le{}& \zeta_1\bigl(x_2^{q_i}(k-\tau_2) - x_2^{*q_i}(k)\bigr) + \zeta_2 u_i(k) + \zeta_2\Bigl(M\bigl(x_2^{q_i}(k-\tau_2) + \varphi_{i,1}\bigl(x_1(k-\tau_1)\bigr)\bigr)\\ &+ \varphi_{i,2}\bigl(x_1(k-\tau_1), x_2(k-\tau_2)\bigr)\Bigr) - 2\zeta_1^{1+q_i},\end{aligned} \tag{10}$$

where $\zeta_1 = x_1(k-\tau_1)$.

The aim of this paper is to develop a new approach to design the smooth dynamic output feedback controller in the form of (3) for the planar switched nonlinear system (1). For this purpose, Lemmas 1, 2 and 3 are introduced as follows [28, 29, 36].

Lemma 1 *For the system state variable $x(k)$ and system output $y(k)$, if there exists a power integrator parameter $q_i \in [1,3,\dots,2n+1]$ such that $|x(k)+y(k)|^{q_i} \le 2^{q_i-1}|x^{q_i}(k)+y^{q_i}(k)|$, then the following inequality holds*:

$$\left|x^{q_i}(k) - y^{q_i}(k)\right| \le q_i\left|x(k)-y(k)\right|\left(x^{q_i-1}(k)+y^{q_i-1}(k)\right).$$

Lemma 2 *For the system state variable $x(k)$ and system output $y(k)$, if there exist the positive scalars a, b, and a nonlinear real-valued function $\Gamma(x,y)$ such that $\Gamma(x,y)>0$, then the following inequality holds*:

$$\left|x(k)\right|^a\left|y(k)\right|^b \le \frac{a\Gamma(x,y)}{a+b}\left|x(k)\right|^{a+b} + \frac{b\Gamma^{-b^{-1}a}}{a+b}(x,y)\left|y(k)\right|^{a+b}.$$

Lemma 3 *For any $w \in \mathbb{R}$ and $c \in \mathbb{R}$, if there exists the power integrator parameter $q_i \in [1,3,\dots,2n+1]$, then the following inequality holds*:

$$\frac{w^{1+q_i}}{2^{q_i-1}} \le -w\left((c-w)^{q_i} - c^{q_i}\right).$$

With Lemma 1, one has

$$\left|x_2^{q_i}(k-\tau_2) - x_2^{*q_i}(k)\right| \le \left|x_2(k-\tau_2) - x_2^*(k)\right| \times \chi_{i,1}\left(x_1^{q_i-1}(k-\tau_1) + x_2^{q_i-1}(k-\tau_2)\right), \tag{11}$$

where

$$\chi_{i,1} \ge \max_{i\in N_S}\left\{q_i M^{q_i-1}, q_i\right\}. \tag{12}$$

With Assumption 1 and (12), inequality (10) is rewritten as follows:

$$\begin{aligned}\Delta V_2(x_1,x_2) \le{}& |\zeta_1|\left|x_2(k-\tau_2) - x_2^*(k)\right| \times \chi_{i,1}\left(x_1^{q_i-1}(k-\tau_1) + x_2^{q_i-1}(k-\tau_2)\right) - 2{\zeta_1}^{1+q_i}\\ &+ \zeta_2\left(u_i(k) + M\left(\varphi_{i,1}\left(x_1(k-\tau_1)\right) + x_2^{q_i}(k-\tau_2)\right)\right.\\ &\left.+ \varphi_{i,2}\left(x_1(k-\tau_1), x_2(k-\tau_2)\right)\right),\end{aligned} \tag{13}$$

where $M(1+a_{i,1}) + a_{i,2} < \delta_{i,1}$.

By considering the construction of the virtual state feedback control law x_2^*, it can be seen that the following inequality holds:

$$\left|x_2(k-\tau_2)\right| \le \left|x_2^*(k)\right| + \left|x_2(k-\tau_2) - x_2^*(k)\right| \le M\left|x_1(k-\tau_1)\right| + \left|x_2(k-\tau_2) - x_2^*(k)\right|.$$

With the above analysis and Lemma 1, one has

$$\begin{aligned}\Delta V_2(x_1,x_2) \le{}& 2^{q_i-2}|\zeta_1|\left|x_2(k-\tau_2) - x_2^*(k)\right|\\ &\times \left|x_2(k-\tau_2) - x_2^*(k)\right|^{q_i-1}\chi_{i,1} - 2{\zeta_1}^{1+q_i}\\ &+ |\zeta_1|\left|x_2(k-\tau_2) - x_2^*(k)\right|\\ &\times \left(1 + 2^{q_i-2}M^{q_i-1}\right)\left|x_1(k-\tau_1)\right|^{q_i-1}\chi_{i,1}\end{aligned}$$

$$
\begin{aligned}
&+ \zeta_2 u_i + 2^{q_i-1}|\zeta_2|\big|x_2(k-\tau_2) - x_2^*(k)\big|^{q_i}\delta_{i,1} \\
&+ |\zeta_2|\big(1 + 2^{q_i-1}M^{q_i-1}\big)\big|x_1(k-\tau_1)\big|^{q_i}\delta_{i,1}.
\end{aligned} \tag{14}
$$

With Lemma 2, there exist the scalars $\tilde{\chi}_{i,1}$ and $\tilde{\delta}_{i,1}$ such that inequality (14) is rewritten as follows:

$$
\begin{aligned}
\Delta V_2(x_1,x_2) \le{}& \zeta_2 u_i(k) - 2\zeta_1^{1+q_i} + \tilde{\chi}_{i,1} m_1|\zeta_1|^{1+q_i} + |\zeta_2|^{1+q_i}\tilde{\delta}_{i,1} + (\tilde{\chi}_{i,1} + \tilde{\delta}_{i,1})m_2|\zeta_1|^{1+q_i} \\
&+ \tilde{\chi}_{i,1}\frac{q_i}{1+q_i}\left(\frac{1}{m_1(q_i+1)}\right)^{q_i^{-1}}|\zeta_2|^{1+q_i} \\
&+ (\tilde{\chi}_{i,1} + \tilde{\delta}_{i,1})\left(\frac{1}{1+q_i}\right)\left(\frac{q_i}{m_2(1+q_i)}\right)^{q_i}|\zeta_2|^{1+q_i} \\
\le{}& \zeta_2^{1+q_i}\sigma_i - \zeta_1^{1+q_i} + \zeta_2 u_i(k),
\end{aligned} \tag{15}
$$

where $\tilde{\chi}_{i,1} = \max_{i\in N_S}\{(2^{q_i-2}M^{q_i-1} + 1)\chi_{i,1}, 2^{q_i-2}\chi_{i,1}\}$, $\tilde{\delta}_{i,1} = \max_{i\in N_S}\{(2^{q_i-1}M^{q_i-1} + 1)\delta_{i,1}, 2^{q_i-1}\delta_{i,1}\}$, m_1 and m_2 are the positive scalars satisfying $m_1 > 0$, $m_2 > 0$, $m_1\tilde{\chi}_{i,1} + m_2(\tilde{\chi}_{i,1} + \tilde{\delta}_{i,1}) \le 1$, and $\sigma_i \ge \frac{q_i}{1+q_i}(\frac{1}{m_1(1+q_i)})^{q_i^{-1}} \times \tilde{\chi}_{i,1} + (\tilde{\chi}_{i,1} + \tilde{\delta}_{i,1})(\frac{1}{1+q_i})(\frac{q_i}{m_2(1+q_i)})^{q_i}$.

With Lemma 2, the feedback control law is designed as follows:

$$
u_i(k) = -(1+\sigma_i)\begin{bmatrix} \zeta_1^{q_i} \\ \zeta_2^{q_i} \end{bmatrix}, \tag{16}
$$

where ζ_1 and ζ_2 are defined in (10) and (9), respectively.

Then, substituting (16) into (15) yields

$$
\Delta V_2(x_1,x_2) \le -\zeta_1^{1+q_i} - \zeta_2^{1+q_i}. \tag{17}
$$

Remark 3 It is worth noting that the time-varying delay $\tau_i(k)$ in (1) depends on the subsystem number N_s absolutely. In practice, the time-delays $\tau_i(k)$ are often different for each subsystem, and many existing results are not suitable for controlling system (1) in this paper. Thus, it is necessary to develop an effective control scheme for system (1). From (6), it can be concluded that the virtual state feedback gain M is a large enough constant satisfying $M \ge \max_{i\in N_S}\{(2 + a_{i,1})^{q_i^{-1}}\}$, which will be determined by the power integrator parameter q_i. From Lemmas 1 and 3, it can be seen that q_i must lie in the proper interval forthe conditions in Lemmas 1 and 3 to hold. For the problem formulated, the nonlinear reduced-order compensator will be designed in Sect. 3.2 to relax the control design conditions and enhance the design flexibility.

3.2 Nonlinear reduced-order compensator design

In this section, the nonlinear reduced-order compensator is designed for the subsystems of the planar switched nonlinear systems (1).

The nonlinear reduced-order compensator is designed as follows:

$$
z(k+1) = -G\big(\varphi_{i,1}\big(x_1(k-\tau_1)\big) + \big(z(k) + Gx_1(k-\tau_1)\big)^{q_i}\big) + u_i(k), \tag{18}
$$

where $G > 0$ is a positive scalar. By letting $e = -Gx_1(k-\tau_1) + x_2(k-\tau_2) - z(k)$, one has

$$\begin{aligned} e(k+1) = {} & -G\bigl(\bigl(Gx_1(k-\tau_1) + z(k) + e\bigr)^{q_i} - \bigl(Gx_1(k-\tau_1) + z(k)\bigr)^{q_i}\bigr) \\ & + \varphi_{i,2}\bigl(x_1(k-\tau_1), x_2(k-\tau_2)\bigr). \end{aligned} \tag{19}$$

For system (19), choose the discrete Lyapunov–Krasovskii functional as follows:

$$V_3(e) = e^2/2. \tag{20}$$

Now, taking the forward difference of $V_3(e)$ yields

$$\begin{aligned} \Delta V_3(e) = {} & e\bigl(G\bigl(\bigl(Gx_1(k-\tau_1) + z(k) + e\bigr)^{q_i} - \bigl(Gx_1(k-\tau_1) + z(k)\bigr)^{q_i}\bigr)\bigr) \\ & + e\varphi_{i,2}\bigl(x_1(k-\tau_1), x_2(k-\tau_2)\bigr). \end{aligned} \tag{21}$$

With Lemmas 1, 3 and (21), one has

$$\begin{cases} \Delta V_3(e) \le -\frac{Ge^{1+q_i}}{2^{q_i-1}} + |e\varphi_{i,2}(x_1(k-\tau_1), x_2(k-\tau_2))|, \\ |x_2(k-\tau_2)|^{q_i} \le 2^{q_i-1}(|x_2^*(k)|^{q_i} + |x_2(k-\tau_2) - x_2^*(k)|^{q_i}). \end{cases} \tag{22}$$

With Assumption 1 and (22), one has

$$\begin{aligned} \Delta V_3(e) & \le -\frac{Ge^{1+q_i}}{2^{q_i-1}} + |e|\bigl(\bigl|x_1(k-\tau_1)\bigr|^{q_i} + \bigl|x_2(k-\tau_2)\bigr|^{q_i}\bigr)a_{i,2} \\ & \le -\frac{Ge^{1+q_i}}{2^{q_i-1}} + |e|\bigl(\bigl|x_1(k-\tau_1)\bigr|^{q_i} + \bigl|x_2(k-\tau_2) - x_2^*(k)\bigr|^{q_i}\bigr)\tilde{a}_{i,2}, \end{aligned} \tag{23}$$

where $\tilde{a}_{i,2} > \max_{i\in N_S}\{1 + 2^{q_i-1}a_{i,2}M^{q_i}, 2^{q_i-1}a_{i,2}\}$.

With Lemma 2, one has

$$\Delta V_3(e) \le |e||\zeta_1|^{q_i}\tilde{a}_{i,2} + |e||\zeta_2|^{q_i}\tilde{a}_{i,2} - \frac{Ge^{1+q_i}}{2^{q_i-1}} \le \frac{(\zeta_1^{1+q_i} + \zeta_2^{1+q_i})}{4} - \frac{Ge^{1+q_i}}{2^{q_i-1}} + e^{1+q_i}J_i, \tag{24}$$

where $J_i \ge \frac{2}{1+q_i}(\frac{4q_i}{1+q_i})^{q_i}\tilde{a}_{i,2}^{1+q_i}$ is a scalar.

Remark 4 The nonlinear reduced-order compensator is designed for the subsystems of the planar switched nonlinear systems (1), and the unmeasurable states of system (1) can be estimated effectively. In addition, from (18), one knows that the transient-state performance of the closed-loop system is determined by the gain parameter G. To obtain better transient-state performance, the design process of the parameter G will be presented in Sect. 3.3.

3.3 Gain parameter *G* design

In this section, the gain parameter G is designed for the nonlinear reduced-order compensator (18).

Since the system state variable x_2 is not available for the feedback control, $Gx_1(k-\tau_1) + z(k)$ can be used instead of $x_2(k-\tau_2)$ in the control law (16). Then the control law u_i in

(18) can be rewritten as follows:

$$u_{i,d} = -(1+\sigma_i)\zeta_2^{q_i} = -(1+\sigma_i)\big(Gx_1(k-\tau_1) - x_2^*(k) + z(k)\big)^{q_i}. \tag{25}$$

By employing the nonlinear reduced-order compensator (18), inequality (17) is rewritten as follows:

$$\begin{aligned}\Delta V_2(x_1,x_2) &\le -\zeta_1^{1+q_i} + \zeta_2\big(u_{i,d} - u_i(k)\big) - \zeta_2^{1+q_i}\\ &= -\zeta_1^{1+q_i} - (1+\sigma_i)\times\zeta_2\big(\big(x_2(k-\tau_2) - x_2^*(k) - e\big)^{q_i} - \big(x_2(k-\tau_2) - x_2^*(k)\big)^{q_i}\big)\\ &\quad - \zeta_2^{1+q_i}.\end{aligned} \tag{26}$$

With Lemma 1, one has

$$\Delta V_2(x_1,x_2) \le \eta_{i,1}(1+\sigma_i)\times|\zeta_2||e|\times\big(\big(x_2(k-\tau_2) - x_2^*(k)\big)^{q_i-1} + e^{q_i-1}\big) - \zeta_2^{1+q_i} - \zeta_1^{1+q_i}, \tag{27}$$

where $\eta_{i,1}$ is a scalar satisfying $1 + 2^{q_i-2} < \eta_{i,1}$. Then, applying Lemma 2 to (27), one has

$$\begin{aligned}\Delta V_2(x_1,x_2) &\le \eta_{i,1}(1+\sigma_i)\times\left(\frac{\Gamma_{i,N_1}|\zeta_2|^{1+q_i}}{1+p_i} + \frac{q_i\Gamma_{i,N_1}^{q_i^{-1}}}{1+p_i}|e|^{1+q_i}\right)\\ &\quad + \eta_{i,1}(1+\sigma_i)\times\left(\frac{\Gamma_{i,N_2}}{1+q_i}|e|^{1+q_i} + \frac{q_i\Gamma_{i,N_2}^{q_i^{-1}}}{1+q_i}|\zeta_2|^{1+q_i}\right) - \zeta_2^{1+q_i} - \zeta_1^{1+q_i}\\ &\le e^{1+q_i}Q_i - \frac{\zeta_2^{1+q_i}}{2} - \zeta_1^{1+q_i},\end{aligned} \tag{28}$$

where $\Gamma_{i,N_1} = \frac{\eta_{i,1}+4q_i}{4}$, $\Gamma_{i,N_2} = \frac{4\eta_{i,1}q_i}{1+q_i}$, and $Q_i = \eta_{i,1}(1+\sigma_i)(\frac{q_i\Gamma_{i,N_1}^{q_i^{-1}}}{1+q_i} + \frac{\Gamma_{i,N_2}}{1+q_i}) \ge 0$.

Taking the sum of (24) and (28) yields

$$\begin{aligned}\Delta V_2(x_1,x_2) + \Delta V_3(e) &\le \frac{(x_1^{1+q_i}(k-\tau_1) + (x_2(k-\tau_2) - x_2^*(k))^{1+q_i})}{4} + e^{1+q_i}Q_i\\ &\quad + e^{1+q_i}J_i - \frac{Ge^{1+q_i}}{2^{q_i-1}} - \zeta_1^{1+q_i} - \frac{\zeta_2^{1+q_i}}{2}\\ &= \left(Q_i + J_i - \frac{G}{2^{q_i-1}}\right)e^{1+q_i} - \frac{3\zeta_1^{1+q_i} + \zeta_2^{1+q_i}}{4}.\end{aligned} \tag{29}$$

For system (1), choose a discrete Lyapunov–Krasovskii functional as follows:

$$V(x_1,x_2,e) = V_2(x_1,x_2) + V_3(e) = \frac{\zeta_1^2 + \zeta_2^2 + e^2}{2}. \tag{30}$$

In this section, the parameter G satisfies $G \ge \max_{i\in N_S}\{2^{q_i-1}(Q_i + J_i) + 2^{q_i-2}\}$. Then, with (24) and (29), one has

$$\Delta V(x_1,x_2,e) \le -\frac{3\zeta_1^{1+q_i} + \zeta_2^{1+q_i} + 2e^{1+q_i}}{4}. \tag{31}$$

Since the Lyapunov–Krasovskii functional $V(x_1, x_2, e)$ is smooth and positive definite, it can be seen that the solutions of the closed-loop switched nonlinear system converge to an adjustable bounded region under arbitrary switchings.

Remark 5 For the smooth dynamic output feedback controller (3), adding a power integrator approach is employed to design the Lyapunov–Krasovskii functional and to construct the virtual state feedback control law for the planar switched nonlinear system. Secondly, the nonlinear reduced-order compensator is designed for the switched nonlinear system to measure the unavailable state vector. Then, the dynamic output feedback controller is constructed based on the virtual state feedback control law and the nonlinear reduced-order compensator. Compared with the previous works, the developed controller in this paper is memoryless and smooth, which only uses the system output. The control design conditions are relaxed because of the developed dynamic compensator. The whole design process is a constructive way almost without any preconditions except for the structure of the subsystems. And it should be pointed out that the structure of the subsystems is necessary for the output feedback stabilization of the inherent nonlinear systems [28, 29]. The objective of this paper is to design the dynamic output feedback controller for a class of discrete-time planar switched nonlinear systems with time-varying delays and multiple subsystems where the solutions of the closed-loop system converge to an adjustable bounded region. Then the obtained results are further extended to the general nonlinear case and one-link manipulator case with the motor dynamics in Sect. 4 to show the effectiveness of the proposed method.

4 Simulations

In this section, two simulation examples are provided to show the effectiveness and applicability of the proposed method.

4.1 Numerical example

Consider the planar switched nonlinear system with time-varying delays and two subsystems as follows:

$$\begin{aligned} &\text{Subsystem (1)}: \begin{cases} x_1(k+1) = x_1(k-\tau_1(k)) + \sin(x_1(k-\tau_1(k))), \\ x_2(k+1) = u_1(k), \\ y(k) = x_1(k). \end{cases} \\ &\text{Subsystem (2)}: \begin{cases} x_1(k+1) = x_2^3(k-\tau_2), \\ x_2(k+1) = u_2(k) + x_2^3(k-\tau_2), \\ y(k) = x_1(k). \end{cases} \end{aligned} \tag{32}$$

For system (32), the virtual state feedback control law is designed as follows:

$$x_2^*(k) = -Mx_1(k-\tau_1), \tag{33}$$

where $M > 0$ is the virtual state feedback gain given as $M = 5$.

With Lemmas 1 and 3, the following inequalities hold:

$$\begin{cases} |x(k)+y(k)|^{q_i} \le 2^{q_i-1}|x^{q_i}(k)+y^{q_i}(k)|, \\ |x^{q_i}(k)-y^{q_i}(k)| \le q_i|x(k)-y(k)|(x^{q_i-1}(k)+y^{q_i-1}(k)), \\ \frac{w^{1+q_i}}{2^{q_i-1}} \le -w((c-w)^{q_i}-c^{q_i}), \end{cases}$$

where $i \in N_S = \{1,2\}$, $q_i \in [1,3,\ldots,2n+1]$ is the power integrator parameter given as $q_1 = q_2 = 3$ in this case.

With Lemma 2, the following inequalities hold:

$$\begin{cases} |x(k)|^a|y(k)|^b \le \frac{a\Gamma(x,y)}{a+b}|x(k)|^{a+b} + \frac{b\Gamma^{-b^{-1}a}}{a+b}(x,y)|y(k)|^{a+b}, \\ m_1\tilde{\chi}_{i,1} + m_2(\tilde{\chi}_{i,1}+\tilde{\delta}_{i,1}) \le 1, \\ \sigma_i \ge \frac{3}{1+3}(\frac{1}{m_1(1+3)})^{\frac{1}{3}} \times \tilde{\chi}_{i,1} + (\tilde{\chi}_{i,1}+\tilde{\delta}_{i,1})(\frac{1}{1+3})(\frac{3}{m_2(1+3)})^3, \end{cases}$$

where a, b, m_1, and m_2 are the positive scalars given as $a = 1$, $b = 1$, $m_1 = 1/4$, and $m_2 = 1/4$.

For system (32), the nonlinear reduced-order compensator is designed as follows:

$$z(k+1) = -G\bigl(\varphi_{i,1}\bigl(x_1(k-\tau_1)\bigr) + \bigl(z(k)+Gx_1(k-\tau_1)\bigr)^{q_i}\bigr) + u_i(k), \tag{34}$$

where G is the gain parameter of the nonlinear reduced-order compensator.

With Lemma 2, the following inequalities hold:

$$\begin{cases} \Delta V_3(e) \le |e||\zeta_1|^3\tilde{a}_{i,2} + |e||\zeta_2|^3\tilde{a}_{i,2} - \frac{4e^{1+3}}{2^{3-1}} \le \frac{(\zeta_1^{1+3}+\zeta_2^{1+3})}{4} - \frac{4e^{1+3}}{2^{3-1}} + e^{1+3}J_i, \\ J_i \ge \frac{2}{1+3}(\frac{4\times 3}{1+3})^3\tilde{a}_{i,2}^{1+2}. \end{cases}$$

With the above analysis, for system (32), the dynamic output feedback controller is designed as follows:

$$\begin{cases} z(k+1) = -G(\varphi_{i,1}(x_1(k-\tau_1)) + (Gx_1(k-\tau_1)+z(k))^{q_i}) + u_i(k), \\ u_i(k) = -(1+\sigma_i)((G-M)x_1(k-\tau_1)+z(k))^{q_i}, \end{cases} \tag{35}$$

where $q_1 = q_2 = 3$ and $1+\sigma_i = 12$.

For the simulations, the initial values of the system state variables are given as $[x_1, x_2] = [1.7, -4.2]^T$. The multiple time-varying delays are given as $\tau_1(k) = 0.25(1+\sin k)$ and $\tau_2(k) = 0.1(1+\sin k)$. The random switching signal in this research is generated by the function "rand" in MATLAB (R2017a). The responses of system state variables x_1 and x_2 are shown in Fig. 2. The responses of system control inputs are shown in Fig. 3. The response of the state feedback control law is shown in Fig. 4. The response of $\Delta V(x_1, x_2, e)$ is shown in Fig. 5. In addition, the switching signal in example 1 is shown in Fig. 6. From Fig. 2, it can be seen that the proposed method is effective and can stabilize the closed-loop system quickly. From Figs. 3–4, it can be seen that the control inputs and state feedback control law are bounded as well. From Fig. 5, it can be seen that $\Delta V(x_1, x_2, e) \le 0$, that is, the closed-loop system is stable.

Remark 6 In this section, the numerical example is used to show the effectiveness of the proposed method. However, only the effectiveness of the proposed method is not enough.

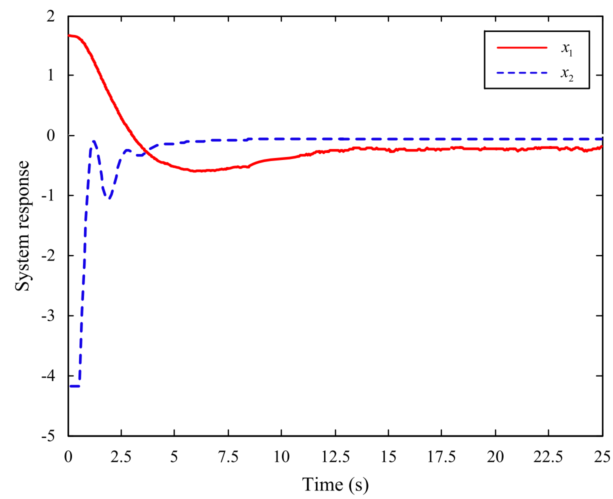

Figure 2 The responses of system state variables x_1 and x_2

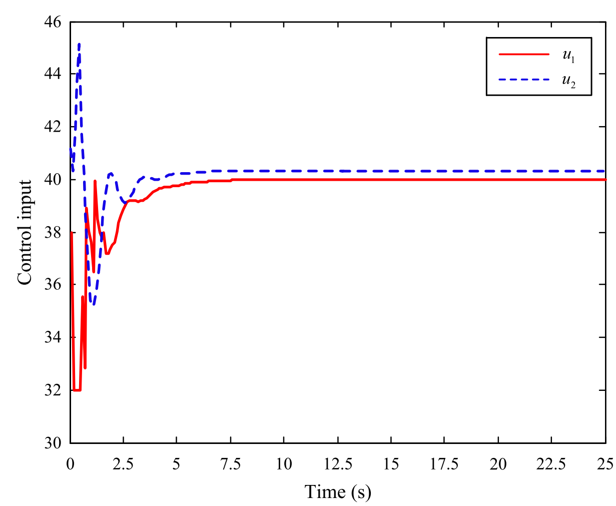

Figure 3 The responses of system control inputs

Therefore, the one-link manipulator example is performed to show the applicability of the proposed method.

4.2 One-link manipulator example

The aim of this example is to show the applicability of the proposed method. Consider a class of one-link manipulators with the motor dynamics as follows [37]:

$$\begin{cases} D\ddot{p} + B\dot{p} + A\sin(p) = \mu + \mu_{1d}, \\ C\dot{\mu} + H\mu = u - K_m\dot{p} + \mu_{2d}, \end{cases} \tag{36}$$

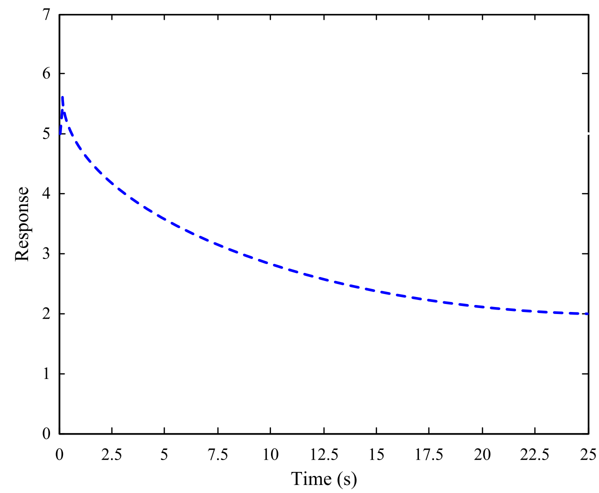

Figure 4 The response of the state feedback control law

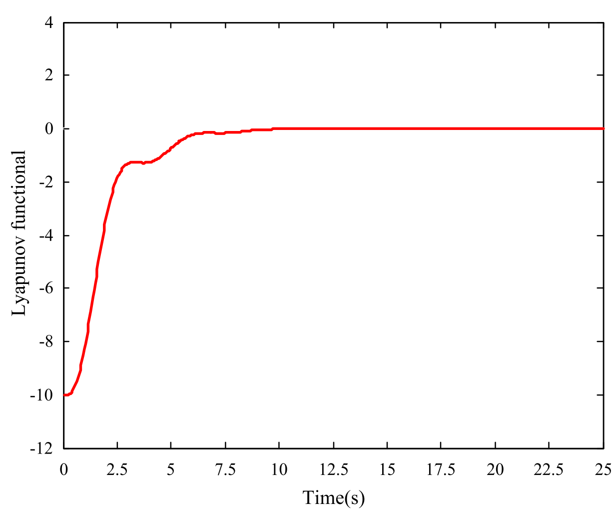

Figure 5 The response of $\Delta V(x_1, x_2, e)$

where p, $\dot{p}$, and $\ddot{p}$ are the position vector, velocity vector, and acceleration vector of the manipulator, respectively. μ is the driving torque of the system. μ_{1d} and μ_{2d} are the nonlinear uncertainties with time-delays [37].

Let $x_1 = p$ and $x_2 = \dot{p}$. Consider the unknown switching behavior and the asymmetric saturation actuators in system (36), system (36) can be rewritten as follows [37]:

$$\begin{cases} x_1(k+1) = g_{1,j}x_2 + f_{1,j}(x_1) + \phi_{1,j}(x_1(k - \tau_{1,j}(k))) + d_{1,j}, & j \in [1,2], \\ x_2(k+1) = g_{2,j}u_i(k) + f_{2,j}(\bar{x}_3) + \phi_{2,j}(\bar{x}_2(k - \tau_{2,j}(k))) + d_{2,j}, & j \in [1,2], \\ y = x_1. \end{cases} \tag{37}$$

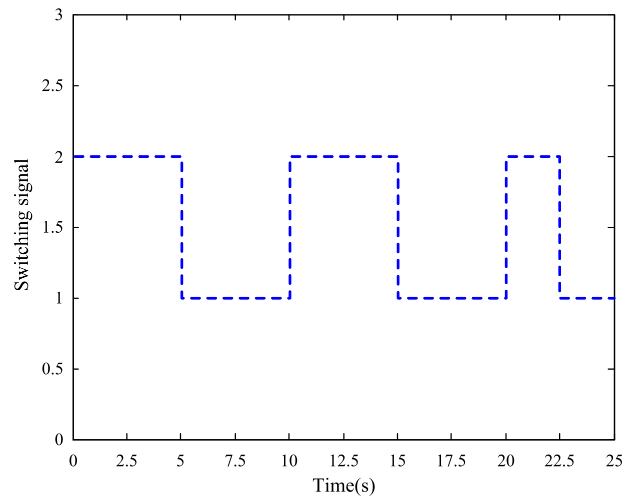

Figure 6 The switching signal in example 1

In this example, we do not consider the torque disturbances, system (37) can be rewritten as follows:

$$\begin{cases} x_1(k+1) = g_{1,j}x_2 + f_{1,j}(x_1) + \phi_{1,j}(x_1(k-\tau_{1,j}(k))), & j \in [1,2], \\ x_2(k+1) = g_{2,j}u_i(k) + f_{2,j}(\bar{x}_3) + \phi_{2,j}(\bar{x}_3(k-\tau_{2,j}(k))), & j \in [1,2]. \\ y = x_1. \end{cases} \tag{38}$$

Then, system (38) is decomposed into two subsystems as follows:

$$\begin{aligned} &\text{Subsystem (1)}: \begin{cases} x_1(k+1) = g_{1,j}x_2 + f_{1,j}(x_1) + \phi_{1,j}(x_1(k-\tau_{1,j}(k))), & j \in [1,2], \\ x_2(k+1) = g_{2,j}u_1(k) + f_{2,j}(\bar{x}_3) + \phi_{2,j}(\bar{x}_3(k-\tau_{2,j}(k))), & j \in [1,2], \\ y = x_1. \end{cases} \\ &\text{Subsystem (2)}: \begin{cases} x_1(k+1) = g_{1,j}x_2 + f_{1,j}(x_1) + \phi_{1,j}(x_1(k-\tau_{1,j}(k))), & j \in [1,2], \\ x_2(k+1) = g_{2,j}u_2(k) + f_{2,j}(\bar{x}_3) + \phi_{2,j}(\bar{x}_3(k-\tau_{2,i}(k))), & j \in [1,2], \\ y = x_1, \end{cases} \end{aligned} \tag{39}$$

where $g_{1,1} = g_{1,2} = 1$, $f_{1,1} = f_{1,2} = \phi_{1,1} = \phi_{1,2} = 0$, $g_{2,1} = g_{2,2} = 1/C$, $f_{2,1} = -\frac{B}{D}x_2 - \frac{A}{D}\sin(x_1)$, $f_{2,2} = -\frac{B}{D}x_2 - \frac{A}{D}\sin(x_1x_2)$, $\phi_{2,1} = 0.5x_1(k-\tau_{2,1}(k))$, and $\phi_{2,2} = 0.3x_1(k-\tau_{2,2}(k))x_2(k-\tau_{2,2}(k))$. The time delays are chosen as $\tau_{1,1} = 1+0.8\sin(k)$, $\tau_{1,2} = 1.5+1.2\sin(k)$, $\tau_{2,1} = 0.8+0.6\sin(k)$, and $\tau_{2,2} = 1.1+\sin(k)$. For the simulations, the parameters in (36) are given as $D = 1$, $B = 1$, $A = 0.5$, $C = 0.05$, $H = 0.5$, $K_m = 10$ [37]. The using details of the parameters in (36) are shown in [37].

For system (39), the virtual state feedback control law is designed as follows:

$$x_2^*(k) = -Mx_1(k-\tau_1), \tag{40}$$

where $M > 0$ is the virtual state feedback gain given as $M = 1$.

With Lemmas 1 and 3, the following inequalities hold:

$$\begin{cases} |x(k)+y(k)|^{q_i} \le 2^{q_i-1}|x^{q_i}(k)+y^{q_i}(k)|, \\ |x^{q_i}(k)-y^{q_i}(k)| \le q_i|x(k)-y(k)|(x^{q_i-1}(k)+y^{q_i-1}(k)), \\ \frac{w^{1+q_i}}{2^{q_i-1}} \le -w((c-w)^{q_i}-c^{q_i}), \end{cases}$$

where $i \in N_S = \{1,2\}$, $q_i \in [1,3,\dots,2n+1]$ is a power integrator parameter given as $q_1 = q_2 = 1$ in this example.

With Lemma 2, the following inequalities hold:

$$\begin{cases} |x(k)|^a|y(k)|^b \le \frac{a\Gamma(x,y)}{a+b}|x(k)|^{a+b} + \frac{b\Gamma^{-b^{-1}a}}{a+b}(x,y)|y(k)|^{a+b}, \\ m_1\tilde{\chi}_{i,1} + m_2(\tilde{\chi}_{i,1}+\tilde{\delta}_{i,1}) \le 1, \\ \sigma_i \ge \frac{1}{1+1}(\frac{1}{m_1(1+1)}) \times \tilde{\chi}_{i,1} + (\tilde{\chi}_{i,1}+\tilde{\delta}_{i,1})(\frac{1}{1+1})(\frac{1}{m_2(1+1)}), \end{cases}$$

where a, b, m_1, and m_2 are positive scalars given as $a = 1$, $b = 1$, $m_1 = 1/2$, and $m_2 = 1/2$.

For system (39), the nonlinear reduced-order compensator is designed as follows:

$$z(k+1) = -G\bigl(\varphi_{i,1}\bigl(x_1(k-\tau_1)\bigr) + \bigl(z(k)+Gx_1(k-\tau_1)\bigr)^{q_i}\bigr) + u_i(k), \tag{41}$$

where G is the gain parameter of the nonlinear reduced-order compensator.

With Lemma 2, the following inequalities hold:

$$\begin{cases} \Delta V_3(e) \le |e||\zeta_1|\tilde{a}_{i,2} + |e||\zeta_2|\tilde{a}_{i,2} - \frac{5e^{1+1}}{2^{1-1}} \le \frac{(\zeta_1^{1+1}+\zeta_2^{1+1})}{4} - \frac{5e^{1+1}}{2^{1-1}} + e^{1+1}J_i, \\ J_i \ge \frac{2}{1+1}(\frac{4\times 1}{1+1})\tilde{a}_{i,2}^{1+1}. \end{cases}$$

With the above analysis, for system (39), the dynamic output feedback controller is designed as follows:

$$\begin{cases} z(k+1) = -G(\varphi_{i,1}(x_1(k-\tau_1)) + (Gx_1(k-\tau_1)+z(k))^{q_i}) + u_i(k), \\ u_i(k) = -(1+\sigma_i)((G-M)x_1(k-\tau_1)+z(k))^{q_i}, \end{cases} \tag{42}$$

where $q_1 = q_2 = 1$ and $1 + \sigma_i = 10$.

For the simulations, the initial values of the system state variables are given as $[x_1, x_2] = [-0.7, 0.2]^T$. The random switching signal is generated by the function "rand" in MATLAB (R2017a). The responses of system state variables are shown in Fig. 7. The responses of system control inputs are shown in Fig. 8. The response of the state feedback control law is shown in Fig. 9. The response of $\Delta V(x_1, x_2, e)$ is shown in Fig. 10. In addition, the switching signal in example 2 is shown in Fig. 11. From Fig. 7, it can be seen that the proposed method is effective and can stabilize the closed-loop system quickly. From Figs. 8–9, it can be seen that the control inputs and state feedback control law are bounded as well. From Fig. 10, it can be seen that $\Delta V(x_1, x_2, e) \le 0$, that is, the closed-loop system is stable.

Remark 7 In this paper, the nonlinear case with multiple time-varying is considered. In fact, the proposed method can be applied to the general nonlinear case without time-delays, while the general nonlinear case should be bounded. The detailed proof process

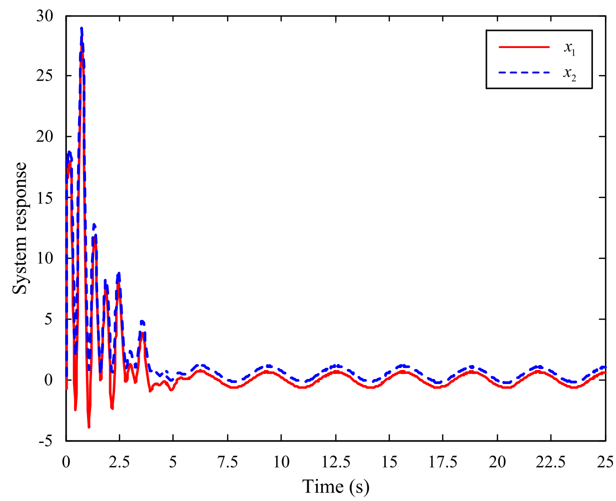

Figure 7 The responses of system state variables x_1 and x_2

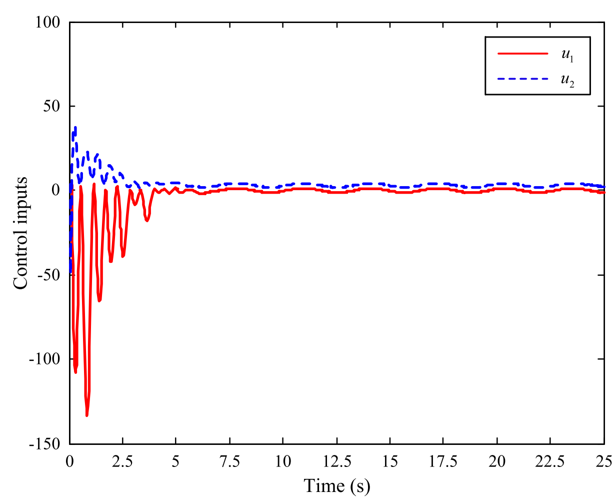

Figure 8 The responses of system control inputs

was shown in [36]. On the other hand, in this section, a class of one-link manipulator systems with the motor dynamics is considered, and the system is decomposed into the nonlinear switched form. The discrete Lyapunov–Krasovskii functional is designed so that (31) holds. If the nonlinear functions satisfy some specific functions as discussed in [38], one can employ the proposed method in [38] for the time-delay control system design. The detailed design process and specific functions were shown in [38].

5 Conclusions

This paper addresses the stability analysis and dynamic output feedback control problem for a class of discrete planar switched nonlinear systems with time-varying delays and multiple subsystems. Not only the time-varying delays, but also the uncontrollable/unobservable Jacobian linearizations are considered in the planar switched nonlinear systems. The virtual state feedback control law is designed and the uncertainties

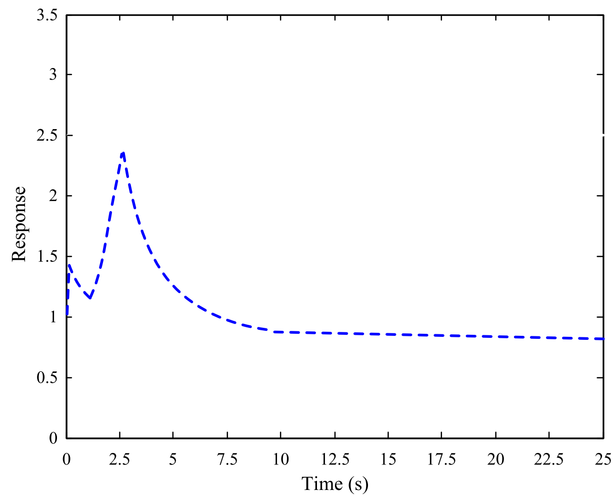

Figure 9 The response of the state feedback control law

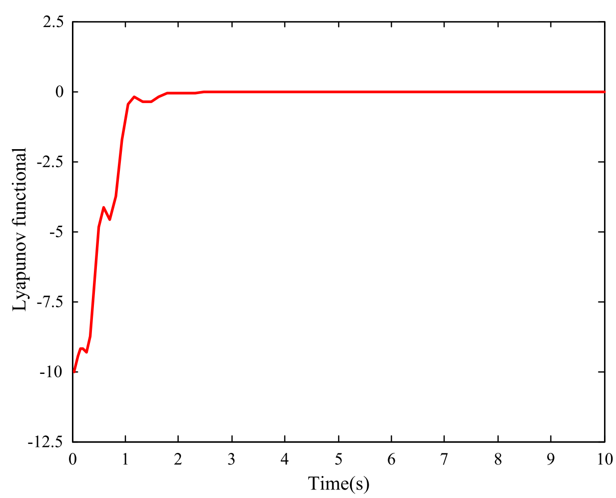

Figure 10 The response of $\Delta V(x_1, x_2, e)$

caused by the time-varying delays are solved effectively. The introduction of adding a power integrator approach leads to a significant relaxation on the system nonlinearities such as feedback linearizability. The nonlinear reduced-order compensator and smooth dynamic output feedback controller are designed, then the control design conditions are relaxed. With the developed new Lyapunov–Krasovskii functional, it can be seen that the solutions of the closed-loop system converge to an adjustable bounded region with the adjustable radius. Finally, two simulation examples are performed to show the effectiveness and applicability of the proposed method. On the other hand, the classical Krasovskii method often requires the time-varying delay $\tau(k)$ to satisfy some conservative conditions such as $0 \leq \tau(k) < \infty$ and $0 \leq \Delta\tau(k) < 1$. However, the aforementioned restrictions of the Lyapunov–Krasovskii method can be avoided by using the Lyapunov–Razumikhin method. Thus, the Lyapunov–Razumikhin method will be considered for the controller design of the switched system in the future.

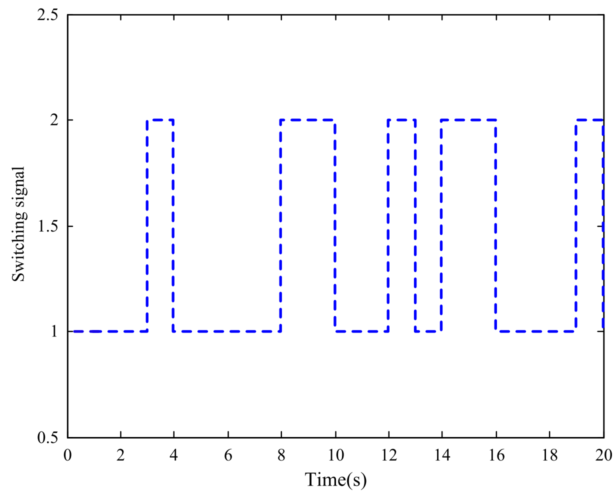

Figure 11 The switching signal in example 2

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Competing interests
The authors declare that there is not conflict of interests regarding the publications of this research.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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