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Absolute stability of large-scale time-delay Lurie indirect control systems with unbounded coefficients

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Abstract

We study the absolute stability of large-scale time-delay Lurie indirect control systems with unbounded coefficients. Based on Lyapunov-Krasovskii functional approach, we derive some novel absolute stability conditions for this class of Lurie systems with a single nonlinearity. These conditions are particularly suitable for large-scale time-delay Lurie systems with unbounded coefficients. At the same time, they are also effective for such systems with bounded or constant coefficients. Furthermore, we extend the results obtained to multiple nonlinearities. The effectiveness of the proposed methods is illustrated via two numerical examples.

Keywords: large-scale time-delay system; lurie indirect control system; absolute stability; Lyapunov second method

1 Introduction

In the early 1940s, the absolute stability concept was defined by Lurie and Postnikov [1]. Since then, the absolute stability analysis for Lurie systems has received considerable attention from the academic community, and there have been a great number of publications on this topic [2–6]. Meanwhile, the time-delay phenomenon frequently appears in practical engineering systems. Its presence can degrade performance of the control system or even lead to instability. Therefore, the research of absolute stability of time-delay Lurie systems is of important significance. In [7], by splitting the whole delay interval into even or uneven subintervals, a new Lyapunov-Krasovskii functional was constructed. Based on Lyapunov second method, some new absolute stability conditions were proposed. In [8], by employing integral-equality technique, the stability criteria for Lurie control systems with multiple delays were obtained. In [9], the absolute stability of Lurie nonlinear systems with time-varying delay was investigated based on augmented Lyapunov-Krasovskii functional and free-weighting matrix approach. At the same time, the absolute stability theory of time-delay Lurie systems was also applied in the study of synchronization problems; some relevant works were presented in [10–14].

It should be pointed out that some practical systems, such as communication systems, electric power systems, and biological systems, can be represented in the form of large-scale systems [15]. Over the past few years, much effort has been devoted to investigating the problem of absolute stability of large-scale Lurie control systems, and many important results have been reported in the open literature. In [16], some sufficient conditions in terms of BMIs were presented to guarantee the absolute stability of interconnected Lurie direct control system. By using the decomposing method of large-scale system and M-matrix property, the authors in [17] considered a class of Lurie indirect control large-scale systems and derived simple stability conditions. Subsequently, the authors in [18] extended the criteria proposed in [17] to the case of large-scale Lurie indirect control systems with multiple operators and unbounded coefficients. By employing a similar method, the absolute stability problem of large-scale Lurie direct control systems with time-varying coefficients was well addressed in [19, 20]. However, to our knowledge, there are few results on the absolute stability of large-scale Lurie systems subject to time-delay. This has motivated our study.

In this paper, we deal with the absolute stability problem of a class of large-scale time-delay Lurie indirect control systems with unbounded coefficients. A Lyapunov-Krasovskii functional-based approach is presented to obtain some new sufficient conditions such that the absolute stability of the system under consideration can be ensured. The main contributions of this paper are as follows: (i) The elements of the system coefficient matrices can be unbounded functions, and the time-delay can be very large under admissible conditions. (ii) The stability criteria proposed are applicable not only to large-scale time-delay Lurie systems with unbounded coefficients but also to this class of systems with bounded or constant coefficients.

Notation: Throughout the paper, $P > 0$ ($P < 0$) means that a matrix P is symmetric positive (negative) definite; $\lambda(A)$ denotes any eigenvalue of square matrix A; $\|x\|$ stands for the Euclidean norm of a vector $x = [x_1\, x_2\, \cdots\, x_n]^T$, that is, $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$; $\|A\|$ is the matrix norm induced by the vector Euclidean norm, that is, $\|A\| = \max_{\|x\|=1} \|Ax\|$, and it is easy to verify that $\|A\| = \sqrt{\lambda_{\max}(A^TA)}$; $\overline{\lim}_{t\to\infty}$ denotes the upper limit. For simplicity of presentation, let $\phi(s) = [x_1^T(t+s)\, x_2^T(t+s)\, \cdots\, x_m^T(t+s)\, \sigma(t)]^T, s \in [-\tau, 0], t \geq 0$; $|\|\phi\|| = \sqrt{\int_{-\tau}^0 \|\phi(s)\|^2\, ds}$.

To obtain our main results, the following lemmas are needed.

Lemma 1 (The Schur complement lemma [21]) *Let M, N, P be constant matrices of appropriate dimensions, where M and N are symmetric. Then*

$$\begin{bmatrix} M & P \\ P^T & N \end{bmatrix} < 0$$

if and only if $N < 0$ and $M - PN^{-1}P^T < 0$.

Consider the following functional differential equations with finite delay:

$$\dot{x}(t) = f(t, x_t), \tag{1}$$

where $x(t) \in R^n$, x_t is a function defined in the interval $[-h, 0]$ as $x_t(\theta) = x(t+\theta)$, $-h \leq \theta \leq 0$, h is the maximum delay, and f is a functional. Let C be the set of all continuous functions defined in the interval $[-h, 0]$. Then the initial condition of (1) can be expressed as $x_{t_0} = \phi$, for $\phi \in C$. With this notation, the domain of definition of f is $R \times C$. A more detailed description can be found in [22].

Denote by K the set of strictly increasing continuous functions $W : R_+ \to R_+$ with $W(0) = 0$. Also, $|\|\phi\|| = \sqrt{\int_{-\tau}^{0} \|\phi(s)\|^2\, ds}$, where $\|\cdot\|$ refers to the usual Euclidean norm. For system (1), we have the following lemma.

Lemma 2 *Suppose that a functional V and functions $W, W_1, W_2, W_3 \in K$ are such that*

(i) $W(\|\phi(0)\|) \le V(t,\phi) \le W_1(\|\phi(0)\|) + W_2(|\|\phi\||)$,

(ii) $\dot{V}(t,\phi) \le -W_3(\|\phi(0)\|)$.

Then the zero solution of (1) *is uniformly asymptotically stable. In addition, if* $\lim_{s\to\infty} W(s) = \infty$, *then it is globally uniformly asymptotically stable.*

Proof Burton in [22, 23] has proved the classical uniform asymptotic stability result. In fact, if $\lim_{s\to\infty} W(s) = \infty$, then, in the proof of Theorem 1 of [23], for arbitrary large $\delta > 0$, there exists $\varepsilon > 0$ such that $W_1(\delta) + W_2([nh\delta^2]^{\frac{1}{2}}) < W(\varepsilon)$, and therefore the global uniform asymptotic stability can be concluded. □

We first analyze large-scale time-delay Lurie indirect control systems with a single nonlinearity, and then we extend the results derived to multiple nonlinearities. For the multiple nonlinearities case, $\sigma(t)$ in $\phi(s)$ is regarded as a vector.

2 Absolute stability of large-scale Lurie systems with a single nonlinearity

Consider the following large-scale time-delay Lurie indirect control system with unbounded coefficients and a single nonlinearity:

$$\begin{cases} \dot{x}_i(t) = \sum_{j=1}^{m} A_{ij}(t)x_j(t) + \sum_{j=1}^{m} B_{ij}(t)x_j\bigl(t-\tau_j(t)\bigr) + b_i(t)f\bigl(\sigma(t)\bigr), \\ \quad i = 1,2,\ldots,m, \\ \dot{\sigma}(t) = \sum_{i=1}^{m} c_i^T(t)x_i(t) - \rho(t)f\bigl(\sigma(t)\bigr), \end{cases} \tag{2}$$

where $x_i(t) \in R^{n_i}$ $(i = 1,2,\ldots,m)$ and $\sigma(t) \in R$ are the state vectors, the vector functions $b_i(t), c_i(t) \in R^{n_i}$ $(i = 1,2,\ldots,m)$ are continuous in $[0,\infty)$, $\sum_{i=1}^{m} n_i = n$; the matrix functions $A_{ij}(t), B_{ij}(t) \in R^{n_i\times n_j}$ $(i,j = 1,2,\ldots,m)$ are continuous in $[0,\infty)$; $\tau_j(t)$ $(j = 1,2,\ldots,m)$ refers to the time-delay; $\rho(t)$ is a continuous function in $[0,\infty)$ and satisfies $\rho(t) \ge \rho > 0$ with constant ρ. The continuous nonlinearity $f(\cdot)$ satisfies the following sector condition:

$$F_{[k_1,k_2]} = \bigl\{f(\cdot)|f(0) = 0; k_1\sigma^2(t) \le \sigma(t)f\bigl(\sigma(t)\bigr) \le k_2\sigma^2(t), \sigma(t) \in R - \{0\}\bigr\},$$

where k_1, k_2 are constants such that $k_2 > k_1 > 0$.

Definition 1 ([24]) System (2) is said to be absolutely stable if its zero solution is globally asymptotically stable for any nonlinearity $f(\cdot) \in F_{[k_1,k_2]}$.

We make the following assumptions for system (2).

A1 The time-delay $\tau_i(t)$ $(i = 1,2,\ldots,m)$ are continuous and piecewise differentiable functions with

$$0 \le \tau_i(t) \le \tau_i, \qquad \dot{\tau}_i(t) \le \alpha_i < 1,$$

where τ_i, α_i ($i = 1, 2, \ldots, m$) are constants. At the nondifferentiability points of $\tau_i(t)$, $\dot{\tau}_i(t)$ represents $\max[\dot{\tau}_i(t-0), \dot{\tau}_i(t+0)]$.

A2 For any $t \in [0, \infty)$, there exist matrices $P_i > 0$, $G_i > 0$ ($i = 1, 2, \ldots, m$) such that

$$\lambda\big(P_iA_{ii}(t) + A_{ii}^T(t)P_i + G_i\big) \le -\delta_i(t) \le -\xi_i < 0,$$

where $\delta_i(t) > 0$, $\xi_i > 0$ ($i = 1, 2, \ldots, m$) are functions and constants, respectively.

A3 For any $t \in [0, \infty)$,

$$\frac{\|P_iA_{ij}(t) + A_{ji}^T(t)P_j\|}{\sqrt{\delta_i(t)\delta_j(t)}} \le \eta_{ij}, \qquad \frac{\|P_iB_{ij}(t)\|}{\sqrt{\delta_i(t)(1-\alpha_j)\lambda_{\min}(G_j)}} \le \gamma_{ij},$$

where η_{ij} ($i, j = 1, 2, \ldots, m$; $i \ne j$), γ_{ij} ($i, j = 1, 2, \ldots, m$) are constants, and $\eta_{ij} = \eta_{ji}$.

A4 For any $t \in [0, \infty)$,

$$\frac{\|P_ib_i(t) + \frac{1}{2}c_i(t)\|}{\sqrt{\delta_i(t)\rho(t)}} \le \mu_i,$$

where μ_i ($i = 1, 2, \ldots, m$) are constants.

To simplify the statements, we define the following auxiliary matrices:

$$D = \begin{bmatrix} -1 & \eta_{12} & \cdots & \eta_{1m} \\ \eta_{21} & -1 & \cdots & \eta_{2m} \\ \vdots & \vdots & & \vdots \\ \eta_{m1} & \eta_{m2} & \cdots & -1 \end{bmatrix}, \qquad R = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} \\ \vdots & \vdots & & \vdots \\ \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mm} \end{bmatrix}, \qquad U = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix}.$$

Notice that by A3 the matrix D is symmetric.

Theorem 1 *Under* A1-A4, *system* (2) *is absolutely stable if* $D + RR^T + UU^T < 0$.

Proof By utilizing P_i, G_i ($i = 1, 2, \ldots, m$) appearing in the assumptions, we choose the following Lyapunov-Krasovskii functional:

$$V(t, \phi) = \sum_{i=1}^m \left(x_i^T(t)P_ix_i(t) + \int_{t-\tau_i(t)}^t x_i^T(s)G_ix_i(s)\,ds \right) + \int_0^{\sigma(t)} f(s)\,ds.$$

We can verify that if $f \in F_{[k_1,k_2]}$, then $\frac{1}{2}k_1\sigma^2(t) \le \int_0^{\sigma(t)} f(s)\,ds \le \frac{1}{2}k_2\sigma^2(t)$. Letting $\tau = \max\{\tau_i, i = 1, 2, \ldots, m\}$, $V(t, \phi)$ satisfies

$$\begin{aligned} &\sum_{i=1}^m \lambda_{\min}(P_i)\|x_i(t)\|^2 + \frac{1}{2}k_1\sigma^2(t) \\ &\quad \le V(t, \phi) \le \sum_{i=1}^m \left(\lambda_{\max}(P_i)\|x_i(t)\|^2 + \lambda_{\max}(G_i) \int_{t-\tau}^t \|x_i(s)\|^2\,ds \right) + \frac{1}{2}k_2\sigma^2(t). \end{aligned}$$

Further, we have

$$\begin{aligned}
&\min\left\{\lambda_{\min}(P_1),\dots,\lambda_{\min}(P_m),\frac{1}{2}k_1\right\}\|\phi(0)\|^2\\
&\quad\le V(t,\phi)\\
&\quad\le \max\left\{\lambda_{\max}(P_1),\dots,\lambda_{\max}(P_m),\frac{k_2}{2}\right\}\|\phi(0)\|^2\\
&\qquad+\max\{\lambda_{\max}(G_1),\dots,\lambda_{\max}(G_m)\}\int_{-\tau}^{0}\|\phi(s)\|^2\,ds.
\end{aligned}$$

Namely, let

$$\begin{aligned}
&W_1(s)=\min\left\{\lambda_{\min}(P_1),\dots,\lambda_{\min}(P_m),\frac{1}{2}k_1\right\}s^2,\\
&W_2(s)=\max\left\{\lambda_{\max}(P_1),\dots,\lambda_{\max}(P_m),\frac{1}{2}k_2\right\}s^2,\\
&W_3(s)=\max\{\lambda_{\max}(G_1),\dots,\lambda_{\max}(G_m)\}\,s^2.
\end{aligned}$$

Then we obtain, for $t\ge 0$, the following inequalities:

$$W_1(\|\phi(0)\|)\le V(t,\phi)\le W_2(\|\phi(0)\|)+W_3(\|\|\phi\|\|).$$

Thus, condition (i) of Lemma 2 is satisfied. Moreover, $\lim_{s\to\infty}W_1(s)=\infty$.

The derivative of $V(t,\phi)$ along the trajectory of system (2) is

$$\begin{aligned}
&\dot V(t,\phi)|_{(2)}\\
&\quad=\sum_{i=1}^{m}\bigl(2x_i^T(t)P_i\dot x_i(t)+x_i^T(t)G_ix_i(t)\bigr)\\
&\qquad-\sum_{i=1}^{m}\bigl(1-\dot\tau_i(t)\bigr)x_i^T\bigl(t-\tau_i(t)\bigr)G_ix_i\bigl(t-\tau_i(t)\bigr)+\dot\sigma(t)f\bigl(\sigma(t)\bigr)\\
&\quad=\sum_{i=1}^{m}x_i^T(t)\bigl(P_iA_{ii}(t)+A_{ii}^T(t)P_i+G_i\bigr)x_i(t)+2\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\ne i}}^{m}x_i^T(t)P_iA_{ij}(t)x_j(t)\\
&\qquad+2\sum_{i=1}^{m}\sum_{j=1}^{m}x_i^T(t)P_iB_{ij}(t)x_j\bigl(t-\tau_j(t)\bigr)+2\sum_{i=1}^{m}x_i^T(t)P_ib_i(t)f\bigl(\sigma(t)\bigr)\\
&\qquad-\sum_{i=1}^{m}\bigl(1-\dot\tau_i(t)\bigr)x_i^T\bigl(t-\tau_i(t)\bigr)G_ix_i\bigl(t-\tau_i(t)\bigr)+\sum_{i=1}^{m}x_i^T(t)c_i(t)f\bigl(\sigma(t)\bigr)\\
&\qquad-\rho(t)f^2\bigl(\sigma(t)\bigr)\\
&\quad=\sum_{i=1}^{m}x_i^T(t)\bigl(P_iA_{ii}(t)+A_{ii}^T(t)P_i+G_i\bigr)x_i(t)+2\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\ne i}}^{m}x_i^T(t)P_iA_{ij}(t)x_j(t)
\end{aligned}$$

$$
\begin{aligned}
&+2\sum_{i=1}^{m}\sum_{j=1}^{m}x_i^T(t)P_iB_{ij}(t)x_j\bigl(t-\tau_j(t)\bigr)+2\sum_{i=1}^{m}x_i^T(t)\biggl(P_ib_i(t)+\frac{1}{2}c_i(t)\biggr)f\bigl(\sigma(t)\bigr)\\
&-\sum_{i=1}^{m}\bigl(1-\dot{\tau}_i(t)\bigr)x_i^T\bigl(t-\tau_i(t)\bigr)G_ix_i\bigl(t-\tau_i(t)\bigr)-\rho(t)f^2\bigl(\sigma(t)\bigr)\\
=&\sum_{i=1}^{m}x_i^T(t)\bigl(P_iA_{ii}(t)+A_{ii}^T(t)P_i+G_i\bigr)x_i(t)+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}x_i^T(t)\bigl(P_iA_{ij}(t)+A_{ji}^T(t)P_j\bigr)x_j(t)\\
&+2\sum_{i=1}^{m}\sum_{j=1}^{m}x_i^T(t)P_iB_{ij}(t)x_j\bigl(t-\tau_j(t)\bigr)+2\sum_{i=1}^{m}x_i^T(t)\biggl(P_ib_i(t)+\frac{1}{2}c_i(t)\biggr)f\bigl(\sigma(t)\bigr)\\
&-\sum_{i=1}^{m}\bigl(1-\dot{\tau}_i(t)\bigr)x_i^T\bigl(t-\tau_i(t)\bigr)G_ix_i\bigl(t-\tau_i(t)\bigr)-\rho(t)f^2\bigl(\sigma(t)\bigr).
\end{aligned}
$$

From A1 and A2, using properties of the matrix norm, we get

$$
\begin{aligned}
&\dot{V}(t,\phi)|_{(2)}\\
&\quad\leq\sum_{i=1}^{m}x_i^T(t)\bigl(P_iA_{ii}(t)+A_{ii}^T(t)P_i+G_i\bigr)x_i(t)+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}x_i^T(t)\bigl(P_iA_{ij}(t)+A_{ji}^T(t)P_j\bigr)x_j(t)\\
&\qquad+2\sum_{i=1}^{m}\sum_{j=1}^{m}x_i^T(t)P_iB_{ij}(t)x_j\bigl(t-\tau_j(t)\bigr)+2\sum_{i=1}^{m}x_i^T(t)\biggl(P_ib_i(t)+\frac{1}{2}c_i(t)\biggr)f\bigl(\sigma(t)\bigr)\\
&\qquad-\sum_{i=1}^{m}(1-\alpha_i)x_i^T\bigl(t-\tau_i(t)\bigr)G_ix_i\bigl(t-\tau_i(t)\bigr)-\rho(t)f^2\bigl(\sigma(t)\bigr)\\
&\quad\leq-\sum_{i=1}^{m}\delta_i(t)\bigl\|x_i(t)\bigr\|^2+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\bigl\|P_iA_{ij}(t)+A_{ji}^T(t)P_j\bigr\|\bigl\|x_i(t)\bigr\|\bigl\|x_j(t)\bigr\|\\
&\qquad+2\sum_{i=1}^{m}\sum_{j=1}^{m}\bigl\|P_iB_{ij}(t)\bigr\|\bigl\|x_i(t)\bigr\|\bigl\|x_j\bigl(t-\tau_j(t)\bigr)\bigr\|\\
&\qquad+2\sum_{i=1}^{m}\biggl\|P_ib_i(t)+\frac{c_i(t)}{2}\biggr\|\bigl\|x_i(t)\bigr\|\bigl|f\bigl(\sigma(t)\bigr)\bigr|\\
&\qquad-\sum_{i=1}^{m}(1-\alpha_i)\lambda_{\min}(G_i)\bigl\|x_i\bigl(t-\tau_i(t)\bigr)\bigr\|^2-\rho(t)f^2\bigl(\sigma(t)\bigr).
\end{aligned}
$$

To fully utilize A3, A4, and the unbounded terms in system coefficients, we take $\sqrt{\delta_i(t)}\|x_i(t)\|$, $\sqrt{(1-\alpha_i)\lambda_{\min}(G_i)}\|x(t-\tau_i(t))\|$ $(i=1,2,\ldots,m)$, and $\sqrt{\rho(t)}|f(\sigma(t))|$ as the variables of the following quadratic form. Then the inequality becomes

$$
\begin{aligned}
&\dot{V}(t,\phi)|_{(2)}\\
&\quad\leq-\sum_{i=1}^{m}\delta_i(t)\bigl\|x_i(t)\bigr\|^2+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\frac{\|P_iA_{ij}(t)+A_{ji}^T(t)P_j\|}{\sqrt{\delta_i(t)\delta_j(t)}}\sqrt{\delta_i(t)}\bigl\|x_i(t)\bigr\|\cdot\sqrt{\delta_j(t)}\bigl\|x_j(t)\bigr\|
\end{aligned}
$$

$$
\begin{aligned}
&+ 2\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\|P_iB_{ij}(t)\|}{\sqrt{\delta_i(t)(1-\alpha_j)\lambda_{\min}(G_j)}}\sqrt{\delta_i(t)}\big\|x_i(t)\big\|\cdot\sqrt{(1-\alpha_j)\lambda_{\min}(G_j)}x_j\big(t-\tau_j(t)\big)\\
&+ 2\sum_{i=1}^{m}\frac{\|P_ib_i(t)+\frac{1}{2}c_i(t)\|}{\sqrt{\delta_i(t)\rho(t)}}\sqrt{\delta_i(t)}\big\|x_i(t)\big\|\cdot\sqrt{\rho(t)}\big|f\big(\sigma(t)\big)\big|\\
&-\sum_{i=1}^{m}(1-\alpha_i)\lambda_{\min}(G_i)\big\|x_i\big(t-\tau_i(t)\big)\big\|^2-\rho(t)f^2\big(\sigma(t)\big)\\
\le{}&-\sum_{i=1}^{m}\delta_i(t)\big\|x_i(t)\big\|^2+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\eta_{ij}\sqrt{\delta_i(t)}\big\|x_i(t)\big\|\cdot\sqrt{\delta_j(t)}\big\|x_j(t)\big\|\\
&+\sum_{i=1}^{m}\sum_{j=1}^{m}2\gamma_{ij}\sqrt{\delta_i(t)}\big\|x_i(t)\big\|\cdot\sqrt{(1-\alpha_j)\lambda_{\min}(G_j)}\big\|x_j\big(t-\tau_j(t)\big)\big\|\\
&+\sum_{i=1}^{m}2\mu_i\sqrt{\delta_i(t)}\big\|x_i(t)\big\|\cdot\sqrt{\rho(t)}\big|f\big(\sigma(t)\big)\big|-\sum_{i=1}^{m}(1-\alpha_i)\lambda_{\min}(G_i)\big\|x_i\big(t-\tau_i(t)\big)\big\|^2\\
&-\rho(t)f^2\big(\sigma(t)\big)\\
={}&Y^T\hat{D}Y,
\end{aligned}
$$

where

$$
Y=\begin{bmatrix}
\sqrt{\delta_1(t)}\|x_1(t)\|\\
\vdots\\
\sqrt{\delta_m(t)}\|x_m(t)\|\\
\sqrt{(1-\alpha_1)\lambda_{\min}(G_1)}\|x_1(t-\tau_1(t))\|\\
\vdots\\
\sqrt{(1-\alpha_m)\lambda_{\min}(G_m)}\|x_m(t-\tau_m(t))\|\\
\sqrt{\rho(t)}|f(\sigma(t))|
\end{bmatrix},
$$

$$
\hat{D}=\begin{bmatrix}
-1 & \eta_{12} & \cdots & \eta_{1m} & \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} & \mu_1\\
\eta_{21} & -1 & \cdots & \eta_{2m} & \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} & \mu_2\\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots\\
\eta_{m1} & \eta_{m2} & \cdots & -1 & \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mm} & \mu_m\\
\gamma_{11} & \gamma_{21} & \cdots & \gamma_{m1} & -1 & 0 & \cdots & 0 & 0\\
\gamma_{12} & \gamma_{22} & \cdots & \gamma_{m2} & 0 & -1 & \cdots & 0 & 0\\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots\\
\gamma_{1m} & \gamma_{2m} & \cdots & \gamma_{mm} & 0 & 0 & \cdots & -1 & 0\\
\mu_1 & \mu_2 & \cdots & \mu_m & 0 & 0 & \cdots & 0 & -1
\end{bmatrix}.
$$

Using the preceding notations, we have

$$
\hat{D}=\begin{bmatrix}
D & R & U\\
R^T & -I & 0\\
U^T & 0 & -1
\end{bmatrix}.
$$

By Lemma 1, $D + RR^T + UU^T < 0$ implies $\hat{D} < 0$. Let $-\beta$ ($\beta > 0$) be the maximum eigenvalue of $\hat{D}$. Then, $\dot{V}(t,\phi)|_{(2)}$ satisfies

$$\begin{aligned}
&\dot{V}(t,\phi)|_{(2)} \\
&\quad \le -\beta\left(\sum_{i=1}^{m}\left(\delta_i(t)\|x_i(t)\|^2 + (1-\alpha_i)\lambda_{\min}(G_i)\|x_i(t-\tau_i(t))\|^2\right) + \rho(t)\left|f(\sigma(t))\right|^2\right) \\
&\quad \le -\beta\left(\sum_{i=1}^{m}\xi_i\|x_i(t)\|^2 + \rho\left|f(\sigma(t))\right|^2\right).
\end{aligned}$$

Since $\sigma(t)f(\sigma(t)) \ge k_1\sigma^2(t)$, we obtain $|f(\sigma(t))| \ge k_1|\sigma(t)|$. Hence,

$$\begin{aligned}
&\dot{V}(t,\phi)|_{(2)} \\
&\quad \le -\beta\left(\sum_{i=1}^{m}\xi_i\|x_i(t)\|^2 + \rho k_1^2\sigma^2(t)\right) \\
&\quad \le -\beta\min\{\xi_1,\dots,\xi_m,\rho k_1^2\}\left(\sum_{i=1}^{m}\|x_i(t)\|^2 + \sigma^2(t)\right).
\end{aligned}$$

Letting $W_4(s) = -\beta\min\{\xi_1,\dots,\xi_m,\rho k_1^2\}s^2$, we have

$$\dot{V}(t,\phi)|_{(2)} \le W_4\left(\|\phi(0)\|\right).$$

This means that condition (ii) of Lemma 2 is satisfied. Thus, by Lemma 2 and Definition 1, system (2) is absolutely stable. This completes the proof. □

A5 For any $t \in [0,\infty)$, there exist matrices $P_i > 0$, $G_i > 0$ ($i = 1,2,\dots,m$) such that

$$\lambda\left(P_iA_{ii}(t) + A_{ii}^T(t)P_i + G_i\right) \le -\delta_i(t) < 0,$$

where $\delta_i(t) > 0$ ($i = 1,2,\dots,m$). Let $\delta(t) = \min\{\delta_1(t),\delta_2(t),\dots,\delta_m(t)\}$ and assume that $\lim_{t\to\infty}\delta(t) = \infty$.

Corollary 1 *Under* A1, A3, A4, *and* A5, *system* (2) *is absolutely stable if* $D + RR^T + UU^T < 0$.

Indeed, by exploiting the limit property, we derive from $\lim_{t\to\infty}\delta(t) = \infty$ that, for any $\xi_i > 0$ ($i = 1,2,\dots,m$) (here let $\xi_i = 1$), there exists $T \ge 0$ such that, for $t > T$,

$$-\delta(t) \le -\xi_i.$$

This implies that

$$\lambda\left(P_iA_{ii}(t) + A_{ii}^T(t)P_i + G_i\right) \le -\delta_i(t) \le -\delta(t) < -\xi_i.$$

The result then follows immediately from Theorem 1.

In fact, we only need to ensure that the conditions mentioned are satisfied when time t is sufficiently large, because asymptotic stability refers to the behavior of the dynamic systems as time tends to infinity. In other words, A2-A5 can be written as follows: There exists $T \geq 0$ such that the corresponding conditions are satisfied for $t > T$. In particular, A3 and A4 can be rewritten as follows.

A6

$$\overline{\lim_{t\to\infty}} \frac{\|P_iA_{ij}(t) + A_{ji}^T(t)P_j\|}{\sqrt{\delta_i(t)\delta_j(t)}} = \eta_{ij}, \qquad \overline{\lim_{t\to\infty}} \frac{\|P_iB_{ij}(t)\|}{\sqrt{\delta_i(t)(1-\alpha_j)\lambda_{\min}(G_j)}} = \gamma_{ij},$$

where η_{ij} $(i,j = 1,2,\ldots,m; i \neq j)$ and γ_{ij} $(i,j = 1,2,\ldots,m)$ are constants, and $\eta_{ij} = \eta_{ji}$.

A7

$$\overline{\lim_{t\to\infty}} \frac{\|P_ib_i(t) + \frac{1}{2}c_i(t)\|}{\sqrt{\delta_i(t)\rho(t)}} = \mu_i,$$

where μ_i $(i = 1,2,\ldots,m)$ are constants.

Corollary 2 *Under* A1, A2, A6, *and* A7, *system* (2) *is absolutely stable if* $D + RR^T + UU^T < 0$.

Corollary 3 *Under* A1, A2, A6, *and* A7, *system* (2) *is absolutely stable if one of the following two conditions is satisfied*:

(I) $\gamma_{ij} = 0$ $(i,j = 1,2,\ldots,m)$ *and* $D + UU^T < 0$.

(II) $\mu_i = 0$ $(i = 1,2,\ldots,m)$ *and* $D + RR^T < 0$.

Corollary 4 *Under* A1, A5, A6, *and* A7, *system* (2) *is absolutely stable if* $\gamma_{ij} = \mu_i = 0$ $(i,j = 1,2,\ldots,m)$ *and* $D < 0$.

The proofs of these corollaries are relatively simple and are omitted here.

Remark 1 It should be pointed out that Lurie system (2) under consideration is an extension of Lurie indirect control systems discussed in [4, 17] since the coefficient matrices are norm-unbounded. This is the main feature of this paper. All the theorems and corollaries are applicable to the large-scale time-delay Lurie systems with unbounded coefficients. Particularly, for this class of systems with bounded or constant coefficients, these results are also effective.

3 Absolute stability of large-scale Lurie systems with multiple nonlinearities

Consider the following large-scale time-delay Lurie indirect control system with unbounded coefficients and multiple nonlinearities:

$$\begin{cases} \dot{x}_i(t) = \sum_{j=1}^m A_{ij}(t)x_j(t) + \sum_{j=1}^m B_{ij}(t)x_j(t-\tau_j(t)) + \sum_{j=1}^r b_{ij}(t)f_j(\sigma_j(t)), \\ \quad i = 1,2,\ldots,m, \\ \dot{\sigma}_l(t) = \sum_{j=1}^m c_{lj}^T(t)x_j(t) - \rho_l(t)f_l(\sigma_l(t)), \quad l = 1,2,\ldots,r, \end{cases} \tag{3}$$

where $x_i(t) \in R^{n_i}$ $(i = 1,2,\ldots,m)$ and $\sigma_l(t) \in R$ $(l = 1,2,\ldots,r)$ are the state vectors; the vector functions $b_{ij}(t) \in R^{n_i}$ $(i = 1,2,\ldots,m; j = 1,2,\ldots,r)$ and $c_{lj}(t) \in R^{n_j}$ $(l = 1,2,\ldots,r; j =$

$1,2,\ldots,m$) are continuous in $[0,\infty)$, $\sum_{i=1}^{m} n_i = n$; the matrix functions $A_{ij}(t), B_{ij}(t) \in R^{n_i \times n_j}$ ($i,j = 1,2,\ldots,m$) are continuous in $[0,\infty)$; $\tau_j(t)$ ($j = 1,2,\ldots,m$) refer to the time-delays; $\rho_l(t)$ ($l = 1,2,\ldots,r$) is continuous in $[0,\infty)$ and satisfies $\rho_l(t) \geq \rho_l > 0$ with constants and ρ_l. The continuous nonlinearities $f_l(\sigma_l)$ ($l = 1,2,\ldots,r$) satisfy the following sector condition:

$$F_{[k_{l1},k_{l2}]} = \{f_l(\cdot)|f_l(0) = 0; k_{l1}\sigma_l^2(t) \leq \sigma_l(t)f_l(\sigma_l(t)) \leq k_{l2}\sigma_l^2(t), \sigma_l(t) \in R - \{0\}\},$$

where k_{l1}, k_{l2} are constants such that $k_{l2} > k_{l1} > 0$.

Definition 2 System (3) is said to be absolutely stable if its zero solution is globally asymptotically stable for any nonlinearity $f_i(\cdot) \in F_{[k_{i1},k_{i2}]}$ ($i = 1,2,\ldots,r$).

The aforementioned assumptions A1-A3 and the following assumptions are critical to system (3).

A8 For any $t \in [0,\infty)$,

$$\frac{\|P_i b_{ij}(t) + \frac{1}{2}c_{ji}(t)\|}{\sqrt{\delta_i(t)\rho_j(t)}} \leq \mu_{ij},$$

where μ_{ij} ($i = 1,2,\ldots,m$; $j = 1,2,\ldots,r$) are constants.

The definitions of matrices D and R are the same as before, whereas the matrix U is redefined as

$$U = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1r} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2r} \\ \vdots & \vdots & & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mr} \end{bmatrix}.$$

Theorem 2 *Under* A1, A2, A3, *and* A8, *system* (3) *is absolutely stable if* $D + RR^T + UU^T < 0$.

Proof Let us construct the following Lyapunov-Krasovskii functional:

$$V(t,\phi) = \sum_{i=1}^{m}\left(x_i^T(t)P_i x_i(t) + \int_{t-\tau_i(t)}^{t} x_i^T(s)G_i x_i(s)\,ds\right) + \sum_{i=1}^{r}\int_0^{\sigma_i(t)} f_i(s)\,ds.$$

As in the proof of Theorem 1, we can show that $V(t,\phi)$ satisfies condition (i) of Lemma 2.

Then, taking the time derivative of $V(t,\phi)$ along the trajectory of system (3) yields

$$\begin{aligned} &\dot{V}(t,\phi)|_{(3)} \\ &\quad= \sum_{i=1}^{m} x_i^T(t)\big(P_iA_{ii}(t) + A_{ii}^T(t)P_i + G_i\big)x_i(t) + \sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m} x_i^T(t)\big(P_iA_{ij}(t) + A_{ji}^T(t)P_j\big)x_j(t) \\ &\qquad + 2\sum_{i=1}^{m}\sum_{j=1}^{m} x_i^T P_i B_{ij}(t)x_j\big(t-\tau_j(t)\big) + 2\sum_{i=1}^{m}\sum_{j=1}^{r} x_i^T(t)\left(P_ib_{ij}(t) + \frac{1}{2}c_{ji}(t)\right)f_j\big(\sigma_j(t)\big) \\ &\qquad - \sum_{i=1}^{m}\big(1-\dot{\tau}_i(t)\big)x_i^T\big(t-\tau_i(t)\big)G_ix_i\big(t-\tau_i(t)\big) - \sum_{i=1}^{r}\rho_i(t)\big(f_i\big(\sigma_i(t)\big)\big)^2. \end{aligned}$$

Analogously, applying A1, A2, and properties of norms, we obtain

$$\begin{aligned}
&\dot{V}(t,\phi)|_{(3)} \\
&\le -\sum_{i=1}^{m}\delta_i(t)\|x_i(t)\|^2+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\|P_iA_{ij}(t)+A_{ji}^T(t)P_j\|\|x_i(t)\|\|x_j(t)\| \\
&\quad+2\sum_{i=1}^{m}\sum_{j=1}^{m}\|P_iB_{ij}(t)\|\|x_i(t)\|\|x_j(t-\tau_j(t))\| \\
&\quad+2\sum_{i=1}^{m}\sum_{j=1}^{r}\left\|P_ib_{ij}(t)+\frac{1}{2}c_{ji}(t)\right\|\|x_i(t)\||f_j(\sigma_j(t))| \\
&\quad-\sum_{i=1}^{m}(1-\alpha_i)\lambda_{\min}(G_i)\|x_i(t-\tau_i(t))\|^2-\sum_{i=1}^{r}\rho_i(t)(f_i(\sigma_i(t)))^2.
\end{aligned}$$

By virtue of $\sqrt{\delta_i(t)}\|x_i(t)\|$, $\sqrt{(1-\alpha_i)\lambda_{\min}(G_i)}\|x(t-\tau_i(t))\|$ $(i=1,2,\ldots,m)$, and $\sqrt{\rho_i(t)}|f_i(\sigma_i(t))|$ $(i=1,2,\ldots,r)$, we can continue estimating the upper bound of $\dot{V}(t,\phi)|_{(3)}$ based on A3 and A8:

$$\begin{aligned}
&\dot{V}(t,\phi)|_{(3)} \\
&\le -\sum_{i=1}^{m}\delta_i(t)\|x_i(t)\|^2+\sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\eta_{ij}\sqrt{\delta_i(t)}\|x_i(t)\|\cdot\sqrt{\delta_j(t)}\|x_j(t)\| \\
&\quad+\sum_{i=1}^{m}\sum_{j=1}^{m}2\gamma_{ij}\sqrt{\delta_i(t)}\|x_i(t)\|\cdot\sqrt{(1-\alpha_j)\lambda_{\min}(G_j)}\|x_j(t-\tau_j(t))\| \\
&\quad+2\sum_{i=1}^{m}\sum_{j=1}^{r}\mu_{ij}\|x_i(t)\||f_j(\sigma_j(t))|-\sum_{i=1}^{m}(1-\alpha_i)\lambda_{\min}(G_i)\|x_i(t-\tau_i(t))\|^2 \\
&\quad-\sum_{i=1}^{r}\rho_i(t)(f_i(\sigma_i(t)))^2 \\
&=Y^T\hat{D}Y,
\end{aligned}$$

where

$$Y=\begin{bmatrix}
\sqrt{\delta_1(t)}\|x_1(t)\| \\
\vdots \\
\sqrt{\delta_m(t)}\|x_m(t)\| \\
\sqrt{(1-\alpha_1)\lambda_{\min}(G_1)}\|x_1(t-\tau_1(t))\| \\
\vdots \\
\sqrt{(1-\alpha_m)\lambda_{\min}(G_m)}\|x_m(t-\tau_m(t))\| \\
\sqrt{\rho_1(t)}|f_1(\sigma_1)| \\
\vdots \\
\sqrt{\rho_r(t)}|f_r(\sigma_r)|
\end{bmatrix},$$

$$\hat{D} = \begin{bmatrix} -1 & \eta_{12} & \cdots & \eta_{1m} & \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1m} & \mu_{11} & \mu_{12} & \cdots & \mu_{1r} \\ \eta_{21} & -1 & \cdots & \eta_{2m} & \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2m} & \mu_{21} & \mu_{22} & \cdots & \mu_{2r} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \eta_{m1} & \eta_{m2} & \cdots & -1 & \gamma_{m1} & \gamma_{m2} & \cdots & \gamma_{mm} & \mu_{m1} & \mu_{m2} & \cdots & \mu_{mr} \\ \gamma_{11} & \gamma_{21} & \cdots & \gamma_{m1} & -1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{m2} & 0 & -1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \gamma_{1m} & \gamma_{2m} & \cdots & \gamma_{mm} & 0 & 0 & \cdots & -1 & 0 & 0 & \cdots & 0 \\ \mu_{11} & \mu_{21} & \cdots & \mu_{m1} & 0 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ \mu_{12} & \mu_{22} & \cdots & \mu_{m2} & 0 & 0 & \cdots & 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \mu_{1r} & \mu_{2r} & \cdots & \mu_{mr} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & -1 \end{bmatrix}.$$

Using the preceding notations, we have

$$\hat{D} = \begin{bmatrix} D & R & U \\ R^T & -I & 0 \\ U^T & 0 & -I \end{bmatrix}.$$

The following proof is similar to that of Theorem 1. System (3) is absolutely stable if $\hat{D} < 0$. Accordingly, the problem comes down to seeking the conditions for $\hat{D} < 0$. By Lemma 1, $\hat{D} < 0$ is equivalent to $D + RR^T + UU^T < 0$. This completes the proof. □

Corollary 5 *Under* A1, A3, A5, *and* A8, *system* (3) *is absolutely stable if* $D + RR^T + UU^T < 0$.

Similarly to the single nonlinearity case, to conclude the absolute stability for system (3), we only need to require that there exists a constant $T \geq 0$ such that the inequality conditions in A2, A3, A5, and A8 are satisfied for $t > T$. Hence, μ_{ij} $(i = 1, 2, \dots, m; j = 1, 2, \dots, r)$ in A8 can be computed by the corresponding upper limit (if the upper limit is a finite value).

A9

$$\overline{\lim_{t\to\infty}} \frac{\|P_i b_{ij}(t) + \frac{1}{2} c_{ji}(t)\|}{\sqrt{\delta_i(t)\rho_j(t)}} = \mu_{ij},$$

where μ_{ij} $(i = 1, 2, \dots, m; j = 1, 2, \dots, r)$ are constants.

Corollary 6 *Under* A1, A5, A6, *and* A9, *system* (3) *is absolutely stable if* $D + RR^T + UU^T < 0$.

Corollary 7 *Under* A1, A2, A6, *and* A9, *system* (3) *is absolutely stable if one of the following two conditions is satisfied*:

(I) $\gamma_{ij} = 0$ $(i, j = 1, 2, \dots, m)$ *and* $D + UU^T < 0$.

(II) $\mu_{ij} = 0$ $(i = 1, 2, \dots, m; j = 1, 2, \dots, r)$ *and* $D + RR^T < 0$.

Corollary 8 *Under* A1, A5, A6, *and* A9, *system* (3) *is absolutely stable if* $\gamma_{ij} = 0$ $(i, j = 1, 2, \dots, m)$, $\mu_{ij} = 0$ $(i = 1, 2, \dots, m; j = 1, 2, \dots, r)$, *and* $D < 0$.

The proofs of these corollaries are relatively simple and are omitted here.

Remark 2 Recently, the absolute stability of time-delay Lurie indirect control systems was studied in [5, 25]. However, these studies were only applicable to the independent Lurie time-delay systems. Although the authors in [18] considered large-scale Lurie indirect control systems, they did not take the time-delay into account. Therefore, the results in this paper have a greater range of applications.

4 Numerical simulation

The following numerical examples are presented to illustrate the effectiveness of the proposed theoretical results.

Example 1 Consider the following large-scale time-delay Lurie indirect control system with a single nonlinearity:

$$\begin{cases} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -2t-\frac{1}{2} & 1 \\ t & -3t-\frac{1}{2} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \sqrt{t} \\ 10 \end{bmatrix} x_3(t) \\ \qquad + \begin{bmatrix} \sqrt{\frac{t}{2}} & 0 \\ 0 & \sqrt{\frac{t}{6}} \end{bmatrix}\begin{bmatrix} x_1(t-\tau_1(t)) \\ x_2(t-\tau_1(t)) \end{bmatrix} + \begin{bmatrix} t^{\frac{1}{3}} \\ 0 \end{bmatrix} x_3(t-\tau_2(t)) \\ \qquad + \begin{bmatrix} -\frac{t}{2} \\ 0 \end{bmatrix} f(\sigma(t)), \\ \dot{x}_3(t) = \begin{bmatrix} \sqrt{t} & \sqrt{t} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} - (t+\frac{1}{2})x_3(t) + \begin{bmatrix} 0 & t^{\frac{1}{4}} \end{bmatrix}\begin{bmatrix} x_1(t-\tau_1(t)) \\ x_2(t-\tau_1(t)) \end{bmatrix} \\ \qquad + \sqrt{\frac{t}{8}} x_3(t-\tau_2(t)) - tf(\sigma(t)), \\ \dot{\sigma}(t) = \begin{bmatrix} t & \sqrt{t} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 2tx_3(t) - (t+1)f(\sigma(t)), \end{cases} \tag{4}$$

where $\tau_1(t) = 3 + 0.5\sin t$, $\tau_2(t) = 7$, and $f(\cdot) \in F_{[0.01,100]}$.

This system is of the form (2) with

$$A_{11}(t) = \begin{bmatrix} -2t-\frac{1}{2} & 1 \\ t & -3t-\frac{1}{2} \end{bmatrix}, \qquad A_{12}(t) = \begin{bmatrix} \sqrt{t} \\ 10 \end{bmatrix},$$

$$A_{21}(t) = \begin{bmatrix} \sqrt{t} & \sqrt{t} \end{bmatrix}, \qquad A_{22}(t) = -t - \frac{1}{2},$$

$$B_{11}(t) = \begin{bmatrix} \sqrt{\frac{t}{2}} & 0 \\ 0 & \sqrt{\frac{t}{6}} \end{bmatrix}, \qquad B_{12}(t) = \begin{bmatrix} t^{\frac{1}{3}} \\ 0 \end{bmatrix}, \qquad B_{21}(t) = \begin{bmatrix} 0 & t^{\frac{1}{4}} \end{bmatrix}, \qquad B_{22}(t) = \sqrt{\frac{t}{8}},$$

$$b_1(t) = \begin{bmatrix} -\frac{t}{2} \\ 0 \end{bmatrix}, \qquad b_2(t) = -t, \qquad c_1(t) = \begin{bmatrix} t \\ \sqrt{t} \end{bmatrix}, \qquad c_2(t) = 2t, \qquad \rho(t) = t + 1.$$

This problem cannot be solved by the method of [18], which did not deal with the time-delay case. Now, we use the criteria proposed in this paper to analyze the absolute stability of this system.

Clearly, A1 is satisfied with $\tau_1 = 3.5$, $\alpha_1 = 0.5$, $\tau_2 = 7$, $\alpha_2 = 0$.

Then, by letting $P_1 = G_1 = I$ we have

$$P_1A_{11}(t) + A_{11}^T(t)P_1 + G_1 = \begin{bmatrix} -4t & t+1 \\ t+1 & -6t \end{bmatrix}$$

and

$$\lambda\big(P_1A_{11}(t)+A_{11}^T(t)P_1+G_1\big)\le -5t+\sqrt{2t^2+2t+1}.$$

Taking $T=\sqrt{2}$, for $t>T$, we obtain

$$\lambda\big(P_1A_{11}(t)+A_{11}^T(t)P_1+G_1\big)<-5t+\sqrt{2}(t+1)<-(4-\sqrt{2})t.$$

Thus, we can take

$$\delta_1(t)=(4-\sqrt{2})t.$$

Notice that, for $t>T$, we have the inequality

$$-\delta_1(t)\le -\xi_1=-(4\sqrt{2}-2).$$

Letting $P_2=G_2=1$, we obtain

$$P_2A_{22}(t)+A_{22}^T(t)P_2+G_2=-2t.$$

Then, we can take

$$\delta_2(t)=2t.$$

Similarly, for $t>T$, we have the inequality

$$-\delta_2(t)\le -\xi_2=-2\sqrt{2}.$$

Hence A2 is satisfied. In addition, we get

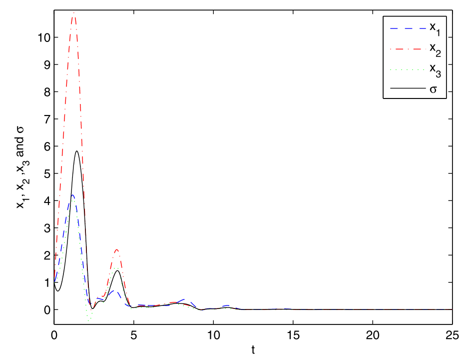
$$\lim_{t\to\infty}\frac{\|P_1A_{12}(t)+A_{21}^T(t)P_2\|}{\sqrt{\delta_1(t)\delta_2(t)}}=0,$$
$$\lim_{t\to\infty}\frac{\|P_2A_{21}(t)+A_{12}^T(t)P_1\|}{\sqrt{\delta_2(t)\delta_1(t)}}=0.$$

Moreover, we have

$$\lim_{t\to\infty}\frac{\|P_1B_{11}(t)\|}{\sqrt{\delta_1(t)(1-\alpha_1)\lambda_{\min}(G_1)}}=\frac{1}{\sqrt{4-\sqrt{2}}},$$
$$\lim_{t\to\infty}\frac{\|P_1B_{12}(t)\|}{\sqrt{\delta_1(t)(1-\alpha_2)\lambda_{\min}(G_2)}}=0,$$
$$\lim_{t\to\infty}\frac{\|P_2B_{21}(t)\|}{\sqrt{\delta_2(t)(1-\alpha_1)\lambda_{\min}(G_1)}}=0,$$
$$\lim_{t\to\infty}\frac{\|P_2B_{22}(t)\|}{\sqrt{\delta_2(t)(1-\alpha_2)\lambda_{\min}(G_2)}}=\frac{1}{4}.$$

Thus, A6 is satisfied with $\eta_{12}=0$, $\eta_{21}=0$, $\gamma_{11}=\frac{1}{\sqrt{4-\sqrt{2}}}$, $\gamma_{12}=0$, $\gamma_{21}=0$, $\gamma_{22}=\frac{1}{4}$.

Figure 1 The state response of system (4).



Furthermore, we derive

$$\lim_{t\to\infty}\frac{\|P_1b_1(t)+\frac{1}{2}c_1(t)\|}{\sqrt{\delta_1(t)\rho(t)}}=0,$$
$$\lim_{t\to\infty}\frac{\|P_2b_2(t)+\frac{1}{2}c_2(t)\|}{\sqrt{\delta_2(t)\rho(t)}}=0.$$

Hence A7 is satisfied with $\mu_1=\mu_2=0$.

Finally, we verify that

$$D+RR^T=\begin{bmatrix}\frac{\sqrt{2}-10}{14} & 0\\ 0 & -\frac{15}{16}\end{bmatrix}<0.$$

Thus, we conclude from Corollary 3 that system (4) is absolutely stable.

For simulation, we choose $f(\sigma(t)) = 2\sigma(t) + \sin\sigma(t)$ and $[x_1(s)\ x_2(s)\ x_3(s)\ \sigma(0)]^T = [1\ 1\ 1\ 1]^T$, $s\in[-7,0]$. The numerical simulation result is shown in Figure 1.

It is seen in Figure 1 that the states of system (4) converge to zero asymptotically. Thus the effectiveness of the proposed criteria is illustrated by the simulation result.

Example 2 Consider the following large-scale time-delay Lurie indirect control system with two nonlinearities:

$$\left\{\begin{aligned}
\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}-4t-\frac{1}{2} & 1\\ 0 & -3t-\frac{1}{2}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}\sqrt{t}\\ 0\end{bmatrix}x_3(t)\\
&\quad+\begin{bmatrix}\sqrt{t} & 0\\ 0 & \sqrt{\frac{t}{6}}\end{bmatrix}\begin{bmatrix}x_1(t-\tau_1(t))\\ x_2(t-\tau_1(t))\end{bmatrix}+\begin{bmatrix}t^{\frac{1}{4}}\\ 10\end{bmatrix}x_3(t-\tau_2(t))\\
&\quad+\begin{bmatrix}-t\\ 0\end{bmatrix}f_1(\sigma_1(t))+\begin{bmatrix}-t\\ 0\end{bmatrix}f_2(\sigma_2(t)),\\
\dot{x}_3(t) &= \begin{bmatrix}1 & \sqrt{t}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}-\left(t+\frac{1}{2}\right)x_3(t)+\begin{bmatrix}0 & t^{\frac{1}{4}}\end{bmatrix}\begin{bmatrix}x_1(t-\tau_1(t))\\ x_2(t-\tau_1(t))\end{bmatrix}\\
&\quad+\frac{\sqrt{t}}{4}x_3(t-\tau_2(t))-tf_1(\sigma_1(t))+2tf_2(\sigma_2(t)),\\
\dot{\sigma}_1(t) &= \begin{bmatrix}2t & 2\sqrt{t}\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+4tx_3(t)-(t+1)f_1(\sigma_1(t)),\\
\dot{\sigma}_2(t) &= \begin{bmatrix}2t & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}-6tx_3(t)-(3t+1)f_2(\sigma_2(t)),
\end{aligned}\right. \tag{5}$$

where $\tau_1(t)=3+0.5\sin t$, $\tau_2(t)=7$, and $f(\cdot)\in F_{[0.01,100]}$.

In comparison with system (3), the coefficient matrices here are

$$
\begin{aligned}
&A_{11}(t)=\begin{bmatrix}-4t-\frac{1}{2} & 1\\ 0 & -3t-\frac{1}{2}\end{bmatrix}, \qquad A_{12}(t)=\begin{bmatrix}\sqrt{t}\\ 0\end{bmatrix},\\
&A_{21}(t)=\begin{bmatrix}1 & \sqrt{t}\end{bmatrix}, \qquad A_{22}(t)=-t-\frac{1}{2},\\
&B_{11}(t)=\begin{bmatrix}\sqrt{t} & 0\\ 0 & \sqrt{\frac{t}{6}}\end{bmatrix}, \qquad B_{12}(t)=\begin{bmatrix}t^{\frac{1}{4}}\\ 10\end{bmatrix}, \qquad B_{21}(t)=\begin{bmatrix}0 & t^{\frac{1}{4}}\end{bmatrix}, \qquad B_{22}(t)=\frac{\sqrt{t}}{4},\\
&b_{11}(t)=\begin{bmatrix}-t\\ 0\end{bmatrix}, \qquad b_{12}(t)=\begin{bmatrix}-t\\ 0\end{bmatrix}, \qquad b_{21}(t)=-t, \qquad b_{22}(t)=2t,\\
&c_{11}(t)=\begin{bmatrix}2t\\ 2\sqrt{t}\end{bmatrix}, \qquad c_{12}(t)=4t, \qquad c_{21}(t)=\begin{bmatrix}2t\\ 0\end{bmatrix}, \qquad c_{22}(t)=-6t,\\
&\rho_1(t)=t+1, \qquad \rho_2(t)=3t+1.
\end{aligned}
$$

Next, we show that this system satisfies the conditions of Corollary 6.

Clearly, A1 is satisfied with $\tau_1 = 3.5$, $\alpha_1 = 0.5$, $\tau_2 = 7$, $\alpha_2 = 0$. Also, letting $P_1 = G_1 = I$, we obtain

$$
P_1A_{11}(t)+A_{11}^T(t)P_1+G_1=\begin{bmatrix}-8t & 1\\ 1 & -6t\end{bmatrix}
$$

and

$$
\lambda\bigl(P_1A_{11}(t)+A_{11}^T(t)P_1+G_1\bigr)\le -7t+\sqrt{t^2+1}.
$$

Taking $T = 1$, for $t > T$, we have

$$
\lambda\bigl(P_1A_{11}(t)+A_{11}^T(t)P_1+G_1\bigr)< -(7-\sqrt{2})t.
$$

Similarly, letting $P_2 = G_2 = 1$, we have

$$
P_2A_{22}(t)+A_{22}^T(t)P_2+G_2=-2t.
$$

Thus A5 is fulfilled with $\delta_1(t) = (7-\sqrt{2})t$, $\delta_2(t) = 2t$. Moreover,

$$
\begin{aligned}
&\lim_{t\to\infty}\frac{\|P_1A_{12}(t)+A_{21}^T(t)P_2\|}{\sqrt{\delta_1(t)\delta_2(t)}}=0,\\
&\lim_{t\to\infty}\frac{\|P_2A_{21}(t)+A_{12}^T(t)P_1\|}{\sqrt{\delta_2(t)\delta_1(t)}}=0.
\end{aligned}
$$

In addition, by simple calculation we get

$$\begin{aligned}
&\lim_{t\to\infty}\frac{\|P_1B_{11}(t)\|}{\sqrt{\delta_1(t)(1-\alpha_1)\lambda_{\min}(G_1)}}=\sqrt{\frac{2}{7-\sqrt{2}}},\\
&\lim_{t\to\infty}\frac{\|P_1B_{12}(t)\|}{\sqrt{\delta_1(t)(1-\alpha_2)\lambda_{\min}(G_2)}}=0,\\
&\lim_{t\to\infty}\frac{\|P_2B_{21}(t)\|}{\sqrt{\delta_2(t)(1-\alpha_1)\lambda_{\min}(G_1)}}=0,\\
&\lim_{t\to\infty}\frac{\|P_2B_{22}(t)\|}{\sqrt{\delta_2(t)(1-\alpha_2)\lambda_{\min}(G_2)}}=\frac{1}{4\sqrt{2}}.
\end{aligned}$$

Hence A6 is satisfied with $\eta_{12}=\eta_{21}=0$, $\gamma_{11}=\sqrt{\frac{2}{7-\sqrt{2}}}$, $\gamma_{12}=0$, $\gamma_{21}=0$, $\gamma_{22}=\frac{1}{4\sqrt{2}}$.

Furthermore, we have

$$\begin{aligned}
&\lim_{t\to\infty}\frac{\|P_1b_{11}(t)+\frac{1}{2}c_{11}(t)\|}{\sqrt{\delta_1(t)\rho_1(t)}}=0,\\
&\lim_{t\to\infty}\frac{\|P_1b_{12}(t)+\frac{1}{2}c_{21}(t)\|}{\sqrt{\delta_1(t)\rho_2(t)}}=0,\\
&\lim_{t\to\infty}\frac{\|P_2b_{21}(t)+\frac{1}{2}c_{12}(t)\|}{\sqrt{\delta_2(t)\rho_1(t)}}=\frac{1}{\sqrt{2}},\\
&\lim_{t\to\infty}\frac{\|P_2b_{22}(t)+\frac{1}{2}c_{22}(t)\|}{\sqrt{\delta_2(t)\rho_2(t)}}=\frac{1}{\sqrt{6}}.
\end{aligned}$$

Then A9 is satisfied with $\mu_{11}=\mu_{12}=0$, $\mu_{21}=\frac{1}{\sqrt{2}}$, $\mu_{22}=\frac{1}{\sqrt{6}}$. At last, we verify that

$$D+RR^T+UU^T=\begin{bmatrix}\frac{\sqrt{2}-5}{7-\sqrt{2}} & 0\\ 0 & -\frac{29}{96}\end{bmatrix}<0.$$

Thus, according to Corollary 6, system (5) is absolutely stable.

For simulation, we choose

$$f_1\bigl(\sigma(t)\bigr)=2\sigma(t)+\sin\sigma(t),\qquad f_2\bigl(\sigma(t)\bigr)=\begin{cases}\sigma(t), & |\sigma(t)|<1,\\ \sigma^3(t), & 1\le|\sigma(t)|\le 2,\\ 4\sigma(t), & |\sigma(t)|>2,\end{cases}$$

and $[x_1(s)\ x_2(s)\ x_3(s)\ \sigma_1(0)\ \sigma_2(0)]^T=[1\ 1\ 1\ 1\ 1]^T$, $s\in[-7,0]$. The response curves of this system are shown in Figure 2.

As depicted in Figure 2, system (5) is asymptotically stable even though the system coefficients are unbounded.

5 Conclusion

In this paper, we have investigated the absolute stability problem of time-varying large-scale time-delay Lurie indirect control systems. Based on the second Lyapunov method,

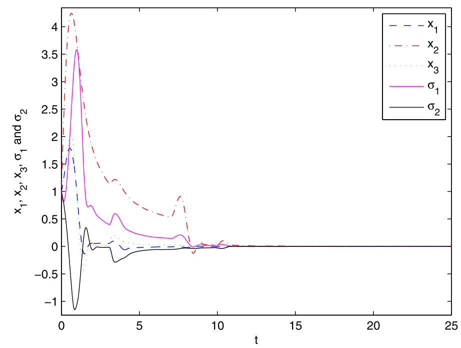


Figure 2 The state response of system (5).

to guarantee absolute stability of this class of systems, some sufficient conditions were formulated by simple inequalities. Finally, two numerical examples were presented to show the effectiveness of the proposed methods.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors have made the same contribution. Both authors read and approved the final manuscript.

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