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Dynamic behaviors of a non-selective harvesting single species stage-structured system incorporating partial closure for the populations

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Abstract

A single species stage-structured system incorporating partial closure for the populations and non-selective harvesting is proposed and studied in this paper. Local and global stability property of the boundary equilibrium and the positive equilibrium are investigated, respectively. Our study shows that the birth rate of the immature species and the fraction of the stocks for harvesting play a crucial role in the dynamic behaviors of the system. If the birth rate of the immature species is too low, then the species will be driven to extinction; also, with the increase in the fraction of the stocks for harvesting, the speed of driving the species to extinction becomes increasing. If the birth rate of the immature species is large enough, then the system always admits a unique globally asymptotically stable positive equilibrium; however, with the increase in the harvesting area, the final density of the species is decreasing. If the birth rate of the immature species lies in an interval, then there exists a threshold m^* such that the species will be driven to extinction for all $m \in (m^*, 1)$, and the system will admit a unique globally asymptotically stable positive equilibrium for all $m \in (0, m^*)$; also, with the increase in the parameter m , the system takes much time to reach its steady-state. For this case, though there are some natural protected areas where the harvesting of the species is forbidden, if the area is too small, the species will still be driven to extinction, that is, the small natural protected area has no influence on the protection of the endangered species. Such a finding maybe useful for human beings to design the protected areas for endangered species. Numeric simulations are carried out to show the feasibility of the main results.

MSC: 34C25; 92D25; 34D20; 34D40

Keywords: Stage structure; Species; Local stability; Lyapunov function; Global stability

1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following single species stage-structured system incorporating partial closure for the populations and non-

selective harvesting:

$$\begin{aligned} \frac{dx_1}{dt} &= \alpha x_2 - \beta x_1 - \delta_1 x_1 - q_1 E m x_1, \\ \frac{dx_2}{dt} &= \beta x_1 - \delta_2 x_2 - \gamma x_2^2 - q_2 E m x_2, \end{aligned} \tag{1.1}$$

where $\alpha, \beta, \delta_1, \delta_2, q_1, q_2, E,$ and γ are all positive constants, $x_1(t)$ and $x_2(t)$ are the densities of the immature and mature species at time t , the following assumptions are made in formulating model (1.1):

1. The per capita birth rate of the immature population is $\alpha > 0$; The per capita death rate of the immature population is $\delta_1 > 0$; The per capita death rate of the mature population is proportional to the current mature population with a proportionality constant $\delta_2 > 0$; $\beta > 0$ denotes the surviving rate of immaturity to reach maturity; The mature species is density dependent with the parameter $\gamma > 0$;
2. E is the combined fishing effort used to harvest and m ($0 < m < 1$) is the fraction of the stock available for harvesting.

During the last decades, many scholars investigated the dynamic behaviors of the stage-structured species, see [1–16] and the references cited therein. Among those works, there are two typical ideas used to establish the model.

(1) Assume that the immature species needs time to grown up, and denote this periodic as τ , this leads to the time delay model. For example, Chen, Chen, et al. [1], Chen, Xie, et al. [2], Chen, Wang, et al. [3], and Ma, Li, et al. [4] studied the dynamic behaviors of the following stage-structured predator–prey model:

$$\begin{aligned} \dot{x}_1(t) &= r_1(t)x_2(t) - d_{11}x_1(t) - r_1(t - \tau_1)e^{-d_{11}\tau_1}x_2(t - \tau_1), \\ \dot{x}_2(t) &= r_1(t - \tau_1)e^{-d_{11}\tau_1}x_2(t - \tau_1) - d_{12}x_2(t) \\ &\quad - b_1(t)x_2^2(t) - c_1(t)x_2(t)y_2(t), \\ \dot{y}_1(t) &= r_2(t)y_2(t) - d_{22}y_1(t) - r_2(t - \tau_2)e^{-d_{22}\tau_2}y_2(t - \tau_2), \\ \dot{y}_2(t) &= r_2(t - \tau_2)e^{-d_{22}\tau_2}y_2(t - \tau_2) - d_{21}y_2(t) \\ &\quad - b_2(t)y_2^2(t) + c_2(t)y_2(t)x_2(t), \end{aligned} \tag{1.2}$$

where $x_1(t)$ and $x_2(t)$ denote the densities of the immature and mature prey species at time t , respectively; $y_1(t)$ and $y_2(t)$ represent the immature and mature population densities of predator species at time t , respectively; $r_i(t), b_i(t), c_i(t)$ ($i = 1, 2$) are all continuous functions bounded above and below by positive constants for all $t \geq 0$. $d_{ij}, \tau_i, i, j = 1, 2$, are all positive constants. They investigated the persistence, extinction, and stability property of the above system. Li, Chen, et al. [5] investigated the stability property of the following mutualism model in a plant-pollinator system with stage structure and the Beddington–DeAngelis functional response:

$$\begin{aligned} \dot{x}_i(t) &= \alpha x_m(t) - \gamma x_i(t) - \alpha e^{-\gamma\tau} x_m(t - \tau), \\ \dot{x}_m(t) &= \alpha e^{-\gamma\tau} x_m(t - \tau) - \beta x_m^2(t) + \frac{m x_m(t) y(t)}{1 + k_1 x_m(t) + k_2 y(t)}, \\ \dot{y}(t) &= \frac{n m x_m(t) y(t)}{1 + k_1 x_m(t) + k_2 y(t)} - d y(t), \end{aligned} \tag{1.3}$$

where $x_i(t)$, $x_m(t)$, $y(t)$ can be described as the immature, mature plant densities, and the pollinators densities at time t , respectively. The authors investigated the persistence, local and global stability of the above system. Lin, Xie, et al. [10] considered the following stage-structured predator–prey model (stage structure for both predator and prey, respectively) with modified Leslie–Gower and Holling-type II schemes:

$$\begin{aligned}
 x_1'(t) &= r_1x_2(t) - d_{11}x_1(t) - r_1e^{-d_{11}\tau_1}x_2(t - \tau_1), \\
 x_2'(t) &= r_1e^{-d_{11}\tau_1}x_2(t - \tau_1) - d_{12}x_2(t) - bx_2^2(t) - \frac{a_1y_2(t)x_2(t)}{x_2(t) + k_1}, \\
 y_1'(t) &= r_2y_2(t) - d_{22}y_1(t) - r_2e^{-d_{22}\tau_2}y_2(t - \tau_2), \\
 y_2'(t) &= r_2e^{-d_{22}\tau_2}y_2(t - \tau_2) - d_{21}y_2(t) - \frac{a_2y_2^2(t)}{x_2(t) + k_2},
 \end{aligned}
 \tag{1.4}$$

where d_{12} and d_{21} represent the death rate of mature prey x_2 and mature predator y_2 , respectively; τ_1 is the time length of prey species from immature ones to mature ones, τ_2 is the time length of predator from immature ones to mature ones. By using the iterative technique method and fluctuation lemma, sufficient conditions which guarantee the global stability of the positive equilibrium and boundary equilibrium are obtained. Their results indicate that for a stage-structured predator–prey community, both stage structure and the death rate of the mature species are the important factors that lead to the permanence or extinction of the system. For more works in this direction, one could refer to [1–16] and the references cited therein. We mention here that the topic of the stability of the equilibrium and the extinction property of the ecosystem are the most important topics in the study of mathematics biology, one could refer to [17–35] for more works in this direction.

(2) Assume that the surviving rate of immaturity to reach maturity is proportional to the number of immature species. For example, Wu and Chen [15] studied the following single species stage-structured ecosystem with both toxicant effect and harvesting:

$$\begin{aligned}
 x_1'(t) &= ax_2 - d_1x_1 - d_2x_1^2 - \beta x_1 - r_1x_1^3, \\
 x_2'(t) &= \beta x_1 - b_1x_2 - c_2Ex_2,
 \end{aligned}
 \tag{1.5}$$

where $x_1(t)$, $x_2(t)$ represent the population density of the immature and the mature at time t , respectively, $r_1x_1^3$ is the effects of toxicant on the immature, E is the harvesting effort, c_2 is the catchability coefficient. They assumed that the immature is density restricted, toxicant affects the immature population and only harvesting the mature species. They showed that toxicant has no influence on the persistence property of the system. They also considered the system with variable harvest effect, and sufficient conditions which ensure the global stability of bionomic equilibrium were obtained. Chen [14] studied the existence and stability of the strictly positive (componentwise) almost periodic solution of the following non-autonomous almost periodic competitive two-species model with

stage structure in one species:

$$\begin{aligned} \dot{x}_1(t) &= -a_1(t)x_1(t) + b_1(t)x_2(t), \\ \dot{x}_2(t) &= a_2(t)x_2(t) - b_2(t)x_2(t) - c(t)x_2^2(t) - \beta_1(t)x_2(t)x_3(t), \\ \dot{x}_3(t) &= x_3(t)(d(t) - e(t)x_3(t) - \beta_2(t)x_2(t)). \end{aligned} \tag{1.6}$$

Here $x_1(t)$ and $x_2(t)$ are immature and mature population densities of one species, respectively, and $x_3(t)$ represents the population density of another species. Khajanchi and Banerjee [16] proposed the following stage-structured predator–prey model with ratio-dependent functional response:

$$\begin{aligned} \frac{dx_i}{dt} &= \alpha x_m(t) - \beta x_i(t) - \delta_1 x_i(t), \\ \frac{dx_m}{dt} &= \beta x_i(t) - \delta_2 x_m(t) - \gamma x_m^2(t) - \frac{\eta(1-\theta)x_m(t)y(t)}{g(1-\theta)x_m(t) + hy(t)}, \\ \frac{dy}{dt} &= \frac{u\eta(1-\theta)x_m(t)y(t)}{g(1-\theta)x_m(t) + hy(t)} - \delta_3 y(t). \end{aligned} \tag{1.7}$$

By constructing a suitable Lyapunov function, the authors obtained a set of sufficient conditions which ensure the uniform persistence and global asymptotic stability of the system. They showed that the constant prey refuge plays an important role in the coexistence of stage-structured predator–prey species. For more works in this direction, one could refer to [14–16] and the references cited therein.

On the other hand, as was pointed out by Chakraborty et al. [20], the study of resource management, including fisheries, forestry, and wildlife management, has great importance, it is necessary to harvest the population but harvesting should be regulated, so that both the ecological sustainability and conservation of the species can be implemented in a long run. Chakraborty et al. [20] proposed the following predator–prey model:

$$\begin{aligned} \frac{dx}{dt} &= rx \left(1 - \frac{x}{K} \right) - \frac{\alpha xy}{a + bx + cy} - q_1 mEx, \\ \frac{dy}{dt} &= sy \left(1 - \frac{y}{L} \right) + \frac{\beta xy}{a + bx + cy} - q_2 mEx. \end{aligned} \tag{1.8}$$

They tried to investigate the existence and stability property of the equilibria of the system; however, since the system is too complicated, they could not give detailed analysis of the influence of parameter m . Recently, many scholars investigated the dynamic behaviors of the non-selective harvesting ecosystem incorporating partial closure. Lin [22] investigated the dynamic behaviors of the following two species commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure:

$$\begin{aligned} \frac{dx}{dt} &= x \left(a_1 - b_1x + \frac{c_1y}{d_1 + y^2} \right) - q_1 Emx, \\ \frac{dy}{dt} &= y(a_2 - b_2y) - q_2 Emy, \end{aligned} \tag{1.9}$$

where $a_i, b_i, q_i, i = 1, 2, c_1, E, m$ ($0 < m < 1$), and d_1 are all positive constants, where E is the combined fishing effort used to harvest and m ($0 < m < 1$) is the fraction of the stock available for harvesting. His study showed that depending on the range of the parameter m , the system may collapse, or partially survive, or the two species could coexist in a stable state. He also showed that if the system admits a unique positive equilibrium, then it is globally asymptotically stable. Chen [21] studied the influence of non-selective harvesting on a Lotka–Volterra amensalism model incorporating partial closure for the populations, and he also found that the dynamic behaviors of the system become complicate.

As was shown above, though there are many works on a stage-structured ecosystem [1–16], seldom did they consider the influence of harvesting [15]. Also, though there are several scholars who investigated the dynamic behaviors of the non-selective harvesting ecosystem incorporating partial closure for the populations (see [20–22, 32, 33, 35]), to this day, still no scholars investigated the influence of non-selective harvesting stage-structured ecosystem incorporating partial closure for the populations. This motivated us to propose system (1.1). We will try to give a thorough analysis of the dynamic behaviors of system (1.1).

The paper is arranged as follows. We investigate the existence and locally stability property of the equilibria of system (1.1) in the next section. In Sect. 3, by constructing some suitable Lyapunov function, we are able to investigate the global stability property of the equilibria. Section 4 presents some numerical simulations to show the feasibility of the main results. We end this paper with a brief discussion.

2 Local stability of the equilibria

The system always admits the boundary equilibrium $O(0, 0)$.

If $\alpha > (\delta_2 + Eq_2m)(1 + \frac{\delta_1 + Eq_1m}{\beta})$ holds, then the system admits a unique positive equilibrium $A(x_1^*, x_2^*)$, where

$$\begin{aligned} x_1^* &= \frac{\alpha x_2^*}{\beta + \delta_1 + q_1Em}, \\ x_2^* &= \frac{\alpha\beta - (\delta_2 + Eq_2m)(\beta + \delta_1 + Eq_1m)}{\gamma(\beta + \delta_1 + Eq_1m)}. \end{aligned} \tag{2.1}$$

We shall now investigate the local stability property of the above equilibria.

The variational matrix of system (1.1) is

$$J(x, y) = \begin{pmatrix} -\beta - \delta_1 - Emq_1 & \alpha \\ \beta & -\delta_2 - 2\gamma x_2 - Emq_2 \end{pmatrix}. \tag{2.2}$$

The characteristic equation of the variational matrix is

$$\lambda^2 - \text{tr}(J)\lambda + \det(J) = 0. \tag{2.3}$$

Obviously, if $\text{tr}(J) < 0$ and $\det(J) > 0$, then both eigenvalues have negative real parts, and the corresponding equilibrium solution is asymptotically stable.

Theorem 2.1 *Assume that*

$$\alpha < (\delta_2 + Eq_2m) \left(1 + \frac{\delta_1 + Eq_1m}{\beta} \right) \tag{2.4}$$

holds, then $O(0,0)$ is locally asymptotically stable.

Proof From (2.2) we could see that the Jacobian matrix of the system about the equilibrium point $O(0,0)$ is given by

$$\begin{pmatrix} -\beta - \delta_1 - Emq_1 & \alpha \\ \beta & -\delta_2 - Emq_2 \end{pmatrix}. \tag{2.5}$$

Hence,

$$\text{tr}(J(0,0)) = -\beta - \delta_1 - Emq_1 - \delta_2 - Emq_2 < 0,$$

and under assumption (2.4), one has

$$\det(J(0,0)) = (-\beta - \delta_1 - Emq_1)(-\delta_2 - Emq_2) - \alpha\beta > 0.$$

Consequently, $O(0,0)$ is locally asymptotically stable. This ends the proof of Theorem 2.1. □

Theorem 2.2 *Assume that*

$$\alpha > (\delta_2 + Eq_2m) \left(1 + \frac{\delta_1 + Eq_1m}{\beta} \right) \tag{2.6}$$

holds, then $A(x^,y^*)$ is locally asymptotically stable.*

Proof From (2.2) we could see that the Jacobian matrix of the system about the equilibrium point $A(x_1^*,x_2^*)$ is given by

$$\begin{pmatrix} -\beta - \delta_1 - Emq_1 & \alpha \\ \beta & -\delta_2 - Emq_2 - 2\gamma x_2^* \end{pmatrix}. \tag{2.7}$$

Hence,

$$\text{tr}(J(x_1^*,x_2^*)) = -\beta - \delta_1 - Emq_1 - \delta_2 - Emq_2 - 2\gamma x_2^* < 0$$

and under assumption (2.6), one has

$$\begin{aligned} \det(J(x_1^*,x_2^*)) &= (-\beta - \delta_1 - Emq_1)(-\delta_2 - Emq_2 - 2\gamma x_2^*) - \alpha\beta \\ &= \alpha\beta - (\beta + \delta_1 + Emq_1)(\delta_2 + Eq_2m) > 0. \end{aligned}$$

Consequently, $A(x_1^*,x_2^*)$ is locally asymptotically stable. This ends the proof of Theorem 2.2. □

3 Global asymptotic stability

This section tries to obtain some sufficient conditions which could ensure the global asymptotic stability of the equilibria.

Theorem 3.1 *Assume that*

$$\alpha < (\delta_2 + Eq_2m) \left(1 + \frac{\delta_1 + Eq_1m}{\beta} \right) \tag{3.1}$$

holds, then $O(0,0)$ is globally asymptotically stable.

Proof Condition (3.1) is equal to

$$\frac{\alpha\beta}{\beta + \delta_1 + q_1Em} - \delta_2 - q_2Em < 0. \tag{3.2}$$

We will prove Theorem 3.1 by constructing some suitable Lyapunov function. Let us define a Lyapunov function

$$V_1(x_1, x_2) = \frac{\beta}{\beta + \delta_1 + q_1Em} x_1 + x_2.$$

One could easily see that the function V_1 is zero at the equilibrium $O(0,0)$ and is positive for all other positive values of x_1 and x_2 . The time derivative of V_1 along the trajectories of (1.1) is

$$\begin{aligned} D^+ V_1(t) &= \frac{\beta}{\beta + \delta_1 + q_1Em} (\alpha x_2 - \beta x_1 - \delta x_1 - q_1Emx_1) \\ &\quad + \beta x_1 - \delta_2 x_2 - \gamma x_2^2 - q_2Emx_2 \\ &= \left(\frac{\alpha\beta}{\beta + \delta_1 + q_1Em} - \delta_2 - q_2Em \right) x_2 - \gamma x_2^2. \end{aligned} \tag{3.3}$$

Obviously, under assumption (3.1), $D^+ V_1(t) < 0$ strictly for all $x_1, x_2 > 0$ except the boundary equilibrium $O(0,0)$, where $D^+ V_1(t) = 0$. Thus, $V_1(x_1, x_2)$ satisfies Lyapunov’s asymptotic stability theorem, and the boundary equilibrium $O(0,0)$ of system (1.1) is globally asymptotically stable.

This completes the proof of Theorem 3.1. □

Theorem 3.2 *Assume that*

$$\alpha > (\delta_2 + Eq_2m) \left(1 + \frac{\delta_1 + Eq_1m}{\beta} \right) \tag{3.4}$$

holds, then $A(x_1^, x_2^*)$ is globally asymptotically stable.*

Proof We will prove Theorem 3.2 by constructing some suitable Lyapunov function. Let us define a Lyapunov function

$$V_1(x_1, x_2) = k_1 \left(x_1 - x_1^* - x_1^* \ln \frac{x_1}{x_1^*} \right) + k_2 \left(x_2 - x_2^* - x_2^* \ln \frac{x_2}{x_2^*} \right),$$

where k_1, k_2 are some positive constants to be determined later.

One could easily see that the function V_2 is zero at the equilibrium $A(x_1^*, x_2^*)$ and is positive for all other positive values of x_1 and x_2 . The time derivative of V_2 along the trajectories of (1.1) is

$$\begin{aligned} D^+ V_2(t) &= k_1 \frac{x_1 - x_1^*}{x_1} \dot{x}_1 + k_2 \frac{x_2 - x_2^*}{x_2} \dot{x}_2 \\ &= k_1 \frac{x_1 - x_1^*}{x_1} (\alpha x_2 - (\beta + \delta_1 + q_1 Em)x_1) \\ &\quad + k_2 \frac{x_2 - x_2^*}{x_2} (\beta x_1 - \delta_2 x_2 - \gamma x_2^2 - q_2 Em x_2). \end{aligned} \tag{3.5}$$

Note that from the relationship of x_1^* and x_2^* , we have

$$\begin{aligned} &\alpha x_2 - (\beta + \delta_1 + q_1 Em)x_1 \\ &= \frac{\alpha}{x_1^*} (x_2 x_1^* - x_1 x_2^*) + \alpha x_1 \frac{x_2^*}{x_1^*} - (\beta + \delta_1 + q_1 Em)x_1 \\ &= \frac{\alpha}{x_1^*} (x_2 x_1^* - x_2 x_1 + x_2 x_1 - x_1 x_2^*) \\ &= \frac{\alpha}{x_1^*} (-x_2(x_1 - x_1^*) + x_1(x_2 - x_2^*)). \end{aligned} \tag{3.6}$$

Also, from the expression of x_2^* , one has

$$\begin{aligned} &\beta x_1 - \delta_2 x_2 - \gamma x_2^2 - q_2 Em x_2 \\ &= \frac{\beta}{x_2^*} (x_1 x_2^* - x_2 x_1^*) + \beta x_2 \frac{x_1^*}{x_2^*} - (\delta_2 + q_2 Em)x_2 - \gamma x_2^2 \\ &= \frac{\beta}{x_2^*} (x_1 x_2^* - x_1 x_2 + x_1 x_2 - x_2 x_1^*) \\ &\quad + \left(\frac{\alpha \beta}{\beta + \delta_1 + q_1 Em} - \delta_2 - q_2 Em \right) x_2 - \gamma x_2^2 \\ &= \frac{\beta}{x_2^*} (x_1(x_2^* - x_2) + x_2(x_1 - x_1^*)) + \gamma x_2^* x_2 - \gamma x_2^2. \end{aligned} \tag{3.7}$$

Applying (3.6) and (3.7) to (3.5) leads to

$$\begin{aligned} D^+ V_2(t) &= k_1 \frac{x_1 - x_1^*}{x_1} \frac{\alpha}{x_1^*} (-x_2(x_1 - x_1^*) + x_1(x_2 - x_2^*)) \\ &\quad + k_2 \frac{x_2 - x_2^*}{x_2} \frac{\beta}{x_2^*} (x_1(x_2^* - x_2) + x_2(x_1 - x_1^*)) \\ &\quad - k_2 \frac{x_2 - x_2^*}{x_2} (\gamma x_2^* x_2 - \gamma x_2^2) \\ &= k_1 \frac{x_1 - x_1^*}{x_1} \frac{\alpha}{x_1^*} (-x_2(x_1 - x_1^*) + x_1(x_2 - x_2^*)) \\ &\quad + k_2 \frac{x_2 - x_2^*}{x_2} \frac{\beta}{x_2^*} (-x_1(x_2 - x_2^*) + x_2(x_1 - x_1^*)) \\ &\quad - k_2 \gamma (x_2 - x_2^*)^2 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{k_1\alpha x_2}{x_1 x_1^*} (x_1 - x_1^*)^2 + \left(\frac{k_1\alpha}{x_1^*} + \frac{k_2\beta}{x_2^*}\right) (x_1 - x_1^*) (x_2 - x_2^*) \\
 &\quad - \frac{k_2\beta x_1}{x_2 x_2^*} (x_2 - x_2^*)^2 - k_2\gamma (x_2 - x_2^*)^2.
 \end{aligned}$$

Now let us choose $k_2 = 1, k_1 = \frac{\beta x_1^*}{x_2^* \alpha}$, then

$$\begin{aligned}
 D^+ V_2(t) &= -\frac{\beta x_2}{x_1 x_2^*} (x_1 - x_1^*)^2 + \frac{2\beta}{x_2^*} (x_1 - x_1^*) (x_2 - x_2^*) \\
 &\quad - \frac{\beta x_1}{x_2 x_2^*} (x_2 - x_2^*)^2 - \gamma (x_2 - x_2^*)^2 \\
 &= -\frac{\beta}{x_2^*} \left[\sqrt{\frac{x_2}{x_1}} (x_1 - x_1^*) - \sqrt{\frac{x_1}{x_2}} (x_2 - x_2^*) \right]^2 \\
 &\quad - \gamma (x_2 - x_2^*)^2. \tag{3.8}
 \end{aligned}$$

Obviously, under assumption (3.1), $D^+ V_2(t) < 0$ strictly for all $x_1, x_2 > 0$ except the positive equilibrium $A(x_1^*, x_2^*)$, where $D^+ V_2(t) = 0$. Thus, $V_2(x_1, x_2)$ satisfies Lyapunov’s asymptotic stability theorem, and the positive equilibrium $A(x_1^*, x_2^*)$ of system (1.1) is globally asymptotically stable.

This completes the proof of Theorem 3.2. □

4 The influence of partial closure

To find out the influence of partial closure, let us consider the single species stage-structured model:

$$\begin{aligned}
 \frac{dx_1}{dt} &= \alpha x_2 - \beta x_1 - \delta_1 x_1, \\
 \frac{dx_2}{dt} &= \beta x_1 - \delta_2 x_2 - \gamma x_2^2,
 \end{aligned} \tag{4.1}$$

where $\alpha, \beta, \delta_1, \delta_2,$ and γ are all positive constants, $x_1(t)$ and $x_2(t)$ are the densities of the immature and mature species at time t . As a direct corollary of Theorems 3.1 and 3.2, we have the following.

Theorem 4.1 *Assume that*

$$\alpha < \delta_2 \left(1 + \frac{\delta_1}{\beta} \right) \tag{4.2}$$

holds, then the boundary equilibrium $O_1(0, 0)$ of system (4.1) is globally stable.

Theorem 4.2 *Assume that*

$$\alpha > \delta_2 \left(1 + \frac{\delta_1}{\beta} \right) \tag{4.3}$$

holds, then the positive equilibrium $B(x_1^{**}, x_2^{**})$ of system (4.1) is globally stable, where

$$\begin{aligned} x_1^{**} &= \frac{\alpha x_2^{**}}{\beta + \delta_1}, \\ x_2^{**} &= \frac{\alpha\beta - \delta_2(\beta + \delta_1)}{\gamma(\beta + \delta_1)}. \end{aligned} \tag{4.4}$$

Now let us discuss the influence of partial closure in three cases.

Case 1. Assume that inequality (4.2) holds, then for all $m \in (0, 1)$, inequality (3.1) holds, that is, if the system without harvesting is extinct, then, for the system with harvesting, despite the partial closure where the species could not be harvested, the species is always driven to extinction. That is, if the birth rate of the immature species is too low, the species will be driven to extinction.

Case 2. Assume that

$$\alpha > (\delta_2 + Eq_2) \left(1 + \frac{\delta_1 + Eq_1}{\beta} \right) \tag{4.5}$$

holds, then for all $m \in (0, 1)$, inequality (3.4) holds. It follows from Theorem 3.2 that the system always admits a unique positive equilibrium which is globally asymptotically stable. That is, if the birth rate of the immature species is large enough such that inequality (4.5) holds, then the partial closure has no influence on the persistence property of the system. However, from (2.1), one could see that

$$\frac{dx_2^*}{dm} = -\frac{\alpha\beta Eq_1}{\gamma(Eq_1 m + \beta + \delta_1)^2} - \frac{Eq_2}{\gamma} < 0 \tag{4.6}$$

and

$$\frac{dx_1^*}{dm} = \frac{\alpha}{\beta + \delta_1 + Eq_1 m} \left(-\frac{\alpha\beta Eq_1}{\gamma(Eq_1 m + \beta + \delta_1)^2} - \frac{Eq_2}{\gamma} \right) - \frac{\alpha Eq_1 x_2^*}{(\beta + \delta_1 + Eq_1 m)^2} < 0. \tag{4.7}$$

Here, with the increase in the harvesting area, the final densities of the immature and mature species are both decreasing.

Case 3. Now let us assume that

$$\delta_2 \left(1 + \frac{\delta_1}{\beta} \right) < \alpha < (\delta_2 + Eq_2) \left(1 + \frac{\delta_1 + Eq_1}{\beta} \right) \tag{4.8}$$

holds, then from (4.3) and Theorem 4.2 we know that the system without harvesting admits a unique positive equilibrium $B(x_1^{**}, x_2^{**})$, which is globally asymptotically stable. In this case, it follows from Theorems 3.1 and 3.2 that there exists a threshold

$$m^* = \frac{-(\beta q_2 + \delta_1 q_2 + \delta_2 q_1) + \sqrt{\Delta}}{2Eq_1 q_2},$$

where

$$\Delta = (\beta q_2 + \delta_1 q_2 + \delta_2 q_1)^2 - 4q_1 q_2 (-\alpha\beta + \beta\delta_2 + \delta_1\delta_2).$$

For all $m \in (0, m^*)$, inequality (3.4) holds, and the system has a unique positive equilibrium $A(x_1^*, x_2^*)$, which is globally asymptotically stable. In this case, by using (2.1), we could see that (4.6) and (4.7) also hold, that is, with the increase in the harvesting area, the final density of the species is decreasing. However, for all $m \in (m^*, 1)$, inequality (3.1) holds, and the species will be driven to extinction. It is well known that m ($0 < m < 1$) is the fraction of the stock available for harvesting, hence, under assumption (4.8), if the harvesting area is too large, despite the fact that there are some areas where the harvesting is forbidden, the species will still be driven to extinction.

5 Numeric simulations

Now let us consider the following examples.

Example 5.1 Let us consider the single species stage-structured system incorporating partial closure for the populations and non-selective harvesting:

$$\begin{aligned} \frac{dx_1}{dt} &= \alpha x_2 - x_1 - x_1 - \frac{1}{2}x_1, \\ \frac{dx_2}{dt} &= x_1 - x_2 - x_2^2 - \frac{1}{2}x_2, \end{aligned} \tag{5.1}$$

here we choose $\beta = \delta_1 = \delta_2 = E = q_1 = q_2 = 1, m = \frac{1}{2}, \gamma = 1$.

(1) From Theorems 2.1 and 3.1, we know that if

$$\alpha < (\delta_2 + Eq_2m) \left(1 + \frac{\delta_1 + Eq_1m}{\beta} \right) = \frac{15}{4}, \tag{5.2}$$

then $O(0, 0)$ is globally attractive, Fig. 1 is numeric simulation for the case $\alpha = 3$;

(2) From Theorems 2.2 and 3.2, we know that if

$$\alpha > (\delta_2 + Eq_2m) \left(1 + \frac{\delta_1 + Eq_1m}{\beta} \right) = \frac{15}{4}, \tag{5.3}$$

then $A(x_1^*, x_2^*)$ is globally asymptotically stable. Now let us take $\alpha = 5$, then the system admits a unique positive equilibrium $(1, 0.5)$. Figure 2 shows that in this case, $(1, 0.5)$ is globally asymptotically stable.

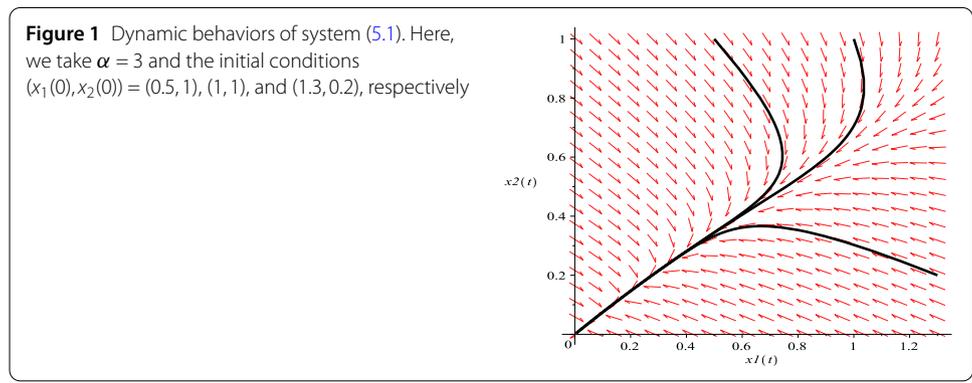


Figure 2 Dynamic behaviors of system (5.1). Here, we take $\alpha = 5$ and the initial conditions $(x_1(0), x_2(0)) = (0.5, 1), (1, 1), (1.4, 0.3), (0.1, 0.1), (0.1, 1), (1.4, 0.1),$ and $(0.1, 0.5),$ respectively

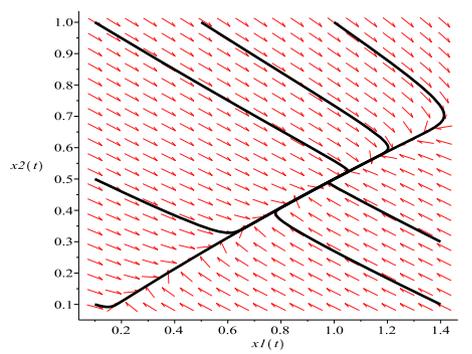
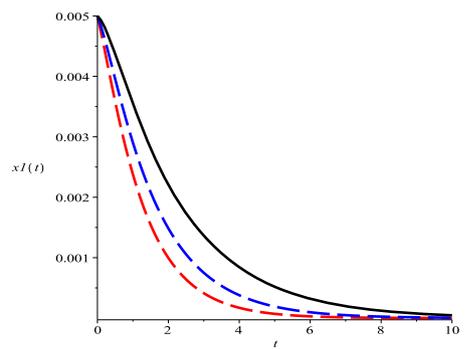


Figure 3 Numeric simulations of the first component $x_1(t)$ of system (5.4), with $m = 0.1, 0.3, 0.5,$ where black curve is the solution of $m = 0.1,$ blue curve is the solution of $m = 0.3,$ and red curve is the solution of $m = 0.5,$ and $(x_1(0), x_2(0)) = (0.005, 0.01)$



Example 5.2 Let us consider the single species stage-structured system incorporating partial closure for the populations and non-selective harvesting:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 - x_1 - x_1 - mx_1, \\ \frac{dx_2}{dt} &= x_1 - x_2 - x_2^2 - mx_2, \end{aligned} \tag{5.4}$$

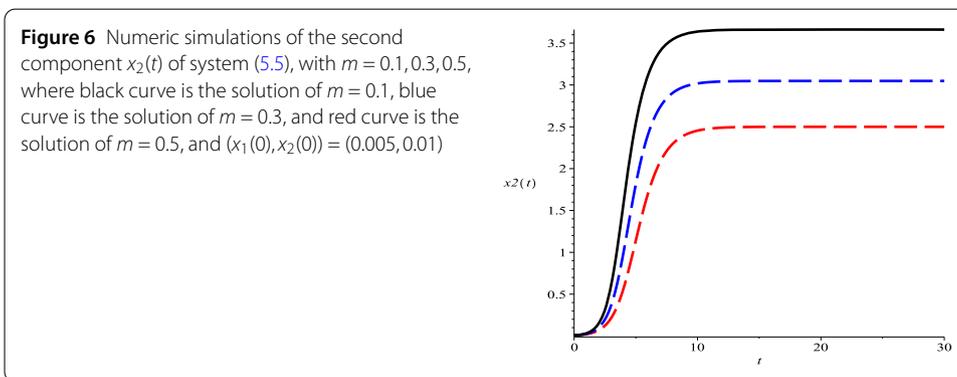
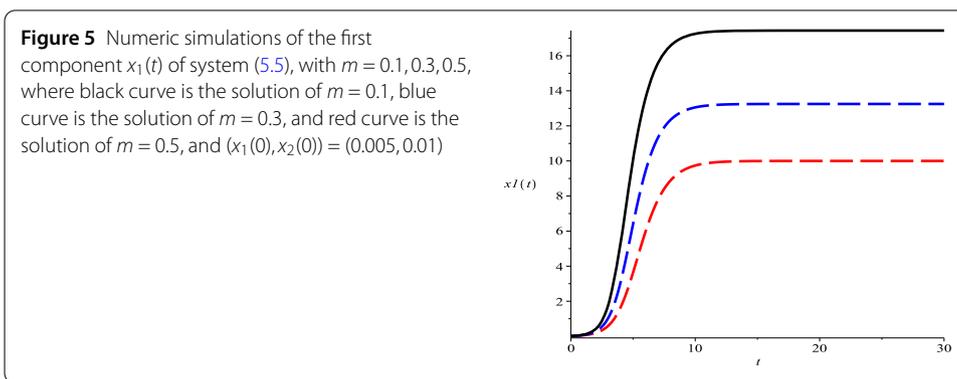
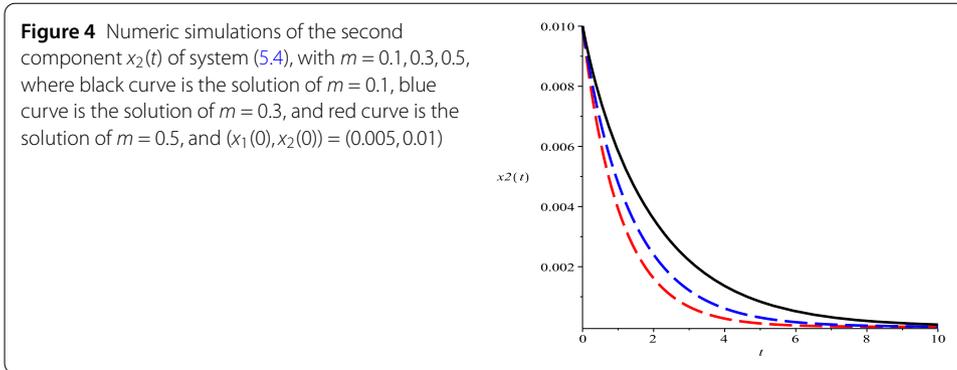
here we choose $\beta = \alpha = \delta_1 = \delta_2 = E = q_1 = q_2 = 1, \gamma = 1.$ In this case, since

$$\alpha = 1 < 2 = \delta_2 \left(1 + \frac{\delta_1}{\beta} \right).$$

It follows from the analysis of *Case 1* in Sect. 4, for all $m \in (0, 1),$ that the species will be driven to extinction. Figure 3 and Fig. 4 show that with the increase in $m,$ the time for the species to go to extinction becomes shorter. That is, with intense harvesting, the chance for the species to be driven to extinction is increasing.

Example 5.3 Let us consider the single species stage-structured system incorporating partial closure for the populations and non-selective harvesting:

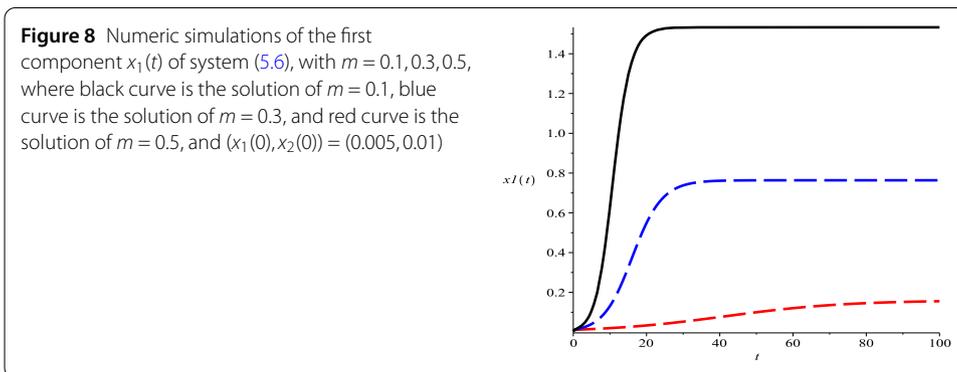
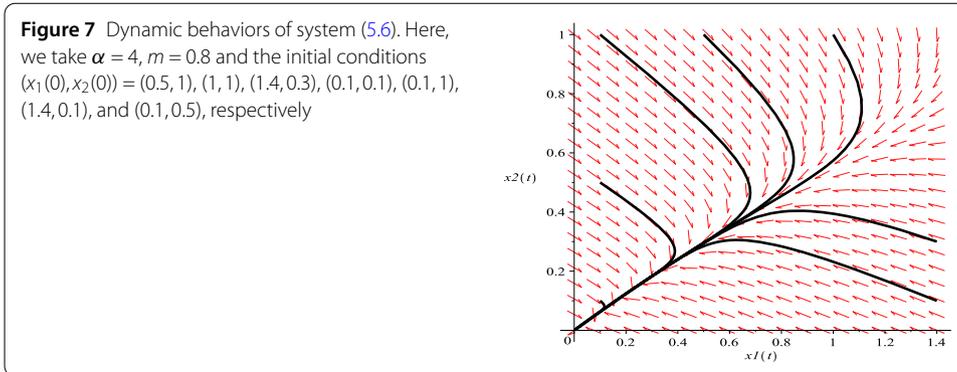
$$\begin{aligned} \frac{dx_1}{dt} &= 10x_2 - x_1 - x_1 - mx_1, \\ \frac{dx_2}{dt} &= x_1 - x_2 - x_2^2 - mx_2, \end{aligned} \tag{5.5}$$



here we choose $\alpha = 10, \beta = \delta_1 = \delta_2 = E = q_1 = q_2 = 1, \gamma = 1$. In this case, since

$$\alpha = 10 > 6 = (\delta_2 + Eq_2) \left(1 + \frac{\delta_1 + Eq_1}{\beta} \right).$$

It follows from the analysis of *Case 2* in Sect. 4, for all $m \in (0, 1)$, that the system always admits a unique positive equilibrium, which is globally asymptotically stable. Figure 5 and Fig. 6 show that with the increase in m , the density of the species becomes decreasing. That is, with intense harvesting, the final density of the species is decreasing.



Example 5.4 Let us consider the single species stage-structured system incorporating partial closure for the populations and non-selective harvesting:

$$\begin{aligned} \frac{dx_1}{dt} &= 4x_2 - x_1 - x_1 - mx_1, \\ \frac{dx_2}{dt} &= x_1 - x_2 - x_2^2 - mx_2, \end{aligned} \tag{5.6}$$

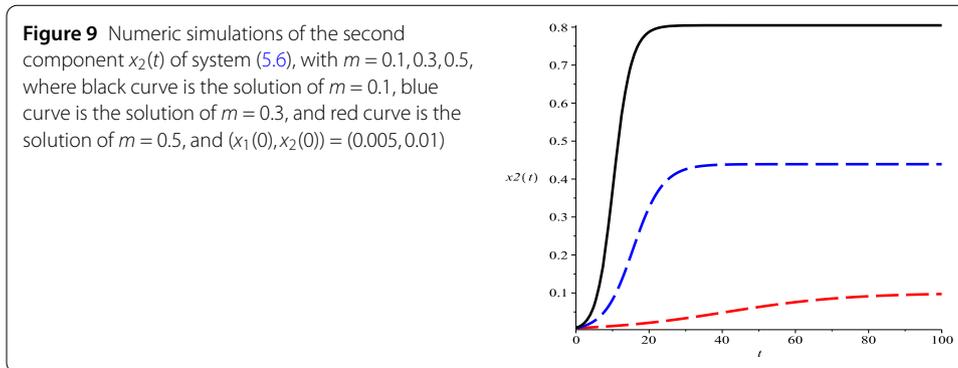
here we choose $\alpha = 4, \beta = \delta_1 = \delta_2 = E = q_1 = q_2 = 1, \gamma = 1.$ In this case, since

$$\delta_2 \left(1 + \frac{\delta_1}{\beta} \right) = 2 < \alpha = 4 < 6 = (\delta_2 + Eq_2) \left(1 + \frac{\delta_1 + Eq_1}{\beta} \right).$$

It follows from the analysis of *Case 3* in Sect. 4, that there exists $m^* \approx 0.5615528128$ such that, for all $m \in (0, m^*),$ the system always admits a unique positive equilibrium, which is globally asymptotically stable, while for $m \in (m^*, 1),$ the system will be driven to extinction. Figure 7 is the case for $m = 0.8.$ Figure 8 and Fig. 9 show that for the case $m \in (0, m^*),$ with the increase in $m,$ the density of the species becomes decreasing. That is, with intense harvesting, the final density of the species is decreasing. Also, from Fig. 8 and Fig. 9, one could see that with the increase in $m,$ the system takes much time to reach its steady-state.

6 Conclusion

Since the pioneering works of Chakraborty et al. [20], many scholars [21, 22, 32, 33, 35] investigated the dynamic behaviors of the non-selective harvesting ecosystem incorporating partial closure for the populations. Though Chakraborty et al. [20] could not give a



distinct analysis of the parameter m , both the works [21] and [22] showed that, depending on the range of the parameter m , the system they considered could collapse, or partially survive, or the two species could coexist in a stable state.

In this paper, we propose a non-selective harvesting single species stage-structured system incorporating partial closure for the populations. Our study shows that the dynamic behaviors of system (1.1) differ from those of the system considered in [21, 22] in the sense that the system could not have a partial survival case. In system (1.1), there are only two possible situations: (1) the boundary equilibrium $O(0, 0)$ is globally asymptotically stable; (2) the positive equilibrium $A(x_1^*, x_2^*)$ is globally asymptotically stable.

Our study shows that with the increase in the harvesting area, the final density of the species becomes decreasing, or it takes much time for the system to approach its steady-state, it is in this sense that the parameter m has the destabilizing effect to the system.

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Competing interests

The authors declare that there is no conflict of interests.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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