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Basic reproductive number for a general hybrid epidemic model

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Abstract

In this paper, a general hybrid epidemic model with multiple and non-periodic pulses in an environmental period is investigated. The definition and computation for the basic reproductive number R_0 are established. The published periodic research model (Yang, Xiao in Nonlinear Anal., Real World Appl. 52:224–234, 2012) can be considered as a special case of the new established model.

Keywords: Hybrid system; Basic reproductive number; Next infection operator

1 Introduction

The basic reproductive number R_0 is a measure of how many secondary hosts will become infected if one initial host is infected in a naïve population [2]. In the past decades, R_0 became one of the most important key parameters in mathematical epidemiology and was widely used in the study of epidemiology and within-host pathogen dynamics [3, 4]. It was established firstly in 1925 by Dublin and Lotka [5] and introduced as the "net reproduction rate" in human demography. For multistate stable population model, Inaba [6] defined R_0 as a spectral radius of the net reproductive matrix. Based on the next generation operator, Diekmann et al. [7] defined R_0 for heterogeneous population in a constant environment. For autonomous epidemic models, Van den Driessche and Watmough [8] investigated a method to calculate R_0 as the spectral radius of the next generation matrix. In 2006, Bacaër and Guernaoui [9] extended R_0 to the case of a periodic environment. Furthermore, Wang and Zhao [10] took the calculation of R_0 given in [8] for periodic systems as the spectral radius of the next infection operator. Based on the main results in [11], Wang and Zhao [12] defined the next infection operator as R_0 for a nonlocal and time-delayed reaction-diffusion model of dengue fever. R_0 and its computation formulae for a large class of time-delayed compartmental population models in a periodic environment were set up in [13].

It is well known that many natural phenomena and human activities have exhibited impulsive effects on the field of epidemiology. Recently, impulsive control problems in the application of epidemics, such as pulse vaccination [14–16] and pulse culling (roguing) [17], have received tremendous attention by many authors. Based on the spectral radius of the next infection operator, Yang and Xiao [18] gave the definition of R_0 for general impulsive epidemic models with periodic pulses on both infected and uninfected compartments. And then they [1] established R_0 and its calculation. However, the pulses are applied with the same time interval. An interesting question arises how R_0 can be defined

and calculated for impulsive epidemic models with pulses at various time intervals in an environmental period. The present research intends to establish the theory of R_0 for such hybrid models.

The paper is organized as follows. In the next section, some basic notations and useful results for a general linear hybrid system are given. In Sect. 3, the definition and computation of R_0 for a general hybrid epidemic model are established. In Sect. 4, two examples to show the application are presented. A brief conclusion is given in the last section.

2 Some useful results for a general linear impulsive system

In order to obtain R_0 for a general nonautonomous impulsive system, we firstly give some useful results for a general linear impulsive system. The homogeneous linear impulsive differential system is considered as follows:

$$\begin{cases} \dot{x}(t) = A(t)x(t), & t \neq t_k, \\ x(t_k^+) = P_k x(t_k), & t = t_k, k \in \mathbb{N}, \mathbb{N} = \{0, 1, \ldots\}, \\ x(t_0^+) = x_0, & t_0 \geq 0, \end{cases} \tag{2.1}$$

which satisfies the following three conditions.

(C_1) $A(\cdot) \in \mathrm{PC}(\mathbb{R}, \mathbb{R}^{n \times n})$, $A(t + \omega) = A(t)$, here $\omega > 0$ is the environmental period, PC means piecewise continuous.

(C_2) $P_k \in \mathbb{R}^{n \times n}$, $\det P_k \neq 0$, $P_0 = I$, where I is an $n \times n$ identity matrix, $t_k < t_{k+1}$, and $\lim_{k \to +\infty} t_k = +\infty$.

(C_3) There exists a positive integer q such that $P_{k+q} = P_k$, $t_{k+q} = t_k + \omega$ for any $k \in \mathbb{N}$.

Set $\Psi_A(t, s)$ $(t \geq s)$ be the evolution operator of the linear ω-periodic system

$$\dot{x}(t) = A(t)x(t), \quad x \in \mathbb{R}^n.$$

That is, for each $s \in \mathbb{R}$, the following equalities hold:

$$\dot{\Psi}_A(t, s) = A(t)\Psi_A(t, s), \qquad \Psi_A(s, s) = I,$$

where I is an $n \times n$ identity matrix.

For any matrix $B \in \mathbb{R}^{n \times n}$, let $r(B)$ be the spectral radius of B. Throughout this paper, we denote

$$\Phi_{AP_k}(\omega) := \prod_{i=1}^{q} \left(P_{q-i+1} \Psi_A(t_{q-i+1}, t_{q-i}) \right). \tag{2.2}$$

Since $\Phi_{AP_k}(\omega)$ is a nonnegative matrix, then $r(\Phi_{AP_k}(\omega))$ is the principal eigenvalue of $\Phi_{AP_k}(\omega)$, in the sense that it is simple and admits an eigenvector $v^* \gg 0$, i.e., $v^* \in \mathrm{Int}\,\mathbb{R}_+^n$. Some useful results for model (2.1) are below.

Theorem 2.1 *If $\eta = (1/\omega) \ln r(\Phi_{AP_k}(\omega))$, then there exists a positive ω-periodic function $v(t)$ such that $\exp(\eta t)v(t)$ is a solution of system* (2.1).

Proof Let $v^* \gg 0$ be an eigenvector associated with the principal eigenvalue $r(\Phi_{AP_k}(\omega))$. By the change of variable $x(t) = \exp(\eta t)v(t)$, the impulsive linear system (2.1) is reduced to the following system:

$$\begin{cases} \dot{v}(t) = A(t)v - \eta v = (A(t) - \eta I)v(t), & t \neq t_k, k \in \mathbb{Z}_+, \\ v(t^+) = P_k v(t), & t = t_k. \\ v(t_0^+) = v^*, & t_0 \geq 0. \end{cases} \tag{2.3}$$

For any $t > t_0$, there exist two nonnegative integers l and m ($0 \leq m < q$) such that $l\omega + t_m < t \leq l\omega + t_{m+1}$. Solving (2.3), for $l\omega + t_m < t \leq l\omega + t_{m+1}$, we get

$$v(t) = \Psi_{A-\eta I}(t, l\omega + t_m) \left(\prod_{i=1}^{m} \left(P_{m-i+1} \Psi_{A-\eta I}(t_{m-i+1}, t_{m-i}) \right) \right)$$
$$\times \left(\Phi_{(A-\eta I)P_k}(\omega) \right)^l v^*. \tag{2.4}$$

It is easy to see that

$$\Phi_{(A-\eta I)P_k}(\omega) = \exp(-\eta\omega)\Phi_{AP_k}(\omega). \tag{2.5}$$

From (2.4), (2.5), and $\eta = (1/\omega)\ln r(\Phi_{AP_k}(\omega))$, the following formula can be derived:

$$\begin{aligned} v((n+1)\omega^+) &= \left(\Phi_{(A-\eta I)P_k}(\omega) \right)^{n+1} v^* \\ &= \left(\Phi_{(A-\eta I)P_k}(\omega) \right)^n \exp(-\eta\omega)\Phi_{AP_k}(\omega)v^* \\ &= \left(\Phi_{(A-\eta I)P_k}(\omega) \right)^n \exp(-\eta\omega)r\left(\Phi_{AP_k}(\omega) \right)v^* \\ &= \left(\Phi_{(A-\eta I)P_k}(\omega) \right)^n v^* \\ &= v(n\omega^+). \end{aligned}$$

Thus, $v(t)$ is a positive ω-periodic solution of system (2.3) and $\exp(\eta t)v(t)$ is a solution of system (2.1). This completes the proof. □

Theorem 2.2 *If $r(\Phi_{AP_k}(\omega)) < 1$, then the trivial solution of system (2.1) is asymptotically stable.*

Using a method similar to that in [19], this result can be easily proved, not shown in this paper.

3 The basic reproductive number for a general hybrid epidemic system

Consider an impulsive differential system with non-periodic pulses in an environment period:

$$\begin{cases} \dot{x}(t) = f(t, x), & t \neq t_k, t \in \mathbb{R}, \\ \Delta x(t_k) = \Theta_k(x(t_k)), & t = t_k, k \in \mathbb{N}, \\ x(t_0^+) = x_0, & t_0 \geq 0, \end{cases} \tag{3.1}$$

where $f : \mathbb{R}^+ \times \Omega \to \mathbb{R}^n$ is an ω-periodic vector function, here $\Omega \subseteq \mathbb{R}^n$ is an open set, $\Delta x(t_k) = x(t_k^+) - x(t_k)$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$.

The existence and uniqueness of the solution for system (3.1) can be found in [20].

We write $x = (x_1, x_2, \ldots, x_n)^T$, here T denotes the transpose. For a compartmental epidemic model, we can split the compartments by two types with the first m compartments $\{x_1, x_2, \ldots, x_m\}$ the infected individuals, and $\{x_{m+1}, x_{m+2}, \ldots, x_n\}$ the uninfected individuals. Denote $X = (x_1, \ldots, x_m)$, $Y = (x_{m+1}, \ldots, x_n)$. Define

$$X_s = \{x \geq 0 \mid x_i = 0, i = 1, \ldots, m\}.$$

Set $x(t_k^+) = x(t_k) + \Theta_k(x(t_k)) := \phi_k(x(t_k))$, where $\phi_k : \Omega \to \Omega, \phi_k \in \mathbb{C}^1(\Omega, \Omega)$.

Taking the symbol in [1, 10], we can rewrite the vector field of (3.1) as

$$\begin{cases} \dot{x}(t) = \mathcal{F}(t, x(t)) - \mathcal{V}(t, x(t)), & t \neq t_k, k \in \mathbb{N}, \\ \left. \begin{array}{l} X(t_k^+) = h_k(x(t_k)), \\ Y(t_k^+) = g_k(x(t_k)), \end{array} \right\} & t = t_k, k \in \mathbb{N}, \\ x(t_0^+) = x_0, & t_0 \geq 0, \end{cases} \qquad (3.2)$$

where $\mathcal{F}(t, x)$ is the newly infected rate, $\mathcal{V}$ represents the net transfer rate out of compartments, $\phi_k = (h_k, g_k)^T$, $h_k = (\phi_{k1}, \ldots, \phi_{km})$, $g_k = (\phi_{k(m+1)}, \ldots, \phi_{kn})$. We assume that system (3.2) has a disease-free periodic solution $x^*(t) = (0, \ldots, 0, x_{m+1}^*(t), \ldots, x_n^*(t))$ over the time interval $(n\omega, (n+1)\omega]$ with $x_i^*(t) > 0$, $m + 1 \leq i \leq n$, for all t. In addition, eight assumptions, similar to those by Yang and Xiao [1], are made as follows:

(H$_1$) If $x_i \geq 0$, then the functions $\mathcal{F}_i(t, x)$, $\mathcal{V}_i^+(t, x)$, and $\mathcal{V}_i^-(t, x)$ are nonnegative and continuous on $R \times R_+^n$ and continuously differential with respect to x for $i = 1, \ldots, n$.

(H$_2$) If $x_i = 0$, then $\mathcal{V}_i^- = 0$. Particularly, if $x \in X_s$, then $\mathcal{V}_i^- = 0$ for $i = 1, \ldots, m$.

(H$_3$) $\mathcal{F}_i = 0$ if $i > m$.

(H$_4$) If $x \in X_s$, then $\mathcal{F}_i(x) = \mathcal{V}_i^+(x) = 0$ for $i = 1, \ldots, m$.

(H$_5$) The pulses on the infected compartments must be uncoupled with the uninfected compartments, that is, $h_k(x(t_k))$ is essentially $h_k(X(t_k))$.

(H$_6$) For any positive integer k, there exists a positive integer q such that $t_{k+q} = t_k + \omega$, $h_{k+q}(x) = h_k(x)$, $g_{k+q}(x) = g_k(x)$, and $h_k(0) = 0$.

(H$_7$) $r(\Phi_{MQ_k}(\omega)) < 1$, where $\Phi_{MQ_k}(\omega) := \prod_{i=1}^q (Q_{q-i+1} \Phi_M(t_{q-i+1}, t_{q-i}))$, and $\Phi_{MQ_k}(t)$ is the fundamental solution matrix of the following impulsive system:

$$\begin{cases} \dot{Z}(t) = M(t)Z, & t \neq t_k, k \in \mathbb{N}, \\ Z(t_k^+) = Q_k Z(t_k), & t = t_k, k \in \mathbb{N}, \end{cases}$$

where

$$M(t) = \left(\frac{\partial f_i(t, x^*(t))}{\partial x_j} \right)_{m+1 \leq i,j \leq n}, \qquad Q_k = \left(\frac{\partial \phi_{ki}(x^*(t))}{\partial x_j} \right)_{m+1 \leq i,j \leq n}.$$

(H$_8$) $r(\Phi_{-VP_k}(\omega)) < 1$, where

$$V(t) = \left(\frac{\partial \mathcal{V}_i(t, x^*(t))}{\partial x_j} \right)_{1 \leq i,j \leq m}, \qquad P_k = \left(\frac{\partial \phi_{ki}(x^*(t))}{\partial x_j} \right)_{1 \leq i,j \leq m}. \qquad (3.3)$$

We let

$$F(t) = \left(\frac{\partial \mathcal{F}_i(t, x^*(t))}{\partial x_j} \right)_{1 \le i,j \le m}. \tag{3.4}$$

In the following, we study the threshold dynamics of model (3.1). It indicates that its basic reproductive number can be defined as the spectral radius of the next infection operator as that in periodic environments, see [10].

Apparently, the matrices P_k and Q_k ($k \in \mathbb{N}$) are constant matrices. $F(t)$ is nonnegative, and $-V(t)$ is cooperative in the sense that the off-diagonal elements of $-V(t)$ are nonnegative.

Denote $Y(t, s)$ to be the evolution operator of the following system:

$$\begin{cases} \dot{y}(t) = -V(t)y, & t \ne t_k, k \in \mathbb{N}, \\ y(t_k^+) = P_k y(t_k), & t = t_k, k \in \mathbb{N}. \end{cases} \tag{3.5}$$

From assumption (H_8) and Theorem 2.2, we know that the trivial solution of the impulsive differential system (3.5) is asymptotically stable.

Define C_ω as the ordered Banach space of all ω-periodic functions from $\mathbb{R}$ to $\mathbb{R}^m$, which is equipped with maximum norm $\| \cdot \|_\infty$. Denote $C_\omega^+ = \{\phi \in C_\omega : \phi(t) \ge 0, t \in \mathbb{R}\}$ as the initial distribution of infectious individuals. Following the idea in [1], we define the next infection operator L:

$$(L\phi)(t) = \lim_{a \to -\infty} \int_a^t Y(t, s)F(s)\phi(s)\, ds, \quad \forall t \in \mathbb{R}, \phi \in C_\omega^+. \tag{3.6}$$

Obviously, the next infection operator L is well defined, positive, continuous, and compact on the domain. Then we define the basic reproductive number of (3.2) as the spectral radius of L, that is,

$$R_0 = r(L).$$

In order to calculate R_0, we consider an auxiliary linear impulsive system with non-periodic pulses in an environmental period

$$\begin{cases} \dot{U}(t) = [-V(t) + \frac{F(t)}{\lambda}]U(t), & t \ne t_k, k \in \mathbb{N}, \\ U(t^+) = P_k U(t), & t = t_k, k \in \mathbb{N}, \end{cases} \tag{3.7}$$

where $\lambda \in (0, \infty)$. Let $U(t, s, \lambda), t \ge s, s \in R$ be the evolution operator of system (3.7). Thus $U(\omega, 0, \lambda) = \Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)$.

By the constant-variation formula for linear impulsive periodic equations (see [10]), we can prove the following result and obtain a numerical method to calculate the basic reproductive number for the hybrid epidemic system (3.1).

Theorem 3.1 *Assume that* (H_1)–(H_8) *hold, we have the following statements.*

(i) *If* $r(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)) = 1$ *has a positive solution* λ_0, *then* λ_0 *is an eigenvalue of* L, *and hence* $R_0 > 0$.

(ii) *If $R_0 > 0$, then $\lambda = R_0$ is the unique solution of $r(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)) = 1$.*

(iii) *$R_0 = 0$ if and only if $r(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)) < 1$ for all $\lambda > 0$.*

According to the results of Theorem 3.1, we obtain that R_0 for the general hybrid epidemic model (3.2) is the positive solution of $r(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)) = 1$. In the following, we show that the basic reproductive number serves as a threshold value for the local stability of the disease-free periodic solution $x^*(t)$ of (3.2).

Theorem 3.2 *Assume that (H_1)–(H_8) hold. The following statements are valid*:

(i) *$R_0 = 1$ if and only if $r(\Phi_{(F-V)P_k}(\omega)) = 1$.*

(ii) *$R_0 > 1$ if and only if $r(\Phi_{(F-V)P_k}(\omega)) > 1$.*

(iii) *$R_0 < 1$ if and only if $r(\Phi_{(F-V)P_k}(\omega)) < 1$.*

The disease-free periodic solution $x^*(t)$ is asymptotically stable if $R_0 < 1$, and unstable if $R_0 > 1$.

Using a similar method of Theorem 2.2 in [10], this result can be easily proved, not shown in this paper.

Remark 3.1　In [1], Yang and Xiao considered a nonlinear impulsive system, in which the pulse period is equal to the environmental period. If we take $q = 1$, then model (3.1) yields the model that was studied in [1]. From (2.2), we have $\Phi_{-VP_k}(\omega) = P_1 \Psi_{-V}(\omega, 0)$ if $q = 1$, which is consistent with the result of the literature. It indicates that here a more general model is established including the previous model as a special case.

4 Examples

We give two examples to illustrate the calculation of R_0 for impulsive nonautonomous model using Theorem 3.1.

Example 1　Barbour's schistosomiasis model with seasonal fluctuations was proposed and studied in [21]. Incorporating impulsive snail-killing strategy, we consider Barbour's schistosomiasis model with periodic infection rates and impulsive control strategy:

$$\begin{cases} \dot{M}(t) = a(t)\Delta y(1 - M) - gM, \\ \dot{y}(t) = b(t)\frac{\Sigma}{\Delta}M(1 - y) - \mu y, \end{cases} \quad t \neq t_k, \\ \begin{cases} M(t^+) = M(t), \\ y(t^+) = (1 - \theta_k)y(t), \end{cases} \quad t = t_k, \tag{4.1}$$

where M and y denote the prevalence of infection in humans and the prevalence of infection in snails, respectively. a and b are infection rates, g is the recovery rate for definitive host infections, Δ and Σ are the densities of infected snails and definitive hosts, respectively. μ is per capita death rate of infected snails. For snail control, we assume spraying pesticide twice a year, that is, $q = 2$ and $\omega = 1$ (year). We also assume that $t_{k+2} = t_k, \theta_{k+2} = \theta_k$, and $a(t)$ and $b(t)$ are nonnegative piecewise continuous functions with period 1.

System (4.1) has a disease-free periodic solution $(0, 0)$. For system (4.1), we have

$$\mathcal{F}(t, x) = \begin{pmatrix} a(t)\Delta y(1 - M) \\ b(t)\frac{\Sigma}{\Delta}M(1 - y) \end{pmatrix}, \qquad \mathcal{V}(t, x) = \begin{pmatrix} gM \\ \mu y \end{pmatrix}.$$

Then, from (3.3) and (3.4), we get

$$F(t) = \begin{pmatrix} 0 & a(t)\Delta \\ b(t)\frac{\Sigma}{\Delta} & 0 \end{pmatrix}, \quad V(t) = \begin{pmatrix} g & 0 \\ 0 & \mu \end{pmatrix},$$

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 - \theta_1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 - \theta_2 \end{pmatrix}.$$

Obviously, assumptions (H_1)–(H_8) hold. According to Theorem 3.1, we know that the basic reproductive number R_0 for system (4.1) is a solution of the polynomial $r(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)) = 1$, where

$$\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega) = P_2 \Phi_{\frac{F}{\lambda} - V}(t_2, t_1) P_1 \Phi_{\frac{F}{\lambda} - V}(t_1, t_0),$$

and $\Phi_{(\frac{F}{\lambda} - V)P_k}(t, s)$ $(t \geq s)$ is the evolution operator of the system

$$\begin{cases} \dot{U}(t) = \begin{bmatrix} -g & \frac{a(t)\Delta}{\lambda} \\ b(t)\frac{\Sigma}{\Delta\lambda} & -\mu \end{bmatrix} U(t), & t \neq t_k, k \in \mathbb{N}, \\ U(t^+) = P_k U(t), & t = t_k, k \in \mathbb{N}. \end{cases}$$

We use the same parameter values in [21] with system (4.1), which are $g = 0.25$, $\mu = 2$, $\Delta = 133.5$, $\Sigma = 0.017$, $a(t) = \frac{\pi}{2} a |\sin(\pi t)|$, $b(t) = \frac{\pi}{2} b |\sin(\pi t)|$, $a = 0.621, b = 49.056$. The control parameter values are chosen as $\theta_1 = 0.6$, $\theta_2 = 0.5$, $t_0 = 0.3, t_1 = 0.5$, by numerical calculation, we get $R_0 = 0.752$.

Example 2 We propose an SIRS epidemic model with pulse vaccination and seasonality, in which non-periodic pulses in an environmental period are considered.

$$\begin{cases} \dot{S}(t) = (\mu - (1 - \sigma)\mu I)(1 - p) - \frac{\beta(t)SI}{1 + \alpha(t)S} - \mu S + \delta R, \\ \dot{I}(t) = \frac{\beta(t)SI}{1 + \alpha(t)S} - \mu I - \gamma I + (1 - \sigma)\mu I, & t \neq t_k, k \in \mathbb{N}, \\ \dot{R}(t) = (\mu - (1 - \sigma)\mu I)p + \gamma I - (\mu + \delta)R, \\ S(t^+) = (1 - \theta_k)S(t), \\ I(t^+) = I(t), & t = t_k, k \in \mathbb{N}, \\ R(t^+) = R(t) + \theta_k S(t), \end{cases} \tag{4.2}$$

where S, I, and R denote the number of susceptible, infected, and removed individuals, respectively. μ is the natural birth rate and death rate, σ is a probability that a child is born from an infectious mother and is also susceptible, γ and δ are the transmission rates, p is the continuous first vaccination rate, $\frac{\beta(t)SI}{1 + \alpha(t)S}$ is the saturation incidence, where $\beta(t)$ (contact rate) and $\alpha(t)$ are two periodic continuous functions with period ω, θ_k is the vaccination rate at time $t = t_k$. We assume that there exists a positive integer q such that $\theta_{k+q} = \theta_k$, $t_{k+q} = t_k + \omega$ for any $k \in \mathbb{N}$. Model (4.2) was proposed in [22] where the pulse vaccination is periodic.

First, we can easily know that system (4.2) has a disease-free periodic solution $(S^*(t), 0, R^*(t))$. And from the definition of $F(t)$, $V(t)$, and the impulsive matrix P_k, we can get

$$F(t) = \frac{\beta(t)S^*(t)}{1 + \alpha(t)S^*(t)}, \qquad V(t) = \gamma + \sigma\mu, \qquad P_k = 1.$$

It follows from Theorem 3.1 that the basic reproductive number R_0 is the solution of the polynomial $r(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)) = 1$, where $\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega) = U(\omega, 0, \lambda)$, and $U(t, 0, \lambda)$ is the evolution operator of the system

$$\dot{U}(t) = \left(-(\gamma + \sigma\mu) + \frac{\beta(t)S^*(t)}{\lambda(1 + \alpha(t)S^*(t))} \right) U(t).$$

Thus,

$$r\big(\Phi_{(\frac{F}{\lambda} - V)P_k}(\omega)\big) = \exp\left(\int_0^\omega \left(\frac{\beta(t)S^*(t)}{\lambda(1 + \alpha(t)S^*(t))} - (\gamma + \sigma\mu) \right) dt \right).$$

Solving the following equation about λ:

$$\exp\left(\int_0^\omega \left(\frac{\beta(t)S^*(t)}{\lambda(1 + \alpha(t)S^*(t))} - (\gamma + \sigma\mu) \right) dt \right) = 1,$$

we have the basic reproductive number

$$R_0 = \frac{1}{\omega(\gamma + \sigma\mu)} \int_0^\omega \frac{\beta(t)S^*(t)}{1 + \alpha(t)S^*(t)} \, dt.$$

This is consistent with the result of [22] when pulse intervals are equal.

5 Conclusion

In this paper, we have given the definition and calculation of the basic reproductive number R_0 for the hybrid epidemic model, in which multiple and non-periodic pulses in an environmental period are taken into consideration. It extends the results of the published research model [1]. Under assumptions (H_1)–(H_8), we have established the criteria (i.e., $R_0 < 1$) to ensure the local stability of the disease-free periodic solution.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The main idea and theoretical proof of this paper were proposed by SG and JW. Programming of numerical simulation was completed by YT. All authors read and approved the final manuscript.

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