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Delay-dependent stability criteria for neutral-type neural networks with interval time-varying delay signals under the effects of leakage delay

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Abstract

We examine the stability problem for delayed neutral-type neural networks (NNs) with interval time-varying delay signals under the effects of leakage term by constructing a suitable Lyapunov–Krasovskii functionals (LKFs) with the triple- and four-integral terms and using the famous Jensen inequality, Wirtinger single integral inequality (WSII), and Wirtinger double integral inequality (WDII), combined with the reciprocally convex approach (RCC) for the stability of addressing NNs. Therefore, the major contribution of this study lies in a consideration of new integral inequalities and improved LKFs, fully taking the relationship between the terms in the Leibniz–Newton formula within the framework of linear matrix inequalities (LMIs). Moreover, we assume that the lower bound of interval time-varying delay is not restricted to zero. Using several examples, we show that the proposed stability criterion is less conservative than previous results. Also, the proposed technique is applied to benchmark problem that is associated with reasonable issues to showing feasibility on a real-world problem, including transporting time delay signals and leakage delay as a process variable in the quadruple-tank process system.

Keywords: Neural network; Neutral type; Leakage delay; Interval time-varying delay; Wirtinger double integral inequality

1 Introduction

During the past few decades, neural networks have been extensively investigated and have been found in a wide range of applications in various science and engineering fields, such as signal processing, target tracking, fault diagnosis, pattern recognition, communication, image processing, parallel computation, and industrial automation. All these applications mainly depend on the dynamical behaviors of the considered NNs and their equilibrium points. Therefore, the study of dynamical behaviors of the delayed NN is an active research topic and has received considerable attention in recent years [1–3]. It is obvious that time-delay naturally exists in many real systems. Moreover, in practice, it is the main reason to affect the stability performances of a system [4, 5]. Therefore, it is necessary and important to investigate the concept of time-delay while discussing the dynamical behaviors of NNs. Recently, much progress has been achieved in the study of NNs with time delays [1–3]. It is well known that, according to dependence on the size of the delays, the stability criteria are usually classified into two types, the delay-independent stability criteria [6, 7] and delay-dependent stability criteria [8–16]. As a result, the delay-dependent stability criteria have received much attention from the researchers because they are concerned with the size of the delay and provide less conservative results than delay-independent criteria.

On the other hand, neutral-type time-delay in the system models are usually encountered in many practical applications, such as population ecology, chemical reactors, water pipes, heat exchangers, and robots in contact with rigid environments [17]. It is well known that the neutral-type time-delay incorporates the time delays both in its state and in the derivatives of state in the system model. Additionally, many dynamical NNs are represented with neutral functional differential equations that combine neutral delay differential equations as their special type. Therefore, these NNs are referred to neutral neural networks or neural networks of neural type. It is obviously known that time-delay and nonlinearity are usually a source of instability and/or poor performance of many systems [18]. Therefore, in stability analysis of NNs with neutral time, delays have been one of the primary research topics, and many remarkable achievements have been explored [19–23].

Recently stability of NNs with leakage delays has become one of impressive research topics and has been widely studied by many researchers. The research on the dynamical behaviors of system models with leakage delay (or forgetting delay), which has been found in the negative feedback term of a dynamical system, can be traced back to 1992. In [24], it was realized that the leakage delay had vast impact research on the dynamical behaviors of the system model. Since then, many researchers have well focused on the systems with leakage delay, and many interesting results have been derived. For example, in [25], a population model with leakage delays was considered, and it was found that the leakage delay can destabilize a system. In [26], the bidirectional associative memory (BAM) neural networks with constant leakage delays were studied extensively based on LKFs and the properties of M-matrices. Since [26], it is more important and necessary to investigate the stability of delayed NNs including leakage effects. For this purpose, recently relevant leakage problems had arisen, and significant progress has been made (see, e.g., [27–29]. Recently, in [27], the global exponential stability for BAM neural networks with time-varying leakage delays was discussed, which extended and improved the main results developed in [25, 26]. In [28], the stability of nonlinear systems with leakage time-varying delay was investigated, and it was proved that the impact of leakage delay cannot be avoided.

Generally speaking, from the available literature we can find that most of the work on NNs with time-varying delays has been studied under the assumption that the range of time-varying delay is from 0 to a certain upper bound. However, in practical world, the time-varying delay may be an interval delay, that is, the lower bound of the delay interval is not restricted to zero. For this case, the stability criteria for dynamical systems proposed in [29, 30] cannot be applied because those results could not consider the case where lower bound of delay is nonzero. Therefore, it is of significant importance to study the stability of NNs containing a nonzero lower bound of the interval time-varying delays, and some of the researchers have reported fruitful results in the previous literature (see [8–13, 16]). Recently, in [8], the asymptotic stability for a class of cellular NNs with interval time-varying delay was studied by combining novel LKFs and delay partitioning approach. New stability criteria for NNs with time-varying interval delays based on a piecewise delay method were proposed in [16]. Recently, the asymptotic stability of delayed NNs have been extensively investigated in [10], where an improved delay-partitioning idea is employed. Very recently, in [13], the exponential stability criteria for cellular NNs with interval time-varying delays were proposed, based on the generalized activation functions. More recently, the authors in [11, 12] investigated the delay-dependent stability criteria for NNs with interval time-varying delays via an augmented LKF and RCC approach. More recently, the authors in [31–34] investigated the stability problems of NNs with time-varying delays, based on the integral inequality technique and also demonstrated that the proposed criteria are applied to the practical application, showing how to derive efficient stability criteria for a real-world problem. The main contributions of this paper are highlighted in the following viewpoints:

- ★ In this paper, as a first attempt, leakage delay is considered in the quadruple-tank process system model to investigate the stability performance in a real-world problem.
- ★ New mathematical technique is adopted together with LKFs when estimating their derivatives to improve stability performance of the neural system (1) with interval time-varying delay signals and leakage delays in the system model.
- ★ Differently from [8–13, 16, 19, 20], several numerical examples are presented to illustrate the validity of the main results with a real-world simulation.
- ★ Additionally, WDII technique is taken into account to bound the time-derivative of triple integral LKFs, which provide more tighter bounding technology for dealing with such LKFs. This technique has never been used in the previous literature [8–13, 16, 19, 20] and plays an important role in reducing conservatism.
- ★ All the sufficient conditions are expressed in terms of LMIs, which can be easily solved by using Matlab software.

The remainder of this paper is structured as follows. The NN model is composed and assumption and some lemmas are presented in Section 2. In Section 3, we derive a new delay-dependent stability criteria for NN model to be asymptotically stable. In Section 4, interesting numerical simulation studies are proposed. Finally, some conclusions and future study directions are made in Section 5.

Notation. The notation used in this paper is quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices; $\|\cdot\|$ refers to the Euclidean vector norm; A^T represents the transpose of a matrix A; I is the identity matrix of compatible dimension; $X > Y$ means that X and Y are symmetric matrices and that $X - Y$ is positive definite; $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the largest and smallest eigenvalues of a given matrix, respectively. The symbol $\maltese$ represents the elements below the main diagonal of a symmetric matrix. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2 Problem formulation and preliminaries

Consider the following neutral-type NNs with leakage term and discrete interval time-varying delays:

$$\begin{aligned} &\dot{e}(t) = -Ae(t-\delta) + W_1 f\big(e(t)\big) + W_2 f\big(e\big(t-\tau(t)\big)\big) + W_3\dot{e}\big(t-h(t)\big), \\ &e(t) = \phi(t), \quad t \in [-\bar{r}, 0], \end{aligned} \tag{1}$$

where $e(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^T \in \mathbb{R}^n$ is the state vector of the network at time t, n corresponds to the number of neurons, $f(e(t)) = [f_1(e_1(t)), f_2(e_2(t)), \ldots, f_n(e_n(t))]^T \in \mathbb{R}^n$ is the neuron activation function. The matrix $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with positive entries $a_i > 0$. W_1, W_2, and W_3 are the interconnection matrices representing the weight coefficients of the neurons, $\phi(t)$ is a vector-valued initial function, and $\bar{r} = \max\{\delta, \tau_M, h\}$, where $\delta \geq 0$ denotes the constant leakage delay. The discrete delay $\tau(t)$ and the neutral delay $h(t)$ are assumed to satisfy

$$0 < \tau_1 \leq \tau(t) \leq \tau_2, \qquad \dot{\tau}(t) \leq \mu, \qquad 0 < h(t) < h, \qquad \dot{h}(t) \leq h_D, \tag{2}$$

where τ_1, τ_2, μ, h, and h_D are known real constants.

Remark 2.1 The first term in the right side of the model (1) variously known as forgetting or leakage term. It is well known from the literature the study of population dynamics (see Gopalsamy [25]) that time delays in the stabilizing negative feedback terms will have a tendency to destabilize a system. The functions $f_j(\cdot)$, $j = 1, 2, \ldots, n$, are signal transmission functions. Furthermore, NNs (1) contains some data about the derivative of the past state to further analysis and model the dynamics of such complex neural responses. Hence NNs (1) have been referred as neutral-type NNs, in which the system has both the state delay and the state derivative with delay, the so-called neutral delay.

Throughout this paper, we assume that each activation function $f_j(\cdot)$ in (1) satisfies the following:

Assumption (H) ([35]) For any $j \in \{1, 2, \ldots, n\}$, $f_j(0) = 0$, and their exist constants F_j^- and F_j^+ such that

$$F_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+ \tag{3}$$

for all $\alpha_1 \neq \alpha_2$, where $\alpha_1, \alpha_2 \in \mathbb{R}$.

Next, we present some preliminary lemmas, which are needed in the proof of our main results.

Lemma 2.1 ([36]) *For any positive definite matrix $M \in \mathbb{R}^{n\times n}$, scalars $h_2 > h_1 > 0$, and a vector function $w : [h_1, h_2] \to \mathbb{R}^n$ such that the integrations concerned are well defined, we have the inequality*

$$-(h_2 - h_1) \int_{t-h_2}^{t-h_1} w^T(s)Mw(s)\,ds \leq -\left(\int_{t-h_2}^{t-h_1} w(s)\,ds\right)^T M \left(\int_{t-h_2}^{t-h_1} w(s)\,ds\right).$$

Lemma 2.2 ([37]) *Let $f_2, f_2, \ldots, f_N : R^m \longmapsto R$ have positive values in an open subset D of R^m. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : R^m \longmapsto R, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}.$$

Lemma 2.3 ([38]) *For a positive definite matrix $M > 0$, we have the following inequality for all continuously differentiable functions $\omega(t)$ in $[a,b] \in \mathbb{R}^n$:*

$$-(b-a)\int_{t-a}^{t-b} \dot{\omega}^T(s)M\dot{\omega}(s)\,ds \le -\begin{bmatrix}\Omega_0\\ \Omega_1\end{bmatrix}^T \begin{bmatrix} M & 0\\ \maltese & 3M\end{bmatrix}\begin{bmatrix}\Omega_0\\ \Omega_1\end{bmatrix},$$

where $\Omega_0 = \omega(b) - \omega(a)$ and $\Omega_1 = \omega(b) + \omega(a) - \frac{2}{(b-a)}\int_a^b \omega(s)\,ds$.

Lemma 2.4 ([39]) *Suppose $r_1 \le r(t) \le r_2$, where $r(\cdot) : \mathbb{R}_+$ (or $\mathbb{Z}_+$) $\to \mathbb{R}_+$ (or $\mathbb{Z}_+$). Then, for any $R = R^T > 0$, we have the integral inequality*:

$$\begin{aligned}-\int_{t-r_2}^{t-r_1} \dot{x}^T(s)R\dot{x}(s)\,ds \le{}& \alpha^T(t)\big[\big(r_2 - r(t)\big)KR^{-1}K^T + \big(r(t)-r_1\big)JR^{-1}J^T\\ &+ [J \quad -J+K \quad -K] + [J \quad -J+K \quad -K]^T\big]\alpha(t),\end{aligned}$$

where $\alpha^T(t) = [x^T(t-r_1)\ x^T(t-r(t))\ x^T(t-r_2)]$, $K = [K_1^T\ K_2^T\ K_3^T]^T$, and $J = [J_1^T\ J_2^T\ J_3^T]^T$.

Lemma 2.5 ([40]) *Let $\mathbb{M} > 0$ be any constant matrix. For given scalars a and b with $a < b$, the following relation is well defined for any differentiable function ω in $[a,b] \to \mathbb{R}^n$:*

$$-\frac{b^2-a^2}{2}\int_{-a}^{-b}\int_{t+u}^{t} \dot{\omega}^T(s)M\dot{\omega}(s)\,ds\,du \le -\begin{bmatrix}\Omega_2\\ \Omega_3\end{bmatrix}^T \begin{bmatrix} M & 0\\ \maltese & 2M\end{bmatrix}\begin{bmatrix}\Omega_2\\ \Omega_3\end{bmatrix},$$

where

$$\Omega_2 = (b-a)\omega(t) - \int_{t-a}^{t-b}\omega(s)\,ds$$

and

$$\Omega_3 = -\frac{(b-a)}{2}\omega(t) - \int_{t-a}^{t-b}\omega(s)\,ds + \frac{3}{b-a}\int_{-a}^{-b}\int_{t+u}^{t}\omega(s)\,ds\,du.$$

Lemma 2.6 ([41]) *Suppose $\gamma_1 \le \gamma(t) \le \gamma_2$, where $\gamma(\cdot) : \mathbb{R}_+$ (or $\mathbb{Z}_+$) $\to \mathbb{R}_+$ (or $\mathbb{Z}_+$). Then, for any constant matrices Ξ_1, Ξ_2, and Ω of proper dimensions, the matrix inequality*

$$\Omega + \big(\gamma(t)-\gamma_1\big)\Xi_1 + \big(\gamma_2-\gamma(t)\big)\Xi_2 < 0$$

holds if and only if

$$\Omega + (\gamma_2-\gamma_1)\Xi_1 < 0, \qquad \Omega + (\gamma_2-\gamma_1)\Xi_2 < 0.$$

3 Main results

This section is devoted to exploring a new stability criterion in terms of LMIs for the designed neural system (1) to be asymptotically stable, based on the conditions developed by newly improved integral inequalities. For simplicity, we denote $F_1 = \operatorname{diag}(F_1^-F_1^+, F_2^-F_2^+, \ldots, F_n^-F_n^+)$ and $F_2 = \operatorname{diag}(\frac{F_1^-+F_1^+}{2}, \frac{F_2^-+F_2^+}{2}, \ldots, \frac{F_n^-+F_n^+}{2})$.

Theorem 3.1 *Assume that Assumption* (H) *holds. For given positive scalars* δ, τ_1, τ_2, μ, h, *and* h_D, *the NN described by* (1) *is asymptotically stable for any time-varying delay* $\tau(t)$ *and* $h(t)$ *satisfying* (2) *if there exist symmetric positive definite matrices* P_i $(i = 1,2,3)$, Q_i $(i = 1,2,3,\ldots,5)$,

$$U = \begin{bmatrix} U_{11} & U_{12} \\ \maltese & U_{22} \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ \maltese & V_{22} \end{bmatrix}, \qquad D = \begin{bmatrix} D_{11} & D_{12} \\ \maltese & D_{22} \end{bmatrix},$$

R_i $(i = 1,2,3)$, $\mathcal{R}_i$ $(i = 1,2,3)$, T_i $(i = 1,2,3)$, U_i $(i = 1,2,3)$, *diagonal matrices* X, Y, *and* Z, *any matrices* G_i $(i = 1,2,\ldots,6)$ *of appropriate dimensions, and any appropriate dimensional matrices* L_i, M_i, N_i, O_i $(i = 1,2,\ldots,4)$ *such that the following LMIs hold*:

$$\begin{bmatrix} D & E \\ \maltese & D \end{bmatrix} \geq 0, \tag{4}$$

$$\begin{bmatrix} \Pi & \sqrt{\tau_1}L & \sqrt{\tau_\rho}M & \sqrt{\tau_\rho}N & \sqrt{\tau_2}O & \tau_\rho\bar{K} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3\tau_\rho \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} \Pi & \sqrt{\tau_1}L & \sqrt{\tau_\rho}M & \sqrt{\tau_\rho}N & \sqrt{\tau_2}O & \tau_\rho\bar{J} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3\tau_\rho \end{bmatrix} < 0, \tag{6}$$

where $\Pi = (\Pi_{i,j})_{18\times 18}$ *with*

$$\Pi_{11} = -P_1A + P_2 + \delta^2P_3 + Q_1 + Q_2 + Q_3 + \tau_1^2U_{11} + \tau_2^2V_{11} + \tau_\rho^2D_{11} - U_{22} - V_{22} - 4R_1,$$

$$\Pi_{12} = -G_1A - \tau_1^2U_{12}A - \tau_2^2V_{12}A - \tau_\rho^2D_{12}A, \qquad \Pi_{14} = L_3^T - M_1 + N_1^T + O_3^T - F_1Z,$$

$$\Pi_{13} = -2R_1 - L_1 + L_2^T + M_1 + O_2^T + U_{22}, \qquad \Pi_{15} = -2R_2 + L_4^T - N_1 - O_1 + O_4^T + V_{22},$$

$$\Pi_{16} = P_1W_1 + G_1W_1 - F_2X - F_2Z + \tau_1^2U_{12}W_1 + \tau_2^2V_{12}W_1 + \tau_\rho^2D_{12}W_1,$$

$$\Pi_{18} = 6\tau_iR_1, \qquad \Pi_{17} = P_1W_2 + G_1W_2 - F_2Z + \tau_1^2U_{12}W_2 + \tau_2^2V_{12}W_2 + \tau_\rho^2D_{12}W_2,$$

$$\Pi_{1_{11}} = 6\tau_2R_2, \qquad \Pi_{1_{12}} = 3S_1 + \frac{\tau_1^2}{2}T_1, \qquad \Pi_{1_{13}} = 3S_3 + \left(\frac{\tau_2^2 - \tau_1^2}{2}\right)T_3,$$

$$\Pi_{1_{14}} = 3S_3 + \left(\frac{\tau_2^2 - \tau_1^2}{2}\right)T_3, \qquad \Pi_{1_{15}} = 3S_2 + \frac{\tau_2^2}{2}T_2, \qquad \Pi_{1_{16}} = AP_1^TA^T,$$

$$\Pi_{1_{17}} = \tau_1^2U_{12}W_3 + \tau_2^2V_{12}W_3 + \tau_\rho^2D_{12}W_3 + G_1W_3, \qquad \Pi_{22} = -P_2 - G_2A - A^TG_2,$$

$$\Pi_{23} = -G_3^TA^T, \qquad \Pi_{24} = -G_4^TA^T, \qquad \Pi_{25} = -G_5^TA^T,$$

$$\Pi_{26} = G_2W_1, \qquad \Pi_{27} = G_2W_2, \qquad \Pi_{2_{17}} = G_2W_3 - A^TG_1,$$

$$\Pi_{33} = -Q_2 + Q_4 - 4R_1 + K_1^T + K_1 - L_2 - L_2^T + M_2 + M_2^T - U_{22} - D_{22},$$

$$\Pi_{34} = K_2^T - K_1 + J_1 - L_3^T - M_2 + M_3^T + N_2 + D_{22}^T - E_2, \qquad \Pi_{36} = G_3 W_1,$$
$$\Pi_{35} = K_3^T - J_1 - L_4^T + M_4^T - N_2 - O_2 + E_2, \qquad \Pi_{37} = G_3 W_2,$$
$$\Pi_{38} = 6\tau_1 R_1, \qquad \Pi_{3_{17}} = G_3 W_3,$$
$$\begin{aligned}\Pi_{44} = {} & -(1-\mu)Q_1 - K_2^T - K_2 + J_2^T + J_2 - M_3 - M_3^T + N_3 + N_3^T \\ & + F_1 Y + F_1 Z - 2D_{22} + E_2 + E_2^T,\end{aligned}$$
$$\Pi_{45} = -K_3^T + J_3^T - J_2 - M_4^T - N_3 + N_4^T - O_3 - D_{22} - E_2, \qquad \Pi_{46} = G_4 W_1 + F_2^T Z^T,$$
$$\Pi_{47} = G_4 W_2 - F_2 Y - F_2 Z, \qquad \Pi_{4_{17}} = G_4 W_3,$$
$$\Pi_{55} = -Q_3 - Q_4 - 4R_2 - J_3^T - J_3 - N_4 - N_4^T + O_4 - O_4^T - V_{22} - D_{22}, \qquad \Pi_{56} = G_5 W_1,$$
$$\Pi_{57} = G_5 W_2, \qquad \Pi_{5_{11}} = 6\tau_2 R_3, \qquad \Pi_{5_{17}} = G_5 W_3, \qquad \Pi_{66} = -X + Z, \qquad \Pi_{67} = -Z,$$
$$\Pi_{6_{16}} = -AP_1^T W_1^T, \qquad \Pi_{6_{17}} = W_1^T G_1^T, \qquad \Pi_{77} = -Y + Z, \qquad \Pi_{7_{16}} = -AP_1^T W_2^T,$$
$$\Pi_{7_{17}} = W_2^T G_1, \qquad \Pi_{88} = -12\tau_1^2 R_1 - 3S_1 - U_{11}, \qquad \Pi_{8_{12}} = \frac{6}{\tau_1} S_1,$$
$$\Pi_{99} = -3S_3 - D_{11}, \qquad \Pi_{9_{13}} = \frac{6}{\tau_\rho} S_3, \qquad \Pi_{10_{10}} = -3S_3 - D_{11}, \qquad \Pi_{10_{14}} = \frac{6}{\tau_\rho} S_3,$$
$$\Pi_{11_{11}} = -12\tau_2^2 R_2 - 3S_2 - V_{11}, \qquad \Pi_{11_{15}} = \frac{6}{\tau_2} S_2, \qquad \Pi_{12_{12}} = -\frac{18}{\tau_1^2} S_1 - T_1,$$
$$\Pi_{13_{13}} = -\frac{18}{\tau_\rho^2} S_3 - T_3, \qquad \Pi_{14_{14}} = -\frac{18}{\tau_\rho^2} S_3 - T_3, \qquad \Pi_{15_{15}} = -\frac{18}{\tau_2^2} S_2 - T_2,$$
$$\Pi_{16_{16}} = -P_3, \qquad \Pi_{16_{17}} = -A^T P_1 W_3,$$
$$\begin{aligned}\Pi_{17_{17}} = {} & Q_5 + \tau_1^2 U_{22} + \tau_2^2 V_{22} + \tau_\rho D_{22} + \tau_1^2 R_1 + \tau_2^2 R_2 + \tau_\rho^2 R_3 \\ & + \tau_1 \mathcal{R}_1 + \tau_\rho \mathcal{R}_2 + \tau_2 \mathcal{R}_3 + \frac{\tau_1^4}{4} T_1 + \frac{\tau_2^4}{4} T_2 + \frac{(\tau_2^2 - \tau_1^2)^2}{4} T_3 \\ & + \left(\frac{\tau_1^3}{6}\right)^2 U_1 + \left(\frac{\tau_2^3}{6}\right)^2 U_2 + \left(\frac{\tau_2^3 - \tau_1^3}{6}\right)^2 U_3 - 2G_1 - 2G_1^T,\end{aligned}$$
$$\Pi_{17_{18}} = G_1 W_3, \qquad \Pi_{18_{18}} = -(1 - h_D)Q_5,$$
$$\bar{J} = [0 \quad 0 \quad J_1 \quad J_2 \quad J_3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$
$$\bar{K} = [0 \quad 0 \quad K_1 \quad K_2 \quad K_3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$
$$E = \begin{bmatrix} E_1 & 0 \\ \maltese & E_2 \end{bmatrix}, \qquad \tau_\rho = \tau_2 - \tau_1.$$

Proof Let us choose the following LKF candidate for NNs (1):

$$V\bigl(e(t), t\bigr) = \sum_{i=1}^{9} V_i\bigl(e(t), t\bigr), \tag{7}$$

where

$$V_1\bigl(e(t), t\bigr) = \left(e(t) - A\int_{t-\delta}^{t} e(s)\,ds\right)^T P_1 \left(e(t) - A\int_{t-\delta}^{t} e(s)\,ds\right),$$
$$V_2\bigl(e(t), t\bigr) = \int_{t-\delta}^{t} e^T(s) P_2 e(s)\,ds + \delta \int_{-\delta}^{0}\int_{t+\theta}^{t} e^T(s) P_3 e(s)\,ds\,d\theta,$$

$$\begin{aligned}
V_3\bigl(e(t),t\bigr) &= \int_{t-\tau(t)}^{t} e^T(s)Q_1e(s)\,ds + \int_{t-\tau_1}^{t} e^T(s)Q_2e(s)\,ds + \int_{t-\tau_2}^{t} e^T(s)Q_3e(s)\,ds \\
&\quad + \int_{t-\tau_2}^{t-\tau_1} e^T(s)Q_4e(s)\,ds, \\
V_4\bigl(e(t),t\bigr) &= \int_{t-h(t)}^{t} \dot{e}^T(s)Q_5\dot{e}(s)\,ds, \\
V_5\bigl(e(t),t\bigr) &= \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \xi^T(s)U\xi(s)\,ds\,d\theta + \tau_2\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \xi^T(s)V\xi(s)\,ds\,d\theta \\
&\quad + \tau_\rho\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \xi^T(s)D\xi(s)\,ds\,d\theta, \\
V_6\bigl(e(t),t\bigr) &= \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)R_1\dot{e}(s)\,ds\,d\theta + \tau_2\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)R_2\dot{e}(s)\,ds\,d\theta \\
&\quad + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{e}^T(s)R_3\dot{e}(s)\,ds\,d\theta, \\
V_7\bigl(e(t),t\bigr) &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)\mathcal{R}_1\dot{e}(s)\,ds\,d\theta + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{e}^T(s)\mathcal{R}_2\dot{e}(s)\,ds\,d\theta \\
&\quad + \int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)\mathcal{R}_3\dot{e}(s)\,ds\,d\theta, \\
V_8\bigl(e(t),t\bigr) &= \frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)T_1\dot{x}(s)\,ds\,du\,d\theta \\
&\quad + \frac{\tau_2^2}{2}\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)T_2\dot{e}(s)\,ds\,du\,d\theta \\
&\quad + \frac{\tau_2^2-\tau_1^2}{2}\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)T_3\dot{e}(s)\,ds\,du\,d\theta, \\
V_9\bigl(e(t),t\bigr) &= \frac{\tau_1^3}{6}\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{u}^{0}\int_{t+\lambda}^{t} \dot{e}^T(s)U_1\dot{e}(s)\,ds\,d\lambda\,du\,d\theta \\
&\quad + \frac{\tau_2^3}{6}\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{u}^{0}\int_{t+\lambda}^{t} \dot{e}^T(s)U_2\dot{e}(s)\,ds\,d\lambda\,du\,d\theta \\
&\quad + \frac{\tau_2^3-\tau_1^3}{6}\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{u}^{0}\int_{t+\lambda}^{t} \dot{e}^T(s)U_3\dot{e}(s)\,ds\,d\lambda\,du\,d\theta, \\
\xi^T(t) &= \operatorname{col}\bigl\{e(t),\dot{e}(t)\bigr\}, \\
U &= \begin{bmatrix} U_{11} & U_{12} \\ \maltese & U_{22} \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ \maltese & V_{22} \end{bmatrix}, \qquad D = \begin{bmatrix} D_{11} & D_{12} \\ \maltese & D_{22} \end{bmatrix}.
\end{aligned}$$

Taking the time derivative of $V(e(t),t)$ along the trajectories of system (1) yields

$$\dot{V}\bigl(e(t),t\bigr) = \sum_{i=1}^{9}\dot{V}_i\bigl(e(t),t\bigr), \tag{8}$$

where

$$\begin{aligned}
\dot{V}_1\bigl(e(t),t\bigr) &= 2\biggl(e(t) - A\int_{t-\delta}^{t} e(s)\,ds\biggr)^T \\
&\quad \times P_1\bigl(-Ae(t) + W_1f\bigl(e(t)\bigr) + W_2f\bigl(e\bigl(t-\tau(t)\bigr)\bigr) + W_3\dot{e}\bigl(t-h(t)\bigr)\bigr),
\end{aligned} \tag{9}$$

$$\dot{V}_2\bigl(e(t),t\bigr) = e^T(t)\bigl(P_2 + \delta^2 P_3\bigr)e(t) + e^T(t-\delta)(-P_2)e(t-\delta) - \delta \int_{t-\delta}^{t} e^T(s)P_3 e(s)\,ds. \tag{10}$$

By Lemma 2.1 we have

$$\begin{aligned}
\dot{V}_2\bigl(e(t),t\bigr) &\le e^T(t)\bigl(P_2 + \delta^2 P_3\bigr)e(t) + e^T(t-\delta)(-P_2)e(t-\delta) \\
&\quad - \biggl(\int_{t-\delta}^{t} e(s)\,ds\biggr)^T P_3 \biggl(\int_{t-\delta}^{t} e(s)\,ds\biggr), \qquad (11)\\
\dot{V}_3\bigl(e(t),t\bigr) &\le e^T(t)(Q_1 + Q_2 + Q_3)e(t) + e^T(t-\tau_1)(-Q_2 + Q_4)e(t-\tau_1) \\
&\quad + e^T(t-\tau_2)(-Q_3 - Q_4)e(t-\tau_2) \\
&\quad + e^T\bigl(t-\tau(t)\bigr)\bigl(-(1-\mu)Q_1\bigr)e\bigl(t-\tau(t)\bigr), \qquad (12)\\
\dot{V}_4\bigl(e(t),t\bigr) &\le \dot{e}^T(t)Q_5\dot{e}(t) + e^T\bigl(t-h(t)\bigr)\bigl(-(1-h_D)Q_5\bigr)e\bigl(t-h(t)\bigr), \qquad (13)\\
\dot{V}_5\bigl(e(t),t\bigr) &= \xi^T(t)\bigl(\tau_1^2 U + \tau_2^2 V + \tau_\rho D\bigr)\xi(t) \\
&\quad - \tau_2 \int_{t-\tau_1}^{t} \xi^T(s)U\xi(s)\,ds - \tau_2 \int_{t-\tau_2}^{t} \xi^T(s)V\xi(s)\,ds \\
&\quad - \tau_\rho \int_{t-\tau_2}^{t-\tau_1} \xi^T(s)D\xi(s)\,ds. \qquad (14)
\end{aligned}$$

Using Lemma 2.1, we get the following inequalities:

$$\begin{aligned}
-\tau_1 \int_{t-\tau_1}^{t} \xi^T(s)U\xi(s)\,ds &\le -\begin{bmatrix} \int_{t-\tau_1}^{t} e(s)\,ds \\ e(t) - e(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} U_{11} & U_{12} \\ \maltese & U_{22} \end{bmatrix} \begin{bmatrix} \int_{t-\tau_1}^{t} e(s)\,ds \\ e(t) - e(t-\tau_1) \end{bmatrix}, \qquad (15)\\
-\tau_2 \int_{t-\tau_2}^{t} \xi^T(s)V\xi(s)\,ds &\le -\begin{bmatrix} \int_{t-\tau_2}^{t} e(s)\,ds \\ e(t) - e(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} V_{11} & V_{12} \\ \maltese & V_{22} \end{bmatrix} \begin{bmatrix} \int_{t-\tau_2}^{t} e(s)\,ds \\ e(t) - e(t-\tau_2) \end{bmatrix}. \qquad (16)
\end{aligned}$$

Using Lemma 2.1 and Lemma 2.2, provided that (4) is satisfied, we can obtain

$$\begin{aligned}
-\tau_\rho \int_{t-\tau_2}^{t-\tau_1} \xi^T(s)D\xi(s)\,ds &\le -\tau_\rho \int_{t-\tau(t)}^{t-\tau_1} \xi^T(s)D\xi(s)\,ds - \tau_\rho \int_{t-\tau_2}^{t-\tau(t)} \xi^T(s)D\xi(s)\,ds \\
&\le -\begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_1} \xi(s)\,ds \\ \int_{t-\tau_2}^{t-\tau(t)} \xi(s)\,ds \end{bmatrix}^T \begin{bmatrix} D & E \\ \maltese & D \end{bmatrix} \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_1} \xi(s)\,ds \\ \int_{t-\tau_2}^{t-\tau(t)} \xi(s)\,ds \end{bmatrix}, \qquad (17)\\
\dot{V}_6\bigl(e(t),t\bigr) &= \dot{e}^T(t)\bigl(\tau_1^2 R_1 + \tau_2^2 R_2 + \tau_\rho^2 R_3\bigr)\dot{e}(t) - \tau_1 \int_{t-\tau_1}^{t} \dot{e}^T(s)R_1\dot{e}(s)\,ds \\
&\quad - \tau_2 \int_{t-\tau_2}^{t} \dot{e}^T(s)R_2\dot{e}(s)\,ds - \int_{t-\tau_2}^{t-\tau_1} \dot{e}^T(s)R_3\dot{e}(s)\,ds. \qquad (18)
\end{aligned}$$

To obtain new bounds for the integral terms in (18), we apply Lemma 2.3 and obtain

$$\begin{aligned}
-\tau_1 \int_{t-\tau_1}^{t} \dot{e}^T(s)R_1\dot{e}(s)\,ds &\le -\begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}^T \begin{bmatrix} R_1 & 0 \\ \maltese & 3R_1 \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}, \qquad (19)\\
-\tau_2 \int_{t-\tau_2}^{t} \dot{e}^T(s)R_2\dot{e}(s)\,ds &\le -\begin{bmatrix} \Psi_3 \\ \Psi_4 \end{bmatrix}^T \begin{bmatrix} R_2 & 0 \\ \maltese & 3R_2 \end{bmatrix} \begin{bmatrix} \Psi_3 \\ \Psi_4 \end{bmatrix}, \qquad (20)
\end{aligned}$$

where

$$\Psi_1 = e(t) - e(t-\tau_1), \qquad \Psi_2 = e(t) + e(t-\tau_1) - \frac{2}{\tau_1}\int_{t-\tau_1}^{t} e(s)\,ds,$$
$$\Psi_3 = e(t) - e(t-\tau_2), \qquad \Psi_4 = e(t) + e(t-\tau_2) - \frac{2}{\tau_2}\int_{t-\tau_2}^{t} e(s)\,ds.$$

Using Lemma 2.4 for the integral terms in (18), we have

$$\begin{aligned}
-\int_{t-\tau_2}^{t-\tau_1} \dot{e}^T(s)R_3\dot{e}(s)\,ds &\le \chi^T(t)\big[\big(\tau_2-\tau(t)\big)JR_3^{-1}J^T + \big(\tau(t)-\tau_1\big)KR_3^{-1}K^T \\
&\quad + [K \quad -K+J \quad -J] + [K \quad -K+J \quad -J]^T\big]\chi(t), \qquad (21)\\
\dot{V}_7\big(e(t),t\big) &= \dot{e}^T(t)(\tau_1\mathcal{R}_1 + \tau_\rho\mathcal{R}_2 + \tau_2\mathcal{R}_3)\dot{e}(t) \\
&\quad - \int_{t-\tau_1}^{t} \dot{e}^T(s)\mathcal{R}_1\dot{e}(s)\,ds - \int_{t-\tau(t)}^{t-\tau_1} \dot{e}^T(s)\mathcal{R}_2\dot{e}(s)\,ds \\
&\quad - \int_{t-\tau_2}^{t-\tau(t)} \dot{e}^T(s)\mathcal{R}_2\dot{e}(s)\,ds - \int_{t-\tau_2}^{t} \dot{e}^T(s)\mathcal{R}_3\dot{e}(s)\,ds, \qquad (22)\\
\dot{V}_8\big(e(t),t\big) &= \dot{e}^T(t)\bigg(\frac{\tau_1^4}{4}T_1 + \frac{\tau_2^4}{4}T_2 + \frac{(\tau_3^2-\tau_1^2)^2}{4}T_3\bigg)\dot{e}(t) - \frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)T_1\dot{e}(s)\,ds\,d\theta \\
&\quad - \frac{\tau_2^2}{2}\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)T_2\dot{e}(s)\,ds\,d\theta - \frac{\tau^2(t)-\tau_1^2}{2}\int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} \dot{e}^T(s)T_3\dot{e}(s)\,ds\,du \\
&\quad - \frac{\tau^2(t)-\tau_1^2}{2}\int_{-\tau_1}^{-\tau(t)}\int_{t+\theta}^{t} \dot{e}^T(s)T_3\dot{e}(s)\,ds\,d\theta. \qquad (23)
\end{aligned}$$

To obtain new bounds for the integral terms in (23), we apply Lemma 2.5 and obtain

$$-\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)T_1\dot{e}(s)\,ds\,d\theta \le -\begin{bmatrix}\Upsilon_1\\ \Upsilon_2\end{bmatrix}^T \begin{bmatrix}T_1 & 0\\ \maltese & 2T_1\end{bmatrix}\begin{bmatrix}\Upsilon_1\\ \Upsilon_2\end{bmatrix}, \qquad (24)$$
$$-\frac{\tau_2^2}{2}\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)T_2\dot{e}(s)\,ds\,d\theta \le -\begin{bmatrix}\Upsilon_3\\ \Upsilon_4\end{bmatrix}^T \begin{bmatrix}T_2 & 0\\ \maltese & 2T_2\end{bmatrix}\begin{bmatrix}\Upsilon_3\\ \Upsilon_4\end{bmatrix}, \qquad (25)$$
$$-\frac{\tau^2(t)-\tau_1^2}{2}\int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} \dot{e}^T(s)T_3\dot{e}(s)\,ds\,d\theta \le -\begin{bmatrix}\Upsilon_5\\ \Upsilon_6\end{bmatrix}^T \begin{bmatrix}T_3 & 0\\ \maltese & 2T_3\end{bmatrix}\begin{bmatrix}\Upsilon_5\\ \Upsilon_6\end{bmatrix}, \qquad (26)$$
$$-\frac{\tau^2(t)-\tau_2^2}{2}\int_{-\tau_2}^{-\tau(t)}\int_{t+\theta}^{t} \dot{e}^T(s)T_3\dot{e}(s)\,ds\,d\theta \le -\begin{bmatrix}\Upsilon_7\\ \Upsilon_8\end{bmatrix}^T \begin{bmatrix}T_3 & 0\\ \maltese & 2T_3\end{bmatrix}\begin{bmatrix}\Upsilon_7\\ \Upsilon_8\end{bmatrix}, \qquad (27)$$

where

$$\Upsilon_1 = \tau_1 e(t) - \int_{t-\tau_1}^{t} e(s)\,ds, \qquad \Upsilon_2 = \frac{\tau_1}{2}e(t) - \int_{t-\tau_1}^{t} e(s)\,ds + \frac{3}{\tau_1}\int_{-\tau_1}^{0}\int_{t+\theta}^{t} e(s)\,ds\,d\theta,$$
$$\Upsilon_3 = \tau_2 e(t) - \int_{t-\tau_2}^{t} e(s)\,ds, \qquad \Upsilon_4 = \frac{\tau_2}{2}e(t) - \int_{t-\tau_2}^{t} e(s)\,ds + \frac{3}{\tau_2}\int_{-\tau_2}^{0}\int_{t+\theta}^{t} e(s)\,ds\,d\theta,$$
$$\Upsilon_5 = \tau_\rho e(t) - \int_{t-\tau(t)}^{t-\tau_1} e(s)\,ds, \qquad \Upsilon_6 = \frac{\tau_\rho}{2}e(t) - \int_{t-\tau(t)}^{t-\tau_1} e(s)\,ds + \frac{3}{\tau_\rho}\int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} e(s)\,ds\,d\theta,$$

$$\Upsilon_7 = \tau_\rho e(t) - \int_{t-\tau_2}^{t-\tau(t)} e(s)\,ds, \qquad \Upsilon_8 = \frac{\tau_\rho}{2} e(t) - \int_{t-\tau_2}^{t-\tau(t)} e(s)\,ds + \frac{3}{\tau_\rho}\int_{-\tau_2}^{-\tau(t)}\int_{t+\theta}^{t} e(s)\,ds\,d\theta.$$

$$\begin{aligned}
\dot{V}_9\bigl(x(t),t\bigr) &= \dot{e}^T(t)\left(\left(\frac{\tau_1^3}{6}\right)^2 U_1 + \left(\frac{\tau_2^3}{6}\right)^2 U_2 + \left(\frac{\tau_2^3-\tau_1^3}{6}\right)^2 U_3\right)\dot{e}(t) \\
&\quad - \frac{\tau_1^3}{6}\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_1\dot{e}(s)\,ds\,du\,d\theta \\
&\quad - \frac{\tau_2^3}{6}\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_2\dot{e}(s)\,ds\,du\,d\theta \\
&\quad - \frac{\tau^3(t)-\tau_1^3}{6}\int_{-\tau(t)}^{-\tau_1}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_3\dot{e}(s)\,ds\,du\,d\theta \\
&\quad - \frac{\tau^3(t)-\tau_1^3}{6}\int_{-\tau_2}^{-\tau(t)}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_3\dot{e}(s)\,ds\,du\,d\theta.
\end{aligned} \tag{28}$$

Using Lemma 2.1, we can rewrite the integral terms in (28) as

$$\begin{aligned}
&-\frac{\tau_1^3}{6}\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_1\dot{e}(s)\,ds\,du\,d\theta \\
&\quad \le -\left(\frac{\tau_1^2}{2}e(t) - \int_{-\tau_1}^{0}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right)^T U_1\left(\frac{\tau_2^2}{2}e(t) - \int_{-\tau_1}^{0}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right), && (29)\\
&-\frac{\tau_2^3}{6}\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_2\dot{e}(s)\,ds\,du\,d\theta \\
&\quad \le -\left(\frac{\tau_2^2}{2}e(t) - \int_{-\tau_2}^{0}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right)^T U_2\left(\frac{\tau_3^2}{2}e(t) - \int_{-\tau_2}^{0}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right), && (30)\\
&-\frac{(\tau^3(t)-\tau_1^3)}{6}\int_{-\tau(t)}^{-\tau_1}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_3\dot{e}(s)\,ds\,du\,d\theta \\
&\quad \le -\left(\left(\frac{\tau_2^2-\tau_1^2}{2}\right)e(t) - \int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right)^T \\
&\qquad \times U_3\left(\left(\frac{\tau_2^2-\tau_1^2}{2}\right)e(t) - \int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right), && (31)\\
&-\frac{(\tau^3(t)-\tau_1^3)}{6}\int_{-\tau_2}^{-\tau(t)}\int_{\theta}^{0}\int_{t+u}^{t} \dot{e}^T(s)U_3\dot{e}(s)\,ds\,du\,d\theta \\
&\quad \le -\left(\left(\frac{\tau_2^2-\tau_1^2}{2}\right)e(t) - \int_{-\tau_2}^{-\tau(t)}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right)^T \\
&\qquad \times U_3\left(\left(\frac{\tau_2^2-\tau_1^2}{2}\right)e(t) - \int_{-\tau_2}^{-\tau(t)}\int_{t+\theta}^{t} e(s)\,ds\,d\theta\right). && (32)
\end{aligned}$$

Furthermore, for any arbitrary matrices G_i $(i = 1, 2, \ldots, 6)$ of compatible dimensions, we have

$$0 = 2\eta_1^T(t)G\bigl[-\dot{e}(t) - Ae(t-\delta) + W_1 f\bigl(e(t)\bigr) + W_2 f\bigl(e\bigl(t-\tau(t)\bigr)\bigr) + W_3\dot{e}\bigl(t-h(t)\bigr)\bigr]. \tag{33}$$

By the Leibniz–Newton formula, the following equations hold for any matrices L_i, M_i, N_i, O_i $(i = 1, 2, \ldots, 4)$ of compatible dimensions:

$$0 = 2\eta_2^T(t)\widehat{L}\left[e(t) - e(t-\tau_1) - \int_{t-\tau_1}^{t} \dot{e}(s)\,ds\right], \tag{34}$$

$$0 = 2\eta_2^T(t)\widehat{M}\left[e(t-\tau_1) - e\bigl(t-\tau(t)\bigr) - \int_{t-\tau(t)}^{t-\tau_1} \dot{e}(s)\,ds\right], \tag{35}$$

$$0 = 2\eta_2^T(t)\widehat{N}\left[e\bigl(t-\tau(t)\bigr) - e(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)} \dot{e}(s)\,ds\right], \tag{36}$$

$$0 = 2\eta_2^T(t)\widehat{O}\left[e(t) - e(t-\tau_2) - \int_{t-\tau_2}^{t} \dot{e}(s)\,ds\right]. \tag{37}$$

By Assumption (H) we can be obtain the following inequalities for any $X, Y, Z \geq 0$:

$$\eta_3^T(t)\Delta_1\eta_3(t) \geq 0, \qquad \eta_3^T\bigl(t-\tau(t)\bigr)\Delta_2\eta_3\bigl(t-\tau(t)\bigr) \geq 0, \qquad \eta_4^T(t)\Delta_3\eta_4(t) \geq 0, \tag{38}$$

where

$$\begin{aligned}
&\eta_1(t) = \begin{bmatrix}\dot{e}^T(t) & e^T(t) & e^T(t-\delta) & e^T(t-\tau_1) & e^T(t-\tau(t)) & e^T(t-\tau_2)\end{bmatrix}, \\
&\eta_2(t) = \begin{bmatrix}e^T(t) & e^T(t-\tau_1) & e^T(t-\tau(t)) & e^T(t-\tau_2)\end{bmatrix}, \\
&\eta_3(t) = \begin{bmatrix}e^T(t) & f^T(e(t))\end{bmatrix}, \qquad \eta_4(t) = \begin{bmatrix}\eta_3^T(t) & \eta_3^T(t-\tau(t))\end{bmatrix}, \\
&\Delta_1 = \begin{bmatrix}-F_1X & F_2X \\ \maltese & -X\end{bmatrix}, \qquad \Delta_2 = \begin{bmatrix}-F_1Y & F_2Y \\ \maltese & -Y\end{bmatrix}, \\
&\Delta_3 = \begin{bmatrix}-F_1Z & F_2Z & F_1Z & -F_2Z \\ \maltese & -Z & -F_2Z & Z \\ \maltese & \maltese & -F_1Z & F_2Z \\ \maltese & \maltese & \maltese & -Z\end{bmatrix}, \\
&\widehat{L} = \begin{bmatrix}L_1 & 0 & L_2 & L_3 & L_4 & \underbrace{0\ldots\ldots\ldots\ldots0}_{12\text{ times}}\end{bmatrix}, \\
&\widehat{M} = \begin{bmatrix}M_1 & 0 & M_2 & M_3 & M_4 & \underbrace{0\ldots\ldots\ldots\ldots0}_{12\text{ times}}\end{bmatrix}, \\
&\widehat{N} = \begin{bmatrix}N_1 & 0 & N_2 & N_3 & N_4 & \underbrace{0\ldots\ldots\ldots\ldots0}_{12\text{ times}}\end{bmatrix}, \\
&\widehat{O} = \begin{bmatrix}O_1 & 0 & O_2 & O_3 & O_4 & \underbrace{0\ldots\ldots\ldots\ldots0}_{12\text{ times}}\end{bmatrix}, \\
&X = \operatorname{diag}\{x_{11}, x_{12}, \ldots, x_{1n}\}, \qquad Y = \operatorname{diag}\{y_{21}, y_{22}, \ldots, y_{2n}\},
\end{aligned}$$

and

$$Z = \operatorname{diag}\{z_{31}, z_{32}, \ldots, z_{3n}\}.$$

By combining from (9) to (38) we get

$$\dot{V}\bigl(e(t), t\bigr) = -\zeta^T(t)\Pi^*\zeta(t), \tag{39}$$

where $\Pi^* = -\Gamma > 0$ with $\Gamma = \Pi + (\tau_2 - \tau(t))JR_3^{-1}J^T + (\tau(t) - \tau_1)KR_3^{-1}K^T$, and we define the augmented state vector

$$\begin{aligned}\zeta^T(t) = \Big[&e^T(t) \quad e^T(t-\delta) \quad e^T(t-\tau_1) \quad e^T\big(t-\tau(t)\big) \quad e^T(t-\tau_2) \\ &f^T\big(e(t)\big) \quad f^T\big(e\big(t-\tau(t)\big)\big) \quad \textstyle\int_{t-\tau_1}^{t} e(s)\,ds \quad \int_{t-\tau(t)}^{t-\tau_1} e^T(s)\,ds \\ &\textstyle\int_{t-\tau_2}^{t-\tau(t)} e^T(s)\,ds \quad \int_{t-\tau_2}^{t} e^T(s)\,ds \quad \int_{-\tau_1}^{0}\int_{t+\theta}^{t} e^T(s)\,ds\,d\theta \\ &\textstyle\int_{-\tau(t)}^{-\tau_1}\int_{t+\theta}^{t} e^T(s)\,ds\,d\theta \quad \int_{-\tau_2}^{-\tau(t)}\int_{t+\theta}^{t} e^T(s)\,ds\,d\theta \quad \int_{-\tau_2}^{0}\int_{t+\theta}^{t} e^T(s)\,ds\,d\theta \\ &\textstyle\int_{t-\delta}^{t} e^T(s)\,ds \quad \dot{e}^T(t) \quad \dot{e}^T\big(t-h(t)\big)\Big].\end{aligned} \tag{40}$$

Thus, we can deduce that

$$V\big(e(t),t\big) + \int_0^t \zeta^T(s)\Pi^*\zeta(s)\,ds \le V\big(e(0),0\big), \quad t \ge 0. \tag{41}$$

Moreover,

$$\begin{aligned}V\big(e(0),0\big) \le \Big\{&2\lambda_{\max}(P_1)\Big(1 + \delta^2 \max_{i\in\Lambda}\Big) + \delta\lambda_{\max}(P_2) + \delta^3\lambda_{\max}(P_3) \\ &+ \tau_2\lambda_{\max}(Q_1) + \tau_1\lambda_{\max}(Q_2) + \tau_2\lambda_{\max}(Q_3) + \tau_\rho\lambda_{\max}(Q_4) + h\lambda_{\max}(Q_5) \\ &+ \tau_1^3\lambda_{\max}(U) + \tau_2^3\lambda_{\max}(V) + \tau_\rho^3\lambda_{\max}(D) + \tau_1^3\lambda_{\max}(R_1) + \tau_2^3\lambda_{\max}(R_2) \\ &+ \tau_\rho^2\lambda_{\max}(R_3) + \tau_1^2\lambda_{\max}(\mathcal{R}_1) + \tau_\rho^2\lambda_{\max}(\mathcal{R}_2) + \tau_2^2\lambda_{\max}(\mathcal{R}_3) \\ &+ \frac{\tau_1^3}{2}\lambda_{\max}(T_1) + \frac{\tau_2^3}{2}\lambda_{\max}(T_2) + \left(\frac{\tau_2^3 - \tau_1^3}{2}\right)\lambda_{\max}(T_3) \\ &+ \frac{\tau_1^4}{6}\lambda_{\max}(U_1) + \frac{\tau_2^4}{6}\lambda_{\max}(U_2) + \left(\frac{\tau_2^4 - \tau_1^4}{6}\right)\lambda_{\max}(U_3)\Big\}\|\phi\|_{\bar{r}}^2 \\ < \infty,&\end{aligned} \tag{42}$$

and the norm is defined by $\|\phi\|_r = \max\{\sup_{\bar{r}\le s\le 0}\|\phi(s)\|, \sup_{-\bar{r}\le s\le 0}\|\dot{\phi}(s)\|\}$. From the definition of $V_2(e(t),t)$ and Lemma 2.1 we know that

$$\begin{aligned}\left\|\int_{t-\delta}^{t} e(s)\,ds\right\|^2 &= \left[\int_{t-\delta}^{t} e(s)\,ds\right]^T\left[\int_{t-\delta}^{t} e(s)\,ds\right] \\ &\le \delta\int_{t-\delta}^{t} e^T(s)e(s)\,ds \le \frac{\delta}{\lambda_{\min}(P_2)}\int_{t-\delta}^{t} e^T(s)e(s)\,ds \\ &\le \frac{\delta}{\lambda_{\min}(P_2)}V\big(e(t),t\big) \le \frac{\delta}{\lambda_{\min}(P_2)}V\big(e(0),0\big),\end{aligned}$$

which, together with the definition of $V_1(e(t),t)$, yields

$$\begin{aligned}\|e(t)\| &\le \left\|\int_{t-\delta}^{t} e(s)\,ds\right\| + \sqrt{\frac{V_1(e(t),t)}{\lambda_{\min}(P_2)}} \le \left\|\int_{t-\delta}^{t} e(s)\,ds\right\| + \sqrt{\frac{V_1(e(0),0)}{\lambda_{\min}(P_2)}} \\ &\le \left\{\sqrt{\sum_{i=1}^{n} a_i\frac{\delta}{\lambda_{\min}(P_2)}} + \sqrt{\frac{1}{\lambda_{\min}(P_2)}}\right\}\sqrt{V\big(e(0),0\big)}.\end{aligned}$$

This implies that the equilibrium point of model (1) is locally stable. Therefore, along the same proof with [42], we will prove that $\|e(t)\| \to 0$ as $t \to \infty$. First, for any constant $\theta \in [0,1]$, from (40), (41), and Lemma 2.1 it follows that

$$\begin{aligned}\|e(t+\theta)-e(t)\|^2 &= \left[\int_t^{t+\theta} \dot{e}(s)\,ds\right]^T \left[\int_t^{t+\theta} \dot{e}(s)\,ds\right] \\ &\le \theta \int_t^{t+\theta} \dot{e}^T(s)\dot{e}(s)\,ds \le \int_t^{t+\theta} \dot{e}^T(s)\dot{e}(s)\,ds \\ &\le \frac{1}{\lambda_{\min}(\Pi^*)} \int_t^{t+1} \zeta^T(s)\Pi^*\zeta(s)\,ds \to 0 \quad \text{as } t \to \infty,\end{aligned}$$

which implies that, for any $\epsilon > 0$, there exists $\mathcal{K}_1 = \mathcal{K}_1(\epsilon) > 0$ such that

$$\|e(t+\theta)-e(t)\| < \frac{\epsilon}{2}, \quad \theta \in [0,1]. \tag{43}$$

Otherwise, from (41) we get

$$\begin{aligned}\left\|\int_t^{t+1} e(s)\,ds\right\|^2 &= \left[\int_t^{t+\theta} e(s)\,ds\right]^T \left[\int_t^{t+\theta} e(s)\,ds\right] \\ &\le \int_t^{t+1} e^T(s)e(s)\,ds \\ &\le \frac{1}{\lambda_{\min}(\Pi^*)} \int_t^{t+1} \zeta^T(s)\Pi^*\zeta(s)\,ds \to 0 \quad \text{as } t \to \infty,\end{aligned}$$

which implies that, for any $\epsilon > 0$, there exists $\mathcal{K}_2 = \mathcal{K}_2(\epsilon) > 0$ such that

$$\left\|\int_t^{t+1} e(s)\,ds\right\| < \frac{\epsilon}{2}, \quad t > \mathcal{K}_2.$$

It should be noted that $e(s)$ is continuous on $[t, t+1]$, $t > 0$. Using the integral mean value theorem, we get that there exists a vector $\vartheta_t = (\vartheta_{t1}, \dots, \vartheta_{tn})^T \in \mathbb{R}^n$, $\vartheta_{tj} \in [t, t+1]$ $(j = 1, \dots, n)$, such that

$$\|e(\vartheta_t)\| = \left\|\int_t^{t+1} e(s)\,ds\right\| < \frac{\epsilon}{2}, \quad t > \mathcal{K}_2. \tag{44}$$

By (43) and (44) we obtain that, for any $\epsilon > 0$, there exists $\mathcal{K} = \max\{\mathcal{K}_1, \mathcal{K}_2\} > 0$ such that $t > \mathcal{K}$ implies

$$\|e(t)\| \le \|e(t)-e(\vartheta_t)\| + \|e(\vartheta(_t))\| \le \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

Therefore, we conclude that model (1) has a unique equilibrium point, which is asymptotically stable. From (39) we have

$$\dot{V}(e(t),t) \le \zeta^T(t)\{\Pi + (\tau_2 - \tau(t))JR_3^{-1}J^T + (\tau(t) - \tau_1)KR_3^{-1}K^T\}\zeta(t) < 0.$$

We can see that if $\tau(t) \in [\tau_1, \tau_2]$, then

$$\Pi + (\tau_2 - \tau(t))JR_3^{-1}J^T + (\tau(t) - \tau_1)KR_3^{-1}K^T < 0. \tag{45}$$

Then by Lemma 2.6 we get the following inequalities:

$$\Pi + (\tau_2 - \tau_1) J R_3^{-1} J^T < 0, \tag{46}$$

$$\Pi + (\tau_2 - \tau_1) K R_3^{-1} K^T < 0. \tag{47}$$

By Schur complement [43] on (46) and (47) we can obtain (5) and (6). This completes the proof. □

Remark 3.1 When $W_3 = 0$, system (1) reduces to the following NN:

$$\begin{aligned} &\dot{e}(t) = -Ae(t-\delta) + W_1 f\bigl(e(t)\bigr) + W_2 f\bigl(e\bigl(t-\tau(t)\bigr)\bigr), \\ &e(t) = \phi(t), \quad t \in \bigl[-\max\{\delta, \tau_M\}, 0\bigr]. \end{aligned} \tag{48}$$

For the deterministic delayed NN (48), we can derive the stability conditions immediately from Theorem 3.1. The details are omitted here, since they are implied by Theorem 3.1 by setting $W_3 = Q_5 = 0$ and removing the 18th row and column in Theorem 3.1. So we have the following corollary.

Corollary 3.1 *Assume that Assumption* (H) *is holds. For given positive scalars* δ, τ_1, τ_2, *and* μ, *the NN described by* (48) *is asymptotically stable for any time-varying delay* $\tau(t)$ *satisfying* (2) *if there exist symmetric positive definite matrices* P_i $(i = 1, 2, 3)$, Q_i $(i = 1, 2, 3, \ldots, 4)$,

$$U = \begin{bmatrix} U_{11} & U_{12} \\ \maltese & U_{22} \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ \maltese & V_{22} \end{bmatrix}, \qquad D = \begin{bmatrix} D_{11} & D_{12} \\ \maltese & D_{22} \end{bmatrix},$$

R_i $(i = 1, 2, 3)$, $\mathcal{R}_i$ $(i = 1, 2, 3)$, T_i $(i = 1, 2, 3)$, U_i $(i = 1, 2, 3)$, *diagonal matrices* X, Y, *and* Z, *any matrices* G_i $(i = 1, 2, \ldots, 6)$ *of appropriate dimensions, and any appropriate dimensional matrices* L_i, M_i, N_i, O_i $(i = 1, 2, \ldots, 4)$ *such that the following LMIs hold*:

$$\begin{bmatrix} D & E \\ \maltese & D \end{bmatrix} \geq 0, \tag{49}$$

$$\begin{bmatrix} \Pi & \sqrt{\tau_1} L & \sqrt{\tau_\rho} M & \sqrt{\tau_\rho} N & \sqrt{\tau_2} O & \tau_\rho \bar{K} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3 \tau_\rho \end{bmatrix} < 0, \tag{50}$$

$$\begin{bmatrix} \Pi & \sqrt{\tau_1} L & \sqrt{\tau_\rho} M & \sqrt{\tau_\rho} N & \sqrt{\tau_2} O & \tau_\rho \bar{J} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3 \tau_\rho \end{bmatrix} < 0, \tag{51}$$

where $\Pi = (\Pi_{i,j})_{17\times 17}$ *and terms are the same as in Theorem* 3.1.

Remark 3.2 When $\delta = 0$, system (1) reduces to the following NN:

$$\begin{aligned} &\dot{e}(t) = -Ae(t) + W_1 f\big(e(t)\big) + W_2 f\big(e\big(t-\tau(t)\big)\big) + W_3\dot{e}\big(t-h(t)\big), \\ &e(t) = \phi(t), \quad t \in \big[-\max\{\tau_M, h\}, 0\big]. \end{aligned} \tag{52}$$

For the deterministic delayed NN (52), we can derive the stability conditions immediately from Theorem 3.1. The details are omitted here, since they are implied by Theorem 3.1 by setting in (7) $P_2 = P_3 = 0$, removing the 2nd and 16th columns and rows in Theorem 3.1. So we have the following corollary.

Corollary 3.2 *Assume that Assumption* (H) *is holds. Then for given positive scalars* τ_1, τ_2, μ, h, *and* h_D, *the NN described by* (52) *is asymptotically stable for any time-varying delay* $\tau(t)$ *and* $h(t)$ *satisfying* (2) *if there exist symmetric positive definite matrices* P_1, Q_i $(i = 1, 2, 3, \dots, 5)$,

$$U = \begin{bmatrix} U_{11} & U_{12} \\ \maltese & U_{22} \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ \maltese & V_{22} \end{bmatrix}, \qquad D = \begin{bmatrix} D_{11} & D_{12} \\ \maltese & D_{22} \end{bmatrix},$$

R_i $(i = 1, 2, 3)$, $\mathcal{R}_i$ $(i = 1, 2, 3)$, T_i $(i = 1, 2, 3)$, U_i $(i = 1, 2, 3)$, *diagonal matrices* X, Y, *and* Z, *any matrices* G_i $(i = 1, 3, \dots, 6)$ *of appropriate dimensions, and any appropriate dimensional matrices* L_i, M_i, N_i, O_i $(i = 1, 2, \dots, 4)$ *such that the following LMIs hold*:

$$\begin{bmatrix} D & E \\ \maltese & D \end{bmatrix} \geq 0, \tag{53}$$

$$\begin{bmatrix} \widetilde{\Pi} & \sqrt{\tau_1}L & \sqrt{\tau_\rho}M & \sqrt{\tau_\rho}N & \sqrt{\tau_2}O & \tau_\rho\bar{K} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3\tau_\rho \end{bmatrix} < 0, \tag{54}$$

$$\begin{bmatrix} \widetilde{\Pi} & \sqrt{\tau_1}L & \sqrt{\tau_\rho}M & \sqrt{\tau_\rho}N & \sqrt{\tau_2}O & \tau_\rho\bar{J} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3\tau_\rho \end{bmatrix} < 0, \tag{55}$$

where $\widetilde{\Pi} = (\widetilde{\Pi}_{i,j})_{16\times 16}$ *and the other terms are the same as in Theorem* 3.1.

Remark 3.3 When $\delta = 0$ and $W_3 = 0$, system (1) reduces to the following NN (56). However, for the general NN, we have achieved some results in [8–13, 16]. We have

$$\begin{aligned} &\dot{e}(t) = -Ae(t) + W_1 f\big(e(t)\big) + W_2 f\big(e\big(t-\tau(t)\big)\big), \\ &e(t) = \phi(t), \quad t \in \big[-\max\{\tau_M\}, 0\big]. \end{aligned} \tag{56}$$

For the deterministic delayed NN (56), we can derive the stability conditions immediately from Corollary 3.1. The details are omitted here, since they are implied by Corollary 3.1, by removing the 2nd and 16th columns and rows of LMIs in Corollary 3.1. So we have the following corollary.

Corollary 3.3 *Assume that Assumption* (H) *is holds. Then for given positive scalars* τ_1, τ_2, *and* μ, *the NN described by* (56) *is asymptotically stable for any time-varying delay* $\tau(t)$ *satisfying* (2) *if there exist symmetric positive definite matrices* P_1, Q_i $(i = 1,2,3,\ldots,4)$,

$$U = \begin{bmatrix} U_{11} & U_{12} \\ \maltese & U_{22} \end{bmatrix}, \qquad V = \begin{bmatrix} V_{11} & V_{12} \\ \maltese & V_{22} \end{bmatrix}, \qquad D = \begin{bmatrix} D_{11} & D_{12} \\ \maltese & D_{22} \end{bmatrix},$$

R_i $(i = 1,2,3)$, $\mathcal{R}_i$ $(i = 1,2,3)$, T_i $(i = 1,2,3)$, U_i $(i = 1,2,3)$, *diagonal matrices* X, Y, *and* Z, *any matrices* G_i $(i = 1,3,\ldots,6)$ *of appropriate dimensions, and any appropriate dimensional matrices* L_i, M_i, N_i, O_i $(i = 1,2,\ldots,4)$ *such that the following LMIs hold*:

$$\begin{bmatrix} D & E \\ \maltese & D \end{bmatrix} \geq 0, \tag{57}$$

$$\begin{bmatrix} \widehat{\widehat{\Pi}} & \sqrt{\tau_1}L & \sqrt{\tau_\rho}M & \sqrt{\tau_\rho}N & \sqrt{\tau_2}O & \tau_\rho\bar{K} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3\tau_\rho \end{bmatrix} < 0, \tag{58}$$

$$\begin{bmatrix} \widehat{\widehat{\Pi}} & \sqrt{\tau_1}L & \sqrt{\tau_\rho}M & \sqrt{\tau_\rho}N & \sqrt{\tau_2}O & \tau_\rho\bar{J} \\ \maltese & -\mathcal{R}_1 & 0 & 0 & 0 & 0 \\ \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 & 0 \\ \maltese & \maltese & \maltese & -\mathcal{R}_2 & 0 & 0 \\ \maltese & \maltese & \maltese & \maltese & -\mathcal{R}_3 & 0 \\ \maltese & \maltese & \maltese & \maltese & \maltese & -R_3\tau_\rho \end{bmatrix} < 0, \tag{59}$$

where $\widehat{\widehat{\Pi}} = (\widehat{\widehat{\Pi}}_{i,j})_{15\times 15}$ *and the terms are the same as in Corollary* 3.1.

Remark 3.4 It is highly pointed out that, in the previous literature [8–13, 16, 19, 20], the condition $F_j^- \leq \frac{f_j(\alpha)}{\alpha} \leq F_j^+$ with $\alpha_2 = 0$, which is a particular case of $F_j^- \leq \frac{f_j(\alpha_1)-f_j(\alpha_2)}{\alpha_1-\alpha_2} \leq F_j^+$, $j = 1,2,\ldots,n$, was usually employed to reduce conservatism. It is worth mentioning that the assumption used in this paper not only considers the terms $F_j^- \leq \frac{f_j(e(t))}{e(t)} \leq F_j^+$ and $F_j^- \leq \frac{f_j(e(t-\tau(t)))}{e(t-\tau(t))} \leq F_j^+$, but also the term $F_j^- \leq \frac{f_j(e(t))-f_j(e(t-\tau(t)))}{f(e(t))-f_j(e(t-\tau(t)))} \leq F_j^+$ has been taken into account in (38), which is helpful to reduce conservatism.

Remark 3.5 So far, in the proof of Theorem 3.1, we have utilized WSII to estimate the derivatives of the LKFs such as $\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)\,ds\,d\theta$, $\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)\,ds\,d\theta$, and $\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)\,ds\,d\theta$. This technique was initially developed by [38], and it was shown to be more tighter than those used in [8–13, 16, 19, 20].

Remark 3.6 Recently, WDII technique was proposed by [40] to reduce conservatism effectively for time-delay systems. Motivated by this study, the developed method of [40] is applied in this paper. Therefore, this may lead to less conservative results.

Remark 3.7 Recently, most of the existing results concerning the stability of delayed NNs of neutral type have not considered time delay in the leakage term. In contrast to the models studied in [8–13, 16, 19, 20], without or with constant delay in the leakage term, we can find that their results cannot be applicable to system (1).

Remark 3.8 In recent years, obtaining better results for time-delay systems has been still ongoing research topic, and it has been received more attention from the researchers. For this purpose, constructing better LKFs and estimating their derivatives by an improved integral inequality technique is the main challenge in the field of time-delay systems. The stability criteria proposed in this paper are obtained from the constructions of several Lyapunov functionals, and, as a result, the proposed LMI conditions are also more complicated and of high computational complexity. To overcome these requirements, recently, some new works have been developed with another type of Lyapunov functions, which is more effective to reduce the computational burden while maintaining less conservative results (see [44, 45]). In the future work, we use this type of new Lyapunov functionals to reduce conservatism together with simple LMI conditions. Therefore, the methods proposed in [44, 45] will be more helpful to reduce the conservatism and computational burden of the time-delay systems with novel Lyapunov functionals.

4 Numerical examples

This section is dedicated to displaying interesting numerical examples to illustrate the effectiveness and applicability of the developed method for the addressed NNs.

Example 4.1 Consider the following NNs of neutral type with time-varying delays. The parametric coefficients are given by

$$\dot{e}(t) = -Ae(t) + W_1 f\bigl(e(t)\bigr) + W_2 f\bigl(e\bigl(t-\tau(t)\bigr)\bigr) + W_3\dot{e}\bigl(t-h(t)\bigr),$$

where

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} \alpha & 0.3 \\ 0.3 & 0.5 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \qquad W_3 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix}.$$

The activation functions are taken as $f_1(e) = f_2(e) = \tanh(e)$. We can verify that Assumption (H) is satisfied with $F_1^- = 0$, $F_1^+ = 1$, $F_2^- = 0$, $F_2^+ = 1$. Thus, $F_1 = \operatorname{diag}(0,0)$, $F_2 = \operatorname{diag}(0.5,0.5)$. For $\mu = 0.9$, $\tau_1 = 0.5$, $\tau_2 = 2.0$, solving LMIs in Corollary 3.2, we obtain that the upper bound of α is 3.94. We apply criteria in [19, 20], and in this work, the maximum value of α for the stability of NN (52) is listed in Table 1. It is easy to see that the proposed stability criterion is much less conservative than that in [19, 20]. Also, it should be highly pointed out that the aforementioned methods and results are demands and not applicable for the cases of $h_D = 0.8$, $h_D \geq 1$, which indicates the merits of the results obtained in this paper.

Table 1 The maximum upper bound of α when $\tau_m = 0.5$, $\tau_M = 2.0$ in Example 4.1

Methods	α ($h_D = 0.5$)	α ($h_D = 0.8$)	α ($h_D \geq 1$)
[19]	1.65	–	–
[20]	2.66	–	–
Corollary 3.2	3.94	3.52	3.43

Table 2 The MADBs of τ_M when $\tau_m = 1$ for different μ in Example 4.2

Methods	$\mu = 0.8$	$\mu = 0.9$	$\mu \geq 1$
[9]	2.5967	2.0443	1.9621
[16] ($m = 2$)	3.8359	2.9234	2.7532
[13] ($m = 2$)	4.0414	3.0250	2.8573
[11]	4.8688	3.8047	3.6001
[12], Theorem 1	5.8868	4.1791	3.9230
[12], Theorem 2	6.1527	4.3379	4.0518
Corollary 3.3	7.2376	5.7526	5.2247

Example 4.2 Consider the following NNs with interval time-varying delays:

$$\dot{e}(t) = -Ae(t) + W_1 f\big(e(t)\big) + W_2 f\big(e\big(t - \tau(t)\big)\big),$$

where

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}.$$

With these parameters, we solve the LMIs in Corollary 3.3. Our aim is to obtain the maximum allowable delay bounds (MADBs) of τ_2 when $\tau_1 = 1$ with different μ for such system (56) to be asymptotically stable. Comparing the results listed in Table 2 with those in the recent literature, we crisply see that the proposed results have less conservatism in this example. Therefore, the resulting stability criterion in this paper is truly much better than those of [9, 11–13, 16].

Example 4.3 Consider the following NN with interval time-varying delays:

$$\dot{e}(t) = -Ae(t) + W_1 f\big(e(t)\big) + W_2 f\big(e\big(t - \tau(t)\big)\big),$$

where

$$A = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.9230 & 0 \\ 0 & 0 & 0 & 0.4480 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},$$

Table 3 The MADBs of τ_2 with $\tau_1 = 3$ for different μ in Example 4.3

Methods	$\mu = 0.1$	$\mu = 0.5$	$\mu = 0.9$	$\mu \geq 1$
[8] ($m = 2$)	3.65	3.32	3.26	3.24
[8] ($m = 4$)	3.71	3.36	3.29	3.28
[10] ($m = 2$)	3.78	3.45	3.39	3.38
[11]	4.1967	3.6246	3.5961	3.5952
[12], Theorem 1	4.3351	3.7706	3.7323	3.7211
[12], Theorem 2	4.3361	3.7723	3.7329	3.7212
Corollary 3.3	5.4735	4.5487	4.4882	4.4531

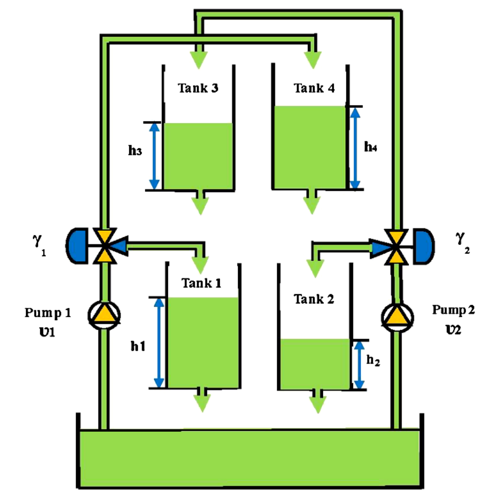

Figure 1 Schematic representation of the QTPS. Source: [46]

$$W_2 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0.1137 & 0 & 0 & 0 \\ 0 & 0.1279 & 0 & 0 \\ 0 & 0 & 0.7994 & 0 \\ 0 & 0 & 0 & 0.2368 \end{bmatrix}.$$

With $F_1 = \operatorname{diag}\{0, 0, 0, 0\}$, by solving LMIs in Corollary 3.3, the purpose of this example is to calculate the MADBs of τ_2 such that the considered delayed system (56) is asymptotically stable for given τ_1 and μ. The comparison of results derived in this paper and those obtained in [8, 10–12] are listed in Table 3. It is clear that the upper bounds τ_2 of the proposed criteria are larger than in those results, which implies that our results are less conservative than those of [8, 10–12]. Furthermore, our proposed method is less conservative than that in [8, 10], and the reduction in the conservatism of the developed method mainly comes from the use of the triple integral term $V_7(e(t), t)$ and augment term $V_5(e(t), t)$ in the LKFs.

Example 4.4 Artificial NNs can be expressed in terms of real biological neurons that are functionally associated with a nervous system. On the other hand, NNs can express not only biological neurons but also other practical system, namely, the quadruple-tank process system (QTPS) as shown in Figure 1.

The setup consists of four interacting tanks, two water pumps and two valves. The two process inputs are the voltages υ_1 and υ_2 supplied to the two pumps. Tank 1 and tank 2 are placed below tank 3 and tank 4 to receive water flow by the action of gravity. As shown in Figure 1, the QTPS can be expressed clearly using the NN model (see, e.g., [31, 33, 34, 46, 47]). The differential equation for the mass balances in the QTPS can be expressed as follows:

$$\dot{\widehat{x}}(t) = \widehat{A_0}\widehat{x}(t) + \widehat{A_1}\widehat{x}(t - \widetilde{\tau}_1) + \widehat{B_0}\widehat{u}(t - \widetilde{\tau}_2) + \widehat{B_1}\widehat{u}(t - \widetilde{\tau}_3), \tag{60}$$

where

$$\widehat{A_0} = \begin{bmatrix} -0.0021 & 0 & 0 & 0 \\ 0 & -0.0021 & 0 & 0 \\ 0 & 0 & -0.0424 & 0 \\ 0 & 0 & 0 & -0.0424 \end{bmatrix},$$

$$\widehat{A_1} = \begin{bmatrix} 0 & 0 & 0.0424 & 0 \\ 0 & 0 & 0 & 0.0424 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{B_0} = \begin{bmatrix} 0.1113\gamma_1 & 0 & 0 & 0 \\ 0 & 0.1042\gamma_2 & 0 & 0 \end{bmatrix}^T,$$

$$\widehat{B_1} = \begin{bmatrix} 0 & 0 & 0 & 0.1113(1-\gamma_1) \\ 0 & 0 & 0.1042(1-\gamma_2) & 0 \end{bmatrix}^T,$$

$$\gamma_1 = 0.333, \qquad \gamma_2 = 0.307,$$

$$\widehat{u} = \widehat{K}x(t), \quad \widehat{K} = \begin{bmatrix} -0.1609 & -0.1765 & -0.0795 & -0.2073 \\ -0.1977 & -0.1579 & -0.2288 & -0.0772 \end{bmatrix}.$$

The differential equations describing the mass balances in the delayed equations. Transport delays are usually included with the delay phenomena in the tank water inlets. Additionally, the transport delays between tanks and valves vary with respect to time. To develop a more interesting realistic problem, leakage delay can be easily added in QTPS by the inlet of water to the tanks and are meaningful in our practical life. However, until now, this has not been explored in the previous literature (see, e.g., [31, 33, 34, 46, 47]). Moreover, in this example, transport delays between valves and tanks being interval time-varying, the following aspects are also taken into account. For clarity, we assumed that $\widetilde{\tau}_1 = 0$, $\widetilde{\tau}_2 = 0$, and $\widetilde{\tau}_3 = \tau(t)$ (since $\tau_1 \leq \tau(t) \leq \tau_2$). Here, the control input $\widehat{u}(t)$ means the amount of water supplied by pumps. Therefore, it is obvious that $\widehat{u}(t)$ has a threshold value due to the constrained area of the hose and the capacity of the pumps. Therefore, it is reasonable to consider $\widehat{u}(t)$ as a nonlinear function:

$$\widehat{u}(t) = \widehat{Kf}\big(\widehat{e}(t)\big),$$
$$\widehat{u}\big(t - \tau(t)\big) = \widehat{Kf}\big(\widehat{e}\big(t - \tau(t)\big)\big),$$

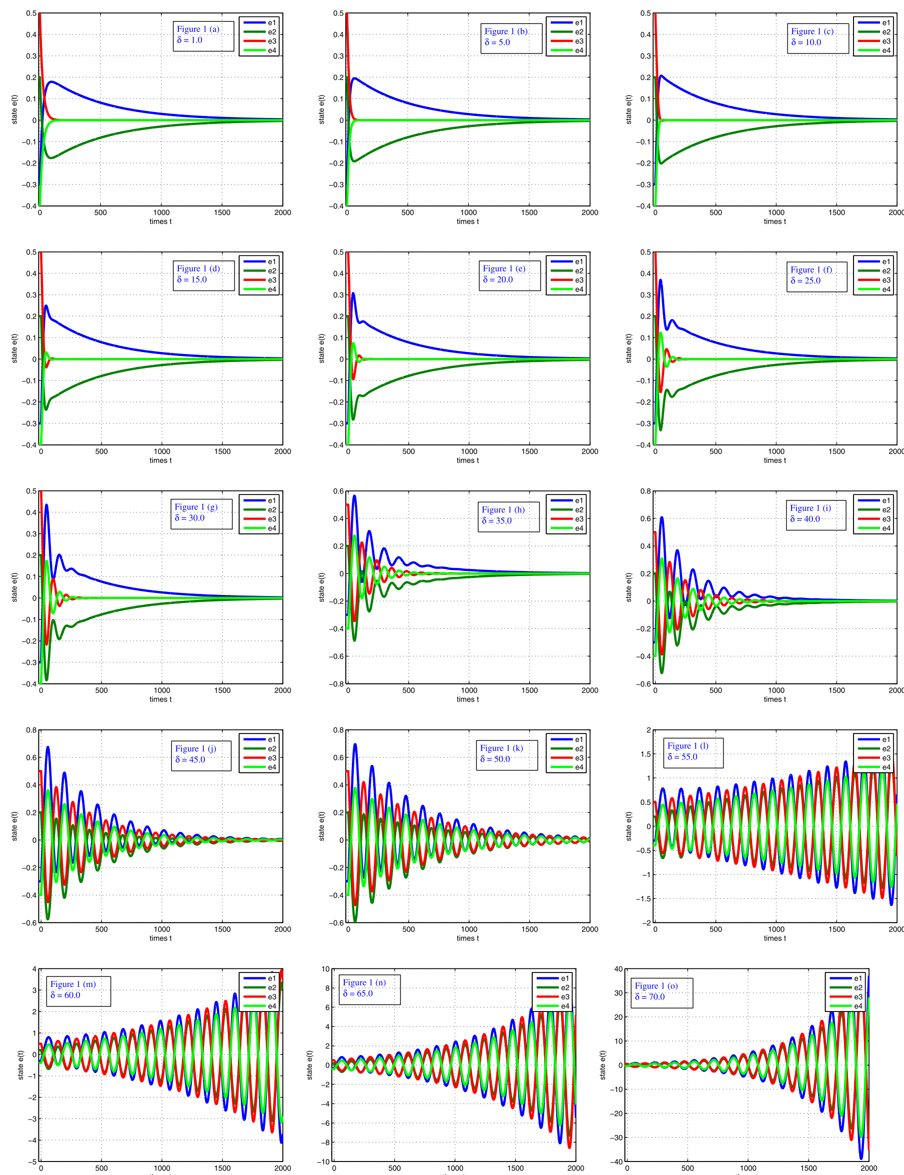

Figure 2 State responses of QTPS (61) in Example 4.4

$$\widehat{f}\big(\widehat{e}(t)\big) = \big[\widehat{f}_1\big(\widehat{e}_1(t)\big), \ldots, \widehat{f}_4\big(\widehat{e}_4(t)\big)\big]^T,$$
$$\widehat{f}_i\big(\widehat{e}_i(t)\big) = 0.01\big(\big|\widehat{e}_i(t) + 1\big| - \big|\widehat{e}_i(t) - 1\big|\big), \quad i = 1, \ldots, 4.$$

Therefore, the QTPS (60) can be rewritten to the form of system (61) as follows:

$$\begin{aligned} \dot{e}(t) &= -Ae(t-\delta) + W_1 f\big(e(t)\big) + W_2 f\big(e\big(t-\tau(t)\big)\big), \\ e(t) &= \phi(t), \quad t \in \big[-\max\{\tau_M\}, 0\big], \end{aligned} \tag{61}$$

where $A = -\widehat{A_0} - \widehat{A_1}$, $W_1 = \widehat{B_0}\widehat{K}$, $W_2 = \widehat{B_1}\widehat{K}$, and $f(\cdot) = \widehat{f}(\cdot)$. In addition, $F_1 = \mathrm{diag}\{0, 0, 0, 0\}$ and $F_2 = \mathrm{diag}\{0.01, 0.01, 0.01, 0.01\}$ with these parameters; we choose $\tau_1 = 1$, $\tau_2 = 5$, and

$\mu = 0.5$ for various δ that secure the feasibility of the LMIs in Corollary 3.1. Using MATLAB LMI control Toolbox and solving the LMIs in Corollary 3.1, we have found that the QTPS (61) is globally asymptotically stable for a small amount of leakage delay as shown in Figure 2 (see Figures 1(a)–1(k)). Taking a large amount of leakage delay δ, we have found that the QTPS (61) is actually unstable and the state trajectories of the QTPS do not converge to the zero equilibrium point as shown in Figure 2 (see Figures 1(l)–1(p)). Therefore, the leakage delay has a significant effect on the dynamical behavior of the QTPS.

5 Conclusions

In this paper, we investigated an improved stability criterion for neutral-type NNs with interval time-varying delay signal and leakage delay. For obtaining less conservative results, some suitable LKFs under the weaker assumptions of neuron activation functions were used to enlarge the feasible region of proposed stability criteria via new technique. In the first place, we derived an improved delay-dependent stability criterion by using the new integral inequality approach. Secondly, we constructed some newly constructed LKFs with triple- and four-integral terms and established less conservative stability criteria in terms of LMIs. Then, the feasibility and applicability of the proposed methods have been shown by numerical simulations. Additionally, the proposed approach is demonstrating the numerical simulation of the QTPS that takes into account transport time delay signals and leakage delay, showing the feasibility on a realistic problem. Therefore, our results have an important significance in theory and in practical applications of NNs with time delays. In addition, the proposed method in this paper can be extendable to many famous dynamical systems, such as cellular NNs [8], state estimation problem [14, 15], filtering problem [48], impulsive problem [49, 50], Markovian jump NNs [34], and sampled data control problem [47]. This will occur in the near future.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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