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Global exponential synchronization of networked dynamical systems under event-triggered control schemes

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Abstract

This paper investigates exponential synchronization of networked dynamical systems under event-triggered control schemes. Two event-triggered sampled-data transmission schemes, which only need the latest observations of their neighborhood and the virtual leader to predict the next observation time, are designed to realize exponential synchronization of networked dynamical systems. That is, the coupled information is updated only when the triggered conditions are violated. Hence, continuous communication can be avoided and the number of information transmission is reduced. A positive lower bound for inter-event intervals is achieved to exclude Zeno behavior. Finally, two numerical simulation examples are provided to illustrate the effectiveness of the proposed results.

Keywords: exponential synchronization; networked dynamical systems; networked dynamical systems; event-triggered; Zeno behavior

1 Introduction

Networked control dynamical systems (NCDSs) have been widely studied over the past decades [1–4]. It is usually investigated in the NCDSs that all oscillators approach a uniform dynamical behavior, that is, all the nodes in the NCDSs reach synchronization eventually. In the process of synchronization, the couplings among nodes and/or external distributed and cooperative control between nodes exist inevitably and meanwhile conflicts may exist due to the limitations of network resources and traffics [5]. In most previous references [6–8], each node received their neighbors' information continuously, which may cost much. As an important component in NCDSs, intermittent sampling has emerged as an interesting topic to avoid communication continuously [9–11]. Nevertheless, sampled-data systems are usually applied periodically in time [12, 13], that is, time-triggered sampling, which might be conservative in terms of the number of control updates.

In order to utilize the real-time information sufficiently and to reduce communication and computation load in NCDSs, aperiodic event-triggered sampling, which is triggered only when measurement error signal violates a prescribed threshold [14–16], is proposed in the last few years. As pointed out in [17], event-triggered sampling was proved to possess a better performance than time-triggered sampling. In event-triggered sampling control, the control law is updated only when some specific significant events occur, other state changes or occurrences of real-time entity are considered insignificant and are neglected

[18–22]. Event-triggered sampling control could adjust task periods to variations in system states adaptively, which produces longer task periods than time-triggered sampling control. In [23], the authors studied the event-triggered distributed average-consensus of discrete-time first-order multi-agent systems with limited communication data rate and general directed network topology. The authors of [24] studied the problem of average consensus over directed and time-varying digital networks of discrete-time first-order multi-agent systems with limited communication data transmission rates. Each agent has a real-valued state but can only exchange binary symbolic sequence with its neighbors due to bandwidth constraints.

Recently, great efforts are still made on applying event-triggered scheme (ETS) to cooperation of multi-agent systems. A key issue of the event-triggered scheme is how to design and optimize event-based conditions and a big challenge is how to prove that inner-event time intervals are positive which can assure the absence of Zeno behavior. In [25], leader-following consensus of general linear multi-agent is investigated by the event-triggered scheme. Three types of schemes, namely, distributed ETS, centralized ETS, and clustered ETS for different network topologies are proposed. All these schemes guarantee that all followers can track the leader eventually. In [26], event-triggered coupling configurations are utilized to realize synchronization of linearly coupled dynamical systems. The diffusion couplings are set up from the latest observations of the nodes and their neighborhood and the next observation time is triggered by the proposed criteria based on the local neighborhood information as well. However, the graph in [26] is undirected and connected.

Motivated by above statement, the objective of this paper is to design two event-triggered schemes for exponential synchronization of networked dynamical systems. The contributions of this paper are listed as follows. First of all, the network topology is directed and contains a directed spanning tree rooted at a virtual node. All the nodes are equipped with nonlinear dynamics. Thus, the model of this paper is more general than in [20, 25, 26]. Second, two distributed event-triggered schemes are proposed to realize exponential synchronization of the networked dynamical systems. The main difficulty of this paper is to prove the Zeno behavior is excluded under the two event-triggered schemes. In addition, to further reduce the number of updatings, two distributed self-triggered schemes are proposed. It is proved that the exponential synchronization can be achieved and the Zeno behavior can be excluded simultaneously under the two self-triggered schemes.

The remainder of this paper is outlined as follows. In Section 2, preliminaries including some necessary definitions and lemmas and the model description are stated. In Section 3, event-triggered schemes are proposed to realize exponential synchronization and the Zeno behavior can be excluded under the proposed schemes. In Section 4, the distributed self-triggered schemes are presented according to the event-triggered schemes. In Section 5, some numerical examples are given to show the effectiveness of the theoretical results. Finally, the conclusion is drawn in Section 6.

Notation Throughout this study, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ represent the set of all n dimensional real column vectors and the set of all $n \times n$ dimensional real matrices. The superscript T represents the transpose. $\|\cdot\|$ denotes the Euclidean norm, that is, for any vector $\xi \in \mathbb{R}^n$, $\|\xi\| = (\xi_1^2, \dots, \xi_n^2)^{\frac{1}{2}}$. $\|x\|_p = (x^T Px)^{\frac{1}{2}}$ for some positive definite matrix $P \in \mathbb{R}^{n\times n}$. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are respectively the maximum and minimum eigenvalues of matrix A. $\otimes$ represents the Kronecker product.

2 Preliminaries and problem formulation

The dynamics of generally networked systems under pinning control can be described as follows:

$$\dot{x}_i(t) = f\bigl(x_i(t)\bigr) + \sum_{j=1}^{N} a_{ij}\Gamma\bigl(x_j(t) - x_i(t)\bigr) + u_i(t), \quad i = 1, \ldots, N, \tag{1}$$

which $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ denotes the state vector of node i and the continuous map $f(\cdot) : \mathbb{R}^N \to \mathbb{R}^N$ denotes the identical node dynamics if there is no coupling. $\Gamma \in \mathbb{R}^{n\times n}$ describes the inner-coupling positive definite matrix between the subsystems. $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix of the directed graph $\mathscr{G} = \{V, E\}$ with the node set V and the link set E: $a_{ij} > 0$ if there exists a directed link from node j to i at time t; $a_{ij} = 0$, otherwise. Define the graph Laplacian matrix of A as $L = (l_{ij}) \in \mathbb{R}^{N\times N}$, in which $l_{ij} = -a_{ij}$, for $i \neq j$, and $a_{ii} = \sum_{j=1}^{N} a_{ij}$. $u_i(t)$ is the pinning control input to be designed. The information in the networks is usually considered to communicate continuously. However, when the nodes are equipped with limited computation capability, limited capability of communication, actuation, and limited onboard energy source, it is not economic to communicate information continuously.

Therefore, an intermittent event-triggered information transmission mechanism emerges reducing the number of communications and computations. In this paper, two distributed event-triggered schemes will be adopted to synchronize the network (1) with a certain desired state $s(t)$ which can be an equilibrium point, periodic orbit or chaotic attractor in the phase space satisfying

$$\dot{s}(t) = f\bigl(s(t)\bigr), \tag{2}$$

where $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T \in \mathbb{R}^n$ is a virtual node. The virtual node and the directed graph constitute an augmented graph $\overline{\mathscr{G}}$.

Then the network model with event-triggered diffusive coupling under pinning control is investigated as follows:

$$\dot{x}_i(t) = f\bigl(x_i(t)\bigr) + \sum_{j=1}^{N} a_{ij}\Gamma\bigl(x_j\bigl(t_{k_i}^i\bigr) - x_i\bigl(t_{k_i}^i\bigr)\bigr) - d_i\Gamma\bigl(x_i\bigl(t_{k_i}^i\bigr) - s\bigl(t_{k_i}^i\bigr)\bigr), \quad t \in \bigl[t_{k_i}^i, t_{k_i+1}^i\bigr), \tag{3}$$

where $t_{k_i}^i$, $i = 1, \ldots, N$, represents the ith node's latest triggering time instant before time t, $t_{k_i+1}^i$ is the next triggering time instant. When the ith node is pinned by the virtual node, $d_i > 0$; otherwise, $d_i = 0$.

For $t \in [t_{k_i}^i, t_{k_i+1}^i)$, $i = 1, \ldots, N$, synchronization error and measurement error of node i are respectively defined as $e_i(t) = x_i(t) - s(t)$, and $\delta_i(t) = x_i(t_{k_i}^i) - x_i(t)$, $\delta_j(t) = x_j(t_{k_i}^i) - x_j(t)$, $j \neq i$, $\delta_0(t) = s(t_{k_i}^i) - s(t)$.

Then one can get

$$\dot{e}_i(t) = f\bigl(e_i(t)\bigr) - \sum_{j=1}^{N} l_{ij}\Gamma e_j\bigl(t_{k_i}^i\bigr) - d_i\Gamma e_i\bigl(t_{k_i}^i\bigr), \tag{4}$$

where $f(e_i(t)) = f(x_i(t)) - f(s(t))$.

The objective of this paper is to design appropriate event-triggered schemes such that (3) and (2) can reach exponential synchronization.

Throughout the rest of the paper, the following assumptions and lemma are needed.

Definition 1 Consider the node dynamics map $f(\cdot):\mathbb{R}^N\to\mathbb{R}^N$. If there exist a positive definite matrix $P\in\mathbb{R}^{n\times n}$, constant $\alpha\in\mathbb{R}$, and positive constant $\beta>0$ such that,

$$(u-v)^TP\bigl[f(u)-f(v)-\alpha\Gamma(u-v)\bigr]\le-\beta(u-v)^T(u-v) \tag{5}$$

holds for all $u,v\in\mathbb{R}^N$. Then we say it belongs to some map class $\mathrm{Quad}(P,\alpha\Gamma,\beta)$.

Assumption 1 There exists a positive constant k such that for any $u,v\in\mathbb{R}^n$,

$$\bigl\|f(u)-f(v)\bigr\|\le k\|u-v\|.$$

Assumption 2 The augmented graph contains a directed span tree rooted at the virtual node.

Lemma 1 *Let L be the Laplacian matrix of a non-negatively weighted digraph $\mathscr{G}$ and $D=\mathrm{diag}\{d_1,\dots,d_N\}$ be a nonnegative diagonal matrix. Then we have the following facts*:

(1) *If $\mathscr{G}$ is balanced, then $\frac{L+L^T}{2}+D>0$ if and only if $\overline{\mathscr{G}}$ is weakly connected.*

(2) *If $\overline{\mathscr{G}}$ has a directed spanning tree, then there exists a positive diagonal matrix $\Xi=\mathrm{diag}\{\xi_1,\dots,\xi_N\}$, such that*

$$\Xi(L+D)+(L+D)^T\Xi>0. \tag{6}$$

Then all the eigenvalues λ_i of $\Xi(L+D)+(L+D)^T\Xi$ are positive. That is, $0<\lambda_1\le\lambda_2\le\cdots\le\lambda_N$.

Remark 1 In [20], the trajectories of all nodes commonly converge to a time-varying weighed average $\bar{x}=\sum_{j=1}^N\xi_jx_j(t)$. All the nodes in this paper could synchronize with an arbitrary desired state $s(t)$ which can be an equilibrium point, periodic orbit or chaotic attractor. In addition, the authors of [20] make use of the general algebraic connectivity to reach global synchronization, which is fit for a strongly connected network. This paper only requires the augmented graph contains a directed spanning tree rooted at the virtual node which does not require the coupling matrix to be symmetric.

3 Event-triggered scheme for pinning synchronization

In this section, pinning synchronization of the considered network (3) with (2) is investigated under event-triggered mechanism. The following algorithm [27] is proposed to determine at least how many and what kinds of nodes should be pinned such that Assumption 2 holds.

Algorithm 1 Find the strongly connected components of $\mathscr{G}(A)$ by using Tarjan's algorithm [28]. Suppose that there are ω strongly connected components for $\mathscr{G}(A)$, labelled as $G_1,G_2,\dots,G_\omega$. Let $m_i=0$, $i=1,\dots,\omega$ and $h=1$.

Step 1: Check whether there exists at least one node n_k belonging to G_h, which is reachable from a node n_g belonging to G_j, $j = 1, 2, \dots, \omega$, $j \neq h$. If it holds, go to Step 2; if it does not hold, go to Step 3.

Step 2: Check whether the following condition holds: $h < \omega$. If so, let $h = h + 1$ and return Step 1; else stop.

Step 3: Arbitrarily having selected one node in G_h and pinned, let $m_h = 1$. Check whether the following condition holds: $h < \omega$. If so, let $h = h + 1$ and return Step 1; else stop.

Remark 2 Using Algorithm 1, we should select at least $\delta = \sum_{i=1}^{\omega} m_i$ nodes in $\mathscr{G}(A)$ to be pinned such that Assumption 2 holds. That is, Assumption 2 can never be ensured if there are only μ nodes to be pinned, where $\mu < \delta$.

Based on the above analysis, one may obtain the following theorem, which summarizes the main result of this section.

Theorem 1 *Suppose Assumptions* 1-2 *are satisfied and* $f \in \mathrm{Quad}(P, \alpha\Gamma, \beta)$ *with a positive matrix* P *and* $\alpha < \frac{\lambda_1}{2\xi_{\max}}$, $\beta > 0$, *and* $P\Gamma$ *is semipositive definite, where* $\xi_{\max} = \max\{\xi_1, \dots, \xi_N\}$. *Pick* $0 < \beta' < \beta$. *Denote* $\kappa_i(t) = \sum_{j=1}^{N} l_{ij}\Gamma\delta_j(t) + d_i\Gamma(\delta_i(t) - \delta_0(t))$. *Then either one of the following two updating rules can guarantee that* (3) *synchronize with* (2) *exponentially.*

(1) *Set* $t^i_{k_i}$ *as the triggering time point by the rule*

$$t^i_{k_i+1} = \max_t \left\{ t \geq t^i_{k_i} : \left\| \kappa_i(t) \right\| \leq \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}} \sqrt{\sum_{i=1}^{N} \xi_i e_i^T(t) P e_i(t)} \right\}. \tag{7}$$

(2) *Set* $t^i_{k_i}$ *as the triggering time point by the rule*

$$t^i_{k_i+1} = \max_t \left\{ t \geq t^i_{k_i} : \left\| \kappa_i(t) \right\| \leq a \exp(-bt) \right\}, \tag{8}$$

where a *is a positive constant and* $0 < b < \beta - \beta'$.

Proof For any $i \in \{1, 2, \dots, N\}$,

$$\begin{aligned} \sum_{j=1}^{N} l_{ij}\Gamma e_j\left(t^i_{k_i}\right) &= \sum_{j=1}^{N} l_{ij}\Gamma\left(x_j\left(t^i_{k_i}\right) - x_j(t) + x_j(t) - s(t) + s(t) - s\left(t^i_{k_i}\right)\right) \\ &= \sum_{j=1}^{N} l_{ij}\Gamma\left(e_j(t) + \delta_j(t)\right), \end{aligned} \tag{9}$$

$$\begin{aligned} d_i\Gamma e_i\left(t^i_{k_i}\right) &= d_i\Gamma\left(x_i\left(t^i_{k_i}\right) - x_i(t) + x_i(t) - s(t) + s(t) - s\left(t^i_{k_i}\right)\right) \\ &= d_i\Gamma\left(e_i(t) + \delta_i(t) - \delta_0(t)\right). \end{aligned}$$

Then, by using (9), (4) can be rewritten as

$$\dot{e}_i(t) = f\left(e_i(t)\right) - \sum_{j=1}^{N} l_{ij}\Gamma\left(e_j(t) + \delta_j(t)\right) - d_i\Gamma\left(e_i(t) + \delta_i(t) - \delta_0(t)\right). \tag{10}$$

Consider the following Lyapunov function:

$$V(t)=\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t)=e^T(t)(\Xi\otimes P)e(t), \tag{11}$$

where P is a positive definite matrix which has been defined in (5) and $\Xi=\mathrm{diag}\{\xi_1,\dots,\xi_N\}$ is the same as in (6).

Taking the derivative of $V(t)$ along the trajectories (10) gives

$$\begin{aligned}
\dot V(t) &= 2\sum_{i=1}^{N}\xi_i e_i^T(t)P\dot e_i(t)\\
&= 2\sum_{i=1}^{N}\xi_i e_i^T(t)P\left(f\big(e_i(t)\big)-\sum_{j=1}^{N}l_{ij}\Gamma\big(e_j(t)+\delta_j(t)\big)-d_i\Gamma\big(e_i(t)+\delta_i(t)-\delta_0(t)\big)\right)\\
&= 2\sum_{i=1}^{N}\xi_i e_i^T(t)P\big(f\big(e_i(t)\big)-\alpha\Gamma e_i(t)\big)+2\alpha\sum_{i=1}^{N}\xi_i e_i^T(t)P\Gamma e_i(t)\\
&\quad -2\sum_{i=1}^{N}\xi_i e_i^T(t)P\left[\sum_{j=1}^{N}l_{ij}\Gamma e_j(t)+d_i\Gamma e_i(t)\right]\\
&\quad -2\sum_{i=1}^{N}\xi_i e_i^T(t)P\left[\sum_{j=1}^{N}l_{ij}\Gamma\delta_j(t)+d_i\Gamma\big(\delta_i(t)-\delta_0(t)\big)\right]\\
&\le -2\beta\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t)+2\alpha\sum_{i=1}^{N}\xi_i e_i^T(t)P\Gamma e_i(t)\\
&\quad -e^T(t)\big\{\big[\Xi(L+D)+(L+D)^T\Xi\big]\otimes P\Gamma\big\}e(t)-2\sum_{i=1}^{N}\xi_i e_i^T(t)P\kappa_i(t)\\
&\le -2\beta V(t)+2\alpha\sum_{i=1}^{N}\xi_i e_i^T(t)P\Gamma e_i(t)\\
&\quad -\lambda_1\sum_{i=1}^{N}e_i^T(t)P\Gamma e_i(t)-2\sum_{i=1}^{N}\xi_i e_i^T(t)P\kappa_i(t).
\end{aligned} \tag{12}$$

Pick a constant $\varepsilon>0$, one has

$$\begin{aligned}
-2\sum_{i=1}^{N}\xi_i e_i^T(t)P\kappa_i(t) &\le \varepsilon\sum_{i=1}^{N}\xi_i e_i^T(t)PP^Te_i(t)+\frac{1}{\varepsilon}\sum_{i=1}^{N}\xi_i\kappa_i^T(t)\kappa_i(t)\\
&\le \varepsilon\lambda_{\max}(P)\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t)+\frac{1}{\varepsilon}\sum_{i=1}^{N}\xi_i\kappa_i^T(t)\kappa_i(t)\\
&= \varepsilon\lambda_{\max}(P)V(t)+\frac{1}{\varepsilon}\sum_{i=1}^{N}\xi_i\big\|\kappa_i(t)\big\|^2,
\end{aligned} \tag{13}$$

where $\lambda_{\max}(P)$ is the maximal eigenvalue of P.

Under the condition $\alpha < \frac{\lambda_1}{2\xi_{\max}}$, one obtains

$$\begin{aligned}\dot{V}(t) &\le -2\beta V(t) + \varepsilon\lambda_{\max}(P)V(t) + \frac{1}{\varepsilon}\sum_{i=1}^{N}\xi_i\|\kappa_i(t)\|^2 \\ &= -2(\beta-\beta')V(t) + (-2\beta' + \varepsilon\lambda_{\max}(P))V(t) + \frac{1}{\varepsilon}\sum_{i=1}^{N}\xi_i\|\kappa_i(t)\|^2, \end{aligned} \tag{14}$$

where $0 < \beta' < \beta$.

(1) If it is guaranteed that

$$\sum_{i=1}^{N}\xi_i\|\kappa_i(t)\|^2 \le \varepsilon(2\beta' - \varepsilon\lambda_{\max}(P))\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t) \tag{15}$$

for some constant $\varepsilon > 0$, then one gets

$$\dot{V}(t) \le -2(\beta-\beta')V(t). \tag{16}$$

That is, the network (3) can synchronize with (2) exponentially fast with the rate of $2(\beta - \beta')$.

A sufficient condition for (15) is that

$$\|\kappa_i(t)\|^2 \le \frac{\varepsilon(2\beta' - \varepsilon\lambda_{\max}(P))}{N\xi_{\max}}\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t). \tag{17}$$

Note $\max_{\varepsilon>0}\{\frac{\varepsilon(2\beta'-\varepsilon\lambda_{\max}(P))}{N\xi_{\max}}\} = \frac{\beta'^2}{N\xi_{\max}\lambda_{\max}(P)}$, when $\varepsilon = \frac{\beta'}{\lambda_{\max}(P)}$.
Then (7) can be guaranteed when $\varepsilon = \beta'/\lambda_{\max}(P)$.

(2) Take $\varepsilon = 2\beta'/\lambda_{\max}(P)$. If it is guaranteed that

$$\|\kappa_i(t)\| \le a\exp(-bt), \tag{18}$$

one gets

$$\dot{V}(t) \le -2(\beta-\beta')V(t) + \frac{a^2\lambda_{\max}(P)\sum_{i=1}^{N}\xi_i}{2\beta'}\exp(-2bt), \tag{19}$$

which implies that $V(t)$ converges to 0 exponentially. □

Remark 3 In Theorem 1, two event-triggered conditions (7) and (8) are proposed. At the triggering time $t^i_{k_i}$, the left-hand term $\|\kappa_i(t^i_{k_i})\| = 0$. In (7), if there exists at least one node which does not synchronize with $s(t)$, the right-hand term must be positive. Therefore, the next triggering time $t^i_{k_i+1}$ must be greater than $t^i_{k_i}$. In (8), the right-hand term must be always positive for all nodes. Thus, the interevent intervals of all nodes are strictly positive. Although the two updating rules (7) and (8) are closely related to each other in some respects, the event-triggered condition in (8) is verified more easily than the condition in (7). Moreover, according to (16) and (19), the convergence under the event-triggered scheme (7) is better than under (8). The number of updating times under the event-triggered scheme (7) is more than (8).

Next, it is proved in detail that under the two updating rules (7) and (8), the inter-event sampling time instants $t^i_{k_i+1} - t^i_{k_i}$ for each node is strictly positive, that is, the coupled network can avoid the Zeno behavior.

Theorem 2 *Suppose Assumptions* 1-2 *are satisfied and* $f \in \mathrm{Quad}(P, \alpha\Gamma, \beta)$ *with a positive matrix* P *and* $\alpha < \frac{\lambda_1}{2\xi_{\max}}$, $\beta > 0$, *and* $P\Gamma$ *is semipositive definite. For any* $0 < \beta' < \beta$ *and any initial condition, the following two propositions are hold.*

(1) *Under the event-triggered scheme* (7), *each node has positive interevent interval which is lower bounded by a constant* τ^i_D.

(2) *Under the event-triggered scheme* (8), *the interevent interval of every node is strictly positive and is lower bounded by a common constant* τ^i_D.

Proof (1) Under the updating rules (7), for $t \in [t^i_{k_i}, t^i_{k_i+1})$, one has

$$V(t) \le V\bigl(t^i_{k_i}\bigr)\exp\bigl(-2\bigl(\beta - \beta'\bigr)\bigl(t - t^i_{k_i}\bigr)\bigr). \tag{20}$$

Note that

$$\dot{e}_i(t) = f\bigl(e_i(t)\bigr) - \sum_{j=1}^{N} l_{ij}\Gamma e_j(t) - d_i\Gamma e_i(t) - \kappa_i(t). \tag{21}$$

Hence, combining with the fact that f satisfies Assumption 1 and $V(t)$ is decreasing, one has

$$\begin{aligned}
\bigl\|\dot{e}_i(t)\bigr\| &\le \bigl\|f\bigl(e_i(t)\bigr)\bigr\| + \|\Gamma\|\sum_{j=1}^{N}|l_{ij}|\bigl\|e_j(t)\bigr\| + d_i\|\Gamma\|\bigl\|e_i(t)\bigr\| + \bigl\|\kappa_i(t)\bigr\| \\
&\le k\bigl\|e_i(t)\bigr\| + \|\Gamma\|\sum_{j=1}^{N}|l_{ij}|\bigl\|e_j(t)\bigr\| + d_i\|\Gamma\|\bigl\|e_i(t)\bigr\| + \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\sqrt{V(t)} \\
&\le \Biggl[\Biggl(k + \|\Gamma\|\sum_{j=1}^{N}|l_{ij}| + d_i\|\Gamma\|\Biggr)\frac{1}{\sqrt{N\xi_{\min}\lambda_{\min}(P)}} + \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\Biggr] \\
&\quad \times \sqrt{V\bigl(t^i_{k_i}\bigr)} = \gamma_i\sqrt{V\bigl(t^i_{k_i}\bigr)},
\end{aligned} \tag{22}$$

where $\gamma_i = (k + \|\Gamma\|\sum_{j=1}^{N}|l_{ij}| + d_i\|\Gamma\|)\frac{1}{\sqrt{N\xi_{\min}\lambda_{\min}(P)}} + \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}$. Furthermore, one can obtain

$$\begin{aligned}
\bigl\|\kappa_i(t)\bigr\| &= \Biggl\|\sum_{j=1}^{N} l_{ij}\Gamma\delta_j(t) + d_i\Gamma\bigl(\delta_i(t) - \delta_0(t)\bigr)\Biggr\| \\
&= \Biggl\|\sum_{j=1}^{N} l_{ij}\Gamma\bigl[\bigl(x_j\bigl(t^i_{k_i}\bigr) - s\bigl(t^i_{k_i}\bigr)\bigr) - \bigl(x_j(t) - s(t)\bigr)\bigr] \\
&\quad + d_i\Gamma\bigl[\bigl(x_i\bigl(t^i_{k_i}\bigr) - s\bigl(t^i_{k_i}\bigr)\bigr) - \bigl(x_i(t) - s(t)\bigr)\bigr]\Biggr\| \\
&= \Biggl\|\sum_{j=1}^{N} l_{ij}\Gamma\bigl[e_j\bigl(t^i_{k_i}\bigr) - e_j(t)\bigr] + d_i\Gamma\bigl[e_i\bigl(t^i_{k_i}\bigr) - e_i(t)\bigr]\Biggr\|
\end{aligned}$$

$$\begin{aligned} &\leq \|\Gamma\|\left(\sum_{j=1}^{N}|l_{ij}|\int_{t_{k_i}^i}^{t}\|\dot{e}_j(\tau)\|\,d\tau + d_i\int_{t_{k_i}^i}^{t}\|\dot{e}_i(\tau)\|\,d\tau\right) \\ &\leq \|\Gamma\|\left(\sum_{j=1}^{N}|l_{ij}|\gamma_j + d_i\gamma_i\right)\sqrt{V(t_{k_i}^i)}\left(t - t_{k_i}^i\right). \end{aligned} \tag{23}$$

Since $f \in \mathrm{Quad}(P, \alpha\Gamma, \beta)$ and f satisfies Assumption 1, there exists some σ (possibly negative) such that

$$(x-y)^T P\bigl(f(x) - f(y)\bigr) \geq \sigma(x-y)^T P(x-y) \tag{24}$$

for all $x, y \in \mathbb{R}^n$.

From (24), one gets

$$\begin{aligned} \dot{V}(t) &\geq 2\sigma V(t) - \lambda_N \sum_{i=1}^{N} e_i^T(t)P\Gamma e_i(t) - \varepsilon\lambda_{\max}(P)\sum_{i=1}^{N}\xi_i e_i^T(t)Pe_i(t) \\ &\quad - \frac{1}{\varepsilon}\sum_{i=1}^{N}\xi_i\kappa_i^T(t)\kappa_i(t) \geq \left(2\sigma - \frac{\lambda_N\|P\Gamma\|}{\xi_{\min}\lambda_{\min}(P)} - 2\beta'\right)V(t) = \varpi V(t), \end{aligned} \tag{25}$$

where $\varpi = 2\sigma - \frac{\lambda_N\|P\Gamma\|}{\xi_{\min}\lambda_{\min}(P)} - 2\beta' < 0$. Thus, one has

$$V(t) \geq V(t_{k_i}^i)\exp\bigl(\varpi(t - t_{k_i}^i)\bigr). \tag{26}$$

Used for the continuity of $\kappa_i(t)$ between the event-triggered time instants $t_{k_i}^i$ and $t_{k_i+1}^i$, the event-triggered time instant $t_{k_i+1}^i$ should satisfy $\|\kappa_i(t_{k_i+1}^i)\| = \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\sqrt{V(t_{k_i+1}^i)}$. Hence, combining with (23) and (26), in order to ensure event-triggered condition (7) is satisfied after instant $t_{k_i}^i$, it is necessary to require the time instant $t_{k_i+1}^i$ to satisfy

$$\begin{aligned} &\frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\sqrt{V(t_{k_i}^i)\exp\bigl(\varpi(t_{k_i+1}^i - t_{k_i}^i)\bigr)} \\ &\quad = \|\Gamma\|\left(\sum_{j=1}^{N}|l_{ij}|\gamma_j + d_i\gamma_i\right)\sqrt{V(t_{k_i}^i)}\left(t_{k_i+1}^i - t_{k_i}^i\right). \end{aligned} \tag{27}$$

That is,

$$\frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\exp\left(\frac{\varpi}{2}\left(t_{k_i+1}^i - t_{k_i}^i\right)\right) = \|\Gamma\|\left(\sum_{j=1}^{N}|l_{ij}|\gamma_j + d_i\gamma_i\right)\left(t_{k_i+1}^i - t_{k_i}^i\right). \tag{28}$$

Therefore, it is concluded that the inter-event time $t_{k_i+1}^i - t_{k_i}^i$ of the node i is lower bounded by

$$\sup\left\{\tau_D^i > 0 : \left(\sum_{j=1}^{N}|l_{ij}|\gamma_j + d_i\gamma_i\right)\|\Gamma\|\tau_D^i \leq \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\exp\left(\frac{\varpi}{2}\tau_D^i\right)\right\}. \tag{29}$$

(2) Under the updating rules (8), for $t \in [t^i_{k_i}, t^i_{k_i+1})$, by (19), one obtains

$$V(t) \leq \rho \exp(-2bt), \tag{30}$$

where $\rho = V(0) + \frac{a^2\lambda_{\max}(P)}{4\beta'(\beta-\beta'-b)} \sum_{i=1}^N \xi_i$.

Then

$$\begin{aligned} \|\dot{e}_i(t)\| &\leq \left[\left(k + \|\Gamma\| \sum_{j=1}^N |l_{ij}| + d_i\|\Gamma\|\right)\sqrt{\frac{\rho}{N\xi_{\min}\lambda_{\min}(P)}} + a\right]\exp(-bt^i_{k_i}) \\ &= \eta_i \exp(-bt^i_{k_i}), \end{aligned} \tag{31}$$

where $\eta_i = (k + \|\Gamma\| \sum_{j=1}^N |l_{ij}| + d_i\|\Gamma\|)\sqrt{\frac{\rho}{N\xi_{\min}\lambda_{\min}(P)}} + a$.

Hence,

$$\|\kappa_i(t)\| \leq \|\Gamma\|\left(\sum_{j=1}^N |l_{ij}|\eta_j + d_i\eta_i\right)\exp(-bt^i_{k_i})(t - t^i_{k_i}). \tag{32}$$

Therefore, (8) can be guaranteed if the following inequality holds:

$$\|\Gamma\|\left(\sum_{j=1}^N |l_{ij}|\eta_j + d_i\eta_i\right)(t - t^i_{k_i}) \leq a\exp[-b(t - t^i_{k_i})]. \tag{33}$$

Since when $t = t^i_{k_i}$, $\|\kappa_i(t)\| = 0$. Under the updating rule (8), the next event will not be triggered until $\|\kappa_i(t)\| = a\exp(-bt)$. Thus, the inter-event time $t^i_{k_i+1} - t^i_{k_i}$ of the node i is lower bounded by the solution τ^i_D of the following equation:

$$\|\Gamma\|\left(\sum_{j=1}^N |l_{ij}|\eta_j + d_i\eta_i\right)\tau^i_D = a\exp(-b\tau^i_D). \tag{34}$$

It can be seen that this equation has a positive solution.

This completes the proof. □

Remark 4 In Theorem 2, it is proved that under the two event-triggered schemes, the Zeno behavior can be excluded. From (28) and (34), it can be seen that the interevent interval of every node is strictly positive and is lower bounded by a common constant τ^i_D.

4 Self-triggered scheme for pinning synchronization

Under the updating rules (7) and (8), one is required to verify the event-triggered condition continuously. To avoid continuously communication among nodes, a self-triggered scheme based on Theorem 1 is proposed. Under the self-triggered scheme, each node in the network can predict next triggered time instant $t^i_{k_i+1}$ only based on the received information at time $t^i_{k_i}$. This scheme does not require one to verify the event-triggered condition continuously and hence more energy can be saved for the network. Inspired by the work of [26, 29], a self-triggered scheme is investigated in the following.

Suppose $t \in [t^i_{k_i}, t^i_{k_i+1})$ and j is the neighbor of the node i. Before the next event-triggered time instant of node j, one can obtain

$$x_j(t) = x_j(t^i_{k_i}) + \int_{t^i_{k_i}}^{t} f(x_j(\tau))\,d\tau - (t - t^i_{k_i})\left[\sum_{m=1}^{N} l_{jm}\Gamma e_m(t^i_{k_i}) + d_j\Gamma e_j(t^i_{k_i})\right] \tag{35}$$

and

$$e_j(t) = e_j(t^i_{k_i}) + \int_{t^i_{k_i}}^{t} (f(x_j(\tau)) - f(s(\tau)))\,d\tau - (t - t^i_{k_i})\left[\sum_{m=1}^{N} l_{jm}\Gamma e_m(t^i_{k_i}) + d_j\Gamma e_j(t^i_{k_i})\right]. \tag{36}$$

Then one has

$$\begin{aligned}\|e_j(t) - e_j(t^i_{k_i})\| &\le k\int_{t^i_{k_i}}^{t} \|e_j(\tau) - e_j(t^i_{k_i})\|\,d\tau \\ &\quad + (t - t^i_{k_i})\left[k\|e_j(t^i_{k_i})\| + \left\|\sum_{m=1}^{N} l_{jm}\Gamma e_m(t^i_{k_i}) + d_j\Gamma e_j(t^i_{k_i})\right\|\right] \\ &= k\int_{t^i_{k_i}}^{t} \|e_j(\tau) - e_j(t^i_{k_i})\|\,d\tau + (t - t^i_{k_i})\varphi_j, \end{aligned} \tag{37}$$

where $\varphi_j = k\|e_j(t^i_{k_i})\| + \|\sum_{m=1}^{N} l_{jm}\Gamma e_m(t^i_{k_i}) + d_j\Gamma e_j(t^i_{k_i})\|$.

Following from the Grönwall inequality, one gets

$$\|e_j(t) - e_j(t^i_{k_i})\| \le \frac{\varphi_j}{k}\left(\exp(k(t - t^i_{k_i})) - 1\right). \tag{38}$$

Therefore, one has

$$\begin{aligned}\|\kappa_i(t)\| &= \left\|\sum_{j=1}^{N} l_{ij}\Gamma[e_j(t^i_{k_i}) - e_j(t)] + d_i\Gamma[e_i(t^i_{k_i}) - e_i(t)]\right\| \\ &\le \left(\sum_{j=1}^{N} |l_{ij}|\varphi_j + d_i\varphi_i\right)\frac{\|\Gamma\|}{k}\left(\exp(k(t - t^i_{k_i})) - 1\right). \end{aligned} \tag{39}$$

In the following, combining with the event-triggered schemes (7) and (8), two self-triggered algorithms are given as follows.

Algorithm 2 (Self-triggered algorithm)

Step 1: Initialization: set $t^i_0 = 0$, for all $i = 1, 2, \dots, N$.

Step 2: At time $t^i_{k_i}$, $k_i \ge 1$, solve the following equation to find the next triggering time $t^i_{k_i+1} = t^i_{k_i} + \tau^i_D$:

$$\begin{aligned}\sup\Bigg\{\tau^i_D \ge 0 : &\frac{(\sum_{j=1}^{N} |l_{ij}|\varphi_j + d_i\varphi_i)\|\Gamma\|}{k}\left(\exp(k\tau^i_D) - 1\right) \\ &\le \frac{\beta'\sqrt{V(t^i_{k_i})}}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\exp\left(\frac{\varpi}{2}\tau^i_D\right)\Bigg\}. \end{aligned} \tag{40}$$

Step 3: If node i does not receive the renewed information from any of its neighbors during $(t^i_{k_i}, t^i_{k_i+1})$, node i is triggered on time instant $t^i_{k_i+1}$.

Step 4: If node i receives the renewed information from its neighbor j at time $t^i_{k'_i} < t^i_{k_i+1}$, compute the new value of ϑ_j and go to Step 2.

Theorem 3 *Suppose Assumptions* 1-2 *are satisfied and* $f \in \mathrm{Quad}(P, \alpha\Gamma, \beta)$ *with a positive matrix* P *and* $\alpha < \frac{\lambda_1}{2\xi_{\max}}$, $\beta > 0$, *and* $P\Gamma$ *is semipositive definite. Then, for any positive* $0 < \beta' < \beta$, *under the self-triggered Algorithm* 1, (3) *can achieve synchronization with* (2) *exponentially. Moreover, the difference of the inter-event sampling time instant* τ^i_D *for node* i *is lower bounded by a common positive instant which is given as*

$$\begin{aligned} &\sup\left\{\tau^i_D \geq 0 : \frac{(\sum_{j=1}^N |l_{ij}|\vartheta_j + d_i\vartheta_i)\|\Gamma\|}{k}\bigl(\exp\bigl(k\tau^i_D\bigr) - 1\bigr) \right. \\ &\quad \left. \leq \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}} \exp\biggl(\frac{\varpi}{2}\tau^i_D\biggr)\right\}, \end{aligned} \tag{41}$$

where $\vartheta_j = (k + \sum_{m=1}^N |l_{jm}|\|\Gamma\| + d_j\|\Gamma\|)\frac{1}{\sqrt{N\xi_{\min}\lambda_{\min}(P)}}$, $\rho = V(0) + \frac{a^2\lambda_{\max}(P)}{4\beta'(\beta-\beta'-b)}\sum_{i=1}^N \xi_i$, *and* $\varpi = 2\sigma - \frac{\lambda_N\|P\Gamma\|}{\xi_{\min}\lambda_{\min}(P)} - 2\beta' < 0$.

Proof Under the self-triggered Algorithm 1, one can have

$$\bigl\|\kappa_i(t)\bigr\| \leq \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\sqrt{\sum_{i=1}^N \xi_i e_i^T(t)Pe_i(t)}.$$

Then, according to Theorem 1, (3) can achieve synchronization with (2) exponentially.

Next, it is only proved that the inter-event interval of node i is strictly positive and has a lower bound τ^i_D, which is given as (41). Furthermore

$$\begin{aligned} \varphi_j &= k\bigl\|e_j\bigl(t^i_{k_i}\bigr)\bigr\| + \left\|\sum_{m=1}^N l_{jm}\Gamma e_m\bigl(t^i_{k_i}\bigr) + d_j\Gamma e_j\bigl(t^i_{k_i}\bigr)\right\| \\ &\leq \left(k + \sum_{m=1}^N |l_{jm}|\|\Gamma\| + d_j\|\Gamma\|\right)\frac{1}{\sqrt{N\xi_{\min}\lambda_{\min}(P)}}\sqrt{V\bigl(t^i_{k_i}\bigr)} = \vartheta_j\sqrt{V\bigl(t^i_{k_i}\bigr)}. \end{aligned} \tag{42}$$

Hence, a sufficient condition to satisfy the updating rule (7) is

$$\left(\sum_{j=1}^N |l_{ij}|\vartheta_j + d_i\vartheta_i\right)\frac{\|\Gamma\|}{k}\bigl(\exp\bigl(k\bigl(t - t^i_{k_d}\bigr)\bigr) - 1\bigr) \leq \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}}\exp\biggl(\frac{\varpi}{2}\tau^i_D\biggr). \tag{43}$$

Thus, for each node, under the self-triggered Algorithm 1, the lower bound of the inter-event sampling time can be given by

$$\begin{aligned} &\sup\left\{\tau^i_D \geq 0 : \frac{(\sum_{j=1}^N |l_{ij}|\vartheta_j + d_i\vartheta_i)\|\Gamma\|}{k}\bigl(\exp\bigl(k\tau^i_D\bigr) - 1\bigr) \right. \\ &\quad \left. \leq \frac{\beta'}{\sqrt{N\xi_{\max}\lambda_{\max}(P)}} \exp\biggl(\frac{\varpi}{2}\tau^i_D\biggr)\right\}. \end{aligned} \tag{44}$$

This completes the proof. □

Remark 5 In (41), if $\tau_D^i = 0$, the left-hand term equals zero, while the right-hand term is nonzero. Therefore, the inter-event interval of node i is strictly positive and has a lower bound τ_D^i, which is given as (41). To predict the next triggering time, each node only requires the states of itself and its neighbors at the last triggering time.

According to the event-triggered scheme (8), one can get the following result.

Theorem 4 *Suppose Assumptions* 1-2 *are satisfied and* $f \in \mathrm{Quad}(P, \alpha\Gamma, \beta)$ *with a positive matrix* P *and* $\alpha < \frac{\lambda_1}{2\xi_{\max}}$, $\beta > 0$, *and* $P\Gamma$ *is semipositive definite. For any positive* $0 < \beta' < \beta$, *set inter-event interval* τ_D^i *by*

$$\tau_D^i = \sup\left\{\tau \geq 0 : \frac{(\sum_{j=1}^N |l_{ij}|\varphi_j + d_i\varphi_i)\|\Gamma\|}{k}\bigl(\exp(k\tau) - 1\bigr) \leq a\exp\bigl(-b\bigl(t_{k_i}^i + \tau\bigr)\bigr)\right\}. \tag{45}$$

The triggering time $t_{k_i}^i$ *is by the following algorithm*:

(1) *Initialization*: *set* $t_0^i = 0$, *for all* $i = 1, 2, \ldots, N$.

(2) *At time* $t_{k_i}^i$, $k_i \geq 1$, *solve the following equation to find the next triggering time* $t_{k_i+1}^i$:

$$\frac{(\sum_{j=1}^N |l_{ij}|\varphi_j + d_i\varphi_i)\|\Gamma\|}{k}\bigl(\exp\bigl(k\tau_D^i\bigr) - 1\bigr) \leq a\exp\bigl(-b\bigl(t_{k_i}^i + \tau_D^i\bigr)\bigr). \tag{46}$$

(3) *Trigger node* i *by changing* $t_{k_i}^i$ *into* $t_{k_i+1}^i = t_{k_i}^i + \tau_D^i$.

Then (3) *can synchronize with* (2) *exponentially and the Zeno behavior can be excluded.*

The proof of this theorem is similar to Theorem 3.

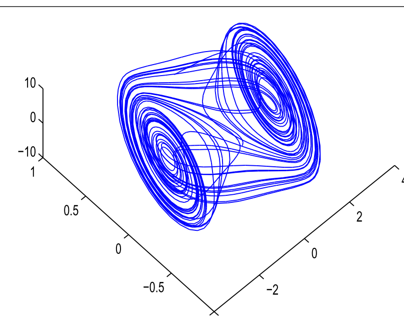
5 An illustrative example

In this section, two numerical simulation examples are given to illustrate the effectiveness of the proposed methods. The system is an array of 3 linearly coupled Chua circuits with the node dynamics

$$f\bigl(x_i(t)\bigr) = \begin{pmatrix} p * (-x_{i1} + x_{i2} - g(x_{i1})) \\ x_{i1} - x_{i2} + x_{i3} \\ -q * x_{i2} \end{pmatrix}, \tag{47}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, and $g(v) = m_1 * v + (m_0 - m_1) * (|v + 1| - |v - 1|)/2$ with the parameters $p = 9.78$, $q = 14.97$, $m_0 = -1.31$, $m_1 = -0.75$. The intrinsic node dynamics

Figure 1 A double-scrolling chaotic attractor of Chua circuit.



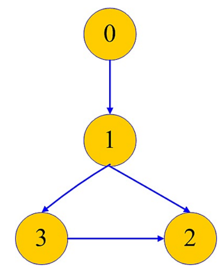


Figure 2 Network topology among the coupled nodes.

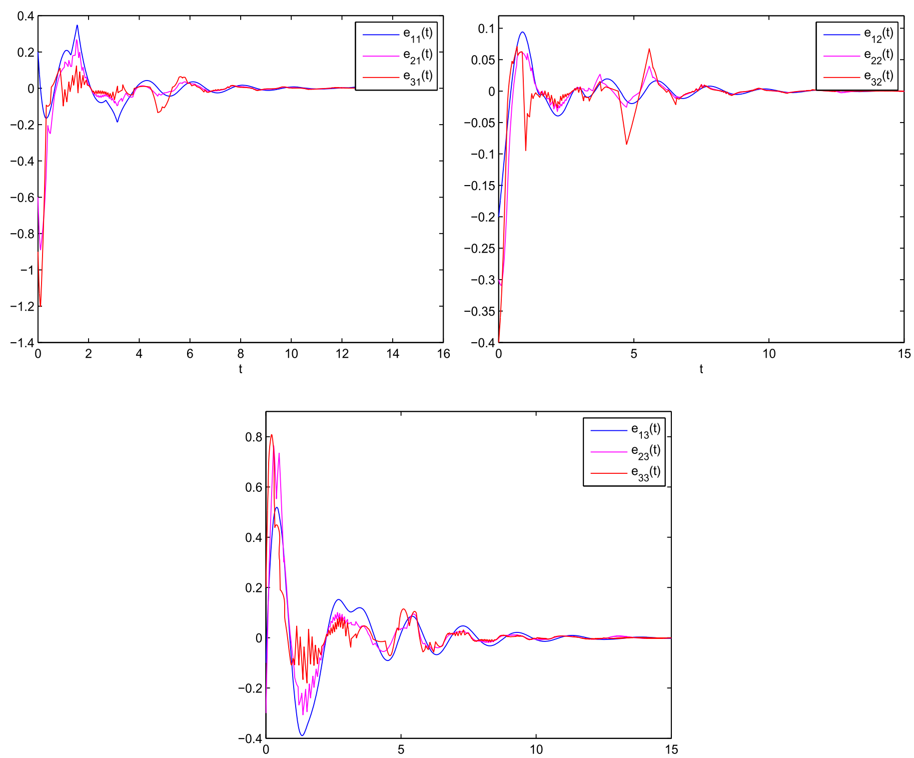


Figure 3 Synchronization errors of the 3 nodes under the updating rule (7).

(without diffusion) has a double-scrolling chaotic attractor [30], which is shown in Figure 1. The coupling graph topology is shown in Figure 2. Node 0 is the virtual node. To validate the Quad condition, let $P = \Gamma = I_3$, where I_3 stands for the identity matrix of three dimensions. Noting the Jacobin matrices of f is one of the following:

$$A_1 = \begin{pmatrix} -2.445 & 9.78 & 0 \\ 1 & -1 & 1 \\ 0 & -14.97 & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 3.0318 & 9.78 & 0 \\ 1 & -1 & 1 \\ 0 & -14.97 & 0 \end{pmatrix}, \tag{48}$$

then we can estimate $\beta' = \alpha - \lambda_{\max}((A_2)^s) = \alpha - 9.1207$, where 9.1207 is the largest eigenvalue of the symmetry parts of all Jacobin matrices of f. In the following, we pick $\alpha = 10$, $\beta = 0.8803$. Thus, f satisfies the Quad condition. Pick $d_1 = 0.62$, $d_2 = d_3 = 0$. Then there exists a diagonal matrix $\Xi = \mathrm{diag}\{1.2, 0.3, 0.4\}$ such that $\Xi(L + D) + (L + D)^T \Xi$ is positive

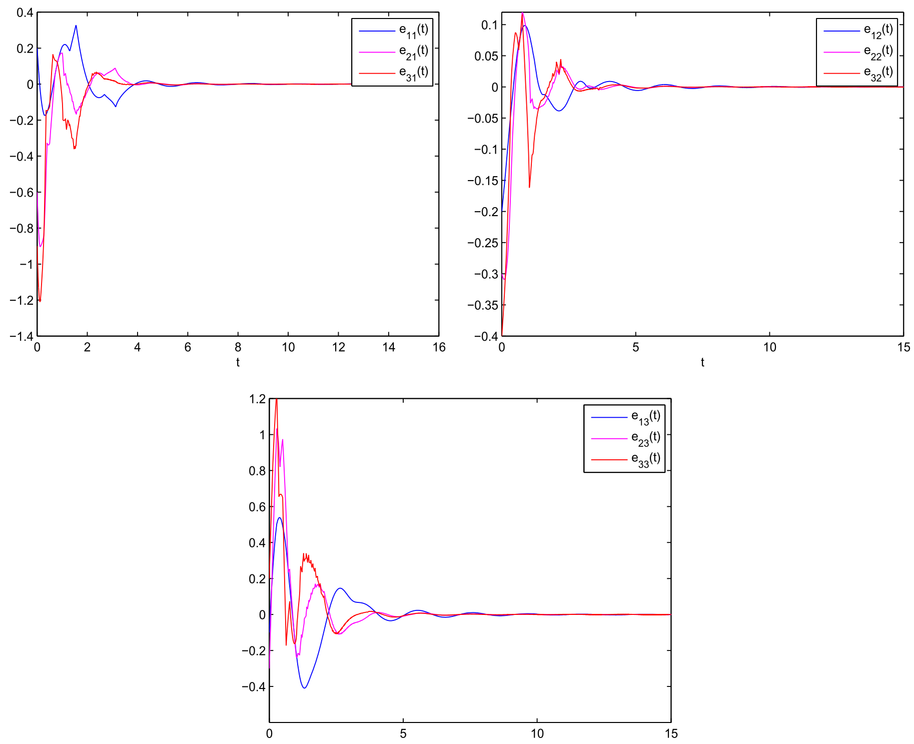


Figure 4 Synchronization errors of the 3 nodes under the updating rule (8).

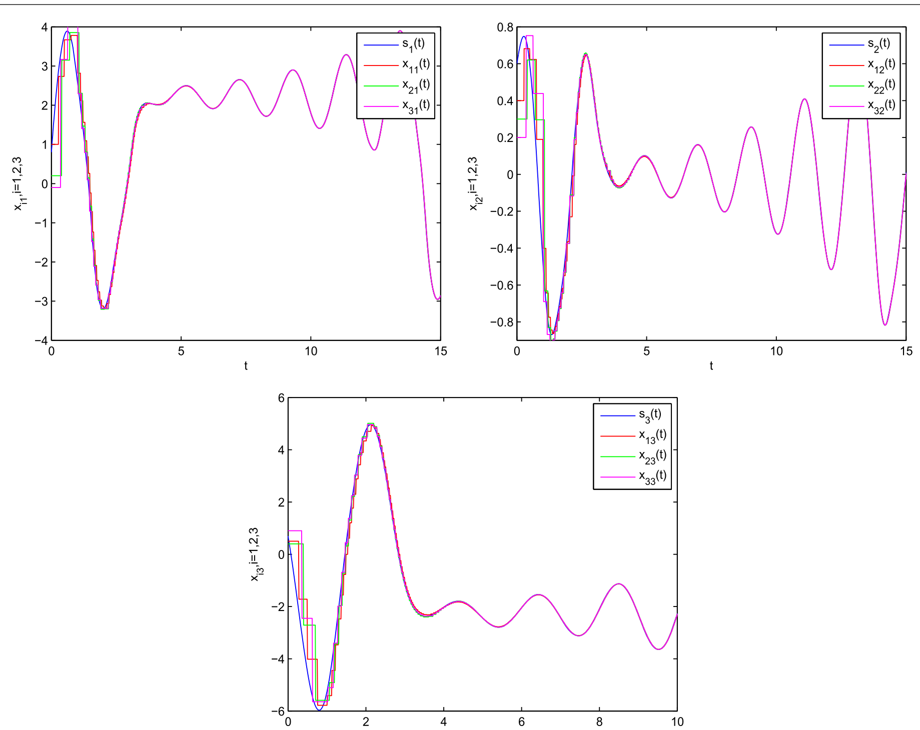


Figure 5 Tracking trajectories between the virtual node and the 3 nodes under the updating rule (41).

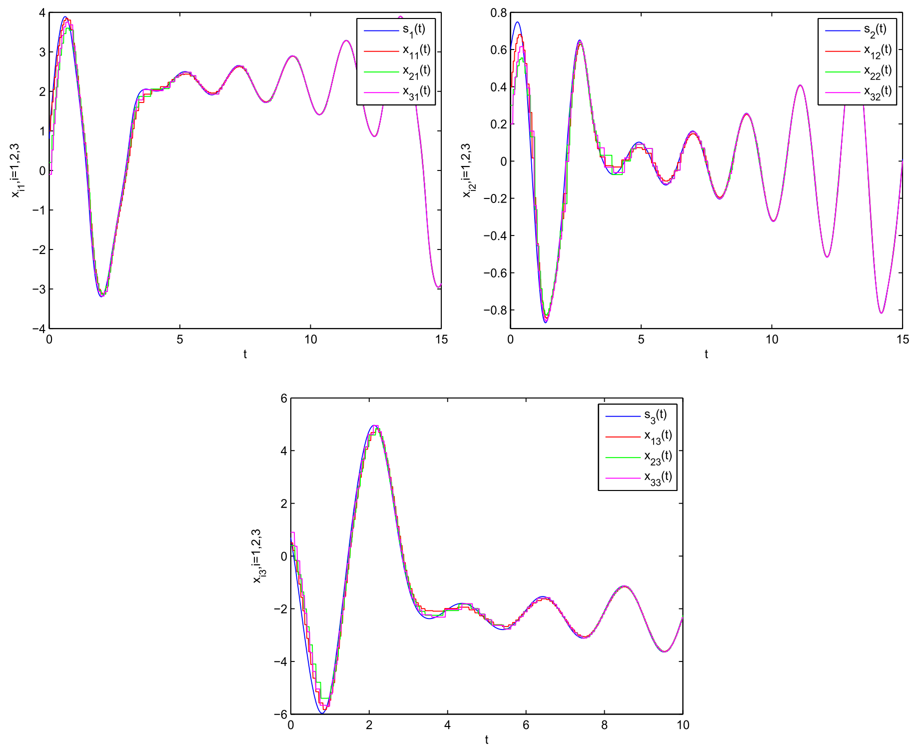


Figure 6 Tracking trajectories between the virtual node and the 3 nodes under the updating rule (45).

with eigenvalues $\lambda_1 = 0.4113$, $\lambda_2 = 1.3487$, $\lambda_3 = 1.7214$. The ordinary differential (2) and (3) are numerically solved by the Runge-Kutta method with a time step 0.01 (seconds) and the time duration of the numerical simulations is $[0, 15]$ (seconds). Under the updating rule (7), Figure 3 shows the synchronization errors of the 3 nodes. We take the same value of d_i as above and $a = 0.5$ and $b = 0.5$. Figure 4 shows the synchronization errors of the 3 nodes under the updating rule (8). The tracking trajectories between the virtual node and the 3 nodes under the updating rule (41) and (45) are shown in Figures 5 and 6, respectively.

6 Conclusions

In this paper, event-triggered schemes and self-triggered schemes are investigated to realize the exponential synchronization of the networked dynamical systems. The coupled information under these schemes is updated only when the triggering conditions are violated. The next observation time for these nodes is predicted only based on the latest observations of their neighborhood and the virtual leader. Thus, continuous communication can be avoided and the number of information transmission is largely reduced. Moreover, a positive lower bound for inter-event intervals is achieved and the Zeno behavior can be excluded. Finally, two numerical simulation examples are provided to illustrate the effectiveness of the proposed results. In the future, we will focus on the related applications of the event-triggered scheme in the coupled neural networks with time-delays and quantization.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the manuscript.

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