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Delay-lower-bound-based adaptive fuzzy memory control for uncertain nonlinear systems with state and input delays

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Abstract

This paper is concerned with the problem of adaptive fuzzy control for a class of uncertain nonlinear strict-feedback systems. The considered systems contain uncertain state delay and input delay simultaneously. By utilizing the mean value theorem, the unknown time-delayed functions related to all state variables are dealt with. A novel adaptive filter is designed to eliminate the effect of the time-varying input delay. Based on the backstepping technique, a delay-lower-bound-based adaptive fuzzy memory control scheme is developed. It is proved that the proposed adaptive memory control method can guarantee that all the signals in the closed-loop system are bounded and the tracking error converges to a small neighborhood of the origin. A simulation example is given to demonstrate the effectiveness of the proposed control scheme.

Keywords: adaptive fuzzy memory control; nonlinear strict-feedback systems; mean value theorem; the backstepping technique

1 Introduction

Over the past decades, adaptive control has become a mature and well-established research area within control systems society. Especially, the adaptive backstepping control technique as a powerful tool has received attractive attention for controlling strict-feedback nonlinear systems. This technique has a number of advantages over the conventional approaches, such as dealing with those nonlinear systems without satisfying the matching condition, providing a promising way to improve the transient performance of adaptive systems by tuning the design parameters. Recently, various considerable and significant results on adaptive backstepping control for nonlinear systems have been reported in [1–6], and the references therein. Among them, [1, 4–6] investigated the adaptive backstepping control technique for single-input and single-out (SISO) nonlinear systems, [2, 3] studied for multiple-input and multiple-output (MIMO) nonlinear systems. Recently, approximation-based adaptive fuzzy control has been developed to deal with the control problem of nonlinear systems with unknown nonlinear functions [7–9]. Reference [7] tried to consider a class of stochastic pure-feedback nonlinear systems with unknown hysteresis by utilizing adaptive fuzzy control method. Reference [8] developed a fuzzy adaptive control approach for nonlinear systems with unknown control gain sign. The backstepping control technique has become one of the most popular design methods for a large class of nonlinear systems. In [10–12], the adaptive fuzzy backstepping control method is applied to deal with switched nonlinear systems. Although the backstepping-based adaptive technique has been used widely and developed extensively, there exists the issue of 'explosion of complexity' in the design process. In order to avoid the problem, dynamic surface control (DSC) technique was first introduced in [13]. The DSC technique was utilized to eliminate the problem of explosion of complexity by introducing a first-order filter at each step of the traditional backstepping approach [14, 15]. Among them, [14] investigated adaptive dynamic surface control method for nonlinear systems with an unknown dead zone in pure feedback form. Reference [15] proposed adaptive fuzzy backstepping dynamic surface control for uncertain nonlinear systems based on filters. More recently, the DSC approach has been further developed in [16, 17].

Time delays are often found in various engineering systems, such as electrical networks, microwave oscillators, nuclear reactors, *etc.* It is well known that time delays may destroy the stability or affect the performance of control systems. Therefore, the investigation of the stability and control design of nonlinear time-delay systems is a challenging and meaningful issue, and has received a great deal of attention in the control community in recent years. In order to deal with the stability analysis and controller design for time-delay systems, the common methods for nonlinear systems are to construct the Lyapunov-Razumikhin functions [18, 19] or appropriate Lyapunov-Krasovskii functionals [20–24]. In [25], the adaptive robust tracking control problem is considered for an Euler-Lagrange system possessing second-order dynamics. In [26], an adaptive fuzzy predictive sliding mode control is presented for nonlinear systems with uncertain dynamics and unknown input delay. In [27], the robust control problem is investigated for a class of uncertain T-S fuzzy systems. Then to improve the transient performance of adaptive systems by tuning the design parameters, by further combining appropriately Lyapunov-Krasovskii functionals with adaptive backstepping technique, adaptive control method is developed for uncertain nonlinear strict-feedback systems with time delays [28–30]. Reference [28] has developed an observer-based adaptive fuzzy control scheme for a class of nonlinear time-delay systems, and [30] tried to solve the time-delay problem in face of a class of perturbed strict-feedback nonlinear systems. Among them, the main idea is to develop the adaptive memoryless controllers, which utilizes appropriately Lyapunov-Krasovskii functionals to compensate for the unknown time-delay terms [29]. Although much progress has been made for the time-delay systems in the adaptive control field, some challenging difficulties still remain. The main difficulties lie in two folds: First, how to develop adaptive backstepping control scheme for nonlinear strict-feedback systems with time delays by relaxing the restrictions of the time-delayed functions with all state variables. Second, how to design adaptive memory-reliable controller with less conservative for a class of nonlinear delay systems.

In the last several years, for the study of stabilization of time-delay systems, both controllers with or without memory were proposed; see [31–34]. The memoryless controllers have feedback of the current state only, and they are designed to guarantee the delay-independent stability of the closed-loop systems. Although the memoryless controllers [31, 32] are easy to implement, they tend to be conservative. In contrast, the memory controllers have a feedback including not only the current states but also the past ones. Hence, it is obvious that the memory controller is less conservative and may achieve a better performance than the memoryless case [34, 35]. Nevertheless, the memory control schemes are only developed for linear time-delay systems [33, 34]. To the best of our knowledge, there are few results for the case where an adaptive fuzzy memory controller is designed for uncertain nonlinear strict-feedback systems with state and input delays.

Motivated by the aforementioned observations, in this paper, the problem of adaptive fuzzy memory control is investigated for a class of uncertain nonlinear strict-feedback systems with state and input delays. Fuzzy logic systems are utilized to approximate the unknown nonlinear functions, and hyperbolic tangent function is introduced to avoid the controller singularity problem. Compared with the existing work, the main advantages of the proposed control scheme are as follows: (i) By using the mean value theorem, delay-lower-bound states are separated from delayed state functions. Based on delay-lower-bound states, a novel adaptive fuzzy memory controller is developed. In contrast with the previous memoryless control approach [30, 36–38], by exploiting the history information of states, therefore, the proposed controller is prone to be less conservative. (ii) Time-varying input delay is considered in this paper. By constructing a novel adaptive filter, the effects from input delay are compensated. Note that in existing results [39–41], the considered delayed input is always time-invariant. Hence the considered systems in this paper are more general and more practical.

Utilizing the mean value theorem, the unknown time-delayed functions related to all state variables are dealt with. A novel adaptive filter is constructed to eliminate the effect of time-varying input delay. For the analysis of stability, *a priori* knowledge of the bound on the control is put forward. Fuzzy logic systems are utilized to approximate the unknown nonlinear functions, and hyperbolic tangent function is introduced to avoid the controller singularity problem. Combining an adaptive control methodology with the backstepping technique, a delay-lower-bound-based adaptive fuzzy memory controller by considering the history information of states is designed. Finally, the proposed adaptive control scheme can guarantee that all the signals in the closed-loop system are bounded and the tracking error converges to a small neighborhood of the origin.

The paper is organized as follows. In Section 2, some preliminaries are presented and the problem is formulated. Adaptive fuzzy memory control design and stability analysis are given in Section 3. A simulation study for a practical example verifies the main results in Section 4. Finally, we conclude this paper in Section 5.

2 Preliminaries and problem statement

2.1 System description

Consider a class of uncertain nonlinear strict-feedback systems with state and input delays in the following form:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + h_1(x_1(t-\tau_1)), \\ \dot{x}_i = x_{i+1} + f_i(\underline{x}_i) + h_i(\underline{x}_i(t-\tau_i)), \quad i = 2, \ldots, n-1, \\ \dot{x}_n = u(t-\tau_0(t)) + f_n(\underline{x}_n) + h_n(\underline{x}_n(t-\tau_n)), \\ y = x_1, \end{cases} \tag{1}$$

where $x = \underline{x}_n = [x_1, \ldots, x_n]^T \in \mathrm{R}^n$ and $y \in \mathrm{R}$ are state vectors and the control output, respectively. $u(t-\tau_0(t)) \in \mathrm{R}$ is the control input, $\tau_0(t)$ is the time-varying input delay. $\underline{x}_i = [x_1, \ldots, x_i]^T \in \mathrm{R}^i$ $(i = 1, \ldots, n)$. $\underline{x}_i(t-\tau_i) = [x_1(t-\tau_1), x_2(t-\tau_2), \ldots, x_i(t-\tau_i)]^T \in \mathrm{R}^i$ are the delayed state vectors, τ_i are unknown delays. $f_i(\cdot) : \mathrm{R}^i \to \mathrm{R}$ are the unknown smooth nonlinear functions, and $h_i(\cdot) : \mathrm{R}^i \to \mathrm{R}$ are the unknown smooth nonlinear time-delay functions satisfying $h_i(0) = 0$. Here states are assumed to be measurable because this paper focuses on uncertain time delays in state and input.

The control objective presented in this paper will design an adaptive fuzzy memory controller to guarantee that all the signals in the closed-loop system are bounded and the tracking error converges to a small neighborhood of the origin.

To facilitate the control system design, the following assumptions are proposed.

Assumption 1 Assume that τ_i $(i = 1, \dots, n)$ are unknowns that satisfy

$$0 < \tau_m \leq \tau_i \leq \tau_M, \tag{2}$$

where τ_m and τ_M are constants. τ_m is the lower bound of the delay and τ_M is the upper bound.

Assumption 2 The input delay satisfies the following restrictions:

$$\tau_0(t) = \tau + \overline{\tau}(t), \tag{3}$$

where $|\overline{\tau}(t)| < \varsigma$, $\tau_0(t) > 0$, $\tau > 0$. τ is a known constant and ς is an unknown small constant.

Assumption 3 The desired reference signal y_r and its derivatives $\dot{y}_r$, $\ddot{y}_r$ are bounded, *i.e.*, there exists a compact set $Y = \{(y_r, \dot{y}_r, \ddot{y}_r) : y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \leq Y_R\}$, where Y_R is a positive constant.

Assumption 4 [39] Assume that *a priori* knowledge of the bound on the control is proposed and the finite integral of past control values is bounded by a known constant, *i.e.*, $\int_{t-\tau}^{t} u(s)\,ds \leq u_0$.

Since h_i $(i = 1, \dots, n)$ are sufficiently smooth functions, one can obtain by using the mean value theorem

$$h_i\bigl(\underline{x}_i(t - \tau_i)\bigr) = h_i\bigl(\underline{x}_i(t - \tau_m)\bigr) + N_i\bigl[\underline{x}_i(t - \tau_i) - \underline{x}_i(t - \tau_m)\bigr], \tag{4}$$

where $N_i = \frac{\partial h_i}{\partial \underline{x}_i}$ $(\underline{x}_i \in \Pi(X_0))$ is unknown constant matrix and satisfies $\|N_i\| \leq L_i$, in which L_i is a well-known positive constant and $\Pi(X_0)$ is a compact set. $\|\cdot\|$ denotes the 2-norm of a vector. Thus, the system (1) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + h_1(x_1(t - \tau_m)) + N_1[x_1(t - \tau_1) - x_1(t - \tau_m)], \\ \dot{x}_i = x_{i+1} + f_i(\underline{x}_i) + h_i(\underline{x}_i(t - \tau_m)) + N_i[\underline{x}_i(t - \tau_i) - \underline{x}_i(t - \tau_m)], \quad 1 = 2, \dots, n-1, \\ \dot{x}_n = u(t - \tau_0(t)) + f_n(\underline{x}_n) + h_n(\underline{x}_n(t - \tau_m)) + N_n[\underline{x}_n(t - \tau_n) - \underline{x}_n(t - \tau_m)], \\ y = x_1. \end{cases} \tag{5}$$

2.2 Fuzzy logic systems

Due to the uncertainty of the considered system, the fuzzy logic system (FLS) is introduced. Construct a fuzzy logic system with the following form of If-then rules:

R^i: If x_1 is F_1^i and x_2 is F_2^i and … and x_n is F_q^i, then y is B^i, $i = 1, 2, \dots, \iota$, where $x = [x_1, \ldots, x_n]^T$ and y are the fuzzy logic system input and output, respectively. The fuzzy sets F_j^i and B^i, associated with the fuzzy functions $\mu_{F_j^i}(x_j)$ and $\mu_{B^i}(y)$, respectively. ι is the rules number. Through the singleton function, the center average defuzzification and product inference [29], the FLS can be formulated as

$$y\bigl(x(t)\bigr) = \frac{\sum_{i=1}^{\iota} \bar{y}_i \prod_{j=1}^{q} \mu_{F_j^i}(x_j)}{\sum_{i=1}^{\iota} (\prod_{j=1}^{q} \mu_{F_j^i}(x_j))}, \tag{6}$$

where $\bar{y}_i = \max_{y \in R} \mu_{B^i}(y)$.

Let $\varphi_i = \frac{\prod_{j=1}^{q} \mu_{F_j^i}(x_j)}{\sum_{i=1}^{\iota} (\prod_{j=1}^{q} \mu_{F_j^i}(x_j))}$, and denote $\theta = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_\iota]^T = [\theta_1, \theta_2, \ldots, \theta_\iota]^T$ and $\varphi^T(x) = [\varphi_1(x), \ldots, \varphi_\iota(x)]$, then FLS (6) can be rewritten as $y(x) = \theta^T \varphi(x)$.

Lemma 1 [42] *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists an FLS such as*

$$\sup_{x \in \Omega} \bigl| f(x) - \theta^T \varphi(x) \bigr| \le \varepsilon. \tag{7}$$

By Lemma 1, fuzzy logic systems are universal approximators, *i.e.*, the y can approximate any smooth function on a compact set, thus we can assume that the nonlinear terms and the time-delay terms in (5) can be approximated by

$$\hat{f}_i(\underline{x}_i | \theta_i) = \theta_i^T \varphi_i(\underline{x}_i), \qquad \hat{h}_i(\underline{x}_{\tau_m} | \theta_i) = \theta_i^T \varphi_i(\underline{x}_{\tau_m}), \quad i = 1, \ldots, n, \tag{8}$$

where $\underline{x}_{\tau_m}$ denotes $\underline{x}(t - \tau_m)$.

Also the fuzzy minimum approximation errors ε_i and δ_i are defined as $\varepsilon_i = f_i(\underline{x}_i) - \hat{f}_i(\underline{x}_i | \theta_{fi})$, $\delta_i = h_i(\underline{x}_{\tau_i}) - \hat{h}_i(\underline{x}_{\tau_i} | \theta_{hi})$, $1 \le i \le n$, where ε_i satisfies $|\varepsilon_i| \le \varepsilon_i^*$, and ε_i^* is an unknown constant and δ_i satisfies $|\delta_i| \le \delta_i^*$, and δ_i^* is an unknown constant.

The system (5) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 + \theta_{f1}^T \varphi_1(x_1) + \varepsilon_1 + \theta_{h1}^T \varphi_1(x_1(t - \tau_m)) + \delta_1 + N_1[x_1(t - \tau_1) - x_1(t - \tau_m)], \\ \dot{x}_i = x_{i+1} + \theta_{fi}^T \varphi_i(\underline{x}_i) + \varepsilon_i + \theta_{hi}^T \varphi_i(\underline{x}_i(t - \tau_m)) + \delta_i + N_i[\underline{x}_i(t - \tau_i) - \underline{x}_i(t - \tau_m)], \\ \quad i = 2, \ldots, n-1, \\ \dot{x}_n = u(t - \tau_0(t)) + \theta_{fn}^T \varphi_n(\underline{x}_n) + \varepsilon_n + \theta_{hn}^T \varphi_n(\underline{x}_n(t - \tau_m)) + \delta_n \\ \qquad + N_n[\underline{x}_n(t - \tau_n) - \underline{x}_n(t - \tau_m)], \\ y = x_1. \end{cases} \tag{9}$$

2.3 Adaptive filter design

Construct a filter with adaptive parameter

$$\dot{\phi} = A\phi + Ky + \hat{\Theta}_f + \hat{\Theta}_h + E_n u(t - \tau), \tag{10}$$

where

$$\phi = [\phi_1, \ldots, \phi_n]^T, \qquad A = \begin{bmatrix} -k_1 & & \\ \vdots & I_{n-1} & \\ -k_n & \cdots & 0 \end{bmatrix}, \qquad K = [k_1, \ldots, k_n]^T,$$

$$\hat{\Theta}_f = \left[\hat{\theta}_{f1}^T\varphi_1(x_1),\ldots,\hat{\theta}_{fn}^T\varphi_n(\underline{x}_n)\right]^T, \qquad \hat{\Theta}_h = \left[\hat{\theta}_{h1}^T\varphi_1(x_{\tau_1}),\ldots,\hat{\theta}_{hn}^T\varphi_n(\underline{x}_{\tau_n})\right]^T,$$

and

$$E_n = [0,0,\ldots,1]^T.$$

A is a Hurwitz matrix. $\hat{\theta}_{f_i}$ is the estimate of θ_{f_i}, $\hat{\theta}_{h_i}$ is the estimate of θ_{h_i}. Thus, for any a given $Q^T = Q > 0$, there exists a positive definite matrix $P^T = P > 0$ such that $A^TP + PA = -Q$. Define $x = \phi + e$, where $e = [e_1,\ldots,e_n]$ is the filtered tracking error and satisfies $|e| \leq e^*$.

Remark 1 Several results have been developed for input-delayed nonlinear systems with exact time-delayed knowledge [43, 44], but few results examine the time-varying input delay problem for uncertain nonlinear systems. The considered system in this paper is with time-varying input delay. The adaptive filter is designed with exact time-delayed knowledge, regardless of the effect of the time-varying input delay.

3 Control design and stability analysis

3.1 Adaptive fuzzy memory control design

In this section, an adaptive fuzzy memory control scheme is designed by combining the backstepping method with the DSC technique, which can guarantee that all the signals in the closed-loop system are bounded and the tracking error is as small as desired.

First of all, the changes of the coordinates are designed as follows:

$$\begin{aligned}
&S_1 = y - y_r,\\
&S_i = x_i - x_{id}, \qquad \chi_i = x_{id} - \alpha_{i-1}, \quad i = 2,\ldots,n-1,\\
&S_n = \phi_n - x_{nd} + \int_{t-\tau}^{t} u(s)\,ds, \qquad \chi_n = x_{nd} - \alpha_{n-1},
\end{aligned} \tag{11}$$

where S_i is called error surface; x_{id} is a state variable, which can be obtained through a first-order filter on the intermediate control function α_{i-1} and χ_i is called the output error of the first-order filter. Define the first-order filter as $\rho_i\dot{x}_{id} + x_{id} = \alpha_{i-1}$, $x_{id}(0) = \alpha_{i-1}(0)$, $i = 2,\ldots,n$.

Step 1: Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}S_1^2 + \frac{1}{2\gamma_{f1}}\tilde{\theta}_{f1}^T\tilde{\theta}_{f1} + \frac{1}{2\gamma_{h1}}\tilde{\theta}_{h1}^T\tilde{\theta}_{h1} + \overline{V}_1, \tag{12}$$

where γ_{f1}, γ_{h1} are positive design constants, $\tilde{\theta}_{f1} = \theta_{f1} - \hat{\theta}_{f1}$, $\tilde{\theta}_{h1} = \theta_{h1} - \hat{\theta}_{h1}$ and $\overline{V}_1$ will be given later. The time derivative of V_1 is

$$\begin{aligned}
\dot{V}_1 &= S_1\dot{S}_1 - \frac{1}{\gamma_{f1}}\tilde{\theta}_{f1}^T\dot{\hat{\theta}}_{f1} - \frac{1}{\gamma_{h1}}\tilde{\theta}_{h1}^T\dot{\hat{\theta}}_{h1} + \dot{\overline{V}}_1\\
&= S_1[\dot{x}_1 - \dot{y}_r] - \frac{1}{\gamma_{f1}}\tilde{\theta}_{f1}^T\dot{\hat{\theta}}_{f1} - \frac{1}{\gamma_{h1}}\tilde{\theta}_{h1}^T\dot{\hat{\theta}}_{h1} + \dot{\overline{V}}_1\\
&= S_1\big[x_2 + \theta_{f1}^T\varphi_1(x_1) + \varepsilon_1 + \theta_{h1}^T\varphi_1\big(x_1(t-\tau_m)\big) + \delta_1\\
&\quad + N_1\big(x_1(t-\tau_1) - x_1(t-\tau_m)\big) - \dot{y}_r\big] - \frac{1}{\gamma_{f1}}\tilde{\theta}_{f1}^T\dot{\hat{\theta}}_{f1} - \frac{1}{\gamma_{h1}}\tilde{\theta}_{h1}^T\dot{\hat{\theta}}_{h1} + \dot{\overline{V}}_1
\end{aligned}$$

$$= S_1\big[S_2 + \chi_2 + \alpha_1 + \theta_{f1}^T\varphi_1(x_1) + \varepsilon_1 + \theta_{h1}^T\varphi_1\big(x_1(t-\tau_m)\big) + \delta_1 + N_1\big[x_1(t-\tau_1) - x_1(t-\tau_m)\big] - \dot{y}_r\big] - \frac{1}{\gamma_{f1}}\tilde{\theta}_{f1}^T\dot{\hat{\theta}}_{f1} - \frac{1}{\gamma_{h1}}\tilde{\theta}_{h1}^T\dot{\hat{\theta}}_{h1} + \dot{\overline{V}}_1. \tag{13}$$

By using the Young inequality and the properties of norms, it follows that

$$\begin{aligned} &S_1\varepsilon_1 + S_1\delta_1 \le S_1^2 + \frac{1}{2}\varepsilon_1^{*2} + \frac{1}{2}\delta_1^{*2}, \\ &S_1N_1\big(x_1(t-\tau_1) - x_1(t-\tau_m)\big) \\ &\quad \le \frac{1}{2}S_1^2 + \frac{1}{2}\|N_1\|^2\big\|x_1(t-\tau_1) - x_1(t-\tau_m)\big\|^2, \\ &\quad \le \frac{1}{2}S_1^2 + L_1^2\big[x_1^2(t-\tau_1) + x_1^2(t-\tau_m)\big]. \end{aligned} \tag{14}$$

Choosing the nonnegative function as

$$\overline{V}_1 = e^{r_1(\tau_M - t)}L_1^2\left[\int_{t-\tau_1}^{t} e^{r_1 s}x_1^2(s)\,ds + \int_{t-\tau_m}^{t} e^{r_1 s}x_1^2(s)\,ds\right]. \tag{15}$$

The time derivative of $\overline{V}_1$ is

$$\dot{\overline{V}}_1 \le -r_1\overline{V}_1 + 2e^{r_1\tau_M}L_1^2x_1^2(t) - L_1^2\big[x_1^2(t-\tau_1) + x_1^2(t-\tau_m)\big]. \tag{16}$$

By substituting (14), (16) into (13), one can obtain

$$\begin{aligned} \dot{V}_1 \le{}& S_1\left[S_2 + \chi_2 + \alpha_1 + \hat{\theta}_{f1}^T\varphi_1(x_1) + \hat{\theta}_{h1}^T\varphi_1\big(x_1(t-\tau_m)\big) - \dot{y}_r + \frac{H_1}{S_1}\right] + \frac{3}{2}S_1^2 + \frac{1}{2}\varepsilon_1^2 \\ &+ \frac{1}{2}\delta_1^2 - r_1\overline{V}_1 + \frac{1}{\gamma_{f1}}\tilde{\theta}_{f1}^T\big[\gamma_{f1}\varphi_1(x_1)S_1 - \dot{\hat{\theta}}_{f1}\big] \\ &+ \frac{1}{\gamma_{h1}}\tilde{\theta}_{h1}^T\big[\gamma_{h1}\varphi_1\big(x_1(t-\tau_m)\big)S_1 - \dot{\hat{\theta}}_{h1}\big], \end{aligned} \tag{17}$$

where $H_1 = 2e^{r_1\tau_M}L_1^2x_1^2(t)$.

Remark 2 Note that H_1/S_1 is discontinuous at $S_1 = 0$. Based on the property of the hyperbolic tangent function, *i.e.*, $\lim_{S_1\to 0}\frac{\tanh^2(S_1/\varepsilon_1)}{S_1} = 0$, the function $\tanh(S_1/\varepsilon_1)$ is introduced to compensate for the singularity problem.

Design the intermediate control function α_1 as follows:

$$\alpha_1 = -c_1S_1 - \hat{\theta}_{f1}^T\varphi_1(x_1) - \hat{\theta}_{h1}^T\varphi_1\big(x_1(t-\tau_m)\big) + \dot{y}_r - \frac{16H_1}{S_1}\tanh^2\left(\frac{S_1}{\varepsilon_1}\right). \tag{18}$$

Choose the adaptive laws as follows:

$$\dot{\hat{\theta}}_{f1} = \gamma_{f1}\varphi_1(x_1)S_1 - \sigma_{f1}\hat{\theta}_{f1}, \qquad \dot{\hat{\theta}}_{h1} = \gamma_{h1}\varphi_1\big(x_1(t-\tau_m)\big)S_1 - \sigma_{h1}\hat{\theta}_{h1}. \tag{19}$$

Substituting (18) and (19) into (17), it follows that

$$\dot{V}_1 \le -\left[c_1 - \frac{3}{2}\right]S_1^2 + S_1[S_2 + \chi_2] + \frac{\sigma_{f1}}{\gamma_{f1}}\tilde{\theta}_{f1}^T\hat{\theta}_{f1} + \frac{\sigma_{h1}}{\gamma_{h1}}\tilde{\theta}_{h1}^T\hat{\theta}_{h1} - r\overline{V}_1 + \lambda_1 + H_1\left[1 - 16\tanh^2\left(\frac{S_1}{\varepsilon_1}\right)\right], \tag{20}$$

where $\lambda_1 = \frac{1}{2}\varepsilon_1^{*2} + \frac{1}{2}\delta_1^{*2}$.

Step i $(2 \le i \le n-2)$: Consider the Lyapunov function candidate

$$V_i = \frac{1}{2}S_i^2 + \frac{1}{2\gamma_{fi}}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} + \frac{1}{2\gamma_{hi}}\tilde{\theta}_{hi}^T\tilde{\theta}_{hi} + \frac{1}{2}\chi_i^2 + \overline{V}_i, \tag{21}$$

where γ_{fi}, γ_{hi} are positive design constants, $\tilde{\theta}_{fi} = \theta_{fi} - \hat{\theta}_{fi}$, $\tilde{\theta}_{hi} = \theta_{hi} - \hat{\theta}_{hi}$, and $\overline{V}_i$ will be given later. The time derivative of V_i is

$$\begin{aligned}\dot{V}_i &= S_i\dot{S}_i - \frac{1}{\gamma_{fi}}\tilde{\theta}_{fi}^T\dot{\hat{\theta}}_{fi} - \frac{1}{\gamma_{hi}}\tilde{\theta}_{hi}^T\dot{\hat{\theta}}_{hi} + \chi_i\dot{\chi}_i + \dot{\overline{V}}_i \\ &= S_i\big[S_{i+1} + \chi_{i+1} + \alpha_i + \theta_{fi}^T\varphi_i(\underline{x}_i) + \theta_{hi}^T\varphi_i\big(\underline{x}_i(t-\tau_m)\big) + \varepsilon_i + \delta_i \\ &\quad + N_i\big(\underline{x}_i(t-\tau_i) - \underline{x}_i(t-\tau_m)\big) - \dot{x}_{id}\big] - \frac{1}{\gamma_{fi}}\tilde{\theta}_{fi}^T\dot{\hat{\theta}}_{fi} - \frac{1}{\gamma_{hi}}\tilde{\theta}_{hi}^T\dot{\hat{\theta}}_{hi} + \chi_i\dot{\chi}_i + \dot{\overline{V}}_i.\end{aligned} \tag{22}$$

By using the Young inequality and the properties of norms, it follows that

$$\begin{aligned}&S_i\varepsilon_i + S_i\delta_i \le S_i^2 + \frac{1}{2}\varepsilon_i^{*2} + \frac{1}{2}\delta_i^{*2}, \\ &S_iN_i\big(\underline{x}_i(t-\tau_i) - \underline{x}_i(t-\tau_m)\big) \\ &\quad \le \frac{1}{2}S_i^2 + \frac{1}{2}\|N_i\|^2\big\|\underline{x}_i(t-\tau_i) - \underline{x}_i(t-\tau_m)\big\|^2 \\ &\quad \le \frac{1}{2}S_i^2 + L_i^2\Sigma_{l=1}^i\big[x_l^2(t-\tau_l) + x_l^2(t-\tau_m)\big].\end{aligned} \tag{23}$$

Choose the nonnegative function

$$\overline{V}_i = e^{r_i(\tau_M - t)}L_i^2\sum_{l=1}^i\left[\int_{t-\tau_l}^t e^{r_l s}x_l^2(s)\,ds + \int_{t-\tau_m}^t e^{r_l s}x_l^2(s)\,ds\right]. \tag{24}$$

The time derivative of $\overline{V}_i$ is

$$\dot{\overline{V}}_i \le -r_i\overline{V}_i + 2e^{r_i\tau_M}L_i^2\sum_{l=1}^i x_l^2(t) - L_i^2\sum_{l=1}^i\big[x_l^2(t-\tau_l) + x_l^2(t-\tau_m)\big]. \tag{25}$$

Choose the intermediate control function α_i and the adaptive laws as follows:

$$\alpha_i = -c_iS_i - \hat{\theta}_{fi}^T\varphi_i(\underline{x}_i) - \hat{\theta}_{hi}^T\varphi_i\big(\underline{x}_i(t-\tau_m)\big) + \dot{x}_{id} - \frac{16H_i}{S_i}\tanh^2\left(\frac{S_i}{\varepsilon_i}\right) - S_{i-1}, \tag{26}$$

$$\dot{\hat{\theta}}_{fi} = \gamma_{fi}\varphi_i(\underline{x}_i)S_i - \sigma_{fi}\hat{\theta}_{fi}, \qquad \dot{\hat{\theta}}_{hi} = \gamma_{hi}\varphi_i\big(\underline{x}_i(t-\tau_m)\big)S_i - \sigma_{hi}\hat{\theta}_{hi}, \tag{27}$$

where $H_i = 2e^{r_i\tau_M}L_i^2\sum_{l=1}^{i}x_l^2(t)$. Substituting (23), (25)-(27) into (22), it follows that

$$\begin{aligned}\dot{V}_i &\le -\left[c_i - \frac{3}{2}\right]S_i^2 + S_iS_{i+1} + S_i\chi_{i+1} + \frac{\sigma_{fi}}{\gamma_{fi}}\tilde{\theta}_{fi}^T\hat{\theta}_{fi} + \frac{\sigma_{hi}}{\gamma_{hi}}\tilde{\theta}_{hi}^T\hat{\theta}_{hi} - r_i\overline{V}_i + \lambda_i \\ &\quad + H_i\left[1 - 16\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)\right] + \chi_i\dot{\chi}_i,\end{aligned} \tag{28}$$

where $\lambda_i = \frac{1}{2}\varepsilon_i^{*2} + \frac{1}{2}\delta_i^{*2}$.

Step $n-1$: Consider the Lyapunov function candidate

$$V_{n-1} = \frac{1}{2}S_{n-1}^2 + \frac{1}{2\gamma_{f_{n-1}}}\tilde{\theta}_{f_{n-1}}^T\tilde{\theta}_{f_{n-1}} + \frac{1}{2\gamma_{h_{n-1}}}\tilde{\theta}_{h_{n-1}}^T\tilde{\theta}_{h_{n-1}} + \frac{1}{2}\chi_{n-1}^2 + \overline{V}_{n-1}, \tag{29}$$

where $\gamma_{f_{n-1}}$, $\gamma_{h_{n-1}}$ are positive design constants, $\tilde{\theta}_{f_{n-1}} = \theta_{f_{n-1}} - \hat{\theta}_{f_{n-1}}$, $\tilde{\theta}_{h_{n-1}} = \theta_{h_{n-1}} - \hat{\theta}_{h_{n-1}}$, and $\overline{V}_{n-1}$ will be given later. The time derivative of V_{n-1} is

$$\begin{aligned}\dot{V}_{n-1} &= S_{n-1}\dot{S}_{n-1} - \frac{1}{\gamma_{f_{n-1}}}\tilde{\theta}_{f_{n-1}}^T\dot{\hat{\theta}}_{f_{n-1}} - \frac{1}{\gamma_{h_{n-1}}}\tilde{\theta}_{h_{n-1}}^T\dot{\hat{\theta}}_{h_{n-1}} + \chi_{n-1}\dot{\chi}_{n-1} + \dot{\overline{V}}_{n-1} \\ &= S_{n-1}\big[\phi_n + e_n + \theta_{f_{n-1}}^T\varphi_{n-1}(\underline{x}_{n-1}) + \theta_{h_{n-1}}^T\varphi_{n-1}\big(\underline{x}_{n-1}(t-\tau_m)\big) + \varepsilon_{n-1} \\ &\quad + \delta_{n-1} - \dot{x}_{n-1d} + N_{n-1}\big(\underline{x}_{n-1}(t-\tau_{n-1}) - \underline{x}_{n-1}(t-\tau_m)\big)\big] - \frac{1}{\gamma_{f_{n-1}}}\tilde{\theta}_{f_{n-1}}^T\dot{\hat{\theta}}_{f_{n-1}} \\ &\quad - \frac{1}{\gamma_{h_{n-1}}}\tilde{\theta}_{h_{n-1}}^T\dot{\hat{\theta}}_{h_{n-1}} + \chi_{n-1}\dot{\chi}_{n-1} + \dot{\overline{V}}_{n-1} \\ &= S_{n-1}\left[S_n + \chi_n + \alpha_{n-1} - \int_{t-\tau}^{t}u(s)\,ds + e_n + \theta_{f_{n-1}}^T\varphi_{n-1}(\underline{x}_{n-1})\right. \\ &\quad + \theta_{h_{n-1}}^T\varphi_{n-1}\big(\underline{x}_{n-1}(t-\tau_m)\big) + \varepsilon_{n-1} + \delta_{n-1} - \dot{x}_{n-1d} + N_{n-1}\big(\underline{x}_{n-1}(t-\tau_{n-1}) \\ &\quad \left. - \underline{x}_{n-1}(t-\tau_m)\big)\right] - \frac{1}{\gamma_{f_{n-1}}}\tilde{\theta}_{f_{n-1}}^T\dot{\hat{\theta}}_{f_{n-1}} - \frac{1}{\gamma_{h_{n-1}}}\tilde{\theta}_{h_{n-1}}^T\dot{\hat{\theta}}_{h_{n-1}} \\ &\quad + \chi_{n-1}\dot{\chi}_{n-1} + \dot{\overline{V}}_{n-1}.\end{aligned} \tag{30}$$

By using the Young inequality, the properties of the norms and Assumption 4, it follows that

$$\begin{aligned}&S_{n-1}\left[-\int_{t-\tau}^{t}u(s)\,ds + e_n + \varepsilon_{n-1} + \delta_{n-1}\right] \le 2S_{n-1}^2 + \frac{1}{2}u_0^2 + \frac{1}{2}e_n^{*2} + \frac{1}{2}\varepsilon_{n-1}^{*2} + \frac{1}{2}\delta_{n-1}^{*2}, \\ &S_{n-1}N_{n-1}\big(\underline{x}_{n-1}(t-\tau_{n-1}) - \underline{x}_{n-1}(t-\tau_m)\big) \\ &\quad \le \frac{1}{2}S_{n-1}^2 + \frac{1}{2}\|N_{n-1}\|^2\big\|\underline{x}_{n-1}(t-\tau_{n-1}) - \underline{x}_{n-1}(t-\tau_m)\big\|^2 \\ &\quad \le \frac{1}{2}S_{n-1}^2 + L_{n-1}^2\Sigma_{l=1}^{n-1}\big[x_l^2(t-\tau_l) + x_l^2(t-\tau_m)\big].\end{aligned} \tag{31}$$

Remark 3 For the design of control and the analysis of the performance, it is always indispensable that *a priori* knowledge is introduced; see [39, 45]. Reference [39] tried to introduce *a priori* knowledge for the design of saturated control for an uncertain nonlinear system with input delay. *A priori* known tracking accuracy is proposed for the design of fuzzy-approximation-based global adaptive control in [45]. In this paper, for the *a priori* knowledge of the bound on the control one is required to develop an adaptive control methodology.

Choosing the nonnegative function

$$\overline{V}_{n-1} = e^{r_{n-1}(\tau_M - t)} L_{n-1}^2 \sum_{l=1}^{n-1} \left[\int_{t-\tau_l}^{t} e^{r_l s} x_l^2(s)\,ds + \int_{t-\tau_m}^{t} e^{r_l s} x_l^2(s)\,ds \right]. \tag{32}$$

The time derivative of $\overline{V}_{n-1}$ is

$$\dot{\overline{V}}_{n-1} \le -r_{n-1}\overline{V}_{n-1} + 2e^{r_{n-1}\tau_M} L_{n-1}^2 \sum_{l=1}^{n-1} x_l^2(t) - L_{n-1}^2 \sum_{l=1}^{n-1} \left[x_l^2(t-\tau_l) + x_l^2(t-\tau_m) \right]. \tag{33}$$

Design the intermediate control function α_{n-1} as follows:

$$\begin{aligned} \alpha_{n-1} = {}& -c_{n-1}S_{n-1} - \hat{\theta}_{f_{n-1}}^T \varphi_{n-1}(x_{n-1}) - \hat{\theta}_{h_{n-1}}^T \varphi_{n-1}\big(x_{n-1}(t-\tau_{n-1})\big) + \dot{x}_{n-1d} \\ & - \frac{16H_{n-1}}{S_{n-1}} \tanh^2\left(\frac{S_{n-1}}{\varepsilon_{n-1}}\right) - S_{n-2} - \frac{1}{2}S_{n-1}. \end{aligned} \tag{34}$$

Choose the adaptive laws as follows:

$$\begin{aligned} \dot{\hat{\theta}}_{f_{n-1}} &= \gamma_{f_{n-1}} \varphi_{n-1}(x_{n-1}) S_{n-1} - \sigma_{f_{n-1}} \hat{\theta}_{f_{n-1}}, \\ \dot{\hat{\theta}}_{h_{n-1}} &= \gamma_{h_{n-1}} \varphi_{n-1}\big(x_{n-1}(t-\tau_{n-1})\big) S_{n-1} - \sigma_{h_{n-1}} \hat{\theta}_{h_{n-1}}. \end{aligned} \tag{35}$$

Substituting (31), (33)-(35) into (30), it follows that

$$\begin{aligned} \dot{V}_{n-1} \le {}& -\left[c_{n-1} - \frac{3}{2}\right] S_{n-1}^2 + S_{n-1}S_n + S_{n-1}\chi_n + \frac{\sigma_{f_{n-1}}}{\gamma_{f_{n-1}}} \tilde{\theta}_{f_{n-1}}^T \hat{\theta}_{f_{n-1}} \\ & + \frac{\sigma_{h_{n-1}}}{\gamma_{h_{n-1}}} \tilde{\theta}_{h_{n-1}}^T \hat{\theta}_{h_{n-1}} - r_{n-1}\overline{V}_{n-1} + \lambda_{n-1} \\ & + H_{n-1}\left[1 - 16\tanh^2\left(\frac{S_{n-1}}{\varepsilon_{n-1}}\right)\right] + \chi_{n-1}\dot{\chi}_{n-1}, \end{aligned} \tag{36}$$

where $H_{n-1} = 2e^{r_{n-1}\tau_M} L_{n-1}^2 \sum_{l=1}^{n-1} x_l^2(t)$, $\lambda_{n-1} = \frac{1}{2}u_0^2 + \frac{1}{2}e_n^{*2} + \frac{1}{2}\varepsilon_{n-1}^{*2} + \frac{1}{2}\delta_{n-1}^{*2}$.

Step n: Consider the Lyapunov functional candidate

$$V_n = \frac{1}{2}S_n^2 + \frac{1}{2\gamma_{f_n}} \tilde{\theta}_{f_n}^T \tilde{\theta}_{f_n} + \frac{1}{2\gamma_{h_n}} \tilde{\theta}_{h_n}^T \tilde{\theta}_{h_n} + \frac{1}{2}\chi_n^2, \tag{37}$$

where γ_{fn}, γ_{hn} are positive design constants, $\tilde{\theta}_{fn} = \theta_{fn} - \hat{\theta}_{fn}$, $\tilde{\theta}_{hn} = \theta_{hn} - \hat{\theta}_{hn}$. The time derivative of V_n is

$$\begin{aligned} \dot{V}_n &= S_n\dot{S}_n - \frac{1}{\gamma_{f_n}} \tilde{\theta}_{f_n}^T \dot{\hat{\theta}}_{f_n} - \frac{1}{\gamma_{h_n}} \tilde{\theta}_{h_n}^T \dot{\hat{\theta}}_{h_n} + \chi_n\dot{\chi}_n \\ &= S_n\big[\dot{\phi}_n - \dot{x}_{nd} + u(t) - u(t-\tau)\big] - \frac{1}{\gamma_{f_n}} \tilde{\theta}_{f_n}^T \dot{\hat{\theta}}_{f_n} - \frac{1}{\gamma_{h_n}} \tilde{\theta}_{h_n}^T \dot{\hat{\theta}}_{h_n} + \chi_n\dot{\chi}_n \end{aligned}$$

$$
\begin{aligned}
&= S_n\big[-k_n\phi_1 + k_n x_1 + u(t-\tau) + \hat{\theta}_{f_n}^T\varphi_n(\underline{x}_n) + \hat{\theta}_{h_n}^T\varphi_n\big(\underline{x}_n(t-\tau_m)\big) - \dot{x}_{nd} + u(t) \\
&\quad - u(t-\tau)\big] + S_n\tilde{\theta}_{f_n}^T\varphi_n(\underline{x}_n) + S_n\tilde{\theta}_{h_n}^T\varphi_n\big(\underline{x}_n(t-\tau_m)\big) - S_n\tilde{\theta}_{f_n}^T\varphi_n(\underline{x}_n) \\
&\quad - S_n\tilde{\theta}_{h_n}^T\varphi_n\big(\underline{x}_n(t-\tau_m)\big) - \frac{1}{\gamma_{f_n}}\tilde{\theta}_{f_n}^T\dot{\hat{\theta}}_{f_n} - \frac{1}{\gamma_{h_n}}\tilde{\theta}_{h_n}^T\dot{\hat{\theta}}_{h_n} + \chi_n\dot{\chi}_n.
\end{aligned} \tag{38}
$$

Take the actual control u as follows:

$$
u(t) = -c_n S_n + k_n\phi_1 - k_n x_1 - \hat{\theta}_{fn}^T\varphi_n(\underline{x}_n) - \hat{\theta}_{hn}^T\varphi_n\big(\underline{x}_n(t-\tau_m)\big) + \dot{x}_{nd} - S_{n-1}. \tag{39}
$$

The adaptive laws are designed as

$$
\dot{\hat{\theta}}_{fn} = \gamma_{fn}\varphi_n(\underline{x}_n)S_n - \sigma_{fn}\hat{\theta}_{fn}, \qquad \dot{\hat{\theta}}_{hn} = \gamma_{hn}\varphi_n\big(\underline{x}_n(t-\tau_m)\big)S_n - \sigma_{hn}\hat{\theta}_{hn}. \tag{40}
$$

By using the Young inequality, it follows that

$$
-S_n\tilde{\theta}_{f_n}^T\varphi_n(\underline{x}_n) - S_n\tilde{\theta}_{h_n}^T\varphi_n\big(\underline{x}_n(t-\tau_m)\big) \le S_n^2 + \frac{1}{2}\tilde{\theta}_{f_n}^T\tilde{\theta}_{f_n} + \frac{1}{2}\tilde{\theta}_{h_n}^T\tilde{\theta}_{h_n}. \tag{41}
$$

By substituting (39)-(41) into (38), one can obtain

$$
\dot{V}_n \le -[c_n - 1]S_n^2 + \frac{\sigma_{f_n}}{\gamma_{f_n}}\tilde{\theta}_{f_n}^T\hat{\theta}_{f_n} + \frac{\sigma_{h_n}}{\gamma_{h_n}}\tilde{\theta}_{h_n}^T\hat{\theta}_{h_n} + \frac{1}{2}\tilde{\theta}_{f_n}^T\tilde{\theta}_{f_n} + \frac{1}{2}\tilde{\theta}_{h_n}^T\tilde{\theta}_{h_n} + \chi_n\dot{\chi}_n. \tag{42}
$$

3.2 Stability analysis

Take the Lyapunov function for a stability analysis in the form of $V = \sum_{i=1}^n V_i$ and its time derivative as

$$
\begin{aligned}
\dot{V} &= \dot{V}_1 + \sum_{i=2}^{n-2}\dot{V}_i + \dot{V}_{n-1} + \dot{V}_n \\
&\le -\sum_{i=1}^{n}\left[c_i - \frac{3}{2}\right]S_i^2 + \sum_{i=1}^{n-1}S_i\chi_{i+1} + \frac{1}{2}\tilde{\theta}_{fn}^T\tilde{\theta}_{fn} + \frac{1}{2}\tilde{\theta}_{hn}^T\tilde{\theta}_{hn} + \sum_{i=1}^{n}\left[\frac{\sigma_{fi}}{\gamma_{fi}}\tilde{\theta}_{fi}^T\hat{\theta}_{fi} + \frac{\sigma_{hi}}{\gamma_{hi}}\tilde{\theta}_{hi}^T\hat{\theta}_{hi}\right] \\
&\quad + \sum_{i=2}^{n}\chi_i\dot{\chi}_i - \sum_{i=1}^{n-1}[r_i\overline{V}_i - \lambda_i] + \sum_{i=1}^{n-1}H_i\left[1 - 16\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)\right].
\end{aligned} \tag{43}
$$

By the definitions of χ_i ($i = 2,\dots,n$) and the first-order filters [46], it yields

$$
\dot{\chi}_i = -\frac{\chi_i}{\rho_i} - \dot{\alpha}_{i-1}. \tag{44}
$$

From the definition and the boundedness of α_i, one can obtain $|\dot{\alpha}_i| \le \Phi_i(S_1,\dots,S_i,\tilde{\theta}_{f1},\dots,\tilde{\theta}_{fi},y_r,\dot{y}_r,\ddot{y}_r,\tilde{\theta}_{h1},\dots,\tilde{\theta}_{hi},\chi_2,\dots,\chi_i)$, where Φ_i ($i = 2,\dots,n-1$) are nonnegative continuous functions.

Given any ψ, the set $C = \{(S_1,\dots,S_i,\tilde{\theta}_{f1},\dots,\tilde{\theta}_{fi},y_r,\dot{y}_r,\ddot{y}_r,\tilde{\theta}_{h1},\dots,\tilde{\theta}_{hi},\chi_2,\dots,\chi_i)^T : \mathcal{V} < \psi\}$ is a compact set. Based on the compact set C and the compact set Y in Assumption 2, the continuous function Φ_i has a maximum B_i on $C \times Y$. It is easy to see that

$$
\dot{\chi}_i \le -\frac{\chi_i}{\rho_i} + B_{i-1}. \tag{45}
$$

Based on Young's inequality, it follows that

$$\begin{aligned} &S_i\chi_{i+1} \le \frac{1}{2}S_i^2 + \frac{1}{2}\chi_{i+1}^2, \qquad \chi_i B_{i-1} \le \frac{\chi_i^2 B_{i-1}^2}{2\pi_i} + \frac{\pi_i}{2}, \\ &\tilde{\theta}_{fi}^T\hat{\theta}_{fi} = \tilde{\theta}_{fi}^T\theta_{fi} - \tilde{\theta}_{fi}^T\tilde{\theta}_{fi} \le -\frac{1}{2}\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} + \frac{1}{2}\theta_{fi}^T\theta_{fi}, \qquad \tilde{\theta}_{hi}^T\hat{\theta}_{hi} \le -\frac{1}{2}\tilde{\theta}_{hi}^T\tilde{\theta}_{hi} + \frac{1}{2}\theta_{hi}^T\theta_{hi}. \end{aligned} \tag{46}$$

By substituting (45), (46) into (43), one can obtain

$$\begin{aligned} \dot{V} \le{}& -\sum_{i=1}^{n}[c_i - 2]S_i^2 - \sum_{i=1}^{n}\left[\frac{\sigma_{fi}}{2\gamma_{fi}} - \frac{1}{2}\right]\tilde{\theta}_{fi}^T\tilde{\theta}_{fi} - \sum_{i=1}^{n}\left[\frac{\sigma_{hi}}{2\gamma_{hi}} - \frac{1}{2}\right]\tilde{\theta}_{hi}^T\tilde{\theta}_{hi} \\ &- \sum_{i=2}^{n}\left[\frac{1}{\rho_i} - \frac{B_{i-1}^2}{2\pi_i} - \frac{1}{2}\right]\chi_i^2 - \sum_{i=1}^{n-1} r_i\overline{V}_i + \lambda + \sum_{i=1}^{n-1} H_i\left[1 - 16\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)\right], \end{aligned} \tag{47}$$

where $\lambda = \sum_{i=1}^{n-1}[\lambda_i + \frac{\pi_i}{2}] + \sum_{i=1}^{n}[\frac{\sigma_{fi}}{2\gamma_{fi}}\theta_{fi}^T\theta_{fi} + \frac{\sigma_{hi}}{2\gamma_{hi}}\theta_{hi}^T\theta_{hi}]$.

From (47), one can obtain

$$\dot{V} \le -cV + \lambda + \sum_{i=1}^{n-1} H_i\left[1 - 16\tanh^2\left(\frac{S_i}{\varepsilon_i}\right)\right], \tag{48}$$

where $c = \min\{2[c_i - 2], 2\gamma_{fi}[\frac{\sigma_{fi}}{2\gamma_{fi}} - \frac{1}{2}], 2\gamma_{hi}[\frac{\sigma_{hi}}{2\gamma_{hi}} - \frac{1}{2}], 2[\frac{1}{\rho_i} - \frac{B_{i-1}^2}{2\pi_i} - \frac{1}{2}], r_i\}$.

For the proof of Theorem 1, the following lemma is introduced.

Lemma 2 *For $1 \le j \le n-1$, consider the set G_ε by $G_\varepsilon = \{S_j||S_j| < 0.2554\varepsilon_j\}$. Then, for $S_j \notin G_{\varepsilon_j}$, the inequality $1 - 16\tanh^2(S_j/\varepsilon_j) < 0$ is satisfied.*

Proof By following the same line as in the stability analysis of [37], it can be shown that all the signals in the closed-loop systems are bounded, and the steady-state error can be made arbitrarily small. □

From the above design procedures and analysis, the following theorem is deduced.

Theorem 1 *Consider the system* (1) *consisting of the adaptive laws* (19), (27), (35), (40), *the virtual control* (18), (26), (34), *and the actual controller* (39). *Under Assumptions* 1-4, *by using the above design procedures, for the bounded initial conditions, the boundedness of all the signals in the closed-loop system can be guaranteed by the proposed control scheme and the tracking error can converge to a small neighborhood of the origin.*

4 Simulation example

Example 1 (Numerical examples) Consider the following uncertain nonlinear strict-feedback systems with state and input delays:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + h_1(x_1(t - \tau_1)), \\ \dot{x}_2 = u(t - \tau_0(t)) + f_2(\underline{x}_2) + h_2(\underline{x}_2(t - \tau_2)), \end{cases}$$

where x_1, x_2 are the state variables, and $y = x_1$ is the system's output. The time delays are selected as $\tau_1 = 3$ s, $\tau_2 = 5$ s and $\tau_0(t) = 0.16 + 0.1\sin(t)$. In order to illustrate the effectiveness of the method in the presence of different uncertainties, two cases of the system will be considered.

Case 1: $f_1(x_1) = 0.1x_1e^{-0.5x_1^2}\cos(x_1)0.1x_1e^{-0.5x_1^2}\cos(x_1)$, $f_2(x_2) = x_2\sin\frac{2}{1+x_1^2}$, $h_1(x_1(t-\tau_1)) = \frac{x_1^3(t-\tau_1)}{1+x_1^2(t-\tau_1)}$, $h_2(x_2(t-\tau_2)) = \frac{x_2^4(t-\tau_2)\sin(x_2)}{1+x_2^2(t-\tau_2)}$.

Case 2: $f_1(x_1) = 0.2x_1$, $f_2(x_2) = 0.2\sin(x_1) + 0.2\cos(t)$, $h_1(x_1(t-\tau_1)) = 0.5x_1(t-\tau_1)$, $h_2(x_2(t-\tau_2)) = 0.8x_1^2(t-\tau_2)$.

The given reference tracking signal is $y_r = \sin(t)$. Choose the fuzzy membership functions as

$$\mu_{F_i^l}(x_i) = \exp\left[-\frac{(x_i-3+l)^2}{4}\right], \quad l = 1,2,3,4,5.$$

Define the fuzzy basis functions as

$$\varphi_{1,l}(x_1) = \frac{\mu_{F_1^l}}{\Sigma_{k=1}^5 \mu_{F_1^k}}, \qquad \varphi_{2,l}(x_1,x_2) = \frac{\mu_{F_1^l}\mu_{F_2^l}}{\Sigma_{k=1}^5 \mu_{F_1^l}\mu_{F_2^k}}, \quad l = 1,2,3,4,5.$$

We can construct intermediate control function α_1 as follows:

$$\alpha_1 = -c_1S_1 - \hat{\theta}_{f1}^T\varphi_1(x_1) - \hat{\theta}_{h1}^T\varphi_1\bigl(x_1(t-\tau_m)\bigr) + \dot{y}_r - \frac{16H_1}{S_1}\tanh^2\left(\frac{S_1}{\varepsilon_1}\right),$$

the actual control u as follows:

$$u(t) = -c_2S_2 + k_2\phi_1 - k_2x_1 - \hat{\theta}_{f2}^T\varphi_2(\underline{x}_2) - \hat{\theta}_{h2}^T\varphi_2\bigl(\underline{x}_2(t-\tau_m)\bigr) + \dot{x}_{2d} - S_1,$$

and the adaptive laws as follows:

$$\dot{\hat{\theta}}_{f1} = \gamma_{f1}\varphi_1(x_1)S_1 - \sigma_{f1}\hat{\theta}_{f1}, \qquad \dot{\hat{\theta}}_{h1} = \gamma_{h1}\varphi_1\bigl(x_1(t-\tau_m)\bigr)S_1 - \sigma_{h1}\hat{\theta}_{h1},$$
$$\dot{\hat{\theta}}_{f2} = \gamma_{f2}\varphi_2(\underline{x}_2)S_2 - \sigma_{f2}\hat{\theta}_{f2}, \qquad \dot{\hat{\theta}}_{h2} = \gamma_{h2}\varphi_2\bigl(\underline{x}_2(t-\tau_m)\bigr)S_2 - \sigma_{h2}\hat{\theta}_{h2}.$$

The initial conditions of states are chosen as $x_1(0) = 0.2$ and $x_2(0) = 0.2$, and the others initial values are chosen as zeros. Choose $\tau_m = 3$ s, $\tau_M = 5$ s.

The simulation results are shown in Figures 1-7.

Figure 1 shows the response trajectories of the control output y (state variable x_1) and the desired reference tracking signal y_r. As can be seen, the control output y can track the reference signal y_r satisfactorily in the presence of different uncertainties.

To illustrate the merits of the proposed method, some simulation results are given for the adaptive memoryless control scheme in [37]. The control input signals u with our control and the method in [37] are shown in Figure 2 and the energy of the control inputs u are shown in Figure 3. Figures 4 and 5 show the trajectories of the tracking error S_1 and the trajectory of the energy of the tracking error S_1, respectively. It is obvious that the tracking error converges to a small neighborhood of the origin, and our proposed memory control methodology does achieve a better performance and requires less control energy than the method in [37].

To further illustrate the effective of the proposed method, some simulation results are given without considering the compensation for the time-varying input delay. Figures 6

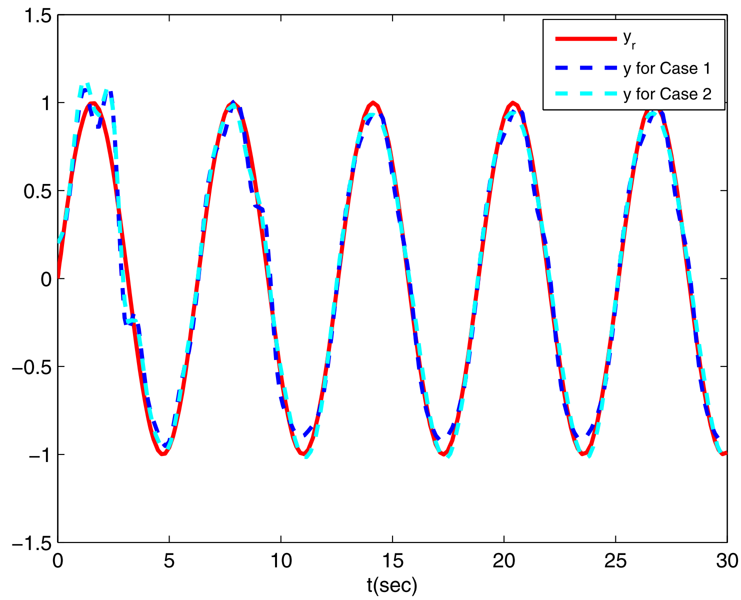

Figure 1 **The trajectories of *y* and y_r.**

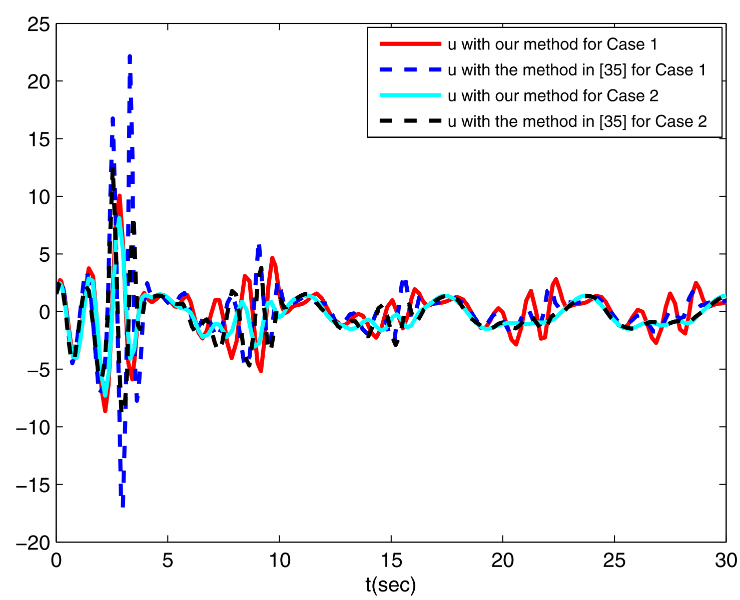

Figure 2 **The trajectory of control *u*.**

and 7 show the trajectory of the tracking error S_1 and control u without compensating the input delay, respectively. Obviously, the input delay can make the systems unstable.

Example 2 (Practical example) Consider the following practical example of a two-stage version of the reactors which are isothermal continuous stirred tanks reactors (CSTR) with delayed recycle streams [36, 38]. Assume that the state delays are bounded and the

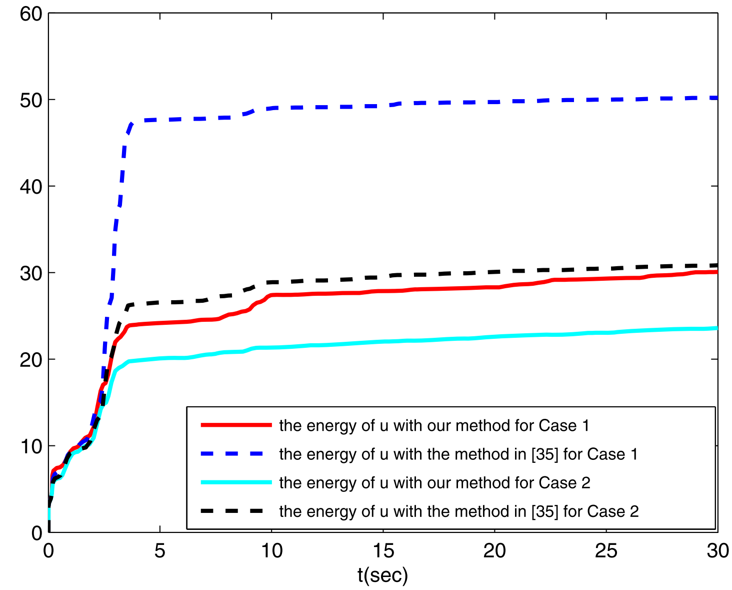

Figure 3 The trajectories of the energy of *u*.

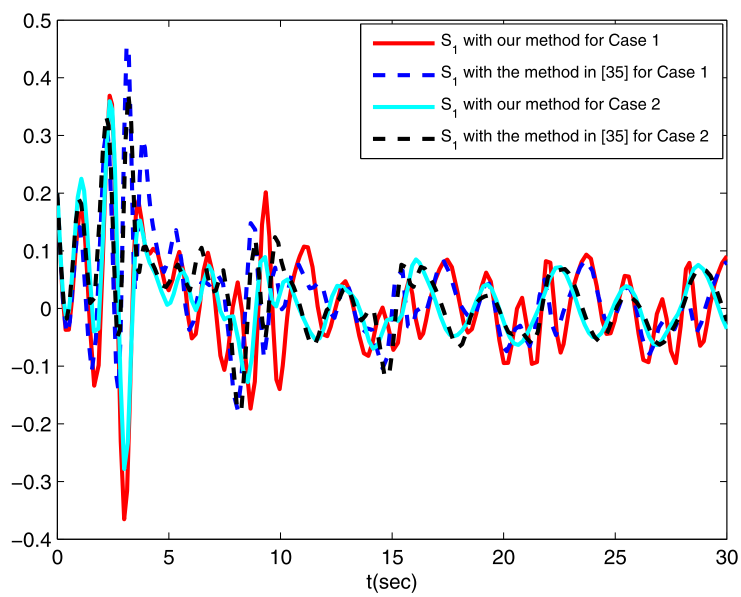

Figure 4 The trajectory of S_1.

input delay is time-varying, and the output is $y = x_1$. The system balance equation is

$$\begin{aligned}
\dot{x}_1(t) &= -\frac{1}{\Theta_1}x_1(t) - \eta_1 x_1(t) + \frac{1-R_2}{V_1}x_2(t) + h_1\bigl(x_1(t-\tau_1)\bigr),\\
\dot{x}_2(t) &= -\frac{1}{\Theta_2}x_2(t) - \eta_1 x_2(t) + \frac{R_1}{V_2}x_1(t-\tau_2) + \frac{F_2}{V_2}u\bigl(t-\tau_0(t)\bigr) + h_2\bigl(x_1(t-\tau_2)\bigr),
\end{aligned}$$

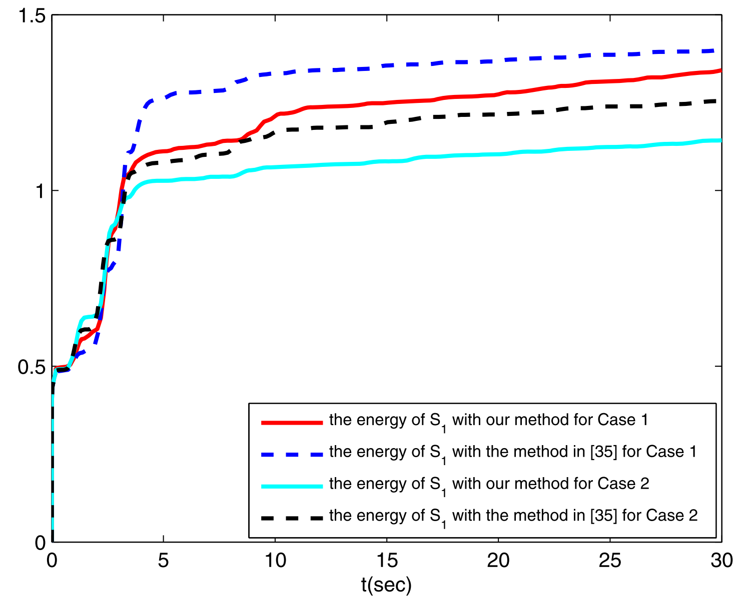

Figure 5 The trajectories of the energy of S_1.

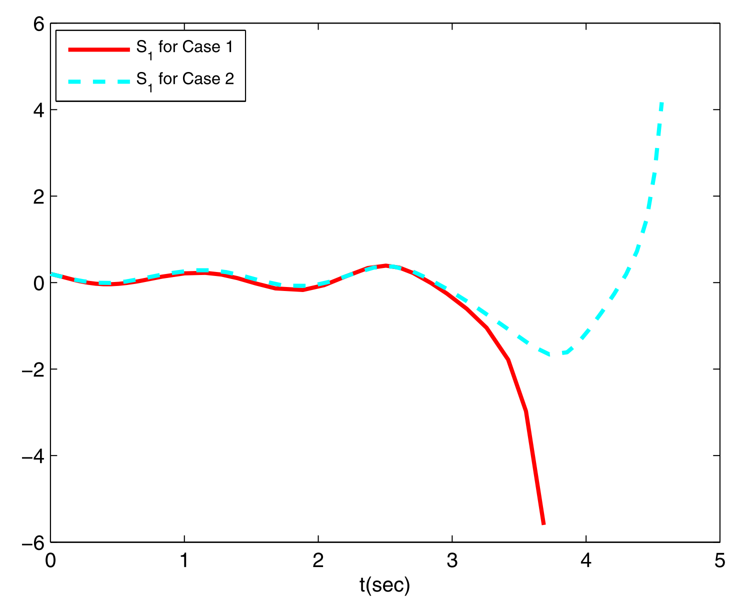

Figure 6 The trajectory of S_1 without compensating input delay.

where $x_1(t)$ and $x_2(t)$ are the compositions, Θ_i are the reactor residence times, η_i are the reaction constants, F_2 is the feed rate, R_i are the recycle flow rates and V_i are reactor volumes, the functions $h_i(t)$ are uncertain nonlinear functions with time delay. The parameters are taken from [36]: $\Theta_i = 2$, $\eta_i = 0.3$, $R_i = 0.5$, $V_i = 0.5$, $F_2 = 0.5$, the nonlinear time-delay functions are selected as $h_1 = 0.5 \sin tx_1^2(t - \tau_1)$, $h_2 = 0.5 \sin tx_1^3(t - \tau_2)$ with τ_1, τ_2 being unknown time delays. The input delay is considered as $\tau_0(t) = 0.16 + 0.1 \sin(t)$. The

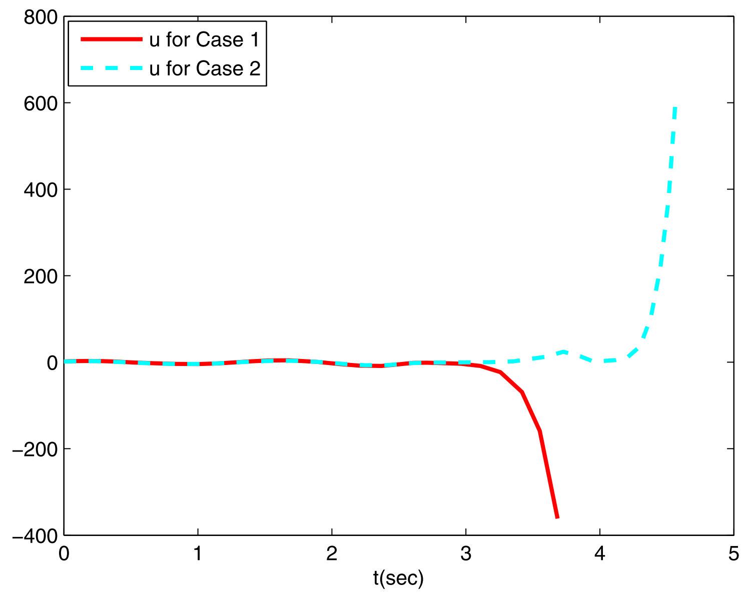

Figure 7 The trajectories of *u* without compensating input delay.

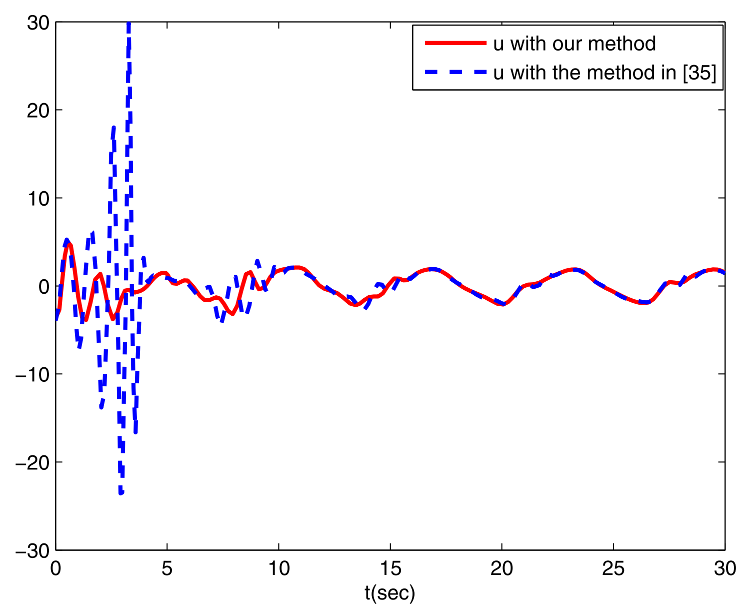

Figure 8 The trajectory of the control *u*.

given reference tracking signal is $y_r = \sin(t)$. We set the control parameters $k_1 = 1$, $k_2 = 1$, $\rho_2 = 0.2$, $c_1 = 2.1$, $c_2 = 5$, $\gamma_{f1} = \gamma_{f2} = \gamma_{h1} = \gamma_{h2} = 4$, $\sigma_{f1} = \sigma_{f2} = \sigma_{h1} = \sigma_{h2} = 2$, $\varepsilon_1 = 5$, $r_1 = 0.1$, $L_1 = 0.035$, $\tau = 0.16$ s.

We show the practical simulation results in Figures 8-11, from which it can be observed that the effectiveness of the proposed approach is confirmed in this paper.

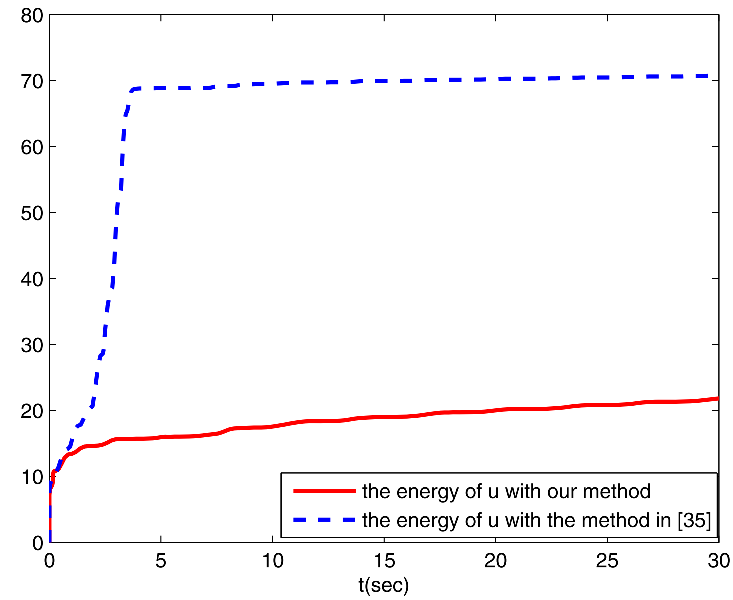

Figure 9 The trajectories of the energy of *u*.

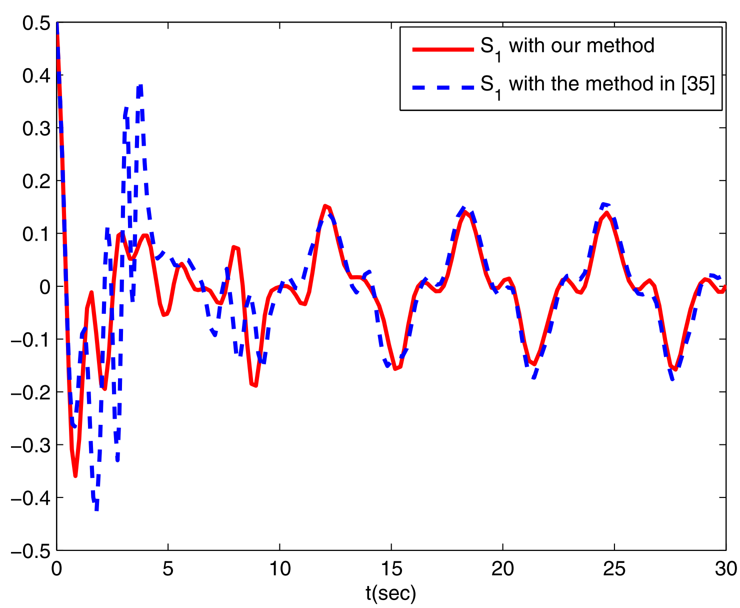

Figure 10 The trajectory of S_1.

5 Conclusion

In this paper, a delay-lower-bound-based adaptive fuzzy memory control scheme has been proposed for a class of uncertain nonlinear systems with unknown time delays in state and input. By utilizing the mean value theorem, the unknown time-delay functions with all state variable have been dealt with. A novel adaptive filter has been designed to eliminate the effect of time-varying input delay. Based on a backstepping technique, a delay-lower-bound-dependent adaptive fuzzy memory controller has been de-

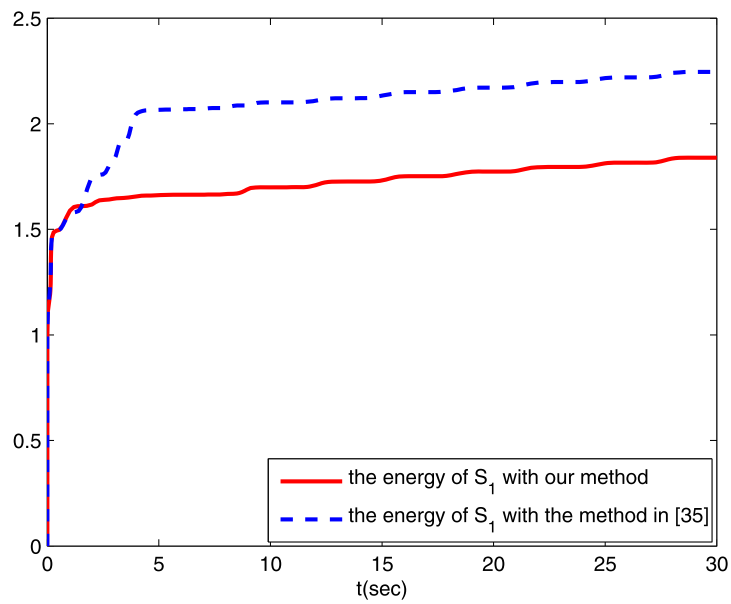

Figure 11 The trajectories of the energy of S_1.

signed. It is analyzed that the proposed control scheme guarantees that all the signals in the closed-loop system are bounded and the tracking error converges to a small neighborhood of the origin. A practical simulation example also demonstrates the theoretical results. Further investigation should include more practical and general systems such as large-scale nonlinear systems and pure-feedback systems, and discrete-time systems.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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