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Global stability of a *SEIR* rumor spreading model with demographics on scale-free networks

Chen Wan, Tao Li* and Zhicheng Sun

*Correspondence:
yuanyzq@163.com
School of Electronics and Information, Yangtze University, Jingzhou, 434023, P.R. China

Abstract

In this paper, a new *SEIR* (susceptible-exposed-infected-removed) rumor spreading model with demographics on scale-free networks is proposed and investigated. Then the basic reproductive number R_0 and equilibria are obtained. The theoretical analysis indicates that the basic reproduction number R_0 has no correlation with the degree-dependent immigration. The globally asymptotical stability of rumor-free equilibrium and the permanence of the rumor are proved in detail. By using a novel monotone iterative technique, we strictly prove the global attractivity of the rumor-prevailing equilibrium.

Keywords: rumor spreading model; scale-free networks; hesitate mechanism; demographics; stability

1 Introduction

With the development of online social networks, rumor has propagated more quickly and widely, coming within people's horizons [1–3]. Rumor propagation may have tremendous negative effects on human lives, such as reputation damage, social panic and so on [4–7]. In order to investigate the mechanism of rumor propagation and effectively control the rumor, lots of rumor spreading models have been studied and analyzed in detail. In 1965, Daley and Kendal first proposed the classical DK model to study the rumor propagation [5]. They divided the population into three disjoint categories, namely, those who who never heard the rumor, those knowing and spreading the rumor, and finally those knowing the rumor but never spreading it. From then on, most rumor propagation studies were based on the DK model [8–14].

In the early stages, most rumor spreading models were established on homogeneous networks [15–18]. However, it is well known that a significant characteristic of social networks is their scale-free property. In networks, the nodes stand for individuals and the contacts stand for various interactions among those individuals. Scale-free networks can be characterized by degree distribution which follows a power-law distribution $P(k) \sim k^{-\gamma}$ $(2 < \gamma \leq 3)$ [19]. Recently, some scholars have studied a variety of rumor spreading models and found that the heterogeneity of the underlying network had a major influence on the dynamic mechanism of rumor spreading [18, 20–26].

It is noteworthy that the influence of hesitation plays a crucial role in process of rumor spreading. Lately, there were a few researchers who have studied the effects of hesitation. For instance, Xia *et al.* [27] proposed a novel *SEIR* rumor spreading model with hesitating mechanism by adding a new exposed group (E) in the classical *SIR* model. Liu *et al.* [28] presented a *SEIR* rumor propagation model on the heterogeneous network. They calculated the basic reproduction number R_0 by using the next generation method, and they found that the basic reproduction number R_0 depends on the fluctuations of the degree distribution. However, in most of the research work mentioned above, the immigration and emigration are not considered when rumor breaks out. Although references [27, 28] proposed a *SEIR* model with hesitating mechanism, neither could serve as a strict proof of globally asymptotically stability of rumor-free equilibrium and the permanence of the rumor. In this paper, considering the immigration and emigration rate, we study and analyze a new *SEIR* model with hesitating mechanism on heterogeneous networks and comprehensively prove the globally asymptotical stability of rumor-free equilibrium and the permanence of rumor in detail.

The rest of this paper is organized as follows. In Section 2, we present a new *SEIR* spreading model with hesitating mechanism on scale-free networks. In Section 3, the basic reproduction number and the two equilibria of the proposed model are obtained. In Section 4, we analyze the globally asymptotic stability of equilibria. Finally, we conclude the paper in Section 5.

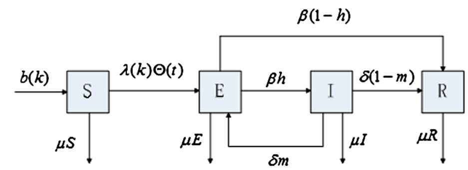
2 Modeling

Consider the whole population as a relevant online network. The *SEIR* rumor spreading model is based on dividing the whole population into four groups, namely: the susceptible, referring to those who have never contacted with the rumor, denoted by S; the exposed, referring to those who have been infected, in a hesitate state not spreading the rumor, denoted by E; the infected, referring to those who have accepted and spread the rumor, denoted by I; the recovered, referring to those who know the rumor but have ceased to spread it, denoted by R. During the period of rumor spreading, we suppose that the individuals with the same number of contacts are dynamically equivalent and belong to the same group in this paper. Let $S_k(t)$, $E_k(t)$, $I_k(t)$ and $R_k(t)$ be the densities of the above-mentioned nodes with the connectivity degree k at time t. Then the aggregate number of population at time is $N(t)$, and the density of the whole population with degree k satisfies

$$N_k(t) = S_k(t) + E_k(t) + I_k(t) + R_k(t). \tag{2.1}$$

The transfer diagram for the *SEIR* rumor propagation model is shown in Figure 1. In this paper, we assume that the degree-dependent parameter $b(k) > 0$ denotes the number of new immigration individuals with degree k per unit time, and each new immigration individual is susceptible. The emigration rate of all individuals is μ. Exposed individuals turn into infected individuals with probability βh due to believing and spreading the rumor. They recover from the rumor with probability $\beta(1-h)$. The infected individuals become exposed individuals with probability δm. They recover from the rumor with probability $\delta(1-m)$.

Figure 1 Transfer diagram for *SEIR* rumor propagation model.



Based on the above hypotheses and notation, the dynamic mean-field reaction rate equations described by

$$\begin{cases} \frac{dS_k(t)}{dt} = b(k) - \lambda(k)\Theta(t)S_k(t) - \mu S_k(t), \\ \frac{dE_k(t)}{dt} = \lambda(k)\Theta(t)S_k(t) - \beta E_k(t) + \delta m I_k(t) - \mu E_k(t), \\ \frac{dI_k(t)}{dt} = \beta h E_k(t) - \delta I_k(t) - \mu I_k(t), \\ \frac{dR_k(t)}{dt} = \beta(1-h)E_k(t) + \delta(1-m)I_k(t) - \mu R_k(t), \end{cases} \tag{2.2}$$

where $\lambda(k) > 0$ is the degree of acceptability of k for individuals for the rumor, and the probability $\Theta(t)$ denotes a link to an infected individual, satisfying

$$\Theta(t) = \frac{1}{\langle k \rangle} \sum_i \frac{\varphi(i)}{i} P(i|k) \frac{I_i(t)}{N_i(t)}. \tag{2.3}$$

Here, $1/i$ represents the probability that one of the infected neighbors of an individual, with degree i, will contact this individual at the present time step; $P(i|k)$ is the probability that an individual of degree k is connected to an individual with degree i. In this paper, we focus on degree uncorrelated networks. Thus, $P(i|k) = iP(i)/\langle k \rangle$, where $\langle k \rangle = \sum_i iP(i)$ is the average degree of the network. For a general function $f(k)$, it is defined as $\langle f(k) \rangle = \sum_i f(i)P(i)$. The function $\varphi(k)$ is the infectivity of an individual with degree k.

Adding the four equations of system (2.2), we have $\frac{dN_k(t)}{dt} = b(k) - \mu N_k(t)$. Then we can obtain $N_k(t) = \frac{b(k)}{\mu}(1 - e^{-\mu t}) + N_k(0)e^{-\mu t}$, where $N_k(0)$ represents the initial density of the whole population with degree k. Hence, $\limsup_{t\to\infty} N_k(t) = b(k)/\mu$, then $N_k(t) = S_k(t) + E_k(t) + I_k(t) + R_k(t) \le b(k)/\mu$ for all $t > 0$. In order to have a population of constant size, we suppose that $S_k(t) + E_k(t) + I_k(t) + R_k(t) = N_k(t) = \eta_k$, where $\eta_k = b(k)/\mu$. Thus, we have

$$\Theta(t) = \frac{1}{\langle k \rangle} \sum_{k=1} \frac{\varphi(k)}{\eta_k} P(k) I_k(t). \tag{2.4}$$

Furthermore,

$$S(t) = \sum_k P(k)S_k(t), \qquad E(t) = \sum_k P(k)E_k(t),$$
$$I(t) = \sum_k P(k)I_k(t), \quad \text{and} \quad R(t) = \sum_k P(k)R_k(t)$$

are the global average densities of the four rumor groups, respectively. From a practical perspective, we only need to consider the case of $P(k) > 0$ for $k = 1, 2, \ldots$. The initial conditions for system (2.2) satisfy

$$\begin{aligned} &0 \le S_k(0), E_k(0), I_k(0), R_k(0) \le \eta_k, \\ &S_k(0) + E_k(0) + I_k(0) + R_k(0) = \eta_k, \qquad \Theta(0) > 0. \end{aligned} \tag{2.5}$$

3 The basic reproduction number and equilibria

In this section, we reveal some properties of the solutions and obtain the equilibria of system (2.2).

Theorem 1 *Define the basic reproduction number* $R_0 = \frac{\beta h}{(\beta+\mu)(\delta+\mu)-\beta h\delta m}\frac{\langle\varphi(k)\lambda(k)\rangle}{\langle k\rangle}$, *then there always exists a rumor-free equilibrium* $E_0(\eta_k,0,0,0)$. *And if* $R_0>1$, *system* (2.2) *has a unique rumor-prevailing equilibrium* $E_+(S_k^\infty,E_k^\infty,I_k^\infty,R_k^\infty)$.

Proof We can easily see that the rumor-free equilibrium $E_0(\eta_k,0,0,0)$ of system (2.2) is always existent. To obtain the equilibrium solution $E_+(S_k^\infty,E_k^\infty,I_k^\infty,R_k^\infty)$, we let the right side of system (2.2) be equal to zero. Thus, we have

$$\begin{cases} b(k)-\lambda(k)\Theta^\infty S_k^\infty-\mu S_k^\infty=0, \\ \lambda(k)\Theta^\infty S_k^\infty-\beta E_k^\infty+\delta m I_k^\infty-\mu E_k^\infty=0, \\ \beta h E_k^\infty-\delta I_k^\infty-\mu I_k^\infty=0, \\ \beta(1-h)E_k^\infty+\delta(1-m)I_k^\infty-\mu R_k^\infty=0, \end{cases}$$

where $\Theta^\infty=\frac{1}{\langle k\rangle}\sum_{k=1}\frac{\varphi(k)}{\eta_k}P(k)I_k^\infty$. One has

$$\begin{cases} E_k^\infty=\frac{(\delta+\mu)}{\beta h}I_k^\infty, \\ S_k^\infty=\frac{(\beta+\mu)(\delta+\mu)-\beta h\delta m}{\beta h\lambda(k)\Theta^\infty}I_k^\infty, \\ R_k^\infty=\frac{(\delta+\mu)(1-h)+h\delta(1-m)}{h\mu}I_k^\infty. \end{cases} \tag{3.1}$$

According to $S_k^\infty+E_k^\infty+I_k^\infty+R_k^\infty=\eta_k$ for all k, we have

$$I_k^\infty=\frac{\mu\beta h\lambda(k)\Theta^\infty\eta_k}{\mu(\beta+\mu)(\delta+\mu)-\mu\beta h\delta m+\lambda(k)\Theta^\infty[(\beta+\mu)(\delta+\mu)-m\beta h\delta]}. \tag{3.2}$$

Inserting equation (3.2) into equation (2.4), we can obtain the self-consistency equation:

$$\begin{aligned} \Theta^\infty &= \frac{1}{\langle k\rangle}\sum_{k=1}\frac{\varphi(k)}{\eta_k} \\ &\quad\times P(k)\frac{\mu\beta h\lambda(k)\Theta^\infty\eta_k}{\mu(\beta+\mu)(\delta+\mu)-\mu\beta h\delta m+\lambda(k)\Theta^\infty[(\beta+\mu)(\delta+\mu)-m\beta h\delta]} \\ &\triangleq f\bigl(\Theta^\infty\bigr). \end{aligned} \tag{3.3}$$

Obviously, $\Theta^\infty=0$ is a solution of (3.3), then $S_k^\infty=\eta_k$ and $E_k^\infty=I_k^\infty=R_k^\infty=0$, which is a rumor-free equilibrium of system (2.2). In order to ensure equation (3.3) has a nontrivial solution, *i.e.*, $0<\Theta^\infty\le 1$, the following conditions must be fulfilled:

$$\frac{df(\Theta^\infty)}{d\Theta^\infty}\bigg|_{\Theta^\infty=0}>1 \quad\text{and}\quad f(1)\le 1.$$

Thus, we can obtain

$$\frac{\beta h}{(\beta+\mu)(\delta+\mu)-\beta h\delta m}\frac{\langle\varphi(k)\lambda(k)\rangle}{\langle k\rangle}>1.$$

Let the base reproduction number as follows:

$$R_0 = \frac{\beta h}{(\beta+\mu)(\delta+\mu) - \beta h\delta m}\frac{\langle\varphi(k)\lambda(k)\rangle}{\langle k\rangle}. \tag{3.4}$$

System (2.2) admits a unique rumor equilibrium $E_+(S_k^\infty, E_k^\infty, I_k^\infty, R_k^\infty)$ satisfying equation (3.1) if and only if $R_0 > 1$. The proof is completed. □

Remark 1 The basic reproductive number R_0 is obtained by equation (3.4), which depends on some model parameters and the fluctuations of the degree distribution. Interestingly, the basic reproductive number R_0 has no correlation with the degree-dependent immigration $b(k)$. According to the form of R_0, we see that increase of the emigration rate μ will make R_0 decrease. If $b(k) = 0$ and $\mu = 0$, then system (2.2) become the network-based *SEIR* model without demographics, and $R_0 = \frac{h}{\delta(1-hm)}\frac{\langle\varphi(k)\lambda(k)\rangle}{\langle k\rangle}$, which is in consistence with reference [28].

4 Discussion

4.1 The stability of the rumor-free equilibrium

Theorem 2 *The rumor-free equilibrium E_0 of SEIR system* (2.2) *is locally asymptotically stable if $R_0 < 1$, and it is unstable if $R_0 > 1$.*

Proof Let $S_k(t) = \eta_k - E_k(t) - I_k(t) - R_k(t)$, where $\eta_k = b(k)/\mu$. Therefore, system (2.2) can be rewritten as

$$\begin{cases} \frac{dE_k(t)}{dt} = \lambda(k)\Theta(t)(\eta_k - E_k(t) - I_k(t) - R_k(t)) - (\beta+\mu)E_k(t) + \delta m I_k(t), \\ \frac{dI_k(t)}{dt} = \beta h E_k(t) - (\delta+\mu)I_k(t), \\ \frac{dR_k(t)}{dt} = \beta(1-h)E_k(t) + \delta(1-m)I_k(t) - \mu R_k(t). \end{cases} \tag{4.1}$$

Then the Jacobian matrix of system (4.1) at $(0,0,0)$ is a $3k_{\max} * 3k_{\max}$ as follows:

$$J = \begin{bmatrix} A_1 & B_{12} & B_{13} & \cdots & B_{1k_{\max}} \\ B_{21} & A_2 & B_{23} & \cdots & B_{2k_{\max}} \\ \vdots & \vdots & \ddots & & \vdots \\ B_{k_{\max}1} & B_{k_{\max}2} & B_{k_{\max}3} & \cdots & A_{k_{\max}} \end{bmatrix},$$

where

$$A_j = \begin{pmatrix} -(\beta+\mu) & \delta m + \frac{\lambda(j)\varphi(j)P(j)}{\langle k\rangle} & 0 \\ \beta h & -(\delta+\mu) & 0 \\ \beta(1-h) & \delta(1-m) & -\mu \end{pmatrix}, \qquad B_{ij} = \begin{pmatrix} 0 & \frac{\lambda(j)\varphi(j)P(j)}{\langle k\rangle} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

By using mathematical induction, the characteristic equation can be calculated as follows:

$$\begin{aligned} &(z+\mu)^{k_{\max}}(z+\beta+\mu)^{k_{\max}-1}(z+\delta+\mu)^{k_{\max}-1} \\ &\quad\times\left((z+\beta+\mu)(z+\delta+\mu) - \beta h\delta m - \beta h\frac{\langle\lambda(k)\varphi(k)\rangle}{\langle k\rangle}\right) = 0, \end{aligned}$$

where

$$\big\langle \lambda(k)\varphi(k)\big\rangle = \lambda(1)\varphi(1)P(1) + \lambda(2)\varphi(2)P(2) + \cdots + \lambda(k_{\max})\varphi(k_{\max})P(k_{\max}).$$

The stability of E_0 is only dependent on

$$(z + \beta + \mu)(z + \delta + \mu) - \beta h\delta m - \beta h\frac{\langle\lambda(k)\varphi(k)\rangle}{\langle k\rangle} = 0. \tag{4.2}$$

Then we have

$$z^2 + (\beta + \delta + 2\mu)z + (\beta + \mu)(\delta + \mu) - \beta h\delta m - \beta h\frac{\langle\lambda(k)\varphi(k)\rangle}{\langle k\rangle} = 0. \tag{4.3}$$

According to equation (4.3), if $R_0 < 1$, we can easily get $(\beta + \mu)(\delta + \mu) - \beta h\delta m - \beta h\frac{\langle\lambda(k)\varphi(k)\rangle}{\langle k\rangle} > 0$, *i.e.*, $z < 0$. Hence, E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. The proof is completed. □

Theorem 3 *The rumor-free equilibrium E_0 of SEIR system* (2.2) *is globally asymptotically stable if $R_0 < 1$.*

Proof First, we define a Lyapunov function $V(t)$ as follows:

$$V(t) = \sum_k \frac{\varphi(k)}{\eta_k}\left[P(k)E_k(t) + \frac{(\beta + \mu)}{\beta h}I_k(t)\right]. \tag{4.4}$$

Then, according to a calculation of the derivative of $V(t)$ along the solution of system (2.2), we have

$$\begin{aligned}
V(t) &= \sum_k \frac{\varphi(k)P(k)}{\eta_k}\left[E_k(t) + \frac{(\beta + \mu)}{\beta h}I_k(t)\right] \\
&= \sum_k \frac{\varphi(k)}{\eta_k}P(k)\bigg[\lambda(k)\Theta(t)S_k(t) - (\beta + \mu)E_k(t) + \delta m I_k(t) \\
&\quad + \frac{(\beta + \mu)}{\beta h}\big(\beta h E_k(t) - (\delta + \mu)I_k(t)\big)\bigg] \\
&\leq \sum_k \frac{\varphi(k)}{\eta_k}P(k)\left[\lambda(k)\Theta(t)\eta_k + \frac{\delta m\beta h - (\beta + \mu)(\delta + \mu)}{\beta h}I_k(t)\right] \\
&= \Theta(t)\sum_k \varphi(k)P(k)\lambda(k) + \frac{\delta m\beta h - (\beta + \mu)(\delta + \mu)}{\beta h}\sum_k \frac{\varphi(k)}{\eta_k}P(k)I_k(t) \\
&= \Theta(t)\big\langle\varphi(k)\lambda(k)\big\rangle + \frac{\delta m\beta h - (\beta + \mu)(\delta + \mu)}{\beta h}\langle k\rangle\Theta(t) \\
&= \Theta(t)\frac{1}{\beta h}\Big[\beta h\big\langle\varphi(k)\lambda(k)\big\rangle + \big[\delta m\beta h - (\beta + \mu)(\delta + \mu)\big]\langle k\rangle\Big] \\
&= \Theta(t)\langle k\rangle\frac{[(\beta + \mu)(\delta + \mu) - \beta h\delta m]}{\beta h}(R_0 - 1).
\end{aligned}$$

When $R_0 < 1$, we can easily find that $V(t) \leq 0$ for all $V(t) \geq 0$, and that $V(t) = 0$ only if $\Theta(t) = 0$, *i.e.*, $I_k(t) = 0$. Thus, by the LaSalle invariance principle [29], this implies the rumor-free equilibrium E_0 of system (2.2) is globally attractive. Therefore, when $R_0 < 1$, the rumor-free equilibrium E_0 of *SEIR* system (2.2) is globally asymptotically stable. The proof is completed. □

4.2 The global attractivity of the rumor-prevailing equilibrium

In this section, the permanent of rumor and the global attractivity of the rumor-prevailing equilibrium are discussed.

Theorem 4 *When $R_0 > 1$, the rumor is permanent on the network, i.e., there exists a positive constant $\varsigma > 0$, such that* $\liminf I(t)_{t\to\infty} = \liminf_{t\to\infty} \sum_k P(k)I_k(t) > \varsigma$.

Proof We desire to use the condition stated in Theorem 4.6 in [30]. Define

$$\begin{aligned}
X &= \bigl\{(S_1, E_1, I_1, R_1, \dots, S_{k_{\max}}, E_{k_{\max}}, I_{k_{\max}}, R_{k_{\max}}) : \\
&\qquad S_k, E_k, I_k, R_k \geq 0 \text{ and } S_k + E_k + I_k + R_k = \eta, k = 1, \dots, k_{\max}\bigr\}, \\
X_0 &= \left\{(S_1, E_1, I_1, R_1, \dots, S_{k_{\max}}, E_{k_{\max}}, I_{k_{\max}}, R_{k_{\max}}) \in X : \sum_k P(k)I_k > 0\right\}, \\
\partial X_0 &= X \setminus X_0.
\end{aligned}$$

In the following, we will explain that system (2.2) is uniformly persistent with respect to $(X_0, \partial X_0)$.

Clearly, X is positively and bounded with respect to system (2.2). Assume that $\Theta(0) = \frac{1}{\langle k\rangle} \sum_{k=1} \frac{\varphi(k)}{\eta_k} P(k)I_k(0) > 0$, then we have $I_k(0) > 0$ for some k. Thus, $I(0) = \sum_{k=1} P(k)I_k(0) > 0$. For $I'(t) = \sum_k P(k)I_k'(t) \geq -(\delta + \mu)\sum_k P(k)I_k(t) = -(\delta + \mu)I(t)$, we have $I(t) \geq I(0)e^{-(\delta+\mu)t} > 0$. Therefore, X_0 is also positively invariant. Furthermore, there exists a compact set B, in which all solutions of system (2.2) initiated in X ultimately enter and remain forever after. The compactness condition (C4.2) of Theorem 4.6 in reference [30] is easily verified for this set B.

Denote

$$\begin{aligned}
M_\partial = {}& \bigl\{\bigl(S_1(0), E_1(0), I_1(0), R_1(0), \dots, S_{k_{\max}}(0), E_{k_{\max}}(0), I_{k_{\max}}(0), R_{k_{\max}}(0)\bigr) : \\
&\bigl(S_1(t), E_1(t), I_1(t), R_1(t), \dots, S_{k_{\max}}(t), E_{k_{\max}}(t), I_{k_{\max}}(t), R_{k_{\max}}(t)\bigr) \in \partial X_0, t \geq 0\bigr\},
\end{aligned}$$

and

$$\begin{aligned}
\Omega = {}& \bigcup \bigl\{\omega\bigl(S_1(0), E_1(0), I_1(0), R_1(0), \dots, S_{k_{\max}}(0), E_{k_{\max}}(0), I_{k_{\max}}(0), R_{k_{\max}}(0)\bigr) : \\
&\bigl(S_1(0), E_1(0), I_1(0), R_1(0), \dots, S_{k_{\max}}(0), E_{k_{\max}}(0), I_{k_{\max}}(0), R_{k_{\max}}(0)\bigr) \in X\bigr\},
\end{aligned}$$

where $\omega(S_1(0), E_1(0), I_1(0), R_1(0), \dots, S_{k_{\max}}(0), E_{k_{\max}}(0), I_{k_{\max}}(0), R_{k_{\max}}(0))$ is the omega limit set of the solutions of system (2.2) starting in $(S_1(0), E_1(0), I_1(0), R_1(0), \dots, S_{k_{\max}}(0), E_{k_{\max}}(0), I_{k_{\max}}(0), R_{k_{\max}}(0))$. Restricting system (2.2) on M_∂, we can obtain

$$\begin{cases}
\frac{dS_k(t)}{dt} = b(k) - \mu S_k(t), \\
\frac{dE_k(t)}{dt} = -(\beta + \mu)E_k(t), \\
\frac{dI_k(t)}{dt} = -(\delta + \mu)I_k(t), \\
\frac{dR_k(t)}{dt} = \beta(1 - h)E_k(t) - \mu R_k(t).
\end{cases} \tag{4.5}$$

Obviously, system (4.5) has a unique equilibrium E_0 in X. Thus, E_0 is the unique equilibrium of system (2.2) in M_∂. We can easily find that E_0 is locally asymptotically stable. For system (4.5) is a linear system; this indicates that E_0 is globally asymptotically stable. Hence $\Omega = \{E_0\}$. And E_0 is a covering of X, which is isolated and acyclic (because there exists no nontrivial solution in M_∂ which links E_0 to itself). Finally, the proof of theorem will be completed if it is shown that E_0 is a weak repeller for X_0, *i.e.*,

$$\limsup_{t\to\infty} \operatorname{dist}\big(\big(S_1(t),E_1(t),I_1(t),R_1(t),\dots,S_{k_{\max}}(t),E_{k_{\max}}(t),I_{k_{\max}}(t),R_{k_{\max}}(t)\big),E_0\big) > 0,$$

where $(S_1(t),E_1(t),I_1(t),R_1(t),\dots,S_{k_{\max}}(t),E_{k_{\max}}(t),I_{k_{\max}}(t),R_{k_{\max}}(t))$ is an arbitrary solution with initial value in X_0. In order to use the method of Leenheer and Smith [31], we need only to prove $W^s(E_0)\cap X_0 = \emptyset$, where $W^s(E_0)$ is the stable manifold of E_0. Assume it is not sure, then there exists a solution $(S_1(t),E_1(t),I_1(t),R_1(t),\dots,S_{k_{\max}}(t),E_{k_{\max}}(t),$ $I_{k_{\max}}(t),R_{k_{\max}}(t))$ in X_0, such that

$$S_k(t)\to\eta_k,\qquad E_k(t)\to 0,\qquad I_k(t)\to 0,\qquad R_k(t)\to 0\quad \text{as } t\to\infty. \tag{4.6}$$

According to $R_0 = \frac{b\beta}{\mu[(\gamma+\varepsilon+\mu)(\beta+\delta+\mu)-\beta\varepsilon]}\frac{\langle\lambda(k)\varphi(k)\rangle}{\langle k\rangle} > 1$, we have

$$\sum_k \frac{\lambda(k)\varphi(k)P(k)}{\langle k\rangle\eta_k}\eta_k > \frac{\mu[(\gamma+\varepsilon+\mu)(\beta+\delta+\mu)-\beta\varepsilon]}{b\beta}.$$

Then we can choose sufficiently small $\xi > 0$ such that

$$\left\langle \frac{\lambda(k)\varphi(k)}{\langle k\rangle\eta_k}(\eta_k-\xi)\right\rangle > \frac{\mu[(\gamma+\varepsilon+\mu)(\beta+\delta+\mu)-\beta\varepsilon]}{b\beta}. \tag{4.7}$$

Since $\xi > 0$, by (4.6) there exists a constant $T > 0$ such that

$$\frac{b}{\mu}-\xi < S_k(t) < \frac{b}{\mu}+\xi,\qquad 0 < E_k(t) < \xi,\qquad 0 < I_k(t) < \xi,\qquad 0 < R_k(t) < \xi \tag{4.8}$$

for all $t \ge T$ and $k = 1,2,\dots,k_{\max}$.

The derivative of $V(t) = \sum_k \frac{\varphi(k)}{\eta_k}[P(k)E_k(t) + \frac{(\beta+\mu)}{\beta h}I_k(t)]$ along the solution of system (2.2) is given by

$$\begin{aligned}
V'(t) &= \sum_k \frac{\varphi(k)}{\eta_k}P(k)\left(E_k'(t)+\frac{(\beta+\mu)}{\beta h}I_k'(t)\right)\\
&= \sum_k \frac{\varphi(k)}{\eta_k}P(k)\left(\lambda(k)\Theta(t)S_k(t)+\delta m I_k(t)-\frac{(\beta+\mu)(\delta+\mu)}{\beta h}I_k(t)\right)\\
&= \sum_k \frac{\varphi(k)}{\eta_k}P(k)\left(\lambda(k)\Theta(t)S_k(t)+\frac{\delta m\beta h-(\beta+\mu)(\delta+\mu)}{\beta h}I_k(t)\right)\\
&> \sum_k \frac{\varphi(k)}{\eta_k}P(k)\left(\lambda(k)\Theta(t)(\eta_k-\xi)+\frac{\delta m\beta h-(\beta+\mu)(\delta+\mu)}{\beta h}I_k(t)\right)
\end{aligned}$$

$$\begin{aligned}
&= \sum_k \frac{\varphi(k)}{\eta_k} P(k)\left(\frac{\lambda(k)(\eta_k - \xi)}{\langle k\rangle} \sum_i \frac{\varphi(i)}{\eta_i} P(i) I_i(t)\right) \\
&\quad + \frac{\delta m\beta h - (\beta+\mu)(\delta+\mu)}{\beta h} \sum_i \frac{\varphi(i)}{\eta_i} P(i) I_i(t) \\
&= \left\langle \frac{\varphi(k)\lambda(k)}{\eta_k \langle k\rangle}(\eta_k - \xi)\right\rangle \sum_{i=1} \frac{\varphi(i)}{\eta_i} P(i) I_i(t) + \frac{\delta m\beta h - (\beta+\mu)(\delta+\mu)}{\beta h} \sum_i \frac{\varphi(i)}{\eta_i} P(i) I_i(t) \\
&= \sum_{i=1}\left[\left\langle \frac{\varphi(k)\lambda(k)}{\eta_k \langle k\rangle}(\eta_k - \xi)\right\rangle - \frac{(\beta+\mu)(\delta+\mu) - \beta h\delta m}{\beta h}\right] \frac{\varphi(i)}{\eta_i} P(i) I_i(t) \\
&> 0.
\end{aligned}$$

Consequently, $V(t) \to \infty$ as $t \to \infty$, which apparently contradicts the boundedness of $V(t)$. This completes the proof. □

Lemma 1 ([32]) *If $a > 0$, $b > 0$ and $\frac{dx(t)}{dt} \geq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we can obtain* $\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}$. *If $a > 0, b > 0$ and $\frac{dx(t)}{dt} \leq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we can obtain* $\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}$.

Next, by using a novel monotone iterative technique in reference [33], we discuss the global attractivity of the rumor-prevailing equilibrium.

Theorem 5 *Suppose that* $(S_k(t), E_k(t), I_k(t), R_k(t))$ *is a solution of system* (2.2), *satisfying the initial condition equation* (2.5). *When $R_0 > 1$, then* $\lim_{t\to\infty}(S_k(t), E_k(t), I_k(t), R_k(t)) = (S_k^\infty, E_k^\infty, I_k^\infty, R_k^\infty)$, *where* $(S_k^\infty, E_k^\infty, I_k^\infty, R_k^\infty)$ *is the unique positive rumor equilibrium of system* (2.2) *satisfying* (3.1) *for $k = 1, 2, \ldots, n$.*

Proof Since the first three equations in system (2.2) are independent of the fourth one, it suffices to consider the following system:

$$\begin{cases}
\frac{dS_k(t)}{dt} = b(k) - \lambda(k)\Theta(t)S_k(t) - \mu S_k(t), \\
\frac{dE_k(t)}{dt} = \lambda(k)\Theta(t)S_k(t) - (\beta+\mu)E_k(t) + \delta m I_k(t), \\
\frac{dI_k(t)}{dt} = \beta h E_k(t) - (\delta+\mu)I_k(t).
\end{cases} \tag{4.9}$$

We assume that k is fixed to be any integer in $\{1, 2, \ldots, n\}$. By Theorem 4, there exist a positive constant $0 < \varepsilon < 1/3$ and a large enough constant $T > 0$ such that $I_k(t) \geq \varepsilon$ for $t > T$. Hence,

$$\Theta(t) = \frac{1}{\langle k\rangle} \sum_{i=1} \frac{\varphi(i)}{\eta_i} P(i) I_i(t) \geq \frac{1}{\langle k\rangle} \frac{\varphi(i_0)P(i_0)}{\eta_{i_0}} \varepsilon = \vartheta\varepsilon > 0,$$

where $\vartheta = \frac{1}{\langle k\rangle} \frac{\varphi(i_0)P(i_0)}{\eta_{i_0}}$. From the first equation of system (4.9), we have

$$\frac{dS_k(t)}{dt} \leq b(k) - \lambda(k)\vartheta\varepsilon S_k(t) - \mu S_k(t), \quad t > T.$$

By Lemma 1, we derive that $\limsup_{t\to+\infty} S_k(t) \leq \frac{\mu\eta_k}{\lambda(k)\vartheta\varepsilon+\mu}$. Then, for arbitrarily given positive constant $0 < \varepsilon_1 < \frac{\lambda(k)\vartheta\varepsilon\eta_k}{2(\lambda(k)\vartheta\varepsilon+\mu)}$, there exists a $t_1 > T$ such that $S_k(t) \leq X_k^{(1)} - \varepsilon_1$ for $t > t_1$, where

$$X_k^{(1)} = \frac{\mu\eta_k}{\lambda(k)\vartheta\varepsilon + \mu} + 2\varepsilon_1 < \eta_k.$$

For $\Theta(t) \leq \frac{1}{\langle k\rangle}\sum_{i=1}\varphi(i)P(i) = M$, from the second equation of system (4.9) for $t > t_1$, we can get

$$\begin{aligned}\frac{dE_k(t)}{dt} &\leq \lambda(k)M\bigl(\eta_k - E_k(k) - I_k(k) - R_k(k)\bigr) - (\beta+\mu)E_k(t) \\ &\quad + \delta m\bigl(\eta_k - E_k(k) - S_k(k) - R_k(k)\bigr) \\ &\leq \lambda(k)M\bigl(\eta_k - E_k(k)\bigr) - (\beta+\mu)E_k(t) + \delta m\bigl(\eta_k - E_k(k)\bigr) \\ &= \eta_k\bigl[\lambda(k)M + \delta m\bigr] - E_k(k)\bigl[\lambda(k)M + \delta m + \beta + \mu\bigr].\end{aligned}$$

Similarly, for arbitrary given positive constant $0 < \varepsilon_2 < \min\{1/2, \varepsilon_1, \frac{(\delta m+\beta+\mu)\eta_k}{2(\lambda(k)M+\delta m+\beta+\mu)}\}$, there exists a $t_2 > t_1$, such that $E_k(t) \leq Y_k^{(1)} - \varepsilon_2$ for $t > t_2$, where

$$Y_k^{(1)} = \frac{\lambda(k)M\eta_k}{\lambda(k)M + \delta m + \beta + \mu} + 2\varepsilon_2 < \eta_k.$$

From the third equation of system (4.9), we have

$$\frac{dI_k(t)}{dt} \leq \beta h\bigl(\eta_k - I_k(t)\bigr) - (\delta+\mu)I_k(t) = \beta h\eta_k - (\delta + \mu + \beta h)I_k(t), \quad t > t_2.$$

Thus, for arbitrary given positive constant $0 < \varepsilon_3 < \min\{1/3, \varepsilon_2, \frac{(\mu+\beta h)\eta_k}{2(\delta+\mu+\beta h)}\}$, there exists a $t_3 > t_2$, such that $I_k(t) \leq Z_k^{(1)} - \varepsilon_3$ for $t > t_3$, where

$$Z_k^{(1)} = \frac{\delta\eta_k}{(\delta + \mu + \beta h)} + 2\varepsilon_3 < \eta_k.$$

On the other hand, from the first equation of system (4.9), we can get

$$\frac{dS_k(t)}{dt} \geq b(k) - \lambda(k)MS_k(t) - \mu S_k(t), \quad t > T.$$

By Lemma 1, we derive that $\liminf_{t\to+\infty} S_k(t) \geq \frac{b(k)}{\lambda(k)M+\mu}$. Then, for arbitrary given positive constant $0 < \varepsilon_4 < \min\{1/4, \varepsilon_3, \frac{b(k)}{2[\lambda(k)M+\mu]}\}$, there exists a $t_4 > t_3$, such that $S_k(t) \geq x_k^{(1)} + \varepsilon_4$, for $t > t_4$, where

$$x_k^{(1)} = \frac{b(k)}{\lambda(k)M + \mu} - 2\varepsilon_4 > 0.$$

It follows that

$$\frac{dE_k(t)}{dt} \geq \lambda(k)\vartheta\varepsilon x_k^{(1)} + \delta m\eta_k - (\beta + \mu + \delta m)E_k(t), \quad t > t_4.$$

Hence, for arbitrary given positive constant $0 < \varepsilon_5 < \min\{1/5, \varepsilon_4, \frac{\lambda(k)\vartheta\varepsilon x_k^{(1)} + \delta m\eta_k}{2(\beta+\mu+\delta m)}\}$, there exists a $t_5 > t_4$, such that $E_k(t) \geq y_k^{(1)} + \varepsilon_5$ for $t > t_5$, where

$$y_k^{(1)} = \frac{\lambda(k)\vartheta\varepsilon x_k^{(1)} + \delta m\eta_k}{(\beta + \mu + \delta m)} - 2\varepsilon_5 > 0.$$

Then

$$\frac{dI_k(t)}{dt} \geq \beta h y_k^{(1)} - (\delta + \mu)I_k(t), \quad t > t_5.$$

Hence, for arbitrary given positive constant $0 < \varepsilon_6 < \min\{1/6, \varepsilon_5, \frac{\beta h y_k^{(1)}}{2(\delta+\mu)}\}$, there exists a $t_6 > t_5$, such that $I_k(t) \geq z_k^{(1)} + \varepsilon_6$ for $t > t_6$, where

$$z_k^{(1)} = \frac{\beta h y_k^{(1)}}{(\delta + \mu)} - 2\varepsilon_6 > 0.$$

Since ε is a small positive constant, we have $0 < x_k^{(1)} < X_k^{(1)} < \eta_k$, $0 < y_k^{(1)} < Y_k^{(1)} < \eta_k$ and $0 < z_k^{(1)} < Z_k^{(1)} < \eta_k$. Let

$$w^{(j)} = \frac{1}{\langle k\rangle}\sum_k \frac{\varphi(k)}{\eta_k}P(k)z_k^j(t), \qquad W^{(j)} = \frac{1}{\langle k\rangle}\sum_k \frac{\varphi(k)}{\eta_k}P(k)Z_k^j(t), \quad j = 1, 2 \ldots.$$

From the above discussion, we found that

$$0 < w^{(1)} \leq \Theta(t) \leq W^{(1)} < M, \quad t > t_6.$$

Again, by system (4.9), we have

$$\frac{dS_k(t)}{dt} \leq b(k) - \lambda(k)w^{(1)}S_k(t) - \mu S_k(t), \quad t > t_6.$$

Hence, for arbitrary given positive constant $0 < \varepsilon_7 < \min\{1/7, \varepsilon_6\}$, there exists a $t_7 > t_6$ such that

$$S_k(t) \leq X_k^{(2)} \triangleq \min\left\{X_k^{(1)} - \varepsilon_1, \frac{b(k)}{\lambda(k)w^{(1)} + \mu} + \varepsilon_7\right\}, \quad t > t_7.$$

Thus,

$$\frac{dE_k(t)}{dt} \leq \lambda(k)W^{(1)}X_k^{(2)} + \delta m Z_k^{(1)} - (\beta + \mu)E_k(t), \quad t > t_7.$$

Therefore, for arbitrary given positive constant $0 < \varepsilon_8 < \min\{1/8, \varepsilon_7\}$, there exists a $t_8 > t_7$, such that

$$E_k(t) \leq Y_k^{(2)} \triangleq \min\left\{Y_k^{(1)} - \varepsilon_2, \frac{\lambda(k)W^{(1)}X_k^{(2)} + \delta m Z_k^{(1)}}{(\beta + \mu)} + \varepsilon_8\right\}.$$

It follows that

$$\frac{dI_k(t)}{dt} \leq \beta h Y_k^{(2)} - (\delta + \mu)I_k(t), \quad t > t_8.$$

So, for arbitrary given positive constant $0 < \varepsilon_9 < \min\{1/9, \varepsilon_8\}$, there exists a $t_9 > t_8$, such that

$$I_k(t) \le Z_k^{(2)} \triangleq \min\left\{Z_k^{(1)} - \varepsilon_3, \frac{\beta h Y_k^{(2)}}{(\delta + \mu)} + \varepsilon_9\right\}, \quad t > t_9.$$

Turning back to system (4.9), we have

$$\frac{dS_k(t)}{dt} \ge b(k) - \lambda(k) W^{(2)} S_k(t) - \mu S_k(t), \quad t > t_9.$$

Hence, for arbitrary given positive constant $0 < \varepsilon_{10} < \min\{1/10, \varepsilon_9, \frac{b(k)}{2(\lambda(k)W^{(2)}+\mu)}\}$, there exists a $t_{10} > t_9$, such that $S_k(t) \ge x_k^{(2)} + \varepsilon_{10}$ for $t > t_{10}$, where

$$x_k^{(2)} = \max\left\{x_k^{(1)} + \varepsilon_4, \frac{b(k)}{\lambda(k) W^{(2)} + \mu} - 2\varepsilon_{10}\right\}.$$

Accordingly, one obtains

$$\frac{dE_k(t)}{dt} \ge \lambda(k) w^{(1)} x_k^{(2)} + \delta m z_k^{(1)} - (\beta + \mu) E_k(t), \quad t > t_{10}.$$

Hence, for arbitrary given positive constant $0 < \varepsilon_{11} < \min\{1/11, \varepsilon_{10}, \frac{\lambda(k)w^{(1)}x_k^{(2)}+\delta m z_k^{(1)}}{2(\beta+\mu)}\}$, there exists a $t_{11} > t_{10}$, such that $E_k(t) \ge y_k^{(2)} + \varepsilon_{11}$ for $t > t_{11}$, where

$$y_k^{(2)} = \max\left\{y_k^{(1)} + \varepsilon_5, \frac{\lambda(k) w^{(1)} x_k^{(2)} + \delta m z_k^{(1)}}{(\beta + \mu)} - 2\varepsilon_{11}\right\}.$$

Thus,

$$\frac{dI_k(t)}{dt} \ge \beta h y_k^{(2)} - (\delta + \mu) I_k(t), \quad t > t_{11}.$$

Hence, for arbitrary given positive constant $0 < \varepsilon_{12} < \min\{1/12, \varepsilon_{11}, \frac{\beta h y_k^{(2)}}{2(\delta+\mu)}\}$, there exists a $t_{12} > t_{11}$, such that $I_k(t) \ge z_k^{(2)} + \varepsilon_{12}$ for $t > t_{12}$, where

$$z_k^{(2)} = \max\left\{z_k^{(1)} + \varepsilon_6, \frac{\beta h y_k^{(2)}}{(\delta + \mu)} - 2\varepsilon_{12}\right\}.$$

In the same way, we can carry out step h ($h = 3, 4, \ldots$) of the calculation and get six sequences: $\{X_k^{(h)}\}$, $\{Y_k^{(h)}\}$, $\{Z_k^{(h)}\}$, $\{x_k^{(h)}\}$, $\{y_k^{(h)}\}$ and $\{z_k^{(h)}\}$. We found that the first three sequences are monotone increasing and the last three sequences are strictly monotone decreasing, and there exists a large positive integer N so that for $h \geq \mathrm{N}$

$$\begin{aligned} X_k^{(h)} &= \frac{b(k)}{\lambda(k)w^{(h-1)} + \mu} + \varepsilon_{6h-5}, \\ Y_k^{(h)} &= \frac{\lambda(k)W^{(h-1)}X_k^{(h)} + \delta m Z_k^{(h-1)}}{(\beta + \mu)} + \varepsilon_{6h-4}, \\ Z_k^{(h)} &= \frac{\beta h Y_k^{(h)}}{(\delta + \mu)} + \varepsilon_{6h-3}, \\ x_k^{(h)} &= \frac{b(k)}{\lambda(k)W^{(h)} + \mu} - 2\varepsilon_{6h-2}, \\ y_k^{(h)} &= \frac{\lambda(k)w^{(h-1)}x_k^{(h)} + \delta m z_k^{(1-1)}}{(\beta + \mu)} - 2\varepsilon_{6h-1}, \\ z_k^{(h)} &= \frac{\beta h y_k^{(h)}}{(\delta + \mu)} - 2\varepsilon_{6h}. \end{aligned} \tag{4.10}$$

Clearly, we found that

$$x_k^{(h)} \leq S_k(t) \leq X_k^{(h)}, \qquad y_k^{(h)} \leq E_k(t) \leq Y_k^{(h)}, \qquad z_k^{(h)} \leq I_k(t) \leq Z_k^{(h)}, \quad t > t_{6h}. \tag{4.11}$$

Owing to the existence of sequential limits of equation (4.10), let $\lim_{t\to\infty} \Omega_k^{(h)} = \Omega_k$, where $\Omega_k^{(h)} \in \{X_k^{(h)}, Y_k^{(h)}, Z_k^{(h)}, x_k^{(h)}, y_k^{(h)}, z_k^{(h)}, W_k^{(h)}, w_k^{(h)}\}$ and $\Omega_k \in \{X_k, Y_k, Z_k, x_k, y_k, z_k, W_k, w_k\}$.

For $0 < \varepsilon_h < 1/h$, one has $\varepsilon_h \to 0$ as $h \to \infty$. Taking $h \to \infty$, by calculating the six sequences of equation (4.10), we can obtain the following form

$$\begin{aligned} X_k &= \frac{b(k)}{\lambda(k)w + \mu}, \qquad Y_k = \frac{\lambda(k)WX_k + \delta m Z_k}{(\beta + \mu)}, \qquad Z_k = \frac{\beta h Y_k}{(\delta + \mu)}, \\ x_k &= \frac{b(k)}{\lambda(k)W + \mu}, \qquad y_k = \frac{\lambda(k)w x_k + \delta m z_k}{(\beta + \mu)}, \qquad z_k = \frac{\beta h y_k}{(\delta + \mu)}. \end{aligned} \tag{4.12}$$

From equation (4.12), a direct computation leads to

$$\begin{aligned} Z_k &= \frac{\beta h \lambda(k) W}{[(\delta + \mu)(\beta + \mu) - \beta h \delta m]} \frac{b(k)}{\lambda(k)w + \mu}, \\ z_k &= \frac{\beta h \lambda(k) w}{[(\delta + \mu)(\beta + \mu) - \beta h \delta m]} \frac{b(k)}{\lambda(k)W + \mu}, \end{aligned} \tag{4.13}$$

where $w = \frac{1}{\langle k\rangle} \sum_k \frac{\varphi(k)}{\eta_k} P(k) z_k$, $W = \frac{1}{\langle k\rangle} \sum_k \frac{\varphi(k)}{\eta_k} P(k) Z_k$.

Further, substituting equation (4.13) into w and W, respectively, we can obtain

$$\begin{aligned} 1 &= \frac{1}{\langle k\rangle} \frac{\beta h}{[(\delta + \mu)(\beta + \mu) - \beta h \delta m]} \sum_k \frac{\varphi(k)}{\eta_k} \frac{P(k)\lambda(k)b(k)}{\lambda(k)W + \mu}, \\ 1 &= \frac{1}{\langle k\rangle} \frac{\beta h}{[(\delta + \mu)(\beta + \mu) - \beta h \delta m]} \sum_k \frac{\varphi(k)}{\eta_k} \frac{P(k)\lambda(k)b(k)}{\lambda(k)w + \mu}. \end{aligned} \tag{4.14}$$

Subtracting the above two equations, a direct computation leads to

$$0 = (w - W) \frac{1}{\langle k\rangle} \frac{\beta h}{[(\delta + \mu)(\beta + \mu) - \beta h \delta m]} \sum_k \frac{\varphi(k)}{\eta_k} \frac{P(k)\lambda(k)b(k)\lambda(k)}{(\lambda(k)W + \mu)(\lambda(k)w + \mu)}.$$

Obviously, it implies that $w = W$. So, $\frac{1}{\langle k\rangle}\sum_k \frac{\varphi(k)}{\eta_k}P(k)(Z_k - z_k) = 0$, which is equivalent to $z_k = Z_k$ for $1 \leq k \leq n$. Then, from equation (4.12) and equation (4.13), it follows that

$$\lim_{t\to 0} S_k(t) = X_k = x_k, \qquad \lim_{t\to 0} E_k(t) = Y_k = y_k, \qquad \lim_{t\to 0} I_k(t) = Z_k = z_k.$$

Finally, by substituting $w = W$ into equation (4.13), in view of equation (3.1) and equation (4.12), it is found that $X_k = S_k^\infty$, $Y_k = E_k^\infty$, $Z_k = R_k^\infty$. This completes the proof. □

5 Conclusions

In this paper, a new *SEIR* rumor spreading model with demographics on scale-free networks is presented. Through the mean-field theory analysis, we obtained the basic reproduction number R_0 and the equilibria. The basic reproduction number R_0 determines the existence of the rumor-prevailing equilibrium, and it depends on the topology of the underlying networks and some model parameters. Interestingly, R_0 bears no relation to the degree-dependent immigration $b(k)$. When $R_0 < 1$, the rumor-free equilibrium E_0 is globally asymptotically stable, *i.e.*, the infected individuals will eventually disappear. When $R_0 > 1$, there exists a unique rumor-prevailing E_+, and the rumor is permanent, *i.e.*, the infected individuals will persist and we have convergence to a uniquely prevailing equilibrium level. The study may provide a reliable tactic basis for preventing the rumor spreading.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
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References

1. Sudbury, A: The proportion of the population never hearing a rumour. J. Appl. Probab. **22**, 443-446 (1985)
2. Centola, D: The spread of behavior in an online social network experiment. Science **329**, 1194-1197 (2010)
3. Garrett, RK: Troubling consequences of online political rumoring. Hum. Commun. Res. **37**, 255-274 (2011)
4. Huo, L, Huang, P: Study on rumor propagation models based on dynamical system theory. Math. Pract. Theory **43**, 1-8 (2013)
5. Daley, DJ, Kendall, DG: Epidemics and rumours. Nature **204**, 1118 (1964)
6. Zanette, DH: Dynamics of rumor propagation on small-world networks. Phys. Rev. E **65**, Article ID 041908 (2002)
7. Pearce, CEM: The exact solution of the general stochastic rumour. Math. Comput. Model. **31**, 289-298 (2000)
8. Zhao, L, Wang, J, Huang, R: 2SI2R rumor spreading model in homogeneous networks. Physica A **441**, 153-161 (2014)
9. Singh, J, Kumar, D, Qurashi, AM, Baleanu, D: A new fractional model for giving up smoking dynamics. Adv. Differ. Equ. **2017**, Article ID 88 (2017)
10. Moreno, Y, Nekovee, M, Pacheco, AF: Dynamics of rumor spreading in complex networks. Phys. Rev. E **69**, Article ID 066130 (2004)
11. Singh, J, Kumar, D, Qurashi, MA, Baleanu, D: Analysis of a new fractional model for damped Berger equation. Open Phys. **15**, 35-41 (2017)
12. Li, X, Ding, D: Mean square exponential stability of stochastic Hopfield neural networks with mixed delays. Stat. Probab. Lett. **126**, 88-96 (2017)
13. Huo, L, Lin, T, Fan, C, Liu, C, Zhao, J: Optimal control of a rumor propagation model with latent period in emergency event. Adv. Differ. Equ. **2015**, Article ID 54 (2015)
14. Wan, C, Li, T, Guan, ZH, Wang, Y, Liu, X: Spreading dynamics of an e-commerce preferential information model on scale-free networks. Physica A **467**, 192-200 (2017)

15. Choudhary, A, Kumar, D, Singh, J: A fractional model of fluid flow through porous media with mean capillary pressure. J. Assoc. Arab Univ. Basic Appl. Sci. **21**, 59-63 (2016)
16. Boccaletti, S, Latora, V, Moreno, Y, Chavez, M, Hwang, DU: Complex networks: structure and dynamics. Phys. Rep. **424**, 175-308 (2006)
17. Gu, J, Li, W, Cai, X: The effect of the forget-remember mechanism on spreading. Eur. Phys. J. B **62**, 247-255 (2008)
18. Zhao, L, Qiu, X, Wang, X, Wang, J: Rumor spreading model considering forgetting and remembering mechanisms in inhomogeneous networks. Physica A **392**, 987-994 (2013)
19. Barabási, AL, Albert, R: Emergence of scaling in random networks. Science **286**, 509-512 (1999)
20. Li, T, Wang, Y, Guan, ZH: Spreading dynamics of a SIQRS epidemic model on scale-free networks. Commun. Nonlinear Sci. Numer. Simul. **19**, 686-692 (2014)
21. Kumar, D, Singh, J, Baleanu, D: A hybrid computational approach for Klein-Gordon equations on Cantor sets. Nonlinear Dyn. **87**, 511-517 (2017)
22. Xu, JP, Zhang, Y: Event ambiguity fuels the effective spread of rumors. Int. J. Mod. Phys. C **26**, Article ID 1550033 (2015)
23. Li, C, Ma, Z: Dynamic analysis of a spatial diffusion rumor propagation model with delay. Adv. Differ. Equ. **2015**, Article ID 364 (2015)
24. Nekovee, M, Moreno, Y, Bianconi, G, Marsili, M: Theory of rumour spreading in complex social networks. Physica A **374**, 457-470 (2007)
25. Xu, J, Zhang, M, Ni, J: A coupled model for government communication and rumor spreading in emergencies. Adv. Differ. Equ. **2016**, Article ID 208 (2016)
26. Srivastava, HM, Kumar, D, Singh, J: An efficient analytical technique for fractional model of vibration equation. Appl. Math. Model. **45**, 192-204 (2017)
27. Xia, LL, Jiang, GP, Song, B, Song, Y: Rumor spreading model considering hesitating mechanism in complex social networks. Physica A **437**, 295-303 (2015)
28. Liu, Q, Li, T, Sun, M: The analysis of an SEIR rumor propagation model on heterogeneous network. Physica A **469**, 372-380 (2017)
29. Hale, JK: Dynamical systems and stability. J. Math. Anal. Appl. **26**, 39-59 (1969)
30. Thieme, HR: Persistence under relaxed point-dissipativity (with application to an endemic model). SIAM J. Math. Anal. **24**, 407-435 (1993)
31. Leenheer, PD, Smith, HL: Virus dynamics: a global analysis. SIAM J. Appl. Math. **63**, 1313-1327 (2003)
32. Chen, F: On a nonlinear nonautonomous predator-prey model with diffusion and distributed delay. J. Comput. Appl. Math. **180**, 33-49 (2005)
33. Zhu, G, Fu, X, Chen, G: Spreading dynamics and global stability of a generalized epidemic model on complex heterogeneous networks. Appl. Math. Model. **36**, 5808-5817 (2012)