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New results on H_∞ state estimation of static neural networks with time-varying delay

Bin Wen^{1*}, Hui Li¹ and Shouming Zhong^{2,3}

*Correspondence:

wenbin_08@126.com

¹School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu, 611731, P.R. China
Full list of author information is available at the end of the article

Abstract

This paper is concerned with the problem of H_∞ state estimation problem for a class of delayed static neural networks. The purpose of the problem is to design a delay-dependent state estimator such that the dynamics of the error system is globally exponentially stable with a prescribed H_∞ performance. Some improved delay-dependent conditions are established by using delay partitioning method and the free-matrix-based integral inequality. The gain matrix and the optimal performance index are obtained via solving a convex optimization problem subject to LMIs (linear matrix inequality). Numerical examples are provided to illustrate the effectiveness of the proposed method comparing with some existing results.

Keywords: static neural networks; H_∞ ; state estimator; Lyapunov-Krasovskii functional

1 Introduction

Neural networks (NNs) have attracted a great deal of attention because of their extensive applications in various fields such as associative memory, pattern recognition, combinatorial optimization, adaptive control, *etc.* [1, 2]. Due to the finite switching speed of the amplifiers, time delays are inevitable in electronic implementations of neural networks (such as Hopfield neural networks, cellular neural networks). Time delay in a system is often a source of instability, and delay systems has been widely studied (see [3–9] and the references therein). The stability and passivity problems of delayed NNs have been widely reported (see [1, 2, 10–14]).

We mainly focus on static neural networks (SNNs) in this paper, which is one type of recurrent neural networks. According to whether the neuron states or the local field states of neurons are chosen as basic variables, the model of neural networks can be classified into static neural networks or local field networks. As mentioned in [15, 16], local field neural networks and SNNs are not always equivalent. In contrast to the local field neural networks, the stability analysis of SNNs has attracted relatively little attention, many interesting results on the stability analysis of SNNs have been addressed in the literature [2, 17–21].

Meanwhile, the state estimation of neural networks is an important issue. In many applications, it is important to estimate the state of neurons to utilize the neural networks.

Recently, some results on the state estimation problem for the neural networks have been investigated in [22–37]. Among them H_∞ state estimation of static neural networks with time delay was studied in [31, 33–37]. In [31], a delay partition approach was proposed to deal with the state estimation problem for a class of static neural networks with time-varying delay. In [33], the state estimation problem of the guaranteed H_∞ and H_2 performance of static neural networks was considered. Further improved results were obtained in [34, 35, 37] by using the convex combination approach. The exponential state estimation of time-varying delayed neural networks was studied in [36]. These literatures all use the Lyapunov-Krasovskii functionals (LKFs) method, conservativeness comes from two things: the choice of functional and the bound on its derivative. A delay partitioning LKF reduce the conservativeness of a simple LKF by containing more delay information. There are two main techniques for dealing with the integrals that appear in the derivative: the free-weighting matrix method and the integral inequality method. However, the inequalities used may lead to conservatism to some extent. Moreover, the information of neuron activation functions has not been adequately taken into account in [31, 33–37]. Therefore, the guaranteed performance state estimation problem has not yet been fully studied and remains a space for improvement. Thus, it remains a space to improve the results.

This paper studies the problem of H_∞ state estimation for a class of delayed SNNs. Our aim is to design a delay-dependent state estimator, such that the filtering error system is globally exponentially stable with a prescribed H_∞ performance. By using delay equal-partitioning method, augmented LKFs are properly constructed. The lower bound is partitioned into several components in [21], so it is impossible to deal the situation when lower bound is zero. Moreover, the effectiveness of delay partitioning method is not evident when lower bound is very small. Different from the delay partitioning method used in [21], we decompose the delay interval into multiple equidistant subintervals, this method is more universal in dealing with the interval time-varying delay. Then the free-weighting matrix technique is used to get a tighter upper bound on the derivatives of the LKFs. As mentioned in Remark 3.3, we also reduce conservatism by taking advantage of the information on activation function. Moreover, the free-matrix-based integral inequality was used to derive improved delay-dependent criteria. Compared with existing results in [34–36], the criteria in this paper lead to less conservatism.

The remainder of this paper is organized as follows. The state estimation problem is formulated in Section 2. Section 3 deals with the design of the state estimator for delayed static neural networks. In Section 4, two numerical examples are provided to show the effectiveness of the results. Finally, some conclusions are made in Section 5.

Notations: The notations used throughout the paper are fairly standard. \mathbb{R}^n denotes the n -dimensional Euclidean space; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices; the notation $A > 0$ (< 0) means A is a symmetric positive (negative) definite matrix; A^{-1} and A^T denote the inverse of matrix A and the transpose of matrix A , respectively; I represents the identity matrix with proper dimensions; a symmetric term in a symmetric matrix is denoted by $(*)$; $\text{sym}\{A\}$ represents $(A + A^T)$; $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem formulation

Consider the delayed static neural network subject to noise disturbances described by

$$\begin{cases} \dot{x}(t) = -Ax(t) + f(Wx(t - h(t)) + J) + B_1w(t), \\ y(t) = Cx(t) + Dx(t - h(t)) + B_2w(t), \\ z(t) = Hx(t), \\ x(t) = \phi(t), \quad t \in [-\tau, 0], \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the neural network, $y(t) \in \mathbb{R}^m$ is the neural network output measurement, $z(t) \in \mathbb{R}^r$ to be estimated is a linear combination of the state, $w(t) \in \mathbb{R}^n$ is the noise input belonging to $L_2[0, \infty)$, $f(\cdot)$ denote the neuron activation function, $A = \text{diag}\{a_1, a_2, \dots, a_n\}$ with $a_i > 0, i = 1, 2, \dots, n$ is the positive diagonal matrix, $B_1, B_2, C, D,$ and H are real known matrices with appropriate dimensions, $W \in \mathbb{R}^{n \times n}$ denote the connection weights, $h(t)$ represent the time-varying delays, J represents the exogenous input vector, the function $\phi(t)$ is the initial condition, and $\tau = \sup_{t \geq 0} \{h(t)\}$.

In this paper, the time-varying delays satisfy

$$0 \leq d_1 \leq h(t) \leq d_2, \quad \dot{h}(t) \leq \mu,$$

where $d_1, d_2,$ and μ are constants.

In order to conduct the analysis, the following assumptions are necessary.

Assumption 2.1 For any $x, y \in \mathbb{R} (x \neq y), i \in \{1, 2, \dots, n\}$, the activation function satisfies

$$k_i^- \leq \frac{f_i(x) - f_i(y)}{x - y} \leq k_i^+, \tag{2}$$

where k_i^-, k_i^+ are constants, and we define $K^- = \text{diag}\{k_1^-, k_2^-, \dots, k_n^-\}, K^+ = \text{diag}\{k_1^+, k_2^+, \dots, k_n^+\}$.

We construct a state estimator for estimation of $z(t)$:

$$\begin{cases} \dot{\hat{x}}(t) = -A\hat{x}(t) + f(W\hat{x}(t - h(t)) + J) + K(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t) + D\hat{x}(t - h(t)), \\ \hat{z}(t) = H\hat{x}(t), \\ \hat{x}(t) = 0, \quad t \in [-\tau, 0], \end{cases} \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state vector of the neural network, $\hat{z}(t)$ and $\hat{y}(t)$ denote the estimated measurement of $z(t)$ and $y(t)$, K is the gain matrix to be determined. Define the error $e(t) = x(t) - \hat{x}(t), \bar{z}(t) = z(t) - \hat{z}(t)$, we can easily obtain the error system:

$$\begin{cases} \dot{e}(t) = -(A + KC)e(t) - KDe(t - h(t)) + g(We(t - h(t))) + (B_1 - KB_2)w(t), \\ \bar{z}(t) = He(t), \end{cases} \tag{4}$$

where $g(We(t)) = f(Wx(t) + J) - f(W\hat{x}(t) + J)$.

Definition 2.1 [35] For any finite initial condition $\phi(t) \in \mathcal{C}^1([-\tau, 0]; \mathbb{R}^n)$, the error system (4) with $w(t) = 0$ is said to be globally exponentially stable with a decay rate β , if there exist constant $\lambda > 0$ and $\beta > 0$ such that

$$\|e(t)\|^2 \leq \lambda e^{-\beta t} \sup_{-\tau < s < 0} \{\|\phi(s)\|^2, \|\dot{\phi}(s)\|^2\}.$$

Given a prescribed level of disturbance attenuation level $\gamma > 0$, the error system is said to be globally exponentially stable with H_∞ performance, when the error system is globally exponentially stable and the response $\bar{z}(t)$ under zero initial condition satisfies

$$\|\bar{z}\|_2^2 \leq \gamma^2 \|w\|_2^2,$$

for every nonzero $w(t) \in L_2[0, \infty)$, where $\|w\|_2 = \sqrt{\int_0^\infty \phi^T(t)\phi(t) dt}$.

Lemma 2.1 ([38]) For any constant matrix $Z \in \mathbb{R}^{n \times n}$, $Z = Z^T > 0$, scalars $h_2 > h_1 > 0$, and vector function $x : [h_1, h_2] \rightarrow \mathbb{R}^n$ such that the following integrations are well defined,

$$-(h_2 - h_1) \int_{t-h_2}^{t-h_1} x^T(s)Zx(s) ds \leq - \int_{t-h_2}^{t-h_1} x^T(s) ds Z \int_{t-h_2}^{t-h_1} x(s) ds.$$

Lemma 2.2 (Schur complement) Given constant symmetric matrices S_1, S_2 , and S_3 , where $S_1 = S_1^T$, and $S_2 = S_2^T > 0$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0.$$

Lemma 2.3 ([39]) For differentiable signal x in $[\alpha, \beta] \rightarrow \mathbb{R}^n$, symmetric matrix $R \in \mathbb{R}^{n \times n}$, $Y_1, Y_3 \in \mathbb{R}^{3n \times 3n}$, any matrices $Y_2 \in \mathbb{R}^{3n \times 3n}$, and $M_1, M_2 \in \mathbb{R}^{3n \times n}$ satisfying

$$\bar{Y} = \begin{bmatrix} Y_1 & Y_2 & M_1 \\ * & Y_3 & M_2 \\ * & * & R \end{bmatrix} \geq 0,$$

the following inequality holds:

$$- \int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s) ds \leq \varpi^T \Omega \varpi, \tag{5}$$

where

$$\begin{aligned} \Omega &= (\beta - \alpha) \left(Y_1 + \frac{1}{3} Y_3 \right) + \text{Sym}\{M_1 \bar{\Pi}_1 + M_2 \bar{\Pi}_2\}, \\ \bar{\Pi}_1 &= \bar{e}_1 - \bar{e}_2, \quad \bar{\Pi}_2 = 2\bar{e}_3 - \bar{e}_1 - \bar{e}_2, \\ \bar{e}_1 &= [I \quad 0 \quad 0], \quad \bar{e}_2 = [0 \quad I \quad 0], \quad \bar{e}_3 = [0 \quad 0 \quad I], \end{aligned}$$

and

$$\varpi = \begin{bmatrix} x^T(\beta) & x^T(\alpha) & \frac{1}{\beta - \alpha} \int_\alpha^\beta x^T(s) ds \end{bmatrix}.$$

3 State estimator design

In this section, a delay partition approach is proposed to design a state estimator for the static neural network (1). For convenience of presentation, we denote

$$\hat{\eta}_1(t) = \begin{bmatrix} e(t-d_1) \\ e(t-d_1-\frac{1}{m}d) \\ e(t-d_1-\frac{2}{m}d) \\ \vdots \\ e(t-d_1-\frac{m-1}{m}d) \end{bmatrix}, \quad \hat{\eta}_2(t) = \begin{bmatrix} g(We(t-d_1)) \\ g(We(t-d_1-\frac{1}{m}d)) \\ g(We(t-d_1-\frac{2}{m}d)) \\ \vdots \\ g(We(t-d_1-\frac{m-1}{m}d)) \end{bmatrix},$$

where $d = d_2 - d_1$.

Theorem 3.1 *Under Assumption 2.1, for given scalars $\mu, 0 \leq d_1 \leq d_2, \gamma > 0, \alpha \geq 0$, and integers $m \geq 1, m \geq k \geq 1$, the system (4) is globally exponentially stable with H_∞ performance γ if there exist matrices $P (\in \mathbb{R}^{n \times n}) > 0, R_i (\in \mathbb{R}^{n \times n}) > 0 (i = 1, 2, \dots, m + 1), Q_1 (\in \mathbb{R}^{2mn \times 2mn}) > 0, Q_2 (\in \mathbb{R}^{2n \times 2n}) > 0$, symmetrical matrices $X_1, X_3, Y_1, Y_3, X_4, X_6, Y_4, Y_6 \in \mathbb{R}^{3n \times 3n}$, positive diagonal matrices $\Gamma, \Lambda_i (i = 1, 2, \dots, m + 1), \Lambda_{m+2}$, and any matrices with appropriate dimensions $M, X_2, X_5, Y_2, Y_5, N_j (j = 1, 2, \dots, 8)$, such that the following LMIs hold:*

$$\begin{bmatrix} \hat{\Xi} & \bar{H}^T \\ * & -I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} X_1 & X_2 & N_1 \\ * & X_3 & N_2 \\ * & * & R_{m+1} \end{bmatrix} \geq 0, \tag{7}$$

$$\begin{bmatrix} Y_1 & Y_2 & N_3 \\ * & Y_3 & N_4 \\ * & * & R_{m+1} \end{bmatrix} \geq 0, \tag{8}$$

$$\begin{bmatrix} X_4 & X_5 & N_5 \\ * & X_6 & N_6 \\ * & * & R_k \end{bmatrix} \geq 0, \tag{9}$$

$$\begin{bmatrix} Y_4 & Y_5 & N_7 \\ * & Y_6 & N_8 \\ * & * & R_k \end{bmatrix} \geq 0, \tag{10}$$

where

$$\begin{aligned} \hat{\Xi} = & \alpha E_{2m+3}^T P E_{2m+3} + \text{sym}\{E_{2m+3}^T P E_{2m+12}\} + \Pi_1^T Q_1 \Pi_1 - e^{-\alpha d/m} \Pi_2^T Q_1 \Pi_2 \\ & + \Pi_3^T Q_2 \Pi_3 - (1 - \mu) e^{-\alpha d_2/m} \Pi_4^T Q_2 \Pi_4 + (d_2 - d_1) E_{2m+12}^T R_{m+1} E_{2m+12} \\ & + (h(t) - d_1) e^{-\alpha d_2} \Pi_5^T \left(X_1 + \frac{1}{3} X_3 \right) \Pi_5 + e^{-\alpha d_2} \text{sym}\{ \Pi_5^T N_1 \Pi_6 + \Pi_5^T N_2 \Pi_7 \} \\ & + (d_2 - h(t)) e^{-\alpha d_2} \Pi_8^T \left(Y_1 + \frac{1}{3} Y_3 \right) \Pi_8 + e^{-\alpha d_2} \text{sym}\{ \Pi_8^T N_3 \Pi_9 + \Pi_8^T N_4 \Pi_{10} \} \end{aligned}$$

$$\begin{aligned}
 & + \left(\frac{d}{m}\right)^2 E_{2m+12}^T \left(\sum_{i=1}^m R_i\right) E_{2m+12} \\
 & + \frac{d}{m} \left(\frac{k}{m}d + d_1 - h(t)\right) e^{-\alpha(d_1+kd/m)} \Pi_{11}^T \left(X_4 + \frac{1}{3}X_6\right) \Pi_{11} \\
 & + \frac{d}{m} e^{-\alpha(d_1+kd/m)} \text{sym}\{\Pi_{11}^T N_5 \Pi_{12} + \Pi_{11}^T N_6 \Pi_{13}\} \\
 & + \frac{d}{m} e^{-\alpha(d_1+kd/m)} \text{sym}\{\Pi_{14}^T N_7 \Pi_{15} + \Pi_{14}^T N_8 \Pi_{16}\} \\
 & + \frac{d}{m} \left(h(t) - \frac{k-1}{m}d - d_1\right) e^{-\alpha(d_1+kd/m)} \Pi_{14}^T \left(Y_4 + \frac{1}{3}Y_6\right) \Pi_{14} \\
 & - \sum_{i=1, i \neq k}^m e^{-\alpha(d_1+id/m)} [E_i^T - E_{i+1}^T] R_i [E_i - E_{i+1}] - 2 \sum_{i=1}^{m+1} (E_{m+i+1}^T \Lambda_i E_{m+i+1}) \\
 & - 2 \sum_{i=1}^{m+1} (E_i^T W^T K^- \Lambda_i K^+ W E_i) + \sum_{i=1}^{m+1} \text{sym}\{E_i^T W^T (K^- + K^+) \Lambda_i E_{m+i+1}\} \\
 & - 2E_{2m+10}^T \Lambda_{m+2} E_{2m+10} - 2E_{2m+4}^T W^T K^- \Lambda_{m+2} K^+ W E_{2m+4} \\
 & + \text{sym}\{E_{2m+4}^T W^T (K^- + K^+) \Lambda_{m+2} E_{2m+10}\} - 2E_{2m+9}^T \Gamma E_{2m+9} \\
 & - 2E_{2m+3}^T W^T K^- \Gamma K^+ W E_{2m+3} + \text{sym}\{E_{2m+3}^T W^T (K^- + K^+) \Gamma E_{2m+9}\} \\
 & - \text{sym}\{E_{2m+3}^T M E_{2m+12}\} - E_{2m+3}^T \text{sym}\{M A\} E_{2m+3} \\
 & - E_{2m+3}^T \text{sym}\{G C\} E_{2m+3} - \text{sym}\{E_{2m+3}^T G D E_{2m+4}\} + \text{sym}\{E_{2m+3}^T M E_{2m+10}\} \\
 & + \text{sym}\{E_{2m+3}^T M B_1 E_{2m+11}\} - \text{sym}\{E_{2m+3}^T G B_2 E_{2m+11}\} \\
 & - E_{2m+12}^T \text{sym}\{M\} E_{2m+12} - \text{sym}\{E_{2m+12}^T M A E_{2m+3}\} \\
 & - \text{sym}\{E_{2m+12}^T G C E_{2m+3}\} - \text{sym}\{E_{2m+12}^T G D E_{2m+4}\} \\
 & + \text{sym}\{E_{2m+12}^T M E_{2m+10}\} + \text{sym}\{E_{2m+12}^T M B_1 E_{2m+11}\} \\
 & - \text{sym}\{E_{2m+12}^T G B_2 E_{2m+11}\} - \gamma^2 E_{2m+11}^T E_{2m+11}, \\
 \Pi_1 & = [E_1^T, E_2^T, E_3^T, \dots, E_m^T, E_{m+2}^T, E_{m+3}^T, \dots, E_{2m+1}^T]^T, \\
 \Pi_2 & = [E_2^T, E_3^T, E_4^T, \dots, E_{m+1}^T, E_{m+3}^T, E_{m+4}^T, \dots, E_{2m+2}^T]^T, \\
 \Pi_3 & = [E_{2m+3}^T, E_{2m+9}^T]^T, \\
 \Pi_4 & = [E_{2m+4}^T, E_{2m+9}^T]^T, \\
 \Pi_5 & = [E_1^T, E_{2m+4}^T, E_{2m+5}^T]^T, \\
 \Pi_6 & = E_1 - E_{2m+4}, \\
 \Pi_7 & = 2E_{2m+5} - E_1 - E_{2m+4}, \\
 \Pi_8 & = [E_{2m+4}^T, E_{m+1}^T, E_{2m+6}^T]^T, \\
 \Pi_9 & = E_{2m+4} - E_{m+1}, \\
 \Pi_{10} & = 2E_{2m+6} - E_{m+1} - E_{2m+4}, \\
 \Pi_{11} & = [E_{2m+4}^T, E_{k+1}^T, E_{2m+7}^T]^T,
 \end{aligned}$$

$$\begin{aligned} \Pi_{12} &= E_{2m+4} - E_{k+1}, \\ \Pi_{13} &= 2E_{2m+7} - E_{k+1} - E_{2m+4}, \\ \Pi_{14} &= [E_k^T, E_{2m+4}^T, E_{2m+8}^T]^T, \\ \Pi_{15} &= E_k - E_{2m+4}, \\ \Pi_{16} &= 2E_{2m+8} - E_k - E_{2m+4}, \\ E_i &= [0_{n \times (i-1)n}, I_n, 0_{n \times (2m+12-i)n}], \quad i = 1, 2, \dots, 2m + 12, \\ \bar{H} &= [0, 0, 0, 0, H, 0, 0, 0, 0, 0, 0, 0, 0], \end{aligned}$$

the estimator gain matrix is given by $K = M^{-1}G$.

Proof Construct a Lyapunov-Krasovskii functional candidate as follows:

$$V(t, e_t) = \sum_{i=1}^5 V_i(t, e_t), \tag{11}$$

where

$$\begin{aligned} V_1(t, e_t) &= e^T(t)Pe(t), \\ V_2(t, e_t) &= \int_{t-\frac{d}{m}}^t e^{-\alpha(t-s)} \begin{bmatrix} \hat{\eta}_1(s) \\ \hat{\eta}_2(s) \end{bmatrix}^T Q_1 \begin{bmatrix} \hat{\eta}_1(s) \\ \hat{\eta}_2(s) \end{bmatrix} ds, \\ V_3(t, e_t) &= \int_{t-h(t)}^t e^{-\alpha(t-s)} \begin{bmatrix} e(s) \\ g(We(s)) \end{bmatrix}^T Q_2 \begin{bmatrix} e(s) \\ g(We(s)) \end{bmatrix} ds, \\ V_4(t, e_t) &= \frac{d}{m} \sum_{i=1}^m \int_{-\frac{i}{m}d-d_1}^{-\frac{i-1}{m}d-d_1} \int_{t+\theta}^t e^{-\alpha(t-s)} \dot{e}^T(s)R_i\dot{e}(s) ds d\theta, \\ V_5(t, e_t) &= \int_{-d_2}^{-d_1} \int_{t+\theta}^t e^{-\alpha(t-s)} \dot{e}^T(s)R_{m+1}\dot{e}(s) ds d\theta, \end{aligned}$$

calculating the derivative of $V(t, e_t)$ along the trajectory of system, we obtain

$$\dot{V}_1(t, e_t) = 2e^T(t)P\dot{e}(t) = \hat{\zeta}_t^T \text{sym}\{E_{2m+3}^T P E_{2m+12}\} \hat{\zeta}_t, \tag{12}$$

$$\begin{aligned} \dot{V}_2(t, e_t) &= -\alpha V_2(t, e_t) + \begin{bmatrix} \hat{\eta}_1(t) \\ \hat{\eta}_2(t) \end{bmatrix}^T Q_1 \begin{bmatrix} \hat{\eta}_1(t) \\ \hat{\eta}_2(t) \end{bmatrix} \\ &\quad - e^{-\alpha d/m} \begin{bmatrix} \hat{\eta}_1(t-d/m) \\ \hat{\eta}_2(t-d/m) \end{bmatrix}^T Q_1 \begin{bmatrix} \hat{\eta}_1(t-d/m) \\ \hat{\eta}_2(t-d/m) \end{bmatrix} \\ &= -\alpha V_2(t, e_t) + \hat{\zeta}_t^T \Pi_1^T Q_1 \Pi_1 \hat{\zeta}_t - e^{-\alpha d/m} \hat{\zeta}_t^T \Pi_2^T Q_1 \Pi_2 \hat{\zeta}_t, \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{V}_3(t, e_t) &= -\alpha V_3(t, e_t) + \begin{bmatrix} e(t) \\ g(We(t)) \end{bmatrix}^T Q_2 \begin{bmatrix} e(t) \\ g(We(t)) \end{bmatrix} \\ &\quad - (1-\mu)e^{-\alpha h(t)} \begin{bmatrix} e(t-h(t)) \\ g(We(t-h(t))) \end{bmatrix}^T Q_2 \begin{bmatrix} e(t-h(t)) \\ g(We(t-h(t))) \end{bmatrix} \end{aligned}$$

$$\leq -\alpha V_3(t, e_t) + \hat{\zeta}_t^T \Pi_3^T Q_2 \Pi_3 \hat{\zeta}_t - (1 - \mu) e^{-\alpha d_2/m} \hat{\zeta}_t^T \Pi_4^T Q_2 \Pi_4 \hat{\zeta}_t, \tag{14}$$

$$\begin{aligned} \dot{V}_4(t, e_t) &\leq -\alpha V_4(t, e_t) + \left(\frac{d}{m}\right)^2 \dot{e}^T(t) \left(\sum_{i=1}^m R_i\right) \dot{e}(t) \\ &\quad - \left(\frac{d}{m}\right) \sum_{i=1}^m \int_{t-\frac{i}{m}d-d_1}^{t-\frac{i-1}{m}d-d_1} e^{-\alpha(d_1+id/m)} \dot{e}^T(s) R_i \dot{e}(s) \, ds, \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{V}_5(t, e_t) &\leq -\alpha V_5(t, e_t) + (d_2 - d_1) \dot{e}^T(t) R_{m+1} \dot{e}(t) \\ &\quad - \int_{t-h(t)}^{t-d_1} e^{-\alpha d_2} \dot{e}^T(s) R_{m+1} \dot{e}(s) \, ds - \int_{t-d_2}^{t-h(t)} e^{-\alpha d_2} \dot{e}^T(s) R_{m+1} \dot{e}(s) \, ds, \end{aligned} \tag{16}$$

according to Lemma 2.3, it follows that

$$\begin{aligned} \dot{V}_5(t, e_t) &\leq -\alpha V_5(t, e_t) + (d_2 - d_1) \hat{\zeta}_t^T E_{2m+12}^T R_{m+1} E_{2m+12} \hat{\zeta}_t \\ &\quad + (h(t) - d_1) e^{-\alpha d_2} \hat{\zeta}_t^T \Pi_5^T \left(X_1 + \frac{1}{3} X_3\right) \Pi_5 \hat{\zeta}_t \\ &\quad + e^{-\alpha d_2} \hat{\zeta}_t^T \text{sym}\{\Pi_5^T N_1 \Pi_6 + \Pi_5^T N_2 \Pi_7\} \hat{\zeta}_t \\ &\quad + (d_2 - h(t)) e^{-\alpha d_2} \hat{\zeta}_t^T \Pi_8^T \left(Y_1 + \frac{1}{3} Y_3\right) \Pi_8 \hat{\zeta}_t \\ &\quad + e^{-\alpha d_2} \hat{\zeta}_t^T \text{sym}\{\Pi_8^T N_3 \Pi_9 + \Pi_8^T N_4 \Pi_{10}\} \hat{\zeta}_t, \end{aligned} \tag{17}$$

there should exist a positive integer $k \in \{1, 2, \dots, m\}$, such that $h(t) \in [\frac{(k-1)}{m}d + d_1, \frac{k}{m}d + d_1]$,

$$\begin{aligned} &-\left(\frac{d}{m}\right) \int_{t-\frac{k}{m}d-d_1}^{t-\frac{k-1}{m}d-d_1} e^{-\alpha(d_1+kd/m)} \dot{e}^T(s) R_k \dot{e}(s) \, ds \\ &\leq \frac{d}{m} \left(\frac{k}{m}d + d_1 - h(t)\right) e^{-\alpha(d_1+kd/m)} \hat{\zeta}_t^T \Pi_{11}^T \left(X_4 + \frac{1}{3} X_6\right) \Pi_{11} \hat{\zeta}_t \\ &\quad + \frac{d}{m} e^{-\alpha(d_1+kd/m)} \hat{\zeta}_t^T \text{sym}\{\Pi_{11}^T N_5 \Pi_{12} + \Pi_{11}^T N_6 \Pi_{13}\} \hat{\zeta}_t \\ &\quad + \frac{d}{m} \left(h(t) - \frac{k-1}{m}d - d_1\right) e^{-\alpha(d_1+kd/m)} \hat{\zeta}_t^T \Pi_{14}^T \left(Y_4 + \frac{1}{3} Y_6\right) \Pi_{14} \hat{\zeta}_t \\ &\quad + \frac{d}{m} e^{-\alpha(d_1+kd/m)} \hat{\zeta}_t^T \text{sym}\{\Pi_{14}^T N_7 \Pi_{15} + \Pi_{14}^T N_8 \Pi_{16}\} \hat{\zeta}_t. \end{aligned} \tag{18}$$

For $i \neq k$, we also have the following inequality by Lemma 2.1:

$$\begin{aligned} &-\left(\frac{d}{m}\right) \sum_{i=1}^m \int_{t-\frac{i}{m}d-d_1}^{t-\frac{i-1}{m}d-d_1} e^{-\alpha(d_1+id/m)} \dot{e}^T(s) R_i \dot{e}(s) \, ds \\ &\leq -\sum_{i=1}^m e^{-\alpha(d_1+id/m)} \left[e^T\left(t - d_1 - \frac{i-1}{m}d\right) - e^T\left(t - d_1 - \frac{i}{m}d\right) \right] R_i \\ &\quad \times \left[e\left(t - d_1 - \frac{i-1}{m}d\right) - e\left(t - d_1 - \frac{i}{m}d\right) \right] \\ &= -\sum_{i=1}^m e^{-\alpha(d_1+id/m)} \hat{\zeta}_t^T [E_i^T - E_{i+1}^T] R_i [E_i - E_{i+1}] \hat{\zeta}_t. \end{aligned} \tag{19}$$

According to Assumption 2.1, we obtain

$$2(g(We(t)) - K^- We(t))^T \Gamma (g(We(t)) - K^+ We(t)) \leq 0, \tag{20}$$

$$2 \sum_{i=1}^{m+1} \left(g \left(We \left(t - d_1 - \frac{i-1}{m} d \right) \right) - K^- We \left(t - d_1 - \frac{i-1}{m} d \right) \right) \Lambda_i \\ \times \left(g \left(We \left(t - d_1 - \frac{i-1}{m} d \right) \right) - K^+ We \left(t - d_1 - \frac{i-1}{m} d \right) \right) \leq 0, \tag{21}$$

$$2(g(We(t-h(t))) - K^- We(t-h(t)))^T \Lambda_{m+1} \\ \times (g(We(t-h(t))) - K^+ We(t-h(t))) \leq 0, \tag{22}$$

where Γ, Λ_i are positive diagonal matrices.

According to the system equation, the following equality holds:

$$2(e^T(t) + \dot{e}^T(t))M \{ -(A + KC)e(t) - KDe(t-h(t)) \\ + g(We(t-h(t))) + (B_1 - KB_2)w(t) - \dot{e}(t) \} = 0. \tag{23}$$

Combining the qualities and inequalities from (12) to (23), we can obtain

$$\bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, x_t) + \alpha V(t, x_t) \leq \hat{\zeta}_t^T (\hat{\Xi}) \hat{\zeta}_t + e^T(t)H^T He(t), \tag{24}$$

where $\hat{\zeta}_t$ is defined as

$$\hat{\zeta}_t^T = \left[\hat{\eta}_1^T(t), e^T(t-d_2), \hat{\eta}_2^T(t), g(We(t-d_2))^T, e^T(t), e^T(t-h(t)), \right. \\ \left. \frac{1}{h(t)-d_1} \int_{t-h(t)}^{t-d_1} e^T(s) ds, \frac{1}{d_2-h(t)} \int_{t-d_2}^{t-h(t)} e^T(s) ds, \frac{1}{d_2-h(t)} \int_{t-d_2}^{t-h(t)} e^T(s) ds, \right. \\ \left. \frac{1}{d_2-h(t)} \int_{t-d_2}^{t-h(t)} e^T(s) ds, g(We(t))^T, g(We(t-h(t)))^T, w^T(t), \dot{e}^T(t) \right].$$

Based on Lemma 2.2, one can deduce that

$$\bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, x_t) + \alpha V(t, x_t) \leq \hat{\zeta}_t^T \begin{bmatrix} \hat{\Xi} & \bar{H}^T \\ * & -I \end{bmatrix} \hat{\zeta}_t, \tag{25}$$

where $\bar{H} = [0, 0, 0, 0, H, 0, 0, 0, 0, 0, 0, 0, 0]$.

If the LMI (6) holds, then

$$\bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, x_t) + \alpha V(t, x_t) < 0, \tag{26}$$

$\alpha V(t, x_t) \geq 0$, so we can obtain

$$\int_0^\infty [\bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t, e(t))] dt < 0,$$

since $V(t, e(t)) > 0$, under the zero initial condition, we have

$$\|\bar{z}(t)\|^2 \leq \gamma^2 \|w(t)\|^2.$$

Therefore, the error system (4) guarantee H_∞ performance γ according to Definition 2.1. In the sequel, we show the globally exponentially stability of the estimation error system with $w(t) = 0$. When $w(t) = 0$, the error system (4) becomes

$$\begin{cases} \dot{e}(t) = -(A + KC)e(t) - KDe(t - h(t)) + g(We(t - h(t))), \\ \bar{z}(t) = He(t), \end{cases} \tag{27}$$

equation (23) becomes

$$2(e^T(t) + \dot{e}^T(t))M\{- (A + KC)e(t) - KDe(t - d) + g(We(t - d)) - \dot{e}(t)\} = 0, \tag{28}$$

considering the same Lyapunov-Krasovskii functional candidate and calculating its time derivative along the solution of (27), we can derive

$$\dot{V}(t, e_t) + \alpha V(t, e_t) \leq \zeta_t^T \Xi \zeta_t, \tag{29}$$

where Ξ is obtained by deleting the terms in $\hat{\Xi}$ associated with $w(t)$,

$$\begin{aligned} \zeta_t^T = & \left[\hat{\eta}_1^T(t), e^T(t - d_2), \hat{\eta}_2^T(t), g(We(t - d_2))^T, e^T(t), e^T(t - h(t)), \right. \\ & \frac{1}{h(t) - d_1} \int_{t-h(t)}^{t-d_1} e^T(s) ds, \frac{1}{d_2 - h(t)} \int_{t-d_2}^{t-h(t)} e^T(s) ds, \frac{1}{d_2 - h(t)} \int_{t-d_2}^{t-h(t)} e^T(s) ds, \\ & \left. \frac{1}{d_2 - h(t)} \int_{t-d_2}^{t-h(t)} e^T(s) ds, g(We(t))^T, g(We(t - h(t)))^T, \dot{e}^T(t) \right]. \end{aligned}$$

Let $G = MK$, it is obvious that if $\hat{\Xi} < 0$, then $\Xi < 0$, so we get

$$\dot{V}(t, e_t) + \alpha V(t, e_t) \leq 0. \tag{30}$$

Integrating the above inequality (30), so we obtain

$$V(t, e_t) \leq e^{-\alpha t} V(0, e_0). \tag{31}$$

From (11), we have

$$V(t, e_t) \geq \lambda_{\min}(P) \|e(t)\|^2, \tag{32}$$

$$V(0, e_0) \leq b \sup_{-\tau < s < 0} \{ \|\phi(s)\|^2, \|\dot{\phi}(s)\|^2 \}, \tag{33}$$

where

$$\begin{aligned} b = & \lambda_{\max}(P) + (1 + \rho^2)d\lambda_{\max}(Q_1) + (1 + \rho^2)d_2\lambda_{\max}(Q_2) \\ & + \frac{d}{m} \sum_{i=1}^m \frac{(\frac{i}{m}d + d_1)^2 - (\frac{i-1}{m}d + d_1)^2}{2} \lambda_{\max}(R_i) + \frac{(d_2)^2 - (d_1)^2}{2} \lambda_{\max}(R_{m+1}), \end{aligned}$$

$$\rho = \max_{1 \leq i \leq 0} (|k_i^-|, |k_i^+|).$$

Combining (31), (32), and (33) yields

$$\|e(t)\|^2 \leq \frac{1}{\lambda_{\min}(P)} V(t, e_t) \leq \frac{b}{\lambda_{\min}(P)} e^{-\alpha t} \sup_{-\tau < s < 0} \{ \|\phi(s)\|^2, \|\dot{\phi}(s)\|^2 \}, \tag{34}$$

hence the error system (4) is globally exponentially stable. Above all, if $\hat{\Xi} < 0$, then the state estimator for the static neural network has the prescribed H_∞ performance and guarantees the globally exponentially stable of the error system. This completes the proof. \square

Remark 3.1 We use two ways to reduce the conservativeness: a good choice of the Lyapunov-Krasovskii functionals, and the use of a less conservative integral inequality. To make the Lyapunov-Krasovskii functionals contain more detailed information of time delay, delay partitioning method is employed. We also use the free-weighting matrix method to obtain a tighter upper bound on the derivative of the LKFs, many free-weighting matrices will be introduced with the increasing number of partitions. That will lead to complexity and computational burden. Hence, the partitioning number should be properly chosen.

Remark 3.2 Condition (6) in Theorem 3.1 depends on the time-varying delay, it is easy to show that the condition is satisfied for all $0 \leq d_1 \leq h(t) \leq d_2$ if $\hat{\Xi}|_{h(t)=d_1} < 0$ and $\hat{\Xi}|_{h(t)=d_2} < 0$.

Remark 3.3 In some previous literature [33, 35, 36], $k_i^- \leq \frac{f_i(x)}{x} \leq k_i^+$, which is a special case of $k_i^- \leq \frac{f_i(x)-f_i(y)}{x-y} \leq k_i^+$ was used to reduce the conservatism. In our proof, not only $k_i^- \leq \frac{g_i(W_i e(t))}{W_i e(t)} \leq k_i^+$, but also $k_i^- \leq \frac{g_i(W_i e(t-h(t)))}{W_i e(t-h(t))} \leq k_i^+$, $k_i^- \leq \frac{g_i(W_i e(t-d_1-\frac{l-1}{m}d))}{W_i e(t-d_1-\frac{l-1}{m}d)} \leq k_i^+$, $j \in \{1, 2, \dots, m+1\}$ have been used, which play an important role in reducing the conservatism.

4 Numerical examples

In this section, numerical examples are provided to illustrate effectiveness of the developed method for the state estimation of static neural networks.

Example 1 Consider the neural networks (1) with the following parameters:

$$A = \begin{pmatrix} 0.96 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1.48 \end{pmatrix}, \quad W = \begin{pmatrix} 0.5 & 0.3 & -0.36 \\ 0.1 & 0.12 & 0.5 \\ -0.42 & 0.78 & 0.9 \end{pmatrix},$$

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0.1 & 0 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$C = (1 \ 0 \ -2), \quad D = (0.5 \ 0 \ -1), \quad B_2 = (-0.1 \ 0 \ 0).$$

To compare with the existing results, we let $\alpha = 0$, $d_1 = 0$. We obtain the optimal H_∞ performance index γ for different values of delay d_2 and μ . It is summarized in Table 1.

From Table 1, it is clear that our results achieve better performance. In addition, the optimal H_∞ performance index γ becomes smaller as the partitioning number is increasing. It shows that the delay partitioning method can reduce the conservatism effectively.

Table 1 The H_∞ performance index γ with different (d_2, μ)

Methods	(0.8, 0.4)	(0.9, 0.7)	(1.1, 0.5)
[34]	1.2197	1.2719	1.8944
[35]	1.2989	1.3164	1.6441
Theorem 3.1 [36]	1.1391	1.1489	1.1538
Theorem 3.2, $m = 5$	0.7196	0.9940	0.7677
Theorem 3.2, $m = 10$	0.7175	0.9916	0.7655

Example 2 Consider the neural networks (1) with the following parameters:

$$A = \begin{pmatrix} 1.56 & 0 & 0 \\ 0 & 2.42 & 0 \\ 0 & 0 & 1.88 \end{pmatrix}, \quad W = \begin{pmatrix} -0.3 & 0.8 & -1.36 \\ 1.1 & 0.4 & -0.5 \\ 0.42 & 0 & -0.95 \end{pmatrix},$$

$$H = \begin{pmatrix} 1 & 0 & 0.5 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.2 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0.2 & 0 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$C = (1 \ 0 \ 0), \quad D = (2 \ 0 \ 0), \quad B_2 = (0.4 \ 0 \ 0).$$

The activation function is $f(x) = \tanh(x)$, it is easy to get $K^- = 0, K^+ = I$. We set $\gamma = 1, \alpha = 0, h(t) = 0.5 + 0.5 \sin(0.8t)$, so the bound of time delay $d_1 = 0, d_2 = 1$, and $\mu = 0.4$. The noise disturbance is assumed to be

$$w(t) = \begin{pmatrix} \frac{1}{0.8+1.2t} \\ 0 \\ 0 \end{pmatrix}.$$

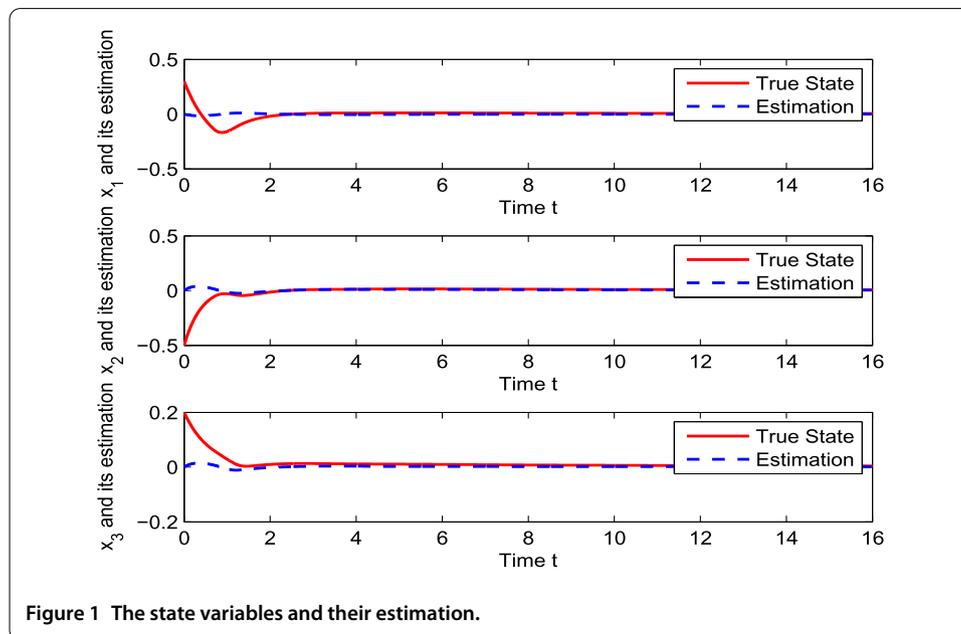
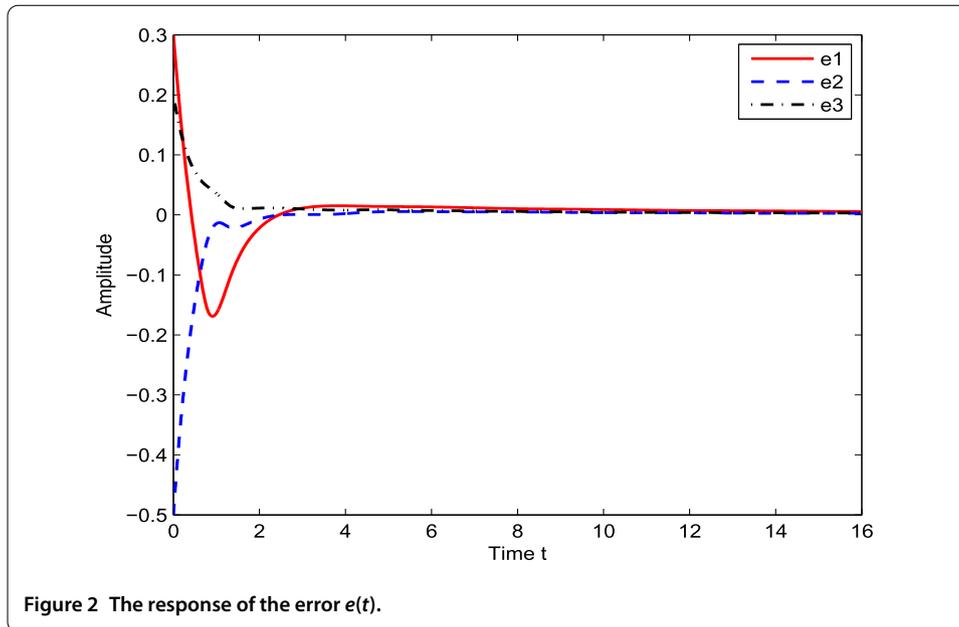


Figure 1 The state variables and their estimation.



By solving through the Matlab LMI toolbox, we obtain the gain matrix of the estimator:

$$K = \begin{pmatrix} -0.0890 \\ 0.2664 \\ 0.0914 \end{pmatrix}.$$

Figure 1 present the state variables and their estimation of the neural network (1) from initial values $[0.3, -0.5, 0.2]^T$. Figure 2 shows the state response of the error system (4) under the initial condition $e(0) = [0.3, -0.5, 0.2]^T$. It is clear that $e(t)$ converges rapidly to zero. The simulation results reveal the effectiveness of the proposed approach to the design of the state estimator for static neural networks.

5 Conclusions

In this paper, we investigated the H_∞ state estimation problem for a class of delayed static neural networks. By constructing the augmented Lyapunov-Krasovskii functionals, new delay-dependent conditions were established. The estimator matrix gain was obtained by solving a set of LMIs, which can guarantee the global exponential stability with a prescribed H_∞ performance level of the error system. In the end, numerical examples were provided to illustrate the effectiveness of the proposed method comparing with some existing results.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have made equal contributions to the writing of this paper. All authors have read and approved the final version of the manuscript.

Author details

¹School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu, 611731, P.R. China. ²School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, 611731, P.R. China. ³Key Laboratory for Neuroinformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu, 611731, P.R. China.

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