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Optimal harvesting control and dynamics of two-species stochastic model with delays

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Abstract

Taking the stochastic effects on growth rate and harvesting effort into account, we propose a stochastic delay model of species in two habitats. The main aim of this paper is to investigate optimal harvesting and dynamics of the stochastic delay model. By using the stochastic analysis theory and differential inequality technology, we firstly obtain sufficient conditions for persistence in the mean and extinction. Furthermore, the optimal harvesting effort and the maximum of expectation of sustainable yield (ESY) are gained by using Hessian matrix, the ergodic method, and optimal harvesting theory of differential equations. To illustrate the performance of the theoretical results, we present a series of numerical simulations of these cases with respect to different noise disturbance coefficients.

Keywords: stochastic delay model; extinction; persistence in the mean; optimal harvesting policy

1 Introduction

The population dynamics could be affected by the process of migration among patches. Due to natural conditions, such as the geology, climate, and hydrology, and the human factors, which include the development of tourism and the locations of industries, the animal habitats have been divided into some small patches. In recent years, many scholars studied the persistence and extinction of species with diffusion [1–6]. The result of [7] shows that the stability of periodic solution has some connection with the diffusion coefficients. The the references mentioned, the models with diffusion affecting the growth rate were studied, whereas Allen [8] proposed a logistic model with diffusion that affects density dependence. It can be described by the following formulation:

$$\frac{\mathrm{d}x_j(t)}{\mathrm{d}t} = x_j(t)[r_j - b_jx_j] + \sum_{k=1,k\neq j}^{m} D_{jk}x_j(t)\big[x_k(t) - \alpha_{jk}x_j(t)\big], \quad j = 1,\dots,m, \tag{1}$$

where x_j ($j = 1,\dots,m$) denotes the density of species x in patch j, $r_j > 0$ is the growth rate of x_j, $b_j > 0$ stands for the density-dependent factor in patch j, $D_{jk} \geq 0$ represent the diffusion coefficient from patch k to patch j, and $\alpha_{jk} \geq 0$, $j,k = 1,\dots,m$, are the diffusion boundary conditions.

It is quite a common phenomenon that the species are always affected by environmental fluctuations. The population growth usually suffers from the disturbance of ecological environment, such as supply of food, climate change, and natural enemies, and the stochasticity can be described by white noise. The stochastic model has attracted the attention of many researchers, and more studies can be found in [9–18]. The growth rates affected by noises can be described by

$$r_i \to r_i + \sigma_{i1}\dot{B}_{i1}(t), \quad i = 1,2, \tag{2}$$

where σ_{i1} is the intensity of $B_{i1}(t)$.

To develop and manage the biological resource, it is important to consider the problem of optimal harvesting. If natural resources management is reasonable, then it can increase sustainable production and equitable profit. Bioeconomic models allow ecological resources or certain benefits to be suitably exploited. A number of researchers have paid particular attention to the study of optimal harvesting policy, and a lot of results have been obtained [19–25]. However, sometimes the harvesting may also be affected by human-caused disturbance, such as the price of labor power, crude oil, and goods, which could also be described as white noise. The proportion coefficient of harvesting can be replaced by

$$E_i \to E_i + \sigma_{i2}\dot{B}_{i2}(t), \quad i = 1,2, \tag{3}$$

where σ_{i2} is the intensity of $B_{i2}(t)$, and $B_{11}(t)$, $B_{12}(t)$, $B_{21}(t)$, and $B_{22}(t)$ are independent standard Brownian motions.

In model (1), we consider the stochastic effects not only on growth rate but also on harvesting effort to investigate the optimal harvesting problem of the following stochastic model:

$$\begin{cases} \mathrm{d}x_1(t) = x_1(t)[r_1 - E_1 - b_1x_1(t)]\,\mathrm{d}t + D_1x_1(t)[\mathrm{e}^{-d_2\tau_2}x_2(t-\tau_2) - \alpha_1x_1(t)]\,\mathrm{d}t \\ \qquad\qquad + \sigma_{11}x_1(t)\,\mathrm{d}B_{11}(t) - \sigma_{12}x_1(t)\,\mathrm{d}B_{12}(t), \\ \mathrm{d}x_2(t) = x_2(t)[r_2 - E_2 - b_2x_2(t)]\,\mathrm{d}t + D_2x_2(t)[\mathrm{e}^{-d_1\tau_1}x_1(t-\tau_1) - \alpha_2x_2(t)]\,\mathrm{d}t \\ \qquad\qquad + \sigma_{21}x_2(t)\,\mathrm{d}B_{21}(t) - \sigma_{22}x_2(t)\,\mathrm{d}B_{22}(t), \end{cases} \tag{4}$$

where $E_i \geq 0$ $(i = 1,2)$ and d_i $(i = 1,2)$ respectively denote harvesting effort and death rate of the species x_i, $\tau_i \geq 0$ $(i = 1,2)$ is the time delay caused by the diffusion of the species x_i $(i = 1,2)$, D_1 is the diffusion coefficient from patch 2 to patch 1,whereas D_2 represents the diffusion coefficient from patch 1 to patch 2, and α_i $(i = 1,2)$ represent the boundary conditions.

2 Stochastic persistence and extinction

For convenience, we define the following notations:

$$\tau = \max\{\tau_1,\tau_2\}, \qquad \langle f(t)\rangle = t^{-1}\int_0^t f(s)\,\mathrm{d}s,$$

$$\langle f\rangle^* = \limsup_{t\to+\infty} t^{-1}\int_0^t f(s)\,\mathrm{d}s, \qquad \langle f\rangle_* = \liminf_{t\to+\infty} t^{-1}\int_0^t f(s)\,\mathrm{d}s,$$

$$M_0 = (b_1 + \alpha_1D_1)(b_2 + \alpha_2D_2), \qquad M = (b_1 + \alpha_1D_1)(b_2 + \alpha_2D_2) - D_1D_2\mathrm{e}^{-(d_1\tau_1+d_2\tau_2)},$$

$$M_1 = k_1(b_2 + \alpha_2 D_2) + k_2 D_1 \mathrm{e}^{-d_2\tau_2}, \qquad M_2 = k_1 D_2 \mathrm{e}^{-d_1\tau_1} + k_2(b_1 + \alpha_1 D_1),$$

where $k_1 = r_1 - E_1 - 0.5\sigma_{11}^2 - 0.5\sigma_{12}^2$ and $k_2 = r_2 - E_2 - 0.5\sigma_{21}^2 - 0.5\sigma_{22}^2$.

To begin with, we introduce some lemmas.

Lemma 2.1 ([21]) *Suppose that $X(t) \in C(\Omega \times [0, +\infty), R_+)$.*

(i) *If there exist two positive constants T and η_0 such that*

$$\ln X(t) \leq \eta t - \eta_0 \int_0^t X(s)\,\mathrm{d}s + \sum_{j=1}^m \sigma_j W_j(t)$$

for all $t \geq T$, where σ_j, $j = 1, \ldots, m$, are constants, then

$$\begin{cases} \langle X(t)\rangle^* \leq \eta/\eta_0 & \text{a.s. if } \eta \geq 0, \\ \lim_{t\to+\infty} X(t) = 0 & \text{a.s. if } \eta < 0. \end{cases}$$

(ii) *If there exist three positive constants T, η, and η_0 such that*

$$\ln X(t) \geq \eta t - \eta_0 \int_0^t X(s)\,\mathrm{d}s + \sum_{j=1}^m \sigma_j W_j(t)$$

for all $t \geq T$, then $\langle X(t)\rangle_* \geq \eta/\eta_0$ a.s.

Lemma 2.2 *For any given initial value $x_0(t) \in C([-\tau, 0], R_+^2)$, there exists a function $x(t)$ that is the unique solution for model* (4) *on $t \geq -\tau$ and remains in R_+^2 with probability* 1.

The proof is similar to that in [17] and it is omitted here.

In order to obtain the properties of model (4), let us first analyze the following auxiliary system:

$$\begin{cases} \mathrm{d}y_1(t) = y_1(t)[r_1 - E_1 - b_1 y_1(t)]\,\mathrm{d}t - \alpha_1 D_1 y_1^2(t)\,\mathrm{d}t \\ \qquad\qquad + \sigma_{11} y_1(t)\,\mathrm{d}B_{11}(t) - \sigma_{12} y_1(t)\,\mathrm{d}B_{12}(t), \\ \mathrm{d}y_2(t) = y_2(t)[r_2 - E_2 - b_2 y_2(t)]\,\mathrm{d}t + D_2 y_2(t)[\mathrm{e}^{-d_1\tau_1} y_1(t-\tau_1) - \alpha_2 y_2(t)]\,\mathrm{d}t \\ \qquad\qquad + \sigma_{21} y_2(t)\,\mathrm{d}B_{21}(t) - \sigma_{22} y_2(t)\,\mathrm{d}B_{22}(t). \end{cases} \tag{5}$$

Lemma 2.3 *If $k_1 = r_1 - E_1 - 0.5\sigma_{11}^2 - 0.5\sigma_{12}^2 > 0$, then any solution $y(t) = (y_1(t), y_2(t))$ of model* (5) *satisfies*

$$\lim_{t\to+\infty} \langle y_1\rangle = \frac{k_1}{b_1 + \alpha_1 D_1} \quad \text{a.s. and} \quad \begin{cases} \lim_{t\to+\infty} y_2(t) = 0 & \text{a.s. if } M_2 < 0, \\ \lim_{t\to+\infty} \langle y_2\rangle = \frac{M_2}{M_0} & \text{a.s. if } M_2 > 0. \end{cases} \tag{6}$$

Proof Applying Itô's formula to system (5) leads to

$$\begin{aligned} \mathrm{d}\big(\ln y_1(t)\big) &= \big[k_1 - b_1 y_1(t) - \alpha_1 D_1 y_1(t)\big]\,\mathrm{d}t + \sigma_{11}\,\mathrm{d}B_{11}(t) - \sigma_{12}\,\mathrm{d}B_{12}(t), \\ \mathrm{d}\big(\ln y_2(t)\big) &= \big[k_2 - b_2 y_2(t) + D_2 \mathrm{e}^{-d_1\tau_1} y_1(t-\tau_1) - \alpha_2 D_2 y_2(t)\big]\,\mathrm{d}t \\ &\quad + \sigma_{21}\,\mathrm{d}B_{21}(t) - \sigma_{22}\,\mathrm{d}B_{22}(t). \end{aligned}$$

Integrating both sides of these two differential equations, we find

$$\ln y_1(t) - \ln y_1(0) = k_1 t - (b_1 + \alpha_1 D_1) \int_0^t y_1(s)\,\mathrm{d}s + \sigma_{11} B_{11}(t) - \sigma_{12} B_{12}(t), \tag{7}$$

$$\begin{aligned} \ln y_2(t) - \ln y_2(0) = {} & k_2 t + D_2 \mathrm{e}^{-d_1\tau_1} \int_0^t y_1(s-\tau_1)\,\mathrm{d}s \\ & - (b_2 + \alpha_2 D_2) \int_0^t y_2(s)\,\mathrm{d}s + \sigma_{21} B_{21}(t) - \sigma_{22} B_{22}(t). \end{aligned} \tag{8}$$

Dividing both sides of (7) and (8) by t, we get

$$t^{-1} \ln \frac{y_1(t)}{y_1(0)} = k_1 - (b_1 + \alpha_1 D_1) t^{-1} \int_0^t y_1(s)\,\mathrm{d}s + t^{-1}\sigma_{11} B_{11}(t) - t^{-1}\sigma_{12} B_{12}(t), \tag{9}$$

$$\begin{aligned} t^{-1} \ln \frac{y_2(t)}{y_2(0)} = {} & k_2 + D_2 \mathrm{e}^{-d_1\tau_1} t^{-1} \int_0^t y_1(s-\tau_1)\,\mathrm{d}s - (b_2 + \alpha_2 D_2) t^{-1} \int_0^t y_2(s)\,\mathrm{d}s \\ & + t^{-1}\sigma_{21} B_{21}(t) - t^{-1}\sigma_{22} B_{22}(t) \\ = {} & k_2 + D_2 \mathrm{e}^{-d_1\tau_1} t^{-1} \int_0^t y_1(s)\,\mathrm{d}s + D_2 \mathrm{e}^{-d_1\tau_1} t^{-1} \left[\int_{-\tau_1}^0 y_1(s)\,\mathrm{d}s - \int_{t-\tau_1}^t y_1(s)\,\mathrm{d}s \right] \\ & - (b_2 + \alpha_2 D_2) t^{-1} \int_0^t y_2(s)\,\mathrm{d}s + t^{-1}\sigma_{21} B_{21}(t) - t^{-1}\sigma_{22} B_{22}(t). \end{aligned} \tag{10}$$

By Lemma 2.1 we derive from (7) that $\langle y_1 \rangle^* \leq \frac{k_1}{(b_1+\alpha_1 D_1)}$ and $\langle y_1 \rangle_* \geq \frac{k_1}{(b_1+\alpha_1 D_1)}$. Hence, we have

$$\lim_{t\to+\infty} \langle y_1 \rangle = \frac{k_1}{b_1 + \alpha_1 D_1} \quad \text{a.s.} \tag{11}$$

Using $\lim_{t\to+\infty} t^{-1} B_{1j}(t) = 0, j = 1, 2$, and substituting (11) into (9) yield

$$\lim_{t\to+\infty} t^{-1} \ln y_1(t) = 0 \quad \text{a.s.} \tag{12}$$

Computing (9) $\times$ $D_2 \mathrm{e}^{-d_1\tau_1}$ + (10) $\times$ $(b_1 + \alpha_1 D_1)$ leads to

$$\begin{aligned} & (b_1 + \alpha_1 D_1) t^{-1} \ln \frac{y_2(t)}{y_2(0)} + D_2 \mathrm{e}^{-d_1\tau_1} t^{-1} \ln \frac{y_1(t)}{y_1(0)} \\ & \quad = M_2 - M_0 \langle y_2 \rangle + (b_1 + \alpha_1 D_1) D_2 \mathrm{e}^{-d_1\tau_1} t^{-1} \left[\int_{-\tau_1}^0 y_1(s)\,\mathrm{d}s - \int_{t-\tau_1}^t y_1(s)\,\mathrm{d}s \right] \\ & \qquad + t^{-1} \left[D_2 \mathrm{e}^{-d_1\tau_1} \big(\sigma_{11} B_{11}(t) - \sigma_{12} B_{12}(t) \big) + (b_1 + \alpha_1 D_1) \big(\sigma_{21} B_{21}(t) \sigma_{22} B_{22}(t) \big) \right]. \end{aligned} \tag{13}$$

From (11) we get

$$\lim_{t\to+\infty} t^{-1} \int_{t-\tau_1}^t y_1(s)\,\mathrm{d}s = \lim_{t\to+\infty} t^{-1} \left[\int_0^t y_1(s)\,\mathrm{d}s - \int_0^{t-\tau_1} y_1(s)\,\mathrm{d}s \right] = 0 \quad \text{a.s.} \tag{14}$$

We can find from (12), (13), and (14) that, when $M_2 < 0$, $\lim_{t\to+\infty} y_2(t) = 0$ a.s., and when $M_2 > 0$, $\lim_{t\to+\infty} \langle y_2 \rangle = \frac{M_2}{M_0}$ a.s. □

Similarly, we give another auxiliary system to help us obtain the main results:

$$\begin{cases} \mathrm{d}z_1(t) = z_1(t)[r_1 - E_1 - b_1 z_1(t)]\,\mathrm{d}t + D_1 z_1(t)[\mathrm{e}^{-d_2\tau_2} z_2(t-\tau_2) - \alpha_1 z_1(t)]\,\mathrm{d}t \\ \qquad + \sigma_{11} z_1(t)\,\mathrm{d}B_{11}(t) - \sigma_{12} z_1(t)\,\mathrm{d}B_{12}(t), \\ \mathrm{d}z_2(t) = z_2(t)[r_2 - E_2 - b_2 z_2(t)]\,\mathrm{d}t - \alpha_2 D_2 z_2^2(t)\,\mathrm{d}t \\ \qquad + \sigma_{21} z_2(t)\,\mathrm{d}B_{21}(t) - \sigma_{22} z_2(t)\,\mathrm{d}B_{22}(t). \end{cases} \tag{15}$$

Similarly to the proof of Lemma 2.3, we get the following:

Lemma 2.4 *If* $k_2 = r_1 - E_1 - 0.5\sigma_{21}^2 - 0.5\sigma_{22}^2 > 0$, *then any solution* $z(t) = (z_1(t), z_2(t))$ *of system* (15) *satisfies*

$$\begin{cases} \lim_{t\to+\infty} z_1(t) = 0 & \textit{a.s. if } M_1 < 0, \\ \lim_{t\to+\infty} \langle z_1\rangle = \frac{M_1}{M_0} & \textit{a.s. if } M_1 > 0, \end{cases} \quad \textit{and} \quad \lim_{t\to+\infty} \langle z_2\rangle = \frac{k_2}{b_2 + \alpha_2 D_2} \quad \textit{a.s.} \tag{16}$$

Moreover, we can derive the following equations:

$$\lim_{t\to+\infty} t^{-1} \ln z_2(t) = 0 \quad \text{a.s.} \tag{17}$$

and

$$\lim_{t\to+\infty} t^{-1} \int_{t-\tau_2}^{t} z_2(s)\,\mathrm{d}s = \lim_{t\to+\infty} t^{-1} \left[\int_0^t z_2(s)\,\mathrm{d}s - \int_0^{t-\tau_2} z_2(s)\,\mathrm{d}s \right] = 0 \quad \text{a.s.} \tag{18}$$

From Lemmas 2.1-2.4 we can obtain the following theorem.

Theorem 2.1 *Suppose that* $M > 0$. *The solution* $x(t) = (x_1(t), x_2(t))$ *of system* (4) *has the following global asymptotic properties*:

(i) *If* $k_1 < 0$ *and* $k_2 < 0$, *then both* x_1 *and* x_2 *go to extinction almost surely* (*a.s.*), *that is,*

$$\lim_{t\to+\infty} x_i(t) = 0 \quad \textit{a.s.}, i = 1, 2.$$

(ii) *If* $k_1 > 0$ *and* $M_2 < 0$, *then* x_1 *is persistent in mean a.s., that is,*

$$\lim_{t\to+\infty} \langle x_1\rangle = \frac{k_1}{b_1 + \alpha_1 D_1} \quad \textit{a.s.},$$

and x_2 *goes to extinction a.s.*

(iii) *If* $k_2 > 0$ *and* $M_1 < 0$, *then* x_1 *goes to extinction a.s., and* x_2 *is persistent in mean a.s., that is,*

$$\lim_{t\to+\infty} \langle x_2\rangle = \frac{k_2}{b_2 + \alpha_2 D_2} \quad \textit{a.s.}$$

(iv) *If* $M_1 > 0$ *and* $M_2 > 0$, *then both* x_1 *and* x_2 *are persistent in mean a.s., that is,*

$$\lim_{t\to+\infty} \langle x_1\rangle = \frac{M_1}{M}, \qquad \lim_{t\to+\infty} \langle x_2\rangle = \frac{M_2}{M} \quad \textit{a.s.} \tag{19}$$

Proof By stochastic comparison theorem we get

$$x_1(t) \le y_1(t), \qquad x_2(t) \le y_2(t). \tag{20}$$

From (12) and (14) we can observe that

$$\lim_{t\to+\infty} t^{-1} \ln x_1(t) = 0 \tag{21}$$

and

$$\lim_{t\to+\infty} t^{-1} \int_{t-\tau_1}^{t} x_1(s)\,\mathrm{d}s = 0. \tag{22}$$

Similarly, we can also obtain

$$x_1(t) \le z_1(t), \qquad x_2(t) \le z_2(t).$$

Then it follows from (17) and (18) that

$$\lim_{t\to+\infty} t^{-1} \ln x_2(t) = 0 \tag{23}$$

and

$$\lim_{t\to+\infty} t^{-1} \int_{t-\tau_2}^{t} x_2(s)\,\mathrm{d}s = 0. \tag{24}$$

Applying Itô's formula to system (4) yields

$$\begin{aligned}
\mathrm{d}\big(\ln x_1(t)\big) &= \big[k_1 - (b_1 + \alpha_1 D_1)x_1(t) + D_1 \mathrm{e}^{-d_2\tau_2} x_2(t-\tau_2)\big]\,\mathrm{d}t + \sigma_{11}\,\mathrm{d}B_{11}(t) - \sigma_{12}\,\mathrm{d}B_{12}(t), \\
\mathrm{d}\big(\ln x_2(t)\big) &= \big[k_2 - (b_2 + \alpha_2 D_2)x_2(t) + D_2 \mathrm{e}^{-d_1\tau_1} x_1(t-\tau_1)\big]\,\mathrm{d}t + \sigma_{21}\,\mathrm{d}B_{21}(t) - \sigma_{22}\,\mathrm{d}B_{22}(t).
\end{aligned}$$

Integrating both sides of these two differential equations, we get

$$\begin{aligned}
\ln\frac{x_1(t)}{x_1(0)} &= k_1 t - (b_1 + \alpha_1 D_1)\int_0^t x_1(s)\,\mathrm{d}s + D_1\mathrm{e}^{-d_2\tau_2}\int_0^t x_2(s-\tau_2)\,\mathrm{d}s \\
&\quad + \sigma_{11}B_{11}(t) - \sigma_{12}B_{12}(t) \\
&= k_1 t - (b_1 + \alpha_1 D_1)\int_0^t x_1(s)\,\mathrm{d}s + D_1\mathrm{e}^{-d_2\tau_2}\int_0^t x_2(s)\,\mathrm{d}s \\
&\quad + D_1\mathrm{e}^{-d_2\tau_2}\left[\int_{-\tau_2}^0 x_2(s)\,\mathrm{d}s - \int_{t-\tau_2}^t x_2(s)\,\mathrm{d}s\right] + \sigma_{11}B_{11}(t) - \sigma_{12}B_{12}(t), \qquad (25) \\
\ln\frac{x_2(t)}{x_2(0)} &= k_2 t - (b_2 + \alpha_2 D_2)\int_0^t x_2(s)\,\mathrm{d}s + D_2\mathrm{e}^{-d_1\tau_1}\int_0^t x_1(s-\tau_1)\,\mathrm{d}s \\
&\quad + \sigma_{21}B_{21}(t) - \sigma_{22}B_{22}(t) \\
&= k_2 t - (b_2 + \alpha_2 D_2)\int_0^t x_2(s)\,\mathrm{d}s + D_2\mathrm{e}^{-d_1\tau_1}\int_0^t x_1(s)\,\mathrm{d}s \\
&\quad + D_2\mathrm{e}^{-d_1\tau_1}\left[\int_{-\tau_1}^0 x_1(s)\,\mathrm{d}s - \int_{t-\tau_1}^t x_1(s)\,\mathrm{d}s\right] + \sigma_{21}B_{21}(t) - \sigma_{22}B_{22}(t). \qquad (26)
\end{aligned}$$

Dividing both sides of (25) and (26) by t, we get

$$\begin{aligned} t^{-1}\ln\frac{x_1(t)}{x_1(0)} &= k_1 - (b_1+\alpha_1 D_1)\langle x_1\rangle + D_1 e^{-d_2\tau_2}\langle x_2\rangle \\ &\quad + D_1 e^{-d_2\tau_2} t^{-1}\left[\int_{-\tau_2}^{0} x_2(s)\,ds - \int_{t-\tau_2}^{t} x_2(s)\,ds\right] \\ &\quad + t^{-1}\bigl(\sigma_{11}B_{11}(t) - \sigma_{12}B_{12}(t)\bigr), \qquad (27) \\ t^{-1}\ln\frac{x_2(t)}{x_2(0)} &= k_2 - (b_2+\alpha_2 D_2)\langle x_2\rangle + D_2 e^{-d_1\tau_1}\langle x_1\rangle \\ &\quad + D_2 e^{-d_1\tau_1} t^{-1}\left[\int_{-\tau_1}^{0} x_1(s)\,ds - \int_{t-\tau_1}^{t} x_1(s)\,ds\right] \\ &\quad + t^{-1}\bigl(\sigma_{21}B_{21}(t) - \sigma_{22}B_{22}(t)\bigr). \qquad (28) \end{aligned}$$

Computing (27) $\times$ $(b_2+\alpha_2 D_2)$ + (28) $\times$ $D_1 e^{-d_2\tau_2}$ results in

$$\begin{aligned} &(b_2+\alpha_2 D_2)t^{-1}\ln\frac{x_1(t)}{x_1(0)} + D_1 e^{-d_2\tau_2} t^{-1}\ln\frac{x_2(t)}{x_2(0)} \\ &\quad = M_1 - M\langle x_1\rangle + D_1 e^{-d_2\tau_2}(b_2+\alpha_2 D_2)t^{-1}\left[\int_{-\tau_2}^{0} x_2(s)\,ds - \int_{t-\tau_2}^{t} x_2(s)\,ds\right] \\ &\qquad + D_1 D_2 e^{-(d_1\tau_1+d_2\tau_2)} t^{-1}\left[\int_{-\tau_1}^{0} x_1(s)\,ds - \int_{t-\tau_1}^{t} x_1(s)\,ds\right] \\ &\qquad + t^{-1}\bigl[(b_2+\alpha_2 D_2)\bigl(\sigma_{11}\,dB_{11}(t) - \sigma_{12}\,dB_{12}(t)\bigr) \\ &\qquad + D_1 e^{-d_2\tau_2}\bigl(\sigma_{21}\,dB_{21}(t) - \sigma_{22}\,dB_{22}(t)\bigr)\bigr]. \qquad (29) \end{aligned}$$

Computing (27) $\times$ $D_2 e^{-d_1\tau_1}$ + (28) $\times$ $(b_1+\alpha_1 D_1)$ leads to

$$\begin{aligned} &D_2 e^{-d_1\tau_1} t^{-1}\ln\frac{x_1(t)}{x_1(0)} + (b_1+\alpha_1 D_1)t^{-1}\ln\frac{x_2(t)}{x_2(0)} \\ &\quad = M_2 - M\langle x_2\rangle + D_1 D_2 e^{-(d_1\tau_1+d_2\tau_2)} t^{-1}\left[\int_{-\tau_2}^{0} x_2(s)\,ds - \int_{t-\tau_2}^{t} x_2(s)\,ds\right] \\ &\qquad + (b_1+\alpha_1 D_1)D_2 e^{-d_1\tau_1} t^{-1}\left[\int_{-\tau_1}^{0} x_1(s)\,ds - \int_{t-\tau_1}^{t} x_1(s)\,ds\right] \\ &\qquad + t^{-1}\bigl[D_2 e^{-d_1\tau_1}\bigl(\sigma_{11}\,dB_{11}(t) - \sigma_{12}\,dB_{12}(t)\bigr) \\ &\qquad + (b_1+\alpha_1 D_1)\bigl(\sigma_{21}\,dB_{21}(t) - \sigma_{22}\,dB_{22}(t)\bigr)\bigr]. \qquad (30) \end{aligned}$$

From the property of limit superior, for sufficiently large t, we can get the following equations from (27) and (28):

$$\begin{aligned} t^{-1}\ln x_1(t) &\le k_1 + \epsilon_1 - (b_1+\alpha_1 D_1)\langle x_1\rangle + D_1 e^{-d_2\tau_2}\langle x_2\rangle^* \\ &\quad + t^{-1}\sigma_{11}B_{11}(t) - t^{-1}\sigma_{12}B_{12}(t), \qquad (31) \\ t^{-1}\ln x_2(t) &\le k_2 + \epsilon_2 - (b_2+\alpha_2 D_2)\langle x_2\rangle + D_2 e^{-d_1\tau_1}\langle x_1\rangle^* \\ &\quad + t^{-1}\sigma_{21}B_{21}(t) - t^{-1}\sigma_{22}B_{22}(t). \qquad (32) \end{aligned}$$

Let $\beta_1 = k_1 + \epsilon_1 + D_1 \mathrm{e}^{-d_2\tau_2}\langle x_2\rangle^*$, $\beta_2 = k_2 + \epsilon_2 + D_2 \mathrm{e}^{-d_1\tau_1}\langle x_1\rangle^*$. So (31) and (32) can be rewritten as

$$t^{-1}\ln x_1(t) \le \beta_1 - (b_1 + \alpha_1 D_1)\langle x_1\rangle + t^{-1}\sigma_{11}B_{11}(t) - t^{-1}\sigma_{12}B_{12}(t), \tag{33}$$

$$t^{-1}\ln x_2(t) \le \beta_2 - (b_2 + \alpha_2 D_2)\langle x_2\rangle + t^{-1}\sigma_{21}B_{21}(t) - t^{-1}\sigma_{22}B_{22}(t). \tag{34}$$

To prove conclusion (i), we suppose that $\langle x_1\rangle^* > 0$. If $\omega \in \{\langle x_2(t,\omega)\rangle^* > 0\}$, then applying Lemma 2.1 to (34) results in

$$\big\langle x_2(t,\omega)\big\rangle^* \le \frac{\beta_2}{b_2 + \alpha_2 D_2} = \frac{k_2 + \epsilon_2 + D_2 \mathrm{e}^{-d_1\tau_1}\langle x_1(t)\rangle^*}{b_2 + \alpha_2 D_2}.$$

For sufficiently large t, substituting (21) and (23) into (30) yields

$$M\big\langle x_2(t,\omega)\big\rangle^* \le M_2 + \epsilon.$$

Since $M > 0$, the left side of the last inequality is positive. Letting ϵ be small enough, we would get $M_2 \ge 0$. Actually, since $k_i < 0$ ($i = 1,2$), we obtain $M_2 < 0$. This is a contradiction. Hence, $\mathcal{P}\{\langle x_2(t,\omega)\rangle^* > 0\} = 0$, so $\langle x_2(t)\rangle^* = 0$ a.s. Then, using it in (31) and noting that $k_1 < 0$, we get that $\lim_{t\to+\infty} x_1(t) = 0$ a.s., which is contradicts the supposition $\langle x_1\rangle^* > 0$. Consequently, we have

$$\langle x_1\rangle^* = 0 \quad \text{a.s.} \tag{35}$$

We are in the position to prove that $\lim_{t\to+\infty} x_2(t) = 0$ a.s. Since $\langle x_1\rangle^* = 0$ a.s., for sufficiently large t, we can derive from (32) that

$$t^{-1}\ln x_2(t) \le k_2 + \epsilon_2 - (b_2 + \alpha_2 D_2)\langle x_2\rangle + t^{-1}\sigma_{21}B_{21}(t) - t^{-1}\sigma_{22}B_{22}(t).$$

Since $k_2 < 0$, applying Lemma 2.1 to the last inequality, we get $\lim_{t\to+\infty} x_2(t) = 0$ a.s.

Now we are ready to prove (ii). Since $k_1 > 0$ and $M_2 < 0$, by Lemma 2.3 we have $\lim_{t\to+\infty} y_2(t) = 0$ a.s. Consequently, by (20), $\lim_{t\to+\infty} x_2(t) = 0$ a.s. Then model (4) is simplified as the following single-species model:

$$\mathrm{d}x_1(t) = x_1(t)\big[r_1 - E_1 - (b_1 + \alpha_1 D_1)x_1(t)\big]\,\mathrm{d}t + \sigma_{11}x_1(t)\,\mathrm{d}B_{11}(t) - \sigma_{12}x_1(t)\,\mathrm{d}B_{12}(t),$$

which coincides with the first equation in (5). Then applying Lemma 2.3 to the last equation leads to

$$\lim_{t\to+\infty}\langle x_1\rangle = \frac{k_1}{b_1 + \alpha_1 D_1} \quad \text{a.s.}$$

Using Lemma 2.4 and (15), then we can prove (iii), and the proof is similar to that of (ii). So the details are omitted.

Let us prove (iv). Substituting (22), (23), and (24) into (29) yields

$$\begin{aligned}
&(b_2 + \alpha_2 D_2)t^{-1}\ln\frac{x_1(t)}{x_1(0)} \\
&\quad \ge M_1 - \epsilon' - M\langle x_1\rangle + t^{-1}\big[(b_2 + \alpha_2 D_2)\big(\sigma_{11}\,\mathrm{d}B_{11}(t) - \sigma_{12}\,\mathrm{d}B_{12}(t)\big) \\
&\qquad + D_1 \mathrm{e}^{-d_2\tau_2}\big(\sigma_{21}\,\mathrm{d}B_{21}(t) - \sigma_{22}\,\mathrm{d}B_{22}(t)\big)\big].
\end{aligned} \tag{36}$$

Noting that $M_1 > 0$, let ϵ' be sufficiently small such that $M_1 - \epsilon' > 0$. Consequently, $\langle x_1\rangle_* \geq \frac{M_1-\epsilon'}{M}$. By the arbitrariness of ϵ' we can observe that

$$\langle x_1\rangle_* \geq \frac{M_1}{M} \quad \text{a.s.} \tag{37}$$

It is not hard to find that $\langle x_1\rangle_* > 0$. Hence, $\beta_1 > 0$. Otherwise, it is easy to see that $\langle x_1\rangle_* = 0$ from inequality (33). Similarly, by using (20), (22), and (24) to (30), we have

$$\begin{aligned}
&(b_1 + \alpha_1 D_1)t^{-1} \ln \frac{x_2(t)}{x_2(0)} \\
&\quad \geq M_2 - \epsilon' - M\langle x_2\rangle + t^{-1}\bigl[D_2 \mathrm{e}^{-d_1\tau_1}\bigl(\sigma_{11}\,\mathrm{d}B_{11}(t) - \sigma_{12}\,\mathrm{d}B_{12}(t)\bigr) \\
&\qquad + (b_1 + \alpha_1 D_1)\bigl(\sigma_{21}\,\mathrm{d}B_{21}(t) - \sigma_{22}\,\mathrm{d}B_{22}(t)\bigr)\bigr].
\end{aligned} \tag{38}$$

According to Lemma 2.1, we would get

$$\langle x_2\rangle_* \geq \frac{M_2}{M} > 0 \quad \text{a.s.} \tag{39}$$

So we have $\beta_2 > 0$. Using Lemma 2.1, from (33) and (34) we have

$$\langle x_1\rangle^* \leq \frac{\beta_1}{b_1 + \alpha_1 D_1}, \qquad \langle x_2\rangle^* \leq \frac{\beta_2}{b_2 + \alpha_2 D_2}.$$

Consequently,

$$\begin{cases} (b_1 + \alpha_1 D_1)\langle x_1\rangle^* - D_1 \mathrm{e}^{-d_2\tau_2}\langle x_2\rangle^* \leq k_1 + \epsilon_1, \\ -D_2 \mathrm{e}^{-d_1\tau_1}\langle x_1\rangle^* + (b_2 + \alpha_2 D_2)\langle x_2\rangle^* \leq k_2 + \epsilon_2. \end{cases} \tag{40}$$

Solving these inequalities and using the arbitrariness of $\epsilon_i (i = 1, 2)$ lead to

$$\langle x_1\rangle^* \leq \frac{M_1}{M}, \qquad \langle x_2\rangle^* \leq \frac{M_2}{M} \quad \text{a.s.}$$

Then (iv) can be proved by combining these inequalities with (37) and (39).

The proof of Theorem 2.1 is complete. □

Remark 1 Similarly to the proof of the Theorem 2.1, we would get:

(v) If $k_1 = 0$, $k_2 < 0$, then x_1 is nonpersistent, that is, $\lim_{t\to+\infty} t^{-1}\int_0^t x_1(s)\,\mathrm{d}s = 0$, and x_2 goes to extinction;

(vi) If $k_1 < 0$, $k_2 = 0$, then x_1 goes to extinction, and x_2 is nonpersistent, that is, $\lim_{t\to+\infty} t^{-1}\int_0^t x_2(s)\,\mathrm{d}s = 0$;

(vii) If $k_1 = 0$ and $k_2 = 0$, then both x_1 and x_2 are nonpersistent.

3 Optimal harvesting

From Section 2 we can observe that both species x_1 and x_2 are persistent in mean if $M_i > 0$, $i = 1, 2$. Our aim in this section is to gain the optimal harvesting effort such that ESY $Y(E) = \lim_{t\to+\infty} \sum_{i=1}^{2} \mathbb{E}(E_i x_i(t))$ can get the maximum when the species are persistent. We first introduce some lemmas.

Lemma 3.1 *Suppose that $x(t)$ is a solution of model* (4) *with any given initial value. For any $q > 0$, there is a $K(q)$ such that*

$$\limsup_{t\to+\infty} \mathbb{E}\left|x(t)\right|^q \leq K(q).$$

Applying Itô's formula to $e^t(x_1^q + x_2^q)$, we would get the conclusion. It is similar to Lemma 3 of [10] and is omitted it here.

Then we can prove the following lemma.

Lemma 3.2 *If $b_1 + \alpha_1 D_1 > D_2 e^{-d_1\tau_1}$, $b_2 + \alpha_2 D_2 > D_1 e^{-d_2\tau_2}$, then model* (4) *is asymptotically stable in distribution, that is, as $t \to +\infty$, for any $\xi(t) \in C([-\tau,0];R_+^2)$, there is a unique probability measure $\nu(\cdot)$ such that the transition probability density $p(t,\xi,\cdot)$ of $x(t)$ converges weakly to $\nu(\cdot)$.*

Proof Let $(x_1(t;\xi), x_2(t;\xi))^T$ and $(x_1(t;\zeta), x_2(t;\zeta))^T$ be two solutions of model (4) with initial values $\xi(\theta) \in C([-\tau,0];R_+^2)$ and $\zeta(\theta) \in C([-\tau,0];R_+^2)$, respectively. Applying Itô's formula to

$$\begin{aligned} V(t) = {} & \sum_{i=1}^{2} \left|\ln x_i(t;\xi) - \ln x_i(t;\zeta)\right| + D_1 e^{-d_2\tau_2} \int_{t-\tau_2}^{t} \left|x_2(s;\xi) - x_2(s;\zeta)\right| \,\mathrm{d}s \\ & + D_2 e^{-d_1\tau_1} \int_{t-\tau_1}^{t} \left|x_1(s;\xi) - x_1(s;\zeta)\right| \,\mathrm{d}s \end{aligned}$$

leads to

$$\begin{aligned} \mathrm{d}^+ V(t) = {} & -\sum_{i=1}^{2} (b_i + \alpha_i D_i)\left|x_i(t;\xi) - x_i(t;\zeta)\right| \,\mathrm{d}t \\ & + \sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} D_i e^{-d_j\tau_j} \left|x_j(t;\xi) - x_j(t;\zeta)\right| \,\mathrm{d}t \\ & - \sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} D_i e^{-d_j\tau_j} \left|x_j(t-\tau_j;\xi) - x_j(t-\tau_j;\zeta)\right| \,\mathrm{d}t \\ & + \sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} D_i e^{-d_j\tau_j} \operatorname{sgn}\bigl(x_i(t;\xi) - x_i(t;\zeta)\bigr)\bigl(x_j(t-\tau_j;\xi) - x_j(t-\tau_j;\zeta)\bigr) \,\mathrm{d}t \\ \leq {} & -\sum_{i=1}^{2} (b_i + \alpha_i D_i)\left|x_i(t;\xi) - x_i(t;\zeta)\right| \,\mathrm{d}t \\ & + \sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} D_i e^{-d_j\tau_j} \left|x_j(t;\xi) - x_j(t;\zeta)\right| \,\mathrm{d}t \\ & - \sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} D_i e^{-d_j\tau_j} \left|x_j(t-\tau_j;\xi) - x_j(t-\tau_j;\zeta)\right| \,\mathrm{d}t \\ & + \sum_{i=1}^{2} \sum_{j=1,j\neq i}^{2} D_i e^{-d_j\tau_j} \left|x_j(t-\tau_j;\xi) - x_j(t-\tau_j;\zeta)\right| \,\mathrm{d}t \end{aligned}$$

$$\begin{aligned}&= -\left(b_1+\alpha_1 D_1-D_2\mathrm{e}^{-d_1\tau_1}\right)\left|x_1(t;\xi)-x_1(t;\zeta)\right|\,\mathrm{d}t\\&\quad-\left(b_2+\alpha_2 D_2-D_1\mathrm{e}^{-d_2\tau_2}\right)\left|x_2(t;\xi)-x_2(t;\zeta)\right|\,\mathrm{d}t.\end{aligned}$$

Consequently,

$$\begin{aligned}\mathbb{E}\big(V(t)\big)&\le V(0)-\left(b_1+\alpha_1 D_1-D_2\mathrm{e}^{-d_1\tau_1}\right)\int_0^t\mathbb{E}\left|x_1(s;\xi)-x_1(s;\zeta)\right|\,\mathrm{d}s\\&\quad-\left(b_2+\alpha_2 D_2-D_1\mathrm{e}^{-d_2\tau_2}\right)\int_0^t\mathbb{E}\left|x_2(s;\xi)-x_2(s;\zeta)\right|\,\mathrm{d}s.\end{aligned}$$

Since $V(t)\ge 0$, it follows from the last inequality that

$$\begin{aligned}&\left(b_1+\alpha_1 D_1-D_2\mathrm{e}^{-d_1\tau_1}\right)\int_0^t\mathbb{E}\left|x_1(s;\xi)-x_1(s;\zeta)\right|\,\mathrm{d}s\\&\quad+\left(b_2+\alpha_2 D_2-D_1\mathrm{e}^{-d_2\tau_2}\right)\int_0^t\mathbb{E}\left|x_2(s;\xi)-x_2(s;\zeta)\right|\,\mathrm{d}s\le V(0)<\infty.\end{aligned}$$

In other words, $\mathbb{E}|x_i(t;\xi)-x_i(t;\zeta)|\in L^1[0,+\infty)$, $i=1,2$.

Moreover, from the first equation of model (4) we have

$$\begin{aligned}\mathbb{E}\big(x_1(t)\big)&=x_1(0)+\int_0^t\big[(r_1-E_1)\mathbb{E}\big(x_1(s)\big)-(b_1+\alpha_1 D_1)\mathbb{E}\big(x_1^2(s)\big)\\&\quad+D_1\mathrm{e}^{-d_2\tau_2}\mathbb{E}\big(x_1(s)x_2(s-\tau_2)\big)\big]\,\mathrm{d}s.\end{aligned}$$

Hence, $\mathbb{E}(x_1(t))$ is a continuously differentiable function. By Lemma 3.1 we can obtain

$$\frac{\mathrm{d}\mathbb{E}(x_1(t))}{\mathrm{d}t}\le(r_1-E_1)\mathbb{E}\big(x_1(t)\big)+D_1\mathrm{e}^{-d_2\tau_2}\mathbb{E}\big(x_1(t)x_2(t-\tau_2)\big)\le K^*,$$

where K^* is a positive constant. Therefore, $\mathbb{E}(x_1(t))$ is uniformly continuous. Applying the same argument to the second equation of model (4), we can obtain that $\mathbb{E}(x_2(t))$ is uniformly continuous. By the conclusion of [26] we can get

$$\lim_{t\to+\infty}\mathbb{E}\left|x_i(t;\xi)-x_i(t;\zeta)\right|=0\quad\text{a.s.}, i=1,2.\tag{41}$$

Note that $p(t,\xi,\mathrm{d}y)$ is the transition probability density of the process $x(t)$ and $P(t,\xi,A)$ denotes the probability of event $x(t;\xi)\in A$ with the initial value $\xi(\theta)\in C([-\tau,0];R_+^2)$. By Lemma 3.1 and Chebyshev's inequlity it follows from [27] that the family of $p(t,\xi,\mathrm{d}y)$ is tight, that is, for any given $\epsilon^*>0$, there is a compact subset $\mathcal{K}\in R_+^2$ such that $P(t,\xi,\mathcal{K})\ge 1-\epsilon^*$.

Let $\mathcal{P}(C([-\tau,0];R_+^2))$ be the probability measures on $C([-\tau,0];R_+^2)$. For any given two measures $P_1,P_2\in\mathcal{P}$, we propose the following metric:

$$d_{\mathbb{L}}(P_1,P_2)=\sup_{g\in\mathbb{L}}\left|\int_{R_+^2}g(x)P_1(\mathrm{d}x)-\int_{R_+^2}g(x)P_2(\mathrm{d}x)\right|,$$

where

$$\mathbb{L}=\left\{g:C\big([-\tau,0];R_+^2\big)\to R\big|\left|g(x)-g(y)\right|\le\|x-y\|,\left|g(\cdot)\right|\le 1\right\}.$$

For any $g \in \mathbb{L}$ and $t, s > 0$, we get

$$\begin{aligned}
&\big|\mathbb{E}g\big(x(t+s;\xi)\big) - \mathbb{E}g\big(x(t;\xi)\big)\big| \\
&\quad = \big|\mathbb{E}\big[\mathbb{E}\big(g\big(x(t+s;\xi)\big)|\mathcal{F}_s\big)\big] - \mathbb{E}g\big(x(t;\xi)\big)\big| \\
&\quad = \left|\int_{R_+^2} \mathbb{E}\big(g\big(x(t;\phi)\big)\big)p(s,\xi,\mathrm{d}\phi) - \mathbb{E}g\big(x(t;\xi)\big)\right| \\
&\quad \le \int_{R_+^2} \big|\mathbb{E}\big(g\big(x(t;\phi)\big)\big) - \mathbb{E}\big(g\big(x(t;\xi)\big)\big)\big| p(s,\xi,\mathrm{d}\phi) \\
&\quad \le 2p\big(s,\xi,U_K^C\big) + \int_{U_K} \big|\mathbb{E}\big(g\big(x(t;\phi)\big)\big) - \mathbb{E}\big(g\big(x(t;\xi)\big)\big)\big| p(s,\xi,\mathrm{d}\phi),
\end{aligned}$$

where $U_K = \{x \in R_+^2 : |x| \le K\}$, and U_K^C is the complementary set of U_K. Since the family of $p(t,\xi,\mathrm{d}y)$ is tight, for any given $s \ge 0$, there exists a sufficiently large K such that $p(s,\xi,U_K^C) < \epsilon^*/4$. By (41) there exists $T > 0$ such that, for $t \ge T$, we have

$$\sup_{g\in\mathbb{L}} \big|\mathbb{E}g\big(x(t;\phi)\big) - \mathbb{E}g\big(x(t;\xi)\big)\big| \le \frac{\epsilon^*}{2}.$$

Hence, it is easy to find that $|\mathbb{E}g(x(t+s;\xi)) - \mathbb{E}g(x(t;\xi))| \le \epsilon^*$. By the arbitrariness of g, we have

$$\sup_{g\in\mathbb{L}} \big|\mathbb{E}g\big(x(t+s;\xi)\big) - \mathbb{E}g\big(x(t;\xi)\big)\big| \le \epsilon^*,$$

that is,

$$d_{\mathbb{L}}\big(p(t+s,\xi,\cdot),p(t,\xi,\cdot)\big) \le \epsilon^*, \quad \forall t \ge T, s > 0.$$

So $\{p(t,0,\cdot) : t \ge 0\}$ is Cauchy in $\mathcal{P}$ with metric $d_{\mathbb{L}}$. There is a unique $\nu(\cdot) \in \mathcal{P}(C([-\tau,0]; R_+^2))$ such that $\lim_{t\to 0} d_{\mathbb{L}}(p(t,0,\cdot),\nu(\cdot)) = 0$. In addition, it follows from (41) that

$$\lim_{t\to 0} d_{\mathbb{L}}\big(p(t,\xi,\cdot),p(t,0,\cdot)\big) = 0.$$

Thus,

$$\lim_{t\to 0} d_{\mathbb{L}}\big(p(t,\xi,\cdot),\nu(\cdot)\big) \le \lim_{t\to 0} d_{\mathbb{L}}\big(p(t,\xi,\cdot),p(t,0,\cdot)\big) + \lim_{t\to 0} d_{\mathbb{L}}\big(p(t,0,\cdot),\nu(\cdot)\big) = 0.$$

This completes the proof of Lemma 3.2. □

For simplicity, we define other notations:

$$B = \begin{pmatrix} b_1 + \alpha_1 D_1 & -D_1 \mathrm{e}^{-d_2\tau_2} \\ -D_2 \mathrm{e}^{-d_1\tau_1} & b_2 + \alpha_2 D_2 \end{pmatrix}, \qquad \Lambda = (\lambda_1,\lambda_2)^T = \big[B\big(B^{-1}\big)^T + I\big]^{-1} Q,$$

where $Q = (r_1 - 0.5\sigma_{11}^2 - 0.5\sigma_{12}^2, r_2 - 0.5\sigma_{21}^2 - 0.5\sigma_{22}^2)^T$, and I is the unit matrix.

Theorem 3.1 *Let $b_1 + \alpha_1 D_1 > D_2 e^{-d_1\tau_1}$, $b_2 + \alpha_2 D_2 > D_1 e^{-d_2\tau_2}$, $M > 0$ and suppose that $B^{-1} + (B^{-1})^T$ is positive definite.*

(i) *If $\lambda_i \geq 0, i = 1, 2$, and $E_i = \lambda_i, i = 1, 2$, we have $M_1 > 0, M_2 > 0$. Then the optimal harvesting effort is $E^* = \Lambda = [B(B^{-1})^T + I]^{-1}Q$, and the maximum of ESY is*

$$Y^* = \Lambda^T B^{-1}(Q - \Lambda). \tag{42}$$

(ii) *When $E_i = \lambda_i, i = 1, 2$, there is $M_i \leq 0$, $i = 1$ or 2, or there is $\lambda_i < 0$, $i = 1$ or 2, then the optimal harvesting policy does not exist.*

Proof Let $G = \{E = (E_1, E_2)^T \in R^2 | M_i > 0, E_i > 0, i = 1, 2\}$. When (19) holds, for every $E \in G$, if the optimal harvesting effort E^* exists, then it must belong to G.

We are in the position to prove (i). It is easy to note that $\Lambda \in G$, so G is ont empty. By (19), for any $E \in G$, we have

$$\lim_{t\to+\infty} t^{-1} \int_0^t E^T x(s)\,\mathrm{d}s = \sum_{i=1}^{2} E_i \lim_{t\to+\infty} t^{-1} \int_0^t x_i(s)\,\mathrm{d}s = E^T B^{-1}(Q - E). \tag{43}$$

According to Lemma 3.2, model (4) has a unique invariant measure $\nu(\cdot)$. By Corollary 3.4.3 in [18] we get that $\nu(\cdot)$ is strong mixing. Moreover, it is ergodic by Theorem 3.2.6 in [28]. Hence, it can be derived from (3.3.2) in [28] that

$$\lim_{t\to+\infty} t^{-1} \int_0^t E^T x(s)\,\mathrm{d}s = \int_{R_+^2} E^T x \nu(\mathrm{d}x). \tag{44}$$

Let $\rho(x)$ represent the stationary probability density of model (4). Then we have

$$Y(E) = \lim_{t\to+\infty} \sum_{i=1}^{2} \mathbb{E}\big(E_i x_i(t)\big) = \lim_{t\to+\infty} \mathbb{E}\big(E^T x(t)\big) = \int_{R_+^2} E^T x \rho(x)\,\mathrm{d}x. \tag{45}$$

Since the invariant measure of model (4) is unique, then by the one-to-one correspondence between $\rho(x)$ and its corresponding invariant measure we have

$$\int_{R_+^2} E^T x \rho(x)\,\mathrm{d}x = \int_{R_+^2} E^T x \nu(\mathrm{d}x). \tag{46}$$

From (43)-(46) we can get

$$Y(E) = E^T B^{-1}(Q - E). \tag{47}$$

Note that $\Lambda = (\lambda_1, \lambda_2)^T$ is the unique solution of the following equation:

$$\begin{aligned} \frac{\mathrm{d}Y(E)}{\mathrm{d}E} &= \frac{\mathrm{d}E^T}{\mathrm{d}E} B^{-1}(Q - E) + \frac{\mathrm{d}}{\mathrm{d}E}\big[(Q - E)^T \big(B^{-1}\big)^T\big]E \\ &= B^{-1}Q - \big[B^{-1} + \big(B^{-1}\big)^T\big]E \\ &= 0. \end{aligned} \tag{48}$$

Hence, $\Lambda = [B(B^{-1})^T + I]^{-1}Q$. We can see that Λ is the unique extreme point of $Y(E)$ because the following Hessian matrix is negative define:

$$\frac{\mathrm{d}}{\mathrm{d}E^T}\left[\frac{\mathrm{d}Y(E)}{\mathrm{d}E}\right] = \left(\frac{\mathrm{d}}{\mathrm{d}E}\left[\left(\frac{\mathrm{d}Y(E)}{\mathrm{d}E}\right)^T\right]\right)^T = \left(\frac{\mathrm{d}}{\mathrm{d}E}\left[Q^T\left(B^{-1}\right)^T - E^T\left[B^{-1} + \left(B^{-1}\right)^T\right]\right]\right)^T$$
$$= -B^{-1} - \left(B^{-1}\right)^T.$$

If $\Lambda \in G$, that is, $\lambda_i \geq 0$ $(i = 1,2)$ and $M_i > 0$ $(i = 1,2)$, then $E^* = \Lambda$, and (41) is the maximum value of ESY.

Now we are going to prove (ii). Suppose that the optimal harvesting effort $\bar{E}^* = (\bar{E}_1^*, \bar{E}_2^*)^T$ exists. So $\bar{E}^* \in G$, that is, $M_i|_{E_i=\bar{E}_i^*,i=1,2} > 0, \bar{E}_i^* \geq 0, i = 1,2$. In other words, if $\bar{E}^*$ is the optimal harvesting effort, then $\bar{E}^*$ must be the unique solution of (48). However, $\Lambda = (\lambda_1, \lambda_2)^T$ is also a solution of (48). Hence, $\lambda_i = \bar{E}_i^* \geq 0$, $i = 1,2$, and $M_i|_{E_i=\lambda_i,i=1,2} = M_i|_{E_i=\bar{E}_i^*,i=1,2} > 0$, $i = 1,2$. It is a contradiction with the condition.

This completes the proof of Theorem 3.1. □

4 Numerical simulations and discussion

Taking white noises into account, in this paper, we consider a stochastic delay model of species in two habitants. Theorem 2.1 describes sufficient conditions for persistence in the mean and extinction, which are derived from the stochastic analysis theory. Furthermore, an ergodic method is applied to show that the stochastic model has a unique stationary distribution. We obtain the optimal harvesting effort and the maximum of ESY in Theorem 3.1 by using Hessian matrix method and optimal harvesting theory of differential equations.

By using the Euler scheme in [29] to illustrate the biological significance of the results, first, it is necessary to discretize model (4). Setting the step size $\triangle t = 0.01$ and the delays $\tau_1 = 2$, $\tau_2 = 1$, respectively, the discretized equations with respect to (4) are sa follows:

$$\begin{cases} x_{1(k+1)} = x_{1k} + x_{1k}[r_1 - E_1 - b_1x_{1k} + D_1\mathrm{e}^{-d_2\tau_2}x_{2(k-100)} - \alpha_1 D_1 x_{1k}]\triangle t \\ \qquad + \sigma_{11}x_{1k}(\triangle B_{11})_k - \sigma_{12}x_{1k}(\triangle B_{12})_k, \\ x_{2(k+1)} = x_{2k} + x_{2k}[r_2 - E_2 - b_2x_{2k} + D_2\mathrm{e}^{-d_1\tau_1}x_{1(k-200)} - \alpha_2 D_2 x_{2k}]\triangle t \\ \qquad + \sigma_{21}x_{2k}(\triangle B_{21})_k - \sigma_{22}x_{2k}(\triangle B_{22})_k, \end{cases}$$

where $(\triangle B_{ij})_k = B_i((k+1)\triangle t) - B_i(k\triangle t)$, $i,j = 1,2, k = 0,1,2,\ldots$. The discretization form is the approximate numerical solution related to (4). Other parameters are set as follows: $r_1 = 0.5$, $r_2 = 0.05$, $b_1 = 0.5$, $b_2 = 0.6$, $d_1 = 0.1$, $d_2 = 0.01$, $\tau_1 = 2$, $\tau_2 = 1$, $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, with initial values $\xi_1(\theta) = 0.3 + 0.03\sin\theta$ and $\xi_2(\theta) = 0.1 + 0.05\cos\theta, \theta \in [-2,0]$.

Notice that

$$\text{(b) } k_1 = -0.025 < 0, \qquad k_2 = -0.0013 < 0;$$
$$\text{(c) } k_1 = 0.0997 > 0, \qquad M_2 = -0.0151 < 0;$$
$$\text{(d) } M_1 = 0.1199 > 0, \qquad M_2 = 0.0149 > 0.$$

In Figure 1(b), $\sigma_{11} = 0.3, \sigma_{12} = 0.4, \sigma_{21} = 0.15, \sigma_{22} = 0.2$, the intensity is large enough such that $k_i < 0, i = 1,2$, and it causes the species extinction. In Figure 1(c), $\sigma_{21} = 0.3, \sigma_{22} = 0.3$ are large such that $M_2 < 0$, so noises affect the persistence of species x_2. When the inten-

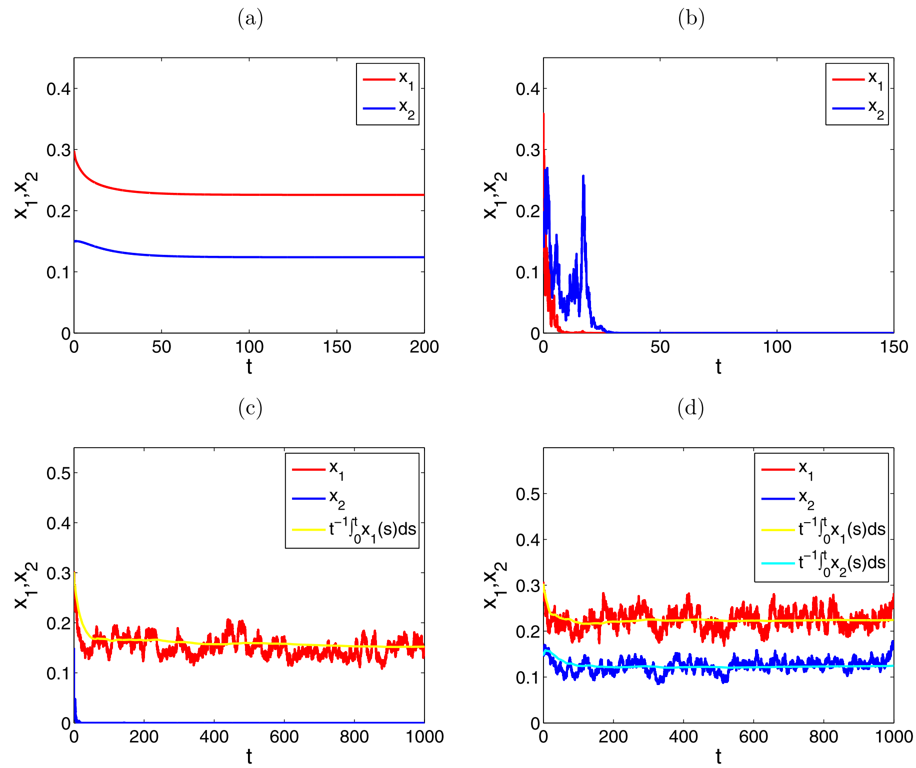


Figure 1 Simulation of the species $x_1(t)$, $x_2(t)$ under stochastic environment. Some paraments are taken: $r_1 = 0.5$, $r_2 = 0.05$, $E_1 = 0.4$, $E_2 = 0.02$, $b_1 = 0.5$, $b_2 = 0.6$, $D_1 = 0.4$, $D_2 = 0.3$, $d_1 = 0.1$, $d_2 = 0.01$, $\tau_1 = 2$, $\tau_2 = 1$; $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\xi_1(\theta) = 0.3 + 0.03\sin\theta$, $\xi_2(\theta) = 0.1 + 0.05\cos\theta$, $\theta \in [-2, 0]$.
(a) $\sigma_{11} = \sigma_{12} = \sigma_{21} = \sigma_{22} \equiv 0$; **(b)** $\sigma_{11} = 0.3$, $\sigma_{12} = 0.4$, $\sigma_{21} = 0.15$, $\sigma_{22} = 0.2$; **(c)** $\sigma_{11} = 0.02$, $\sigma_{12} = 0.01$, $\sigma_{21} = 0.3$, $\sigma_{22} = 0.3$; **(d)** $\sigma_{11} = 0.02$, $\sigma_{12} = 0.01$, $\sigma_{21} = 0.01$, $\sigma_{22} = 0.02$.

sity of the white noise is small, the species can still be persistent just as the deterministic model; see Figure 1(d). Hence, it shows that noise with small intensity can allow the species to preserve the prosperity, whereas noise with large intensity may be a cause of species extinction.

In Figure 2(a), notice that $D_1 = D_2 \equiv 0$ and by computing we can obtain that $k_1 = 0.0997 > 0, k_2 = -0.0004 < 0$, and $M_2 = -0.0002 < 0$; we can derive from Theorem 2.1 that x_1 is persistent in the mean and x_2 goes to extinction. On the contrary, let $D_1 = 0.4$ and $D_2 = 0.3$, which means that there exists diffusion between patches. Then species in patch 1 would move to patch 2. We see that the parameter k_2 is still negative, but $M_2 = 0.0488$ becomes a positive constant. Therefore, x_2 turns into persistence in the mean (see Figure 2(b)). It shows that diffusion is beneficial to the persistence of population.

In Figure 3, we choose $D_1 = 0.2, D_2 = 0.3$. By computing we get $b_1 + \alpha_1 D_1 = 0.58 > D_2 e^{-d_1\tau_1} = 0.2456$ and $b_2 + \alpha_2 D_2 = 0.69 > D_1 e^{-d_2\tau_2} = 0.198$. It is not hard to estimate that $B^{-1} + (B^{-1})^T$ is positive definite. From $\Lambda = [B(B^{-1})^T + I]^{-1}Q$ we see $\Lambda = (\lambda_1, \lambda_2)^T = (0.2451, 0.0369)^T$. Then we get $M_1 = 0.1783 > 0$ and $M_2 = 0.07 > 0$. By Theorem 3.1 we obtain

$$E_1^* = \lambda_1 = 0.2451, \qquad E_2^* = \lambda_2 = 0.0369; \qquad Y^* = \Lambda^T B^{-1}(Q - \Lambda) = 0.1316,$$

whereas E is different from E^*, and the ESY satisfies $Y(E) < Y^*$.

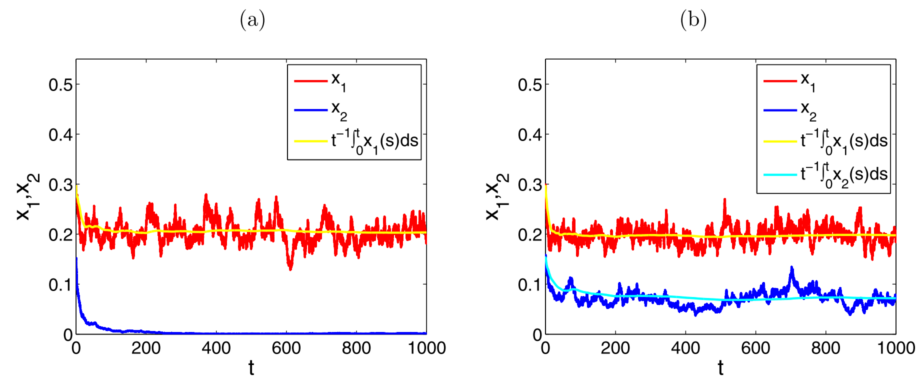
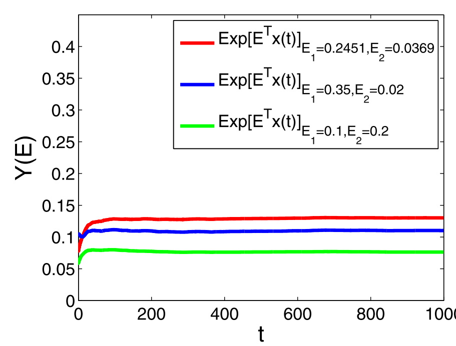


Figure 2 Simulation of the species $x_1(t)$, $x_2(t)$ in two habitants. Some paraments are taken: $r_1 = 0.5$, $r_2 = 0.05$, $E_1 = 0.4$, $E_2 = 0.05$, $b_1 = 0.5$, $b_2 = 0.6$, $d_1 = 0.1$, $d_2 = 0.01$, $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\sigma_{11} = 0.02$, $\sigma_{12} = 0.01$, $\sigma_{21} = 0.02$, $\sigma_{22} = 0.02$, $\xi_1(\theta) = 0.3 + 0.03 \sin\theta$, $\xi_2(\theta) = 0.1 + 0.05 \cos\theta$, $\theta \in [-2, 0]$. **(a)** $D_1 = D_2 \equiv 0$; **(b)** $D_1 = 0.4$, $D_2 = 0.3$.

Figure 3 The optimal harvesting effort and the maximum of ESY. Some paraments are taken: $r_1 = 0.5$, $r_2 = 0.05$, $b_1 = 0.5$, $b_2 = 0.6$, $D_1 = 0.2$, $D_2 = 0.3$, $d_1 = 0.2$, $d_2 = 0.5$, $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\sigma_{11} = 0$, $\sigma_{12} = 0$, $\sigma_{21} = 0$, $\sigma_{22} = 0$, $\xi_1(\theta) = 0.3 + 0.03 \sin\theta$, $\xi_2(\theta) = 0.1 + 0.05 \cos\theta$, $\theta \in [-2, 0]$. Red line is with $E_1 = E_1^* = 0.2451$, $E_2 = E_2^* = 0.0369$, blue line is with $E_1 = 0.35$, $E_2 = 0.02$, and green line is with $E_1 = 0.1$, $E_2 = 0.2$.



Based on theoretical analysis and numerical simulations, we present the main results in this paper:

(1) Comparing with deterministic models [8], our work extends the related results. It reveals that environment disturbance tends to have negative effects on the persistence of population. That is to say, if the intensity of noise is sufficiently large,then the species may suffer extinction, whereas the prosperity of permanence can be preserved under noise with small intensity.

(2) The Fokker-Planck equation is a classical method to handle stochastic optimal harvesting policy [25]. In this paper, we adopt a new approach, namely ergodic theory, to deal with the optimal harvesting problem, which can avoid solving the corresponding Fokker-Planck equation.

(3) Most of the existing works [14, 16, 23] considered the effects of white noise on the growth rate, whereas we have studied not only environment disturbance on that but also harvesting effort affected by human and social factors.

The research results of this paper provide theoretic reference for some modern fields, such as fishery management. It is beneficial for people to make a rational exploitation and derive maximum profit. Some interesting topics in this direction deserve further development. We can consider diffusion coefficients disturbed by white noises or extend the present work into generalized forms, namely a multidimensional stochastic model. Another interesting problem is that we can consider a more realistic but complicated model under a stochastic environment with Lévy jumps or Markovian switching.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to the writing of this paper. Both authors read and approved the final manuscript.

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