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Existence of almost periodic solution for neutral Nicholson blowflies model

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Abstract

This paper is concerned with a class of neutral Nicholson blowflies models with leakage delays and linear harvesting terms. Under appropriate conditions, some criteria are established for the existence and global exponential stability of almost periodic solutions for the model by applying exponential dichotomy theory. An example is provided to illustrate the effectiveness of the results.

MSC: 34A38; 34C11; 34C35

Keywords: neutral Nicholson blowflies model; almost periodic solution; linear harvesting terms; leakage delays

1 Introduction

The delay differential equation model

$$x'(t) = -\delta x(t) + px(t-\tau)e^{-\gamma x(t-\tau)} \tag{1.1}$$

and its analogous models have been proposed by Nicholson [1] and Gurney *et al.* [2], to describe the dynamics of Nicholson's blowflies model. In [1], $x(t)$ is the size of the population at time t; p is the maximum per capital daily egg production, $\frac{1}{\gamma}$ is the size at which the population reproduces at its maximum rate, δ is the per capita daily adult death rate, and τ is the generation time. Since then, Nicholson's blowflies model and its modifications have attracted much attention, some researchers obtained numerous interesting results such as permanence, extinction and stability [3–7].

It is well known that periodically and almost periodically environmental variabilities are important foundations to be considered for the theory of natural selection. Compared with periodic effects, almost functional differential equations are more frequent in some ecological models. Therefore, in recent years, there has been considerable interest in the existence and stability of almost periodic type solutions for Nicholson's blowflies models [8–12].

According to the fact that the harvests of population species commonly appear in fishery, forestry and wildlife management, the study of population dynamics with harvesting is an important subject in mathematical bioeconomics [13–15]. Recently, assuming that the harvesting function is a function of the delayed estimate of the true population, Berezansky *et al.* [3] studied the following Nicholson blowflies model with a linear harvesting term:

$$x'(t) = -\delta x(t) + Px(t-\tau)e^{-\gamma x(t-\tau)} - Hx(t-\sigma), \quad \delta, p, \tau, H, \sigma \in (0, +\infty). \tag{1.2}$$

Meanwhile, an open problem of linear harvest terms was put forward in [3]. Since then, there have been some interesting results about the influence of dynamic models with linear terms.

In [16], Liu and Meng proposed a class of non-autonomous Nicholson-type delay systems with linear harvesting terms, some criteria were established to ensure the existence and exponential convergence of the almost periodic solution of Nicholson-type systems. Recently, Wang [17] investigated the existence and convergence of positive almost periodic solutions of the following Nicholson blowflies model with path structure and multiple linear harvesting terms:

$$\begin{aligned} x_i'(t) &= -\alpha_i(t)x_i(t) + \sum_{j=1}^{n} \beta_{ij}(t)x_j(t) + \sum_{j=1}^{m} c_{ij}(t)x_i\bigl(t-\tau_{ij}(t)\bigr)e^{-\gamma_{ij}(t)x_i(t-\tau_{ij}(t))} \\ &\quad - \sum_{j=1}^{l} H_{ij}(t)x_i\bigl(t-\sigma_{ij}(t)\bigr), \quad i = 1, 2, \ldots, n. \end{aligned} \tag{1.3}$$

Some researchers focus on typical time delays named leakage delays which may exist in many real systems and also have a great impact on the dynamic systems. Recently, neutral type time delay has drawn much attention [18, 19], however, to the best of our knowledge, there are few papers published on neutral Nicholson's blowflies model with leakage delays and linear harvesting terms. Thus, it is necessary and important to consider the existence and stability of almost periodic solutions to neutral Nicholson's blowflies models with leakage delays and linear harvesting terms. The main purpose of this paper is to study the existence and stability of almost periodic solutions of the following neutral delay Nicholson blowflies models with leakage delays and linear harvesting terms:

$$\begin{aligned} x_i'(t) &= -\alpha_i(t)x_i\bigl(t-a_i(t)\bigr) + \sum_{j=1}^{n} \beta_{ij}(t)x_j\bigl(t-b_{ij}(t)\bigr) + \sum_{j=1}^{n} \delta_{ij}(t)x_j'\bigl(t-\eta_{ij}(t)\bigr) \\ &\quad + \sum_{j=1}^{m} c_{ij}(t)x_j\bigl(t-\tau_{ij}(t)\bigr)e^{-\gamma_{ij}(t)x_i(t-\tau_{ij}(t))} - \sum_{j=1}^{l} H_{ij}(t)x_i\bigl(t-\sigma_{ij}(t)\bigr). \end{aligned} \tag{1.4}$$

For equation (1.4), we assume that the following assumption holds:

(H1) $\alpha_i, a_i, \beta_{ij}, b_{ij}, \delta_{ij}, \eta_{ij}, c_{ik_1}, \tau_{ik_1}, \gamma_{ik_1}, H_{ik_2}, \sigma_{ik_2} : R \to R_+ = [0, +\infty)$ are almost periodic functions and $i, j = 1, 2, \ldots, n$; $k_1 = 1, 2, \ldots, m$; $k_2 = 1, 2, \ldots, l$. To simplify the notation and without loss of generality, for all $t \in R$ and $i = 1, 2, \ldots, n$, we will assume $\beta_{ii} = 0$ and $\delta_{ii} = 0$.

For convenience, we set

$$f^+ = \sup_{t \in R} f(t), \qquad f^- = \inf_{t \in R} f(t),$$

where f is a bounded continuous function on R.

By applying the exponential dichotomy of linear differential equations and some analysis techniques, we investigate the existence and global exponential stability of the almost periodic solutions to equation (1.4). Let $X = \{\phi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T | \varphi_i \in C^1(R, R_+), \varphi_i$ are almost periodic functions on $R, i = 1, 2, \ldots, n\}$ with the norm $\|\phi\| = \max\{|\varphi|_0, |\varphi'|_0\}$, where $|\varphi|_0 = \max_{1\le i\le n} \varphi_i^+$, $|\varphi'|_0 = \max_{1\le i\le n}(\varphi_i')^+$ and $C^1(R, R_+)$ is the collection of continuous functions with continuous derivatives on R. It is easy to see that X is a Banach space. Throughout this paper, a fixed initial time $t_0 \in R$ is chosen and the following initial value of equation (1.4) is satisfied:

$$x_i(t_0 + s) = \varphi_i(s), \quad s \in [-r, 0], i = 1, 2, \ldots, n, \tag{1.5}$$

where each φ_i is a nonnegative function on $[-r, 0]$ and $\varphi_i(0) > 0$,

$$r = \max\{a_i, b_{ij}, \eta_{ij}, \tau_{ik_1}, \sigma_{ik_2} : i, j = 1, 2, \ldots, n; k_1 = 1, 2, \ldots, m; k_2 = 1, 2, \ldots, l\}.$$

The rest of the paper is organized as follows. In Section 2, some definitions and lemmas which play important roles in proofs of the results will be introduced. In Section 3, some criteria for the existence and stability of almost periodic solutions to equation (1.4) are established. An example is provided to illustrate the effectiveness of the proposed results in Section 4.

2 Preliminary

Now let us recall the following definitions and lemmas, which will be useful in proving the main results.

Definition 2.1 ([20, 21]) A function $f : R \to R^n$ is said to be almost periodic, if for any $\varepsilon > 0$, there is a constant $l(\varepsilon) > 0$, such that in any interval of length $l(\varepsilon)$ there exists τ such that the inequality

$$\|f(t + \tau) - f(t)\| < \varepsilon$$

is satisfied for all $t \in R$. The number τ is called an ε-*translation* number of $f(t)$.

Definition 2.2 ([20, 21]) Let $f(t) : R \to R^n$ is continuously differentiable in t, $u(t)$ and $u'(t)$ are almost periodic on R, then $u(t)$ is said to be a continuously differentiable almost periodic function.

Definition 2.3 ([20, 21]) Let $x \in R^n$ and $Q(t)$ be a $n \times n$ continuously matrix defined on R. The linear system

$$x'(t) = A(t)x(t) \tag{2.1}$$

is said to admit an exponential dichotomy on R if there exist constants λ, k, projection P and the fundamental solution matrix $X(t)$ of equation (2.1) satisfying

$$\|X(t)PX^{-1}(s)\| \le \lambda e^{-k(t-s)}, \quad t \ge s; \qquad \|X(t)(I - P)X^{-1}(s)\| \le \lambda e^{-k(s-t)}, \quad t \le s.$$

Lemma 2.1 ([20, 21]) *If the linear system* (2.1) *admits an exponential dichotomy, the almost periodic system*

$$x'(t) = A(t)x(t) + g(t)$$

has a unique almost periodic solution $x(t)$, *and*

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)f(s)\,ds - \int_{t}^{\infty} X(t)(I-P)X^{-1}(s)f(s)\,ds,$$

where $X(t)$ *is the fundamental solution matrix of equation* (2.1).

Lemma 2.2 ([21, 22]) *Assume* $c_i(t)$ $(i = 1, 2, \ldots, n)$ *are almost periodic on* R, *and*

$$M[c_i] = \lim_{T\to+\infty} \frac{1}{T} \int_{t}^{t+T} c_i(s)\,ds > 0, \quad i = 1, 2, \ldots, n.$$

Then the linear system $x' = C(t)x(t)$ *admits an exponential dichotomy on* R, *where* $C(t) = \operatorname{diag}(-c_1(t), -c_2(t), \ldots, -c_n(t))$.

3 Main results

In this section, we will state and prove our main results concerning the existence and stability of almost solutions of equation (1.4). The following assumptions are satisfied:

(H2) $\alpha_i^- > 0$, $i = 1, 2, \ldots, n$.

(H3) There exists a positive constant L satisfying $\gamma_{ij}^+ L < 1$.

(H4) $M := \max_{1\le i\le n}\{\frac{\Theta_i+\Lambda_i}{\alpha_i^-}, (1 + \frac{a_i^+}{\alpha_i^-})(\Theta_i + \Lambda_i)\} < 1$, where $\Theta_i = a_i^+\alpha_i^+ + \sum_{j=1}^n \beta_{ij}^+ + \delta_{ij}^+$, $\Lambda_i = \sum_{j=1}^m c_{ij}^+ + \sum_{j=1}^l H_{ij}^+$.

Theorem 3.1 *Assume that the conditions* (H1)-(H4) *hold. Then equation* (1.4) *has a unique almost solution in* $X_0 = \{\phi \in X \mid \|\phi\| \le L, \phi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T\}$.

Proof Rewrite equation (1.4) in the form

$$\begin{aligned} x_i'(t) ={}& -\alpha_i(t)x_i(t) + \alpha_i(t)\int_{t-a_i(t)}^{t} x_i'(s)\,ds \\ &+ \sum_{j=1}^{n} \beta_{ij}(t)x_j\bigl(t-b_{ij}(t)\bigr) + \sum_{j=1}^{n} \delta_{ij}(t)x_j'\bigl(t-\eta_{ij}(t)\bigr) \\ &+ \sum_{j=1}^{m} c_{ij}(t)x_i\bigl(t-\tau_{ij}(t)\bigr)e^{-\gamma_{ij}(t)x_i(t-\tau_{ij}(t))} \\ &- \sum_{j=1}^{l} H_{ij}(t)x_i\bigl(t-\sigma_{ij}(t)\bigr), \quad i = 1, 2, \ldots, n. \end{aligned} \tag{3.1}$$

For any given $\phi \in X$, we consider the following auxiliary equation:

$$x_i'(t) = -\alpha_i(t)x_i(t) + f_i(t,\varphi) + g_i(t,\varphi), \tag{3.2}$$

where $i = 1, 2, \ldots, n$,

$$
\begin{aligned}
f_i(t,\varphi) = {} & \alpha_i(t)\int_{t-a_i(t)}^{t} \varphi_i'(s)\,ds + \sum_{j=1}^{n}\beta_{ij}(t)\varphi_j\bigl(t-b_{ij}(t)\bigr) \\
& + \sum_{j=1}^{n}\delta_{ij}(t)\varphi_j'\bigl(t-\eta_{ij}(t)\bigr),
\end{aligned}
\tag{3.3}
$$

and

$$
g_i(t,\varphi) = \sum_{j=1}^{m} c_{ij}(t)\varphi_i\bigl(t-\tau_{ij}(t)\bigr)e^{-\gamma_{ij}(t)\varphi_i(t-\tau_{ij}(t))} - \sum_{j=1}^{l} H_{ij}(t)\varphi_i\bigl(t-\sigma_{ij}(t)\bigr).
\tag{3.4}
$$

Since $\alpha_i^- > 0$ and due to Lemma 2.2, the linear system

$$
x_i'(t) = -\alpha_i(t)x_i(t), \quad i = 1, 2, \ldots, n,
$$

admits an exponential dichotomy on R. Therefore, by Lemma 2.1, we see that system (3.2) has a unique almost periodic solution as follows:

$$
x_i^{\varphi} = \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\bigl(f_i(s,\varphi) + g_i(s,\varphi)\bigr)\,ds, \quad i = 1, 2, \ldots, n.
\tag{3.5}
$$

Define the following nonlinear operator:

$$
\Phi : X_0 \to X_0, \qquad (\varphi_1, \varphi_2, \ldots, \varphi_n)^T \to \bigl(x_1^{\varphi}, x_2^{\varphi}, \ldots, x_n^{\varphi}\bigr)^T,
\tag{3.6}
$$

where x_i^{φ} $(i = 1, 2, \ldots, n)$ are given by equation (3.5). We will prove that the operator Φ is a contraction mapping.

Firstly, for any $\phi \in X_0$, we show that $\Phi\phi \in X_0$. From (3.3), for $i = 1, 2, \ldots, n$, we get

$$
\begin{aligned}
\bigl|f_i(t,\varphi)\bigr| = {} & \left|\alpha_i(t)\int_{t-a_i(t)}^{t}\varphi_i'(s)\,ds + \sum_{j=1}^{n}\beta_{ij}(t)\varphi_j\bigl(t-b_{ij}(t)\bigr) + \sum_{j=1}^{n}\delta_{ij}(t)\varphi_j'\bigl(t-\eta_{ij}(t)\bigr)\right| \\
\le {} & \alpha_i(t)\int_{t-a_i(t)}^{t}\bigl|\varphi_i'(s)\bigr|\,ds + \sum_{j=1}^{n}\beta_{ij}(t)\bigl|\varphi_j\bigl(t-b_{ij}(t)\bigr)\bigr| \\
& + \sum_{j=1}^{n}\delta_{ij}(t)\bigl|\varphi_j'\bigl(t-\eta_{ij}(t)\bigr)\bigr| \\
\le {} & \alpha_i^+ \alpha_i^+ \bigl|\varphi'\bigr|_0 + \sum_{j=1}^{n}\beta_{ij}^+|\varphi|_0 + \sum_{j=1}^{n}\delta_{ij}^+\bigl|\varphi'\bigr|_0 \\
\le {} & \left(\alpha_i^+\alpha_i^+ + \sum_{j=1}^{n}\beta_{ij}^+ + \sum_{j=1}^{n}\delta_{ij}^+\right)\|\varphi\| \\
\le {} & \left(\alpha_i^+\alpha_i^+ + \sum_{j=1}^{n}\beta_{ij}^+ + \sum_{j=1}^{n}\delta_{ij}^+\right)L = \Theta_i L.
\end{aligned}
\tag{3.7}
$$

Since $\varphi_i \in C^1(R, R_+)$ $(i = 1, 2, \ldots, n)$, similarly, we obtain

$$\begin{aligned}
\left|g_i(t,\varphi)\right| &\le \sum_{j=1}^{m} c_{ij}(t)\varphi_i\big(t-\tau_{ij}(t)\big)e^{-\gamma_{ij}^{-}\varphi_i(t-\tau_{ij}(t))} + \sum_{j=1}^{l} H_{ij}(t)\varphi_i\big(t-\sigma_{ij}(t)\big) \\
&\le \sum_{j=1}^{m} c_{ij}(t)\varphi_i\big(t-\tau_{ij}(t)\big) + \sum_{j=1}^{l} H_{ij}^{+}\|\varphi\| \\
&\le \left(\sum_{j=1}^{m} c_{ij}^{+} + \sum_{j=1}^{l} H_{ij}^{+}\right)\|\varphi\| \\
&\le \left(\sum_{j=1}^{m} c_{ij}^{+} + \sum_{j=1}^{l} H_{ij}^{+}\right)L = \Lambda_i L, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.8}$$

By (3.5)-(3.8), we get

$$\begin{aligned}
\left|\Phi\phi(t)\right| &\le \left|\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du} f_i(s,\varphi)\,ds\right| + \left|\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du} g_i(s,\varphi)\,ds\right| \\
&\le \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|f_i(s,\varphi)\,ds\right| + \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|g_i(s,\varphi)\,ds\right| \\
&\le \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\Theta_i L\,ds + \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\Lambda_i L\,ds \\
&\le \frac{\Theta_i L}{\alpha_i^-} + \frac{\Lambda_i L}{\alpha_i^-} = \frac{\Theta_i + \Lambda_i}{\alpha_i^-}L, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.9}$$

Similarly, by (3.5)-(3.8), we have

$$\begin{aligned}
\left|(\Phi\phi)'(t)\right| &\le \left|\left(\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du} f_i(s,\varphi)\,ds\right)'\right| + \left|\left(\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du} g_i(s,\varphi)\,ds\right)'\right| \\
&\le \left|f_i(t,\varphi) - \alpha_i(t)\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du} f_i(s,\varphi)\,ds\right| \\
&\quad + \left|g_i(t,\varphi) - \alpha_i(t)\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du} g_i(s,\varphi)\,ds\right| \\
&\le \left|f_i(t,\varphi)\right| + \alpha_i(t)\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|f_i(s,\varphi)\,ds\right| \\
&\quad + \left|g_i(t,\varphi)\right| + \alpha_i(t)\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|g_i(s,\varphi)\,ds\right| \\
&\le \Theta_i L + \frac{\alpha_i^+}{\alpha_i^-}\Theta_i L + \Lambda_i L + \frac{\alpha_i^+}{\alpha_i^-}\Lambda_i L \\
&= \left(1 + \frac{\alpha_i^+}{\alpha_i^-}\right)(\Theta_i + \Lambda_i)L, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.10}$$

It follows from (H4), (3.9) and (3.10) that

$$\|\Phi\phi\| = \max_{1\le i\le n}\left\{\frac{\Theta_i + \Lambda_i}{\alpha_i^-}L, \left(1 + \frac{\alpha_i^+}{\alpha_i^-}\right)(\Theta_i + \Lambda_i)L\right\} \le L,$$

therefore, $\Phi\phi \in X_0$. Next, we show that Φ is a contraction.

For $\phi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T$, $\tilde{\phi} = (\tilde{\varphi}_1, \tilde{\varphi}_2, \ldots, \tilde{\varphi}_n)^T \in X_0$, we have

$$\begin{aligned}
&\bigl|f_i(s,\varphi) - f_i(s,\tilde{\varphi})\bigr| \\
&\quad = \left|\alpha_i(s)\int_{s-a_i(s)}^{s}\bigl(\varphi_i'(u) - \tilde{\varphi}'_i(u)\bigr)\,du + \sum_{j=1}^{n}\beta_{ij}(s)\bigl(\varphi_j\bigl(s-b_{ij}(s)\bigr) - \tilde{\varphi}_j\bigl(s-b_{ij}(s)\bigr)\bigr)\right. \\
&\qquad \left. + \sum_{j=1}^{n}\delta_{ij}(s)\bigl(\varphi_j'\bigl(s-\eta_{ij}(s)\bigr) - \tilde{\varphi}'_j\bigl(s-\eta_{ij}(s)\bigr)\bigr)\right| \\
&\quad \le \alpha_i(s)\int_{s-a_i(s)}^{s}\bigl|\varphi_i'(u) - \tilde{\varphi}'_i(u)\bigr|\,du + \sum_{j=1}^{n}\beta_{ij}(s)\bigl|\varphi_j\bigl(s-b_{ij}(s)\bigr) - \tilde{\varphi}_j\bigl(s-b_{ij}(s)\bigr)\bigr| \\
&\qquad + \sum_{j=1}^{n}\delta_{ij}(s)\bigl|\varphi_j'\bigl(s-\eta_{ij}(s)\bigr) - \tilde{\varphi}'_j\bigl(s-\eta_{ij}(s)\bigr)\bigr| \\
&\quad \le \alpha_i^+ a_i^+\|\varphi - \tilde{\varphi}\| + \sum_{j=1}^{n}\beta_{ij}^+\|\varphi - \tilde{\varphi}\| + \sum_{j=1}^{n}\delta_{ij}^+\|\varphi - \tilde{\varphi}\| \\
&\quad = \left(\alpha_i^+ a_i^+ + \sum_{j=1}^{n}\beta_{ij}^+ + \sum_{j=1}^{n}\delta_{ij}^+\right)\|\varphi - \tilde{\varphi}\| \\
&\quad = \Theta_i\|\varphi - \tilde{\varphi}\|, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.11}$$

For $0 < rL < 1$, and $|xe^{-rx} - ye^{-ry}| = e^{-r\xi}|1 - r\xi||x - y| = (1 - r\xi)e^{-r\xi}|x - y| \le |x - y|$, where ξ is between in x and y.

Since $\gamma_{ij}^+ L < 1$, we obtain

$$\begin{aligned}
&\bigl|g_i(s,\varphi) - g_i(s,\tilde{\varphi})\bigr| \\
&\quad = \left|\sum_{j=1}^{m}c_{ij}(s)\bigl(\varphi_i\bigl(s-\tau_{ij}(s)\bigr)e^{-\gamma_{ij}(s)\varphi_i(s-\tau_{ij}(s))} - \tilde{\varphi}_i\bigl(s-\tau_{ij}(s)\bigr)e^{-\gamma_{ij}(s)\tilde{\varphi}_i(s-\tau_{ij}(s))}\bigr)\right. \\
&\qquad \left. + \sum_{j=1}^{l}H_{ij}(s)\bigl(\varphi_i\bigl(s-\sigma_{ij}(s)\bigr) - \tilde{\varphi}_i\bigl(s-\sigma_{ij}(s)\bigr)\bigr)\right| \\
&\quad \le \sum_{j=1}^{m}c_{ij}(s)\bigl|\varphi_i\bigl(s-\tau_{ij}(s)\bigr)e^{-\gamma_{ij}(s)\varphi_i(s-\tau_{ij}(s))} - \tilde{\varphi}_i\bigl(s-\tau_{ij}(s)\bigr)e^{-\gamma_{ij}(s)\tilde{\varphi}_i(s-\tau_{ij}(s))}\bigr| \\
&\qquad + \sum_{j=1}^{l}H_{ij}(s)\bigl|\varphi_i\bigl(s-\sigma_{ij}(s)\bigr) - \tilde{\varphi}_i\bigl(s-\sigma_{ij}(s)\bigr)\bigr| \\
&\le \sum_{j=1}^{m}c_{ij}^+\|\varphi - \tilde{\varphi}\| + \sum_{j=1}^{l}H_{ij}^+\|\varphi - \tilde{\varphi}\| \\
&\quad = \left(\sum_{j=1}^{m}c_{ij}^+ + \sum_{j=1}^{l}H_{ij}^+\right)\|\varphi - \tilde{\varphi}\| \\
&\quad = \Lambda_i\|\varphi - \tilde{\varphi}\|, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{3.12}$$

By (H4), (3.5)-(3.6) and (3.11)-(3.12), we have

$$
\begin{aligned}
&\left|(\Phi\phi - \Phi\tilde{\phi})(t)\right| \\
&\quad \le \left|\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\big(f_i(s,\varphi) - f_i(s,\tilde{\varphi})\big)\,ds\right| \\
&\qquad + \left|\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\big(g_i(s,\varphi) - g_i(s,\tilde{\varphi})\big)\,ds\right| \\
&\quad \le \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|f_i(s,\varphi) - f_i(s,\tilde{\varphi})\right|ds + \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|g_i(s,\varphi) - g_i(s,\tilde{\varphi})\right|ds \\
&\quad \le \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\Theta_i\|\varphi - \tilde{\varphi}\|\,ds + \int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\Lambda_i\|\varphi - \tilde{\varphi}\|\,ds \\
&\quad \le \frac{\Theta_i}{\alpha_i^-}\|\varphi - \tilde{\varphi}\| + \frac{\Lambda_i}{\alpha_i^-}\|\varphi - \tilde{\varphi}\| \\
&\quad = \frac{\Theta_i + \Lambda_i}{\alpha_i^-}\|\varphi - \tilde{\varphi}\|, \quad i = 1, 2, \dots, n.
\end{aligned}
\tag{3.13}
$$

Similarly, by (3.5)-(3.6) and (3.11)-(3.12), we have

$$
\begin{aligned}
&\left|(\Phi\phi - \Phi\tilde{\phi})'(t)\right| \\
&\quad \le \left|\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\big(f_i(s,\varphi) - f_i(s,\tilde{\varphi})\big)'\,ds\right| \\
&\qquad + \left|\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\big(g_i(s,\varphi) - g_i(s,\tilde{\varphi})\big)'\,ds\right| \\
&\quad \le \left|f_i(t,\varphi) - f_i(t,\tilde{\varphi})\right| + \alpha_i(t)\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|f_i(s,\varphi) - f_i(s,\tilde{\varphi})\right|ds \\
&\qquad + \left|g_i(t,\varphi) - g_i(t,\tilde{\varphi})\right| + \alpha_i(t)\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\left|g_i(s,\varphi) - g_i(s,\tilde{\varphi})\right|ds \\
&\quad \le \Theta_i\|\varphi - \tilde{\varphi}\| + \alpha_i^+\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\Theta_i\|\varphi - \tilde{\varphi}\|\,ds \\
&\qquad + \Lambda_i\|\varphi - \tilde{\varphi}\| + \alpha_i^+\int_{-\infty}^{t} e^{-\int_s^t \alpha_i(u)\,du}\Lambda_i\|\varphi - \tilde{\varphi}\|\,ds \\
&\quad \le \Theta_i\|\varphi - \tilde{\varphi}\| + \frac{\alpha_i^+}{\alpha_i^-}\Theta_i\|\varphi - \tilde{\varphi}\| + \Lambda_i\|\varphi - \tilde{\varphi}\| + \frac{\alpha_i^+}{\alpha_i^-}\Lambda_i\|\varphi - \tilde{\varphi}\| \\
&\quad = \left(1 + \frac{\alpha_i^+}{\alpha_i^-}\right)(\Theta_i + \Lambda_i)\|\varphi - \tilde{\varphi}\|, \quad i = 1, 2, \dots, n.
\end{aligned}
\tag{3.14}
$$

It follows from (3.13) and (3.14) that $\|\Phi\phi - \Phi\tilde{\phi}\| < M\|\phi - \tilde{\phi}\|$. Thus, this implies that the mapping Φ is a contraction. Therefore, by the fixed point theorem of Banach space, Φ has a fixed point $\phi^* \in X_0$ such that $\Phi\phi^* = \phi^*$. That is to say, (3.1) has a unique almost periodic solution $\phi^* \in X_0$. So, there exists a unique almost periodic solution ϕ^* of equation (1.4) in X_0. The proof is complete. □

Next, we show that the solution of equation (1.4) is globally exponentially stable. In order to obtain the result, we make the following assumption:

(H5) $\alpha_i^+ a_i^+ < 1$ for $i = 1, 2, \ldots, n$, and there exist positive constants $\zeta_1, \zeta_2, \ldots, \zeta_n$ and Υ_* such that, for $t > 0$ and $i = 1, 2, \ldots, n$,

$$\begin{aligned} -\Upsilon_* > {} & -\bigl[\alpha_i(t)\bigl(1 - 2\alpha_i^+ a_i^+\bigr) - \bigl|\alpha_i(t) - \bigl(1 - a_i'(t)\bigr)\alpha_i\bigl(t - a_i(t)\bigr)\bigr|\bigr]\frac{\zeta_i}{1 - \alpha_i^+ a_i^+} \\ & + \sum_{j=1}^{n}\bigl(\beta_{ij}(t) + \delta_{ij}(t)\bigr)\frac{\zeta_i}{1 - \alpha_j^+ a_j^+} \\ & + \left(\sum_{j=1}^{m} c_{ij}(t) + \sum_{j=1}^{l} H_{ij}(t)\right)\frac{\zeta_i}{1 - \alpha_i^+ a_i^+}. \end{aligned}$$

It follows from (H5) that we can choose $0 < r < \alpha_i^-$ and Υ such that

$$\begin{aligned} -\Upsilon > {} & -\bigl[\bigl(\alpha_i(t) - r\bigr)\bigl(1 - 2\alpha_i^+ \alpha_i^+\bigr) - \bigl|\alpha_i(t)e^{ra_i(t)} - \bigl(1 - a_i'(t)\bigr)\alpha_i\bigl(t - a_i(t)\bigr)\bigr|\bigr]\frac{\zeta_i}{1 - \alpha_i^+ a_i^+} \\ & + \sum_{j=1}^{n}\bigl(\beta_{ij}(t)e^{rb_{ij}(t)} + \delta_{ij}(t)e^{r\eta_{ij}(t)}\bigr)\frac{\zeta_j}{1 - \alpha_j^+ a_j^+} \\ & + \left(\sum_{j=1}^{m} c_{ij}(t)e^{r\tau_{ij}(t)} + \sum_{j=1}^{l} H_{ij}(t)e^{r\sigma_{ij}(t)}\right)\frac{\zeta_i}{1 - \alpha_i^+ a_i^+}, \quad t \geq 0, i = 1, 2, \ldots, n. \end{aligned}$$

Theorem 3.2 *Let $Z^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ be the almost periodic solution of equation* (1.4) *in the region X_0. Suppose that the conditions* (H1)-(H5) *are satisfied. Then the almost solution $Z^*(t)$ of equation* (1.4) *is globally exponentially stable.*

Proof Define $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T = (x_1(t) - x_1^*(t), x_2(t) - x_2^*(t), \ldots, x_n(t) - x_n^*(t)) = Z(t) - Z^*(t)$, where $Z(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is an arbitrary solution of equation (1.4). It follows that

$$\begin{aligned} y_i'(t) = {} & -\alpha_i(t)y_i\bigl(t - a_i(t)\bigr) + \sum_{j=1}^{n}\beta_{ij}(t)y_i\bigl(t - b_{ij}(t)\bigr) \\ & + \sum_{j=1}^{n}\delta_{ij}(t)y_j'\bigl(t - \eta_{ij}(t)\bigr) \\ & + \sum_{j=1}^{m} c_{ij}(t)\bigl[x_i\bigl(t - \tau_{ij}(t)\bigr)e^{-\gamma_{ij}(t)x_i(t-\tau_{ij}(t))} - x_i^*\bigl(t - \tau_{ij}(t)\bigr)e^{-\gamma_{ij}(t)x_i^*(t-\tau_{ij}(t))}\bigr] \\ & - \sum_{j=1}^{l} H_{ij}(t)y_i\bigl(t - \sigma_{ij}(t)\bigr). \end{aligned}$$

We obtain

$$\begin{aligned} Y_i'(t) = {} & -\bigl(\alpha_i(t) - r\bigr)Y_i(t) - \bigl(\alpha_i(t) - r\bigr)\int_{t-a_i(t)}^{t}\alpha_i(s)e^{rs}y_i(s)\,ds \\ & - \bigl[\alpha_i(t) - \bigl(1 - a_i'(t)\bigr)\alpha_i\bigl(t - a_i(t)\bigr)e^{-ra_i(t)}\bigr]e^{rt}y_i\bigl(t - a_i(t)\bigr) \\ & + e^{rt}\left\{\sum_{j=1}^{n}\beta_{ij}(t)y_j\bigl(t - b_{ij}(t)\bigr) + \sum_{j=1}^{n}\delta_{ij}(t)y_j'\bigl(t - \eta_{ij}(t)\bigr)\right. \end{aligned}$$

$$+\sum_{j=1}^{m} c_{ij}(t)\big[x_i\big(t-\tau_{ij}(t)\big)e^{-\gamma_{ij}(t)x_i(t-\tau_{ij}(t))} - x_i^*\big(t-\tau_{ij}(t)\big)e^{-\gamma_{ij}(t)x_i^*(t-\tau_{ij}(t))}\big]$$

$$-\sum_{j=1}^{l} H_{ij}(t)y_i\big(t-\sigma_{ij}(t)\big)\Bigg\}, \tag{3.15}$$

where $Y_i(t) = e^{rt}y_i(t) - \int_{t-a_i(t)}^{t} \alpha_i(s)e^{rs}y_i(s)\,ds$, $i = 1,2,\ldots,n$.

Denote $M = \max\{\max_{1\le i\le n}(\sup_{s\in(-\infty,0]}|Y_i(s)|), \max_{1\le i\le n}(\sup_{s\in(-\infty,0]}|Y_i'(s)|)\}$.

There exists $K > 0$ such that $|Y_i(t)| \le M < K\zeta_i$, $|Y_i'(t)| \le M < K\zeta_i$ for all $t \in (-\infty,0]$, and $i = 1,2,\ldots,n$.

We claim that $|Y_i(t)| < K\zeta_i$, $|Y_i'(t)| < K\zeta_i$ for $t \in (0,\infty)$, and $i = 1,2,\ldots,n$. Otherwise, there exists $i \in \{1,2,\ldots,n\}$, $\theta \in (0,+\infty)$ such that $|Y_i(\theta)| = K\zeta_i$, $|Y_k(t)| < K\zeta_k$ and $|Y_i'(\theta)| = K\zeta_i$, $|Y_k'(t)| < K\zeta_k$ for $t \in (-\infty,\theta)$, $k \in \{1,2,\ldots,n\}$.

It follows that, for $t \in (-\infty,\theta)$, $k \in \{1,2,\ldots,n\}$,

$$\begin{aligned} e^{rt}\big|y_k(t)\big| &\le \left|e^{rt}y_k(t) - \int_{t-a_i(t)}^{t} \alpha_i(s)e^{rs}y_i(s)\,ds\right| \\ &\quad + \left|\int_{t-a_i(t)}^{t} \alpha_i(s)e^{rs}y_i(s)\,ds\right| \\ &\le K\zeta_k + \alpha_k^+ a_k^+ \sup_{s\in(-\infty,\theta]} e^{rs}\big|y_k(s)\big| \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} e^{rt}\big|y_k'(t)\big| &\le \left|e^{rt}y_k'(t) - \int_{t-a_i(t)}^{t} \alpha_i(s)e^{rs}y_i'(s)\,ds\right| \\ &\quad + \left|\int_{t-a_i(t)}^{t} \alpha_i(s)e^{rs}y_i'(s)\,ds\right| \\ &\le K\zeta_k + \alpha_k^+ a_k^+ \sup_{s\in(-\infty,\theta]} e^{rs}\big|y_k'(s)\big|. \end{aligned} \tag{3.17}$$

Thus

$$e^{rt}\big|y_k(t)\big| \le \sup_{s\in(-\infty,\theta]} e^{rs}\big|y_k(s)\big| \le \frac{K\zeta_k}{1-\alpha_k^+ a_k^+} \tag{3.18}$$

and

$$e^{rt}\big|y_k'(t)\big| \le \sup_{s\in(-\infty,\theta]} e^{rs}\big|y_k(s)\big| \le \frac{K\zeta_k}{1-\alpha_k^+ a_k^+}. \tag{3.19}$$

Now we calculate the upper left derivative of $Y_i(t)$. From (H5) and (3.15)-(3.19), we have

$$\begin{aligned} 0 &\le D^-\big|Y_i(\theta)\big| \\ &\le -\big(\alpha_i(\theta)-r\big)Y_i(\theta) + \left|-\big(\alpha_i(\theta)-r\big)\int_{\theta-a_i(\theta)}^{\theta} \alpha_i(s)e^{rs}y_i(s)\,ds\right| \\ &\quad + \Big|-\big[\alpha_i(\theta) - \big(1-a_i'(\theta)\big)\alpha_i\big(\theta-a_i(\theta)\big)e^{-ra_i(\theta)}\big]e^{r\theta}y_i\big(\theta-a_i(\theta)\big) \end{aligned}$$

$$
\begin{aligned}
&+ e^{r\theta}\Biggl\{\sum_{j=1}^{n}\beta_{ij}(\theta)y_j\bigl(\theta-b_{ij}(\theta)\bigr)+\sum_{j=1}^{n}\delta_{ij}(\theta)y_j'\bigl(\theta-\eta_{ij}(\theta)\bigr)\\
&+\sum_{j=1}^{m}c_{ij}(\theta)\bigl[x_i\bigl(\theta-\tau_{ij}(\theta)\bigr)e^{-\gamma_{ij}(\theta)x_i(\theta-\tau_{ij}(\theta))}-x_i^*\bigl(\theta-\tau_{ij}(\theta)\bigr)e^{-\gamma_{ij}(\theta)x_i^*(\theta-\tau_{ij}(\theta))}\bigr]\\
&-\sum_{j=1}^{l}H_{ij}(\theta)y_i\bigl(\theta-\sigma_{ij}(\theta)\bigr)\Biggr\}\Biggr|\\
\leq{}& -\bigl(\alpha_i(\theta)-r\bigr)\biggl|e^{r\theta}y_i(\theta)-\int_{\theta-a_i(\theta)}^{t}\alpha_i(s)e^{rs}y_i(s)\,ds\biggr|+\bigl(\alpha_i(\theta)-r\bigr)\frac{K\zeta_i}{1-\alpha_i^+a_i^+}\\
&+\bigl|\alpha_i(\theta)-\bigl(1-a_i'(\theta)\bigr)\alpha_i\bigl(\theta-a_i(\theta)\bigr)e^{-ra_i(\theta)}\bigr|e^{ra_i(\theta)}e^{r(\theta-a_i(\theta))}\bigl|y_i\bigl(\theta-a_i(\theta)\bigr)\bigr|\\
&+\sum_{j=1}^{n}\beta_{ij}(\theta)e^{rb_{ij}(\theta)}e^{r(\theta-b_{ij}(\theta))}\bigl|y_i\bigl(\theta-b_{ij}(\theta)\bigr)\bigr|\\
&+\sum_{j=1}^{n}\delta_{ij}(\theta)e^{r\eta_{ij}(\theta)}e^{r(\theta-\eta_{ij}(\theta))}\bigl|y_i'\bigl(\theta-\eta_{ij}(\theta)\bigr)\bigr|\\
&+\sum_{j=1}^{m}c_{ij}(\theta)e^{r\tau_{ij}(\theta)}e^{r(\theta-\tau_{ij}(\theta))}\bigl|y_i\bigl(\theta-\tau_{ij}(\theta)\bigr)\bigr|\\
&+\sum_{j=1}^{l}H_{ij}(\theta)e^{r\sigma_{ij}(\theta)}e^{r(\theta-\sigma_{ij}(\theta))}\bigl|y_i\bigl(\theta-\sigma_{ij}(\theta)\bigr)\bigr|\\
\leq{}& -\bigl[\bigl(\alpha_i(\theta)-r\bigr)\bigl(1-2\alpha_i^+a_i^+\bigr)-\bigl|\alpha_i(\theta)e^{ra_i(\theta)}-\bigl(1-a_i'(\theta)\bigr)\alpha_i\bigl(\theta-a_i(\theta)\bigr)\bigr|\bigr]\frac{K\zeta_i}{1-\alpha_i^+a_i^+}\\
&+\sum_{j=1}^{n}\bigl(\beta_{ij}(\theta)e^{rb_{ij}(\theta)}+\delta_{ij}(\theta)e^{r\eta_{ij}(\theta)}\bigr)\frac{K\zeta_i}{1-\alpha_i^+a_i^+}\\
&+\left(\sum_{j=1}^{m}c_{ij}(\theta)e^{r\tau_{ij}(\theta)}+\sum_{j=1}^{l}H_{ij}(\theta)e^{r\sigma_{ij}(\theta)}\right)\frac{K\zeta_i}{1-\alpha_i^+\alpha_i^+}\\
={}&\Biggl\{-\bigl[\bigl(\alpha_i(\theta)-r\bigr)\bigl(1-2\alpha_i^+\alpha_i^+\bigr)-\bigl|\alpha_i(\theta)e^{ra_i(\theta)}-\bigl(1-a_i'(\theta)\bigr)\alpha_i\bigl(\theta-a_i(\theta)\bigr)\bigr|\bigr]\frac{\zeta_i}{1-\alpha_i^+a_i^+}\\
&+\sum_{j=1}^{n}\bigl(\beta_{ij}(\theta)e^{rb_{ij}(\theta)}+\delta_{ij}(\theta)e^{r\eta_{ij}(\theta)}\bigr)\frac{\zeta_j}{1-\alpha_j^+a_j^+}\\
&+\left(\sum_{j=1}^{m}c_{ij}(\theta)e^{r\tau_{ij}(\theta)}+\sum_{j=1}^{l}H_{ij}(\theta)e^{r\sigma_{ij}(\theta)}\right)\frac{\zeta_i}{1-\alpha_i^+a_i^+}\Biggr\}K\\
<{}& -\Upsilon K\\
<{}&\ 0,
\end{aligned}
$$

which is a contradiction. So, for all $t \in (0,+\infty)$ and $i = 1,2,\dots,n$, $|Y_i(t)| < K\zeta_i$ holds. According to the above, it is easy to obtain $|x_i(t) - x_i^*(t)| \leq \frac{K\zeta_j}{1-\alpha_i^+a_i^+}e^{-rt}$ for all $t \in (0,+\infty)$ and $i = 1,2,\dots,n$. Therefore, the solution of equation (1.4) is globally exponential stable. This completes the proof of Theorem 3.2. □

4 An example

In this section, an example is given to demonstrate the results obtained in Section 3.

Example 4.1 Consider the following neutral Nicholson blowflies model with leakage delays and linear harvesting terms:

$$\begin{cases} \dot{x}_1(t) = -(11+\cos^2 t)x_1(t-0.01|\sin t|) + (0.01+0.03\sin^2 t)x_2(t-0.02|\cos\sqrt{2}t|) \\ \qquad + (0.02+0.05\sin^2 t)x_2'(t-0.07|\cos\sqrt{3}t|) \\ \qquad + (0.03+0.07\sin^2 t)x_1(t-0.05|\cos\sqrt{5}t|) \\ \qquad \times e^{-0.05x_1(t-0.03|\cos\sqrt{5}t|)} \\ \qquad + (0.04+0.09\sin^2 t)x_1(t-0.07|\cos\sqrt{7}t|)e^{-0.007x_1(t-0.005|\cos\sqrt{7}t|)} \\ \qquad - 0.06\cos^2 t x_1(t-0.06|\cos\sqrt{6}t|), \\ \dot{x}_2(t) = -(11+\sin^2 t)x_1(t-0.01|\cos t|) + (0.01+0.03\cos^2 t)x_2(t-0.02|\sin\sqrt{2}t|) \\ \qquad + (0.02+0.05\cos^2 t)x_1'(t-0.07|\sin\sqrt{3}|t) \\ \qquad + (0.03+0.07\cos^2 t)x_1(t-0.05|\sin\sqrt{5}t|) \\ \qquad \times e^{-0.005x_1(t-0.03|\sin\sqrt{5}t|)} \\ \qquad + (0.04+0.09\cos^2 t)x_1(t-0.07|\sin\sqrt{7}t|)e^{-0.07x_1(t-0.05|\sin\sqrt{7}t|)} \\ \qquad - 0.08\sin^2 t x_1(t-0.08|\sin\sqrt{8}t|). \end{cases}$$

We take $L = e^2$, $\zeta_1 = \zeta_2 = 0.5$. It is not difficult to check that the assumptions (H2)-(H5) are all satisfied, respectively. Hence, from Theorem 3.1 and Theorem 3.2, the above system has exactly one almost periodic solution, which is globally exponentially stable.

Remark 4.1 In this paper, a class of neutral delay Nicholson's blowflies model with leakage delays and linear harvesting terms is considered. To the best of our knowledge, no results can be associated with equation (1.4). So, the results of this paper are novel. Moreover, the obtained results are dependent on the leakage delay. As for future work, it would be an interesting topic to investigate the Nicholson blowflies model dependent on a probability distribution leakage term.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors declare that the work was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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