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Dynamics of a predator-prey model with impulsive biological control and unilaterally impulsive diffusion

Jianjun Jiao¹, Shaohong Cai¹, Limei Li² and Yujuan Zhang^{3*}

*Correspondence:

zhangyujuan1968@163.com

³College of Mathematics and Information Science, Anshan Normal University, Anshan, 114007, P.R. China

Full list of author information is available at the end of the article

Abstract

In this paper, we establish a predator-prey model with impulsive biological control and unilaterally impulsive diffusion. This predator-prey model for two regions, which are connected by diffusion of predator population, portrays the evolution of the population. We study the model for biological pest control in which a pest population is controlled by a program of periodic releases of a fixed yield of predators that prey on the pest population. We prove that there exists a globally asymptotically stable prey-extinction boundary periodic solution. The condition for permanence is also obtained. Simulations are also employed to verify our results. We conclude that the impulsive diffusion and releasing predator provide reliable tactic basis for pest management.

Keywords: predator-prey model; unilaterally impulsive diffusion; impulsive biological control; extinction; permanence

1 Introduction

The warfare between humans and pests has persisted for thousands of years. In the past few decades, man has adopted some advanced and modern weapons for instance chemical pesticides, biological pesticides, remote sensing and measure, computers, atomic energy *etc.* Some brilliant achievements have been obtained. However, the warfare will never be over. Although a great deal of pesticides were used to control pests, the insect pests impairing crops are increasing because of the resistance to the pesticide. With pesticides employed, the residual pests breed a large number of pests with resistance to pesticides. So the pesticide is invalid in a sense. Moreover, insect pests will remain. On the other hand, the chemical pesticides kill not only pests but also their natural enemies. Therefore, insect pests are rampant again. Then the effect of chemical control was challenged. Furthermore, the practice proves that long-term adopting chemical control may give rise to disastrous results, for example, we witness environmental contamination and toxicosis of man and animal, and so on.

The use of a natural enemy to suppress pests is one of the most important approaches in pest control. Biological control [1–8] is one of the reductions in pest populations from the actions of other living organisms, often called natural enemies or beneficial species. It is the purposeful introduction and establishment of one or more natural enemies from the

region of origin of an exotic pest, specifically for the purpose of suppressing the abundance of the pest in a new target region to a level at which it no longer causes economic damage. Jiao *et al.* [9] analyzed the dynamics of a stage-structured Holling mass defence predator-prey model with impulsive perturbations on predators,

$$\left\{ \begin{array}{l} \frac{dx_1(t)}{dt} = rx_2(t) - re^{-w\tau_1}x_2(t - \tau_1) - wx_1(t), \\ \frac{dx_2(t)}{dt} = re^{-w\tau_1}x_2(t - \tau_1) - \frac{\beta x_2(t)}{1+ax_2+bx_2^2}x_3(t) - d_3x_2(t) - d_4x_2^2(t), \\ \frac{dx_3(t)}{dt} = \frac{k\beta x_2(t)}{1+ax_2+bx_2^2}x_3(t) - dx_3(t), \end{array} \right\} \quad t \neq n\tau, \quad (1.1)$$

$$\left\{ \begin{array}{l} \Delta x_1(t) = 0, \\ \Delta x_2(t) = 0, \\ \Delta x_3(t) = \mu, \end{array} \right\} \quad t = n\tau, n = 1, 2, \dots,$$

$$(\varphi_1(\zeta), \varphi_2(\zeta), \varphi_3(\zeta)) \in C_+ = C([- \tau_1, 0], R_+^3), \quad \varphi_i(0) > 0, i = 1, 2, 3,$$

where $x_1(t)$, $x_2(t)$ represent the immature and mature pest densities, respectively. $x_3(t)$ denotes the density of nature enemy. The biological meanings of the parameters can be found in [9].

The dispersal is a ubiquitous phenomenon in the natural world. It is important for us to understand the ecological and evolutionary dynamics of populations mirrored by the large number of mathematical models devoted to it in the scientific literature [10–13]. In recent years, the analysis of these models focused on the coexistence of populations and the local (or global) stability of the equilibria [14–20]. Spatial factors play a fundamental role on the persistence and stability of the population, although the complete results have not yet been obtained even in the simplest one-species case. If the population dynamics with the effects of spatial heterogeneity is modeled by a diffusion process, most previous papers focused on the population dynamical system modeled by the ordinary differential equations. But in practice, it is often the case that diffusion occurs in a regular pulse. For example, when winter comes, birds will migrate between patches in search for a better environment, whereas they do not diffuse in other seasons, and the excursion of foliage seeds occurs at a fixed period of time every year. Thus impulsive diffusion provides a more natural description. Lately theories of impulsive differential equations [21] have been introduced into population dynamics. Jiao *et al.* [22] proposed and investigated the dynamical behaviors of a stage-structured predator-prey model with prey impulsively diffusing between two patches

$$\left\{ \begin{array}{l} \frac{dx_1(t)}{dt} = x_1(t)(a_1 - b_1x_1(t)), \\ \frac{dx_2(t)}{dt} = x_2(t)(a_2 - b_2x_2(t)) - \alpha x_2(t)y_2(t), \\ \frac{dy_1(t)}{dt} = k\alpha x_2(t)y_2(t) - k\alpha e^{-w\tau_1}x_2(t - \tau_1)y_2(t - \tau_1) - wy_1(t), \\ \frac{dy_2(t)}{dt} = k\alpha e^{-w\tau_1}x_2(t - \tau_1)y_2(t - \tau_1) - dy_2(t), \end{array} \right\} \quad t \neq n\tau, \quad (1.2)$$

$$\left\{ \begin{array}{l} \Delta x_1(t) = d_1(x_2(t) - x_1(t)), \\ \Delta x_2(t) = d_2(x_1(t) - x_2(t)), \\ \Delta y_1(t) = 0, \\ \Delta y_2(t) = 0, \end{array} \right\} \quad t = n\tau, n = 1, 2, \dots,$$

where we suppose that the system is composed of two patches connected by diffusion and occupied by a single species x_i ($i = 1, 2$) is the density of species in the i th patch. $y_1(t)$,

$y_2(t)$ represent the densities of the immature individual predator and mature individual predator at time t in the second patch. The biological meanings of the parameters can be found in [22].

Theories of impulsive differential equations have been introduced into population dynamics lately [23–28]. Impulsive equations are found in almost every domain of applied science and have been studied in many investigations [28–33], they generally describe phenomena which are subject to steep or instantaneous changes. The theories of population dynamical systems and their applications have achieved many good results. However, the oasis vegetation degradation combined with a dynamical system has been considered very little. In this paper, we will investigate an impulsive dispersal on SIR model on restricting infected individuals boarding transports. We expect to obtain some dynamical properties of the investigated system. We also expect that impulsive dispersal will provide a reliable tactic for controlling epidemic.

The organization of this paper is as follows. In the next section, we introduce the model and background concepts. In Section 3, some important lemmas are presented. We give the globally asymptotically stable conditions of the prey-extinction boundary periodic solution of System (2.1), and the permanent condition of System (2.1). In Section 4 is a simulation analysis, and a brief discussion are given in the last section to conclude this work.

2 The model

In this paper, we establish a predator-prey model with impulsive biological control and unilaterally impulsive diffusion.

$$\left. \begin{aligned} \frac{dx_1(t)}{dt} &= x_1(t)(a_1 - b_1x_1(t)) - \beta_1x_1(t)y_1(t), \\ \frac{dy_1(t)}{dt} &= k_1\beta_1x_1(t)y_1(t) - d_1y_1(t), \\ \frac{dy_2(t)}{dt} &= y_2(t)(a_2 - b_2y_2(t)), \\ \Delta x_1(t) &= 0, \\ \Delta y_1(t) &= Dy_2(t), \\ \Delta y_2(t) &= -Dy_2(t), \end{aligned} \right\} \quad \begin{aligned} &t \neq (n+l)\tau, t \neq (n+1)\tau, \\ &t = (n+l)\tau, n \in \mathbb{Z}^+, \\ &t = (n+1)\tau, n \in \mathbb{Z}^+, \end{aligned} \quad (2.1)$$

where we suppose that the system is composed of two patches connected by diffusion. These two patches are separated by rivers or highways or railways. The predator population can transverse the rivers or highways or railways, while the prey population cannot. $x_1(t)$ and $y_1(t)$ represent the numbers of preys and predators in the populations in patch 1 at time t . $y_2(t)$ represents the number of predators in the population in patch 2 at time t . $a_1 > 0$ represents the intrinsic growth rate of the prey population in patch 1. $b_1 > 0$ represents the coefficient of the intraspecific competition of the prey population in patch 1. $a_2 > 0$ represents the intrinsic growth rate of the predator population in patch 2. $b_2 > 0$ represents the coefficient of the intraspecific competition of the predator population in patch 2. The predator consumes the prey according to $\beta_1x_1(t)$, in patch 1 at time t . k_1 is the rate of conversion of nutrients into the reproduction of the predator in patch 1. d_1 represents the death rate of the predator population in patch 1. The pulse diffusion occurs every τ period (τ is a positive constant), the system evolves from

its initial state without being further affected by diffusion until the next pulse appears; $\Delta y_i((n+l)\tau) = y_i((n+l)\tau^+) - y_i((n+l)\tau)$ where $y_i((n+l)\tau^+)$ represents the density of population in the i th patch immediately after the n th diffusion pulse at time $t = (n+l)\tau$, while $y_i((n+l)\tau)$ represents the density of population in the i th patch before the n th diffusion pulse at time $t = (n+l)\tau$, $0 < l < 1$, $n \in \mathbb{Z}_+$. $0 < D < 1$ represents the diffusive rate between two patches. $\Delta y_1((n+1)\tau) = y_1((n+1)\tau^+) - y_1((n+1)\tau)$ and μ represent the releasing amount of predator population at $t = (n+1)\tau$, $n \in \mathbb{Z}_+$, in patch 1.

3 The lemmas

The solution of (2.1), denoted by $X(t) = (x_1(t), y_1(t), y_2(t))^T$, is a piecewise continuous function $X: \mathbb{R}_+ \rightarrow \mathbb{R}_+^3$, $X(t)$ is continuous on $(n\tau, (n+l)\tau]$ and $((n+l)\tau, (n+1)\tau]$, $n \in \mathbb{Z}_+$ and $X(n\tau^+) = \lim_{t \rightarrow n\tau^+} X(t)$, $X((n+l)\tau^+) = \lim_{t \rightarrow (n+l)\tau^+} X(t)$ exist. Obviously the global existence and uniqueness of solutions of (2.1) is guaranteed by the smoothness properties of f , which denotes the mapping defined by right side of System (2.1) [21].

Let $V: \mathbb{R}_+ \times \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$, then V is said to belong to class V_0 , if:

- (i) V is continuous in $(n\tau, (n+l)\tau] \times \mathbb{R}_+^3$ and $((n+l)\tau, (n+1)\tau] \times \mathbb{R}_+^3$, for each $z \in \mathbb{R}_+^3$, $n \in \mathbb{Z}_+$, $V(n\tau^+, z) = \lim_{(t,y) \rightarrow (n\tau^+, z)} V(t, y)$, $V((n+l)\tau^+, z) = \lim_{(t,y) \rightarrow ((n+l)\tau^+, y)} V(t, y)$ exist.
- (ii) V is locally Lipschitzian in z .

Definition 3.1 $V \in V_0$, then, for $(t, z) \in (n\tau, (n+l)\tau] \times \mathbb{R}_+^3$ and $((n+l)\tau, (n+1)\tau] \times \mathbb{R}_+^3$, the upper right derivative of $V(t, z)$ with respect to the impulsive differential System (2.1) is defined as

$$D^+ V(t, z) = \limsup_{h \rightarrow 0} \frac{1}{h} [V(t+h, z+hf(t, z)) - V(t, z)].$$

Since $\frac{dx_i(t)}{dt} = 0$, when $x_i(t) = 0$; $\frac{dy_i(t)}{dt} = 0$, when $y_i(t) = 0$, and $\Delta y_1(t) = \mu > 0$, when $t = (n+1)\tau$, we can easily obtain the following lemma.

Lemma 3.2 Suppose $X(t)$ is a solution of (2.1) with $X(0^+) \geq 0$, then $X(t) \geq 0$ for $t \geq 0$. Furthermore, $X(t) > 0$ ($t \geq 0$) for $X(0^+) > 0$.

Lemma 3.3 [21] Let the function $m \in PC'[R^+, R]$ satisfy the inequalities

$$\begin{cases} m'(t) \leq p(t)m(t) + q(t), & t \geq t_0, t \neq t_k, k = 1, 2, \dots, \\ m(t_k^+) \leq d_k m(t_k) + b_k, & t = t_k, \end{cases} \quad (3.1)$$

where $p, q \in PC[R^+, R]$ and $d_k \geq 0$, b_k are constants; then

$$\begin{aligned} m(t) &\leq m(t_0) \prod_{t_0 < t_k < t} d_k \exp\left(\int_{t_0}^t p(s) ds\right) \\ &+ \sum_{t_0 < t_k < t} \left(\prod_{t_k < t_j < t} d_j \exp\left(\int_{t_0}^t p(s) ds\right) \right) b_k \\ &+ \int_{t_0}^t \prod_{s < t_k < t} d_k \exp\left(\int_s^t p(\sigma) d\sigma\right) q(s) ds, \quad t \geq t_0. \end{aligned}$$

Now, we show that all solutions of (2.1) are uniformly ultimately bounded.

Lemma 3.4 *There exists a constant $M > 0$ such that $x_1(t) \leq M$, $y_1(t) \leq M$, $y_2(t) \leq M$ for each solution $(x_1(t), y_1(t), y_2(t))$ of (2.1) with all t large enough.*

Proof Define

$$V(t) = k_1 x_1(t) + y_1(t) + y_2(t),$$

then $t \neq n\tau$, $t \neq (n+1)\tau$, we have

$$\begin{aligned} D^+ V(t) + d_1 V(t) &= k_1 x_1(t) [(a_1 + d_1) - b_1 x_1(t)] + y_2(t) [(a_2 + d_1) - b_2 y_2(t)] \\ &= -k_1 b_1 \left(x_1(t) - \frac{a_1 + d_1}{2b_1} \right)^2 + \frac{k_1 (a_1 + d_1)^2}{4b_1} - b_2 \left(y_2(t) - \frac{a_2 + d_1}{2b_2} \right)^2 + \frac{(a_2 + d_1)^2}{4b_2} \\ &\leq \frac{k_1 (a_1 + d_1)^2}{4b_1} + \frac{(a_2 + d_1)^2}{4b_2} \triangleq \zeta. \end{aligned}$$

When $t = (n+1)\tau$, $V((n+1)\tau^+) = k_1 x_1((n+1)\tau^+) + y_1((n+1)\tau^+) + y_2((n+1)\tau^+) = k_1 x_1((n+1)\tau) + y_1((n+1)\tau) + D y_2((n+1)\tau) + (1-D)y_2((n+1)\tau) = V((n+1)\tau)$. When $t = (n+1)\tau$, $V((n+1)\tau^+) = k_1 x_1((n+1)\tau^+) + y_1((n+1)\tau^+) + y_2((n+1)\tau^+) = k_1 x_1((n+1)\tau) + y_1((n+1)\tau) + \mu + y_2((n+1)\tau) = V((n+1)\tau) + \mu$. By Lemma 3.3, for $t \in (n\tau, (n+1)\tau]$, we have

$$\begin{aligned} V(t) &\leq V(0^+) e^{-\lambda t} + \frac{\zeta}{d_1} (1 - e^{-d_1 t}) + \mu \frac{e^{-d_1(t-\tau)}}{1 - e^{-d_1 \tau}} + \mu \frac{e^{d_1 \tau}}{e^{d_1 \tau} - 1} \\ &\rightarrow \frac{\zeta}{d_1} + \mu \frac{e^{d_1 \tau}}{e^{d_1 \tau} - 1}, \quad \text{as } t \rightarrow \infty. \end{aligned}$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, we see that there exists a constant $M > 0$ such that $x_1(t) \leq M$, $y_1(t) \leq M$, $y_2(t) \leq M$ for t large enough. The proof is complete. \square

If $x_1(t) = 0$, we have the following subsystem of (2.1):

$$\left\{ \begin{array}{l} \frac{dy_1(t)}{dt} = -d_1 y_1(t), \\ \frac{dy_2(t)}{dt} = y_2(t)(a_2 - b_2 y_2(t)), \end{array} \right\} \quad t \neq (n+1)\tau, t \neq (n+1)\tau, \\ \left\{ \begin{array}{l} \Delta y_1(t) = D y_2(t), \\ \Delta y_2(t) = -D y_2(t), \end{array} \right\} \quad t = (n+1)\tau, \\ \left\{ \begin{array}{l} \Delta y_1(t) = \mu, \\ \Delta y_2(t) = 0, \end{array} \right\} \quad t = (n+1)\tau. \quad (3.2)$$

We can easily obtain the analytic solution of (3.2) between pulses as follows:

$$\left\{ \begin{array}{l} y_1(t) = \begin{cases} y_1(n\tau^+) e^{-d_1(t-n\tau)}, & t \in [n\tau, (n+1)\tau), \\ y_1((n+1)\tau^+) e^{-d_1(t-(n+1)\tau)}, & t \in [(n+1)\tau, (n+1)\tau), \end{cases} \\ y_2(t) = \begin{cases} \frac{a_2 y_2(n\tau^+) e^{a_2(t-n\tau)}}{a_2 + b_2 y_2(n\tau^+) (e^{a_2(t-n\tau)} - 1)}, & t \in [n\tau, (n+1)\tau), \\ \frac{a_2 y_2((n+1)\tau^+) e^{a_2(t-(n+1)\tau)}}{a_2 + b_2 y_2((n+1)\tau^+) (e^{a_2(t-(n+1)\tau)} - 1)}, & t \in [(n+1)\tau, (n+1)\tau). \end{cases} \end{array} \right. \quad (3.3)$$

Considering the third and fourth equations of (3.2), we have

$$\begin{cases} y_1((n+l)\tau^+) = e^{-d_1 l \tau} y_1(n\tau^+) + D \times \frac{a_2 y_2(n\tau^+) e^{a_2 l \tau}}{a_2 + b_2 y_2(n\tau^+) (e^{a_2 l \tau} - 1)}, \\ y_2((n+l)\tau^+) = (1-D) \times \frac{a_2 y_2(n\tau^+) e^{a_2 l \tau}}{a_2 + b_2 y_2(n\tau^+) (e^{a_2 l \tau} - 1)}. \end{cases} \quad (3.4)$$

Considering the fifth and sixth equations of (3.2), we also have

$$\begin{cases} y_1((n+1)\tau^+) = e^{-d_1(1-l)\tau} y_1((n+l)\tau^+) + \mu, \\ y_2((n+1)\tau^+) = \frac{a_2 e^{a_2(1-l)\tau} y_2((n+l)\tau^+)}{a_2 + b_2 y_2((n+l)\tau^+) (e^{a_2(1-l)\tau} - 1)}. \end{cases} \quad (3.5)$$

Substituting (3.4) into (3.5), we have the stroboscopic map of (3.2),

$$\begin{cases} y_1((n+1)\tau^+) = e^{-d_1 \tau} y_1(n\tau^+) + \frac{D a_2 e^{[a_2 l - d_1(1-l)]\tau} y_2(n\tau^+)}{a_2 + b_2 (e^{a_2 l \tau} - 1) y_2(n\tau^+)} + \mu, \\ y_2((n+1)\tau^+) = \frac{(1-D) a_2 e^{a_2 \tau} y_2(n\tau^+)}{a_2 + b_2 (e^{a_2 l \tau} - 1) [1 + (1-D) e^{a_2 l \tau}] y_2(n\tau^+)}. \end{cases} \quad (3.6)$$

Choosing notations $A = e^{-d_1 \tau} < 1$, $B_1 = D a_2 e^{[a_2 l - d_1(1-l)]\tau} > 0$, $B_2 = (1-D) a_2 e^{a_2 \tau} > 0$, $C_1 = b_2 (e^{a_2 l \tau} - 1) > 0$, $C_2 = b_2 (e^{a_2 l \tau} - 1) [1 + (1-D) e^{a_2 l \tau}] > 0$, and

$$B_2 - a_2 > 0 \quad \Leftrightarrow \quad 0 < D < 1 - e^{-a_2 \tau},$$

we can rewrite (3.6) as

$$\begin{cases} y_1((n+1)\tau^+) = A y_1(n\tau^+) + \frac{B_1 y_2(n\tau^+)}{a_2 + C_1 y_2(n\tau^+)} + \mu, \\ y_2((n+1)\tau^+) = \frac{B_2 y_2(n\tau^+)}{a_2 + C_2 y_2(n\tau^+)}. \end{cases} \quad (3.7)$$

Equation (3.7) has two fixed points $(y_1^*, 0)$ and (y_1^{**}, y_2^{**}) , where

$$\begin{cases} y_1^* = \frac{\mu}{1-A} > 0, \\ y_1^{**} = \frac{B_1(B_2 - a_2)}{(1-A)[a_2 C_2 + C_1(B_2 - a_2)]} + \frac{\mu}{1-A} > 0, & 0 < D < 1 - e^{-a_2 \tau}, \\ y_2^{**} = \frac{B_2 - a_2}{C_2} > 0, & 0 < D < 1 - e^{-a_2 \tau}. \end{cases} \quad (3.8)$$

Lemma 3.5

- (i) If $1 - e^{-a_2 \tau} < D < 1$ holds, the fixed point $(y_1^*, 0)$ of (3.6) or (3.7) is globally asymptotically stable.
- (ii) If $0 < D < 1 - e^{-a_2 \tau}$ holds, the fixed point (y_1^{**}, y_2^{**}) of (3.6) or (3.7) is globally asymptotically stable.

Proof For convenience, we take the notation $(y_1^n, y_2^n) = (y_1(n\tau^+), y_2(n\tau^+))$. The linear form of (3.5) can be written as

$$\begin{pmatrix} y_1^{n+1} \\ y_2^{n+1} \end{pmatrix} = M \begin{pmatrix} y_1^n \\ y_2^n \end{pmatrix}. \quad (3.9)$$

Obviously, the near dynamics of $(y_1^*, 0)$ and (y_1^{**}, y_2^{**}) are determined by the linear System (3.6) or (3.7). The stabilities of $(y_1^*, 0)$ and (y_1^{**}, y_2^{**}) are determined by the eigenvalue of

M less than 1. If M satisfies the *Jury* criterion [34], we can know the eigenvalue of M less than 1,

$$1 - \operatorname{tr} M + \det M > 0. \quad (3.10)$$

- (i) If $1 - e^{-a_2\tau} < D < 1$ holds, that is, $\frac{B_2}{a_2} < 1$, we can easily see that $(y_1^*, 0)$ is a unique fixed point of (3.6) or (3.7), and

$$M = \begin{pmatrix} A & \frac{B_1}{a_2} \\ 0 & \frac{B_2}{a_2} \end{pmatrix}. \quad (3.11)$$

For

$$\begin{aligned} 1 - \operatorname{tr} M + \det M &= 1 - \left(A + \frac{B_2}{a_2} \right) + \left(A \times \frac{B_2}{a_2} \right) \\ &= (1 - A) \left(1 - \frac{B_2}{a_2} \right) > 0. \end{aligned}$$

From the *Jury* criterion, $(y_1^*, 0)$ is locally stable, then it is globally asymptotically stable.

- (ii) If $0 < D < 1 - e^{-a_2\tau}$ holds, that is, $\frac{a_2}{B_2} < 1$, we can easily see that (y_1^{**}, y_2^{**}) is a unique fixed point of (3.6) or (3.7), and

$$M = \begin{pmatrix} A & \frac{B_1 a_2}{(a_2 + C_1 y_2^{**})^2} \\ 0 & \frac{B_2 a_2}{(a_2 + C_2 y_2^{**})^2} \end{pmatrix}. \quad (3.12)$$

For

$$\begin{aligned} 1 - \operatorname{tr} M + \det M &= 1 - \left(A + \frac{B_2 a_2}{(a_2 + C_2 y_2^{**})^2} \right) + \left(A \times \frac{B_2 a_2}{(a_2 + C_2 y_2^{**})^2} \right) \\ &= (1 - A) \left[1 - \frac{B_2 a_2}{(a_2 + C_2 y_2^{**})^2} \right] \\ &= (1 - A) \left(1 - \frac{a_2}{B_2} \right) > 0. \end{aligned}$$

From the *Jury* criterion, (y_1^{**}, y_2^{**}) is locally stable, then it is globally asymptotically stable. This completes the proof. \square

Lemma 3.6

- (i) If $0 < D < 1 - e^{-a_2\tau}$ holds, the periodic solution $(\widetilde{y_1}(t), \widetilde{y_2}(t))$ of System (3.2) is globally asymptotically stable, where

$$\begin{cases} \widetilde{y_1}(t) = \begin{cases} y_1^{**} e^{-d_1(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ y_1^{***} e^{-d_1(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{y_2}(t) = \begin{cases} \frac{a_2 y_2^{**} e^{a_2(t-n\tau)}}{a_2 + b_2 y_2^{**} (e^{a_2(t-n\tau)} - 1)}, & t \in [n\tau, (n+l)\tau), \\ \frac{a_2 y_2^{***} e^{a_2(t-(n+l)\tau)}}{a_2 + b_2 y_2^{***} (e^{a_2(t-(n+l)\tau)} - 1)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \quad (3.13)$$

where y_1^{**} and y_2^{**} are determined as (3.8), y_1^{***} and y_2^{***} are defined as

$$\begin{cases} y_1^{***} = e^{-d_1 l \tau} y_1^{**} + D \times \frac{a_2 y_2^{**} e^{a_2 l \tau}}{a_2 + b_2 y_2^{**} (e^{a_2 l \tau} - 1)}, \\ y_2^{***} = (1 - D) \times \frac{a_2 y_2^{**} e^{a_2 l \tau}}{a_2 + b_2 y_2^{**} (e^{a_2 l \tau} - 1)}. \end{cases} \quad (3.14)$$

(ii) If $1 - e^{-a_2 \tau} < D < 1$ holds, the periodic solution $(\widehat{y_1(t)}, 0)$ of System (3.2) is globally asymptotically stable, where

$$\widehat{y_1(t)} = \begin{cases} y_1^* e^{-d_1(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ y_1^{****} e^{-d_1(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \quad (3.15)$$

where $y_1^{****} = e^{-d_1 \tau} y_1^*$, and y_1^* is determined as (3.8).

Therefore, if $y_1(0) = 0.5$, $y_2(0) = 0.5$, $d_1 = 0.6$, $a_2 = 0.1$, $b_2 = 0.2$, $\mu = 0.4$, $l = 0.25$, $\tau = 1$, $D = 0.15$, then $0.15 = D > D^* = 1 - e^{-a_2 \tau} \doteq 0.0952$; the predator population $y_2(t)$ will go into extinction. Its dynamical behaviors can be seen in Figure 1. If $y_1(0) = 0.5$, $y_2(0) = 0.5$, $d_1 = 0.6$, $a_2 = 0.1$, $b_2 = 0.2$, $\mu = 0.4$, $l = 0.25$, $\tau = 1$, $D = 0.05$, then $0.05 = D < D^* \doteq 0.0952$; the predator population are permanent. Its dynamical behaviors can be seen in Figure 2. Obviously, the diffusion rate between the patches affects the dynamics of System (3.2).

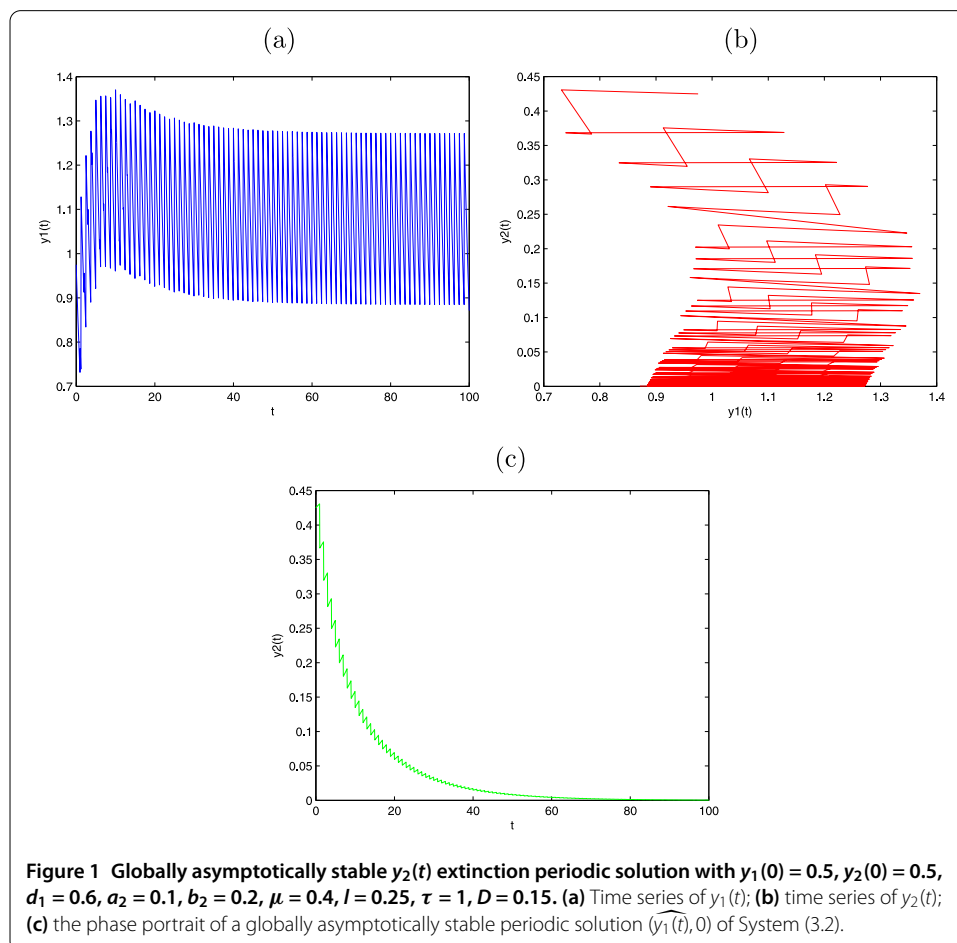
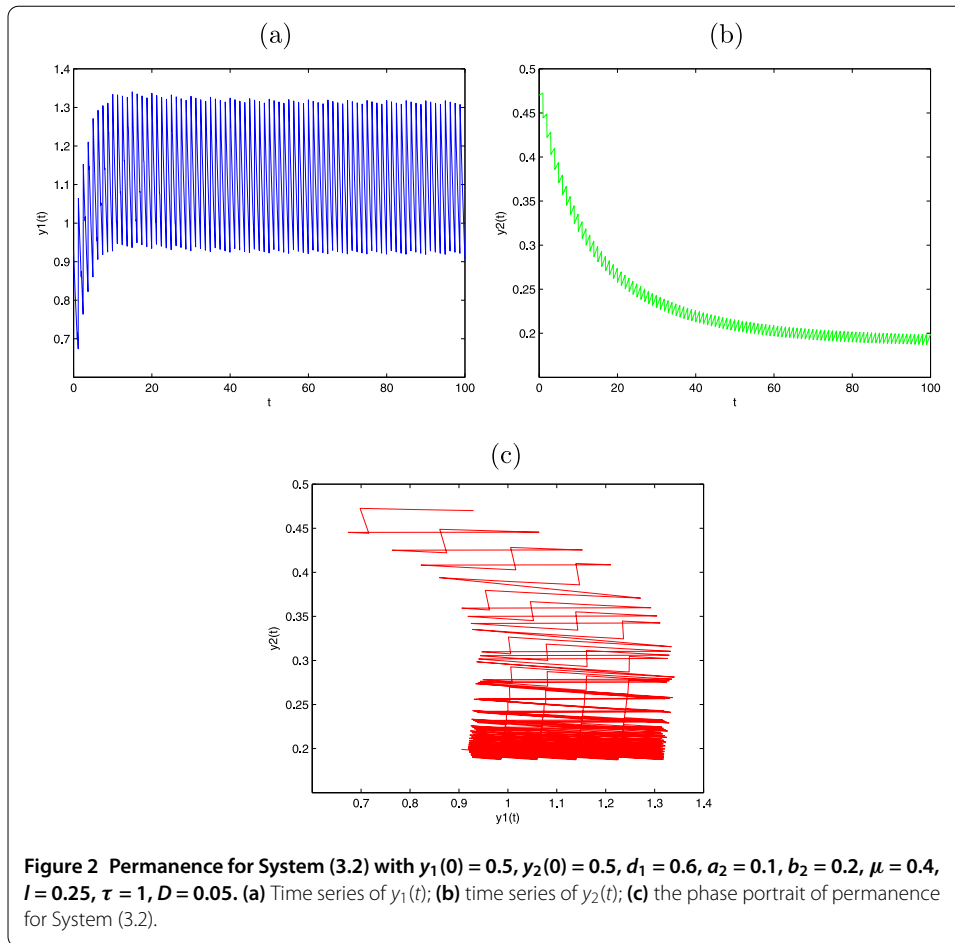


Figure 1 Globally asymptotically stable $y_2(t)$ extinction periodic solution with $y_1(0) = 0.5$, $y_2(0) = 0.5$, $d_1 = 0.6$, $a_2 = 0.1$, $b_2 = 0.2$, $\mu = 0.4$, $l = 0.25$, $\tau = 1$, $D = 0.15$. (a) Time series of $y_1(t)$; (b) time series of $y_2(t)$; (c) the phase portrait of a globally asymptotically stable periodic solution $(\widehat{y_1(t)}, 0)$ of System (3.2).



4 The dynamics

Theorem 4.1 *If*

$$1 - e^{-a_2\tau} < D < 1 \quad (4.1)$$

and

$$a_1\tau < \frac{\beta_1}{d_1} [y_1^*(1 - e^{-d_1 l\tau}) + y_1^{****}(e^{-d_1 l\tau} - e^{-d_1\tau})] \quad (4.2)$$

hold, the prey-extinction boundary periodic solution $(0, \widehat{y_1(t)}, 0)$ of (2.1) is globally asymptotically stable, where y_1^* and y_1^{****} are defined as (3.8) and (3.15).

Proof First, we prove the local stability of the prey-extinction boundary periodic solution $(0, \widehat{y_1(t)}, 0)$ of (2.1). Defining $x_1(t) = x_1(t)$, $y_{11}(t) = y_1(t) - \widehat{y_1(t)}$, $y_2(t) = y_2(t)$, then we have the following linearly similar system for (2.1) which concerns one periodic solution $(0, \widehat{y_1(t)}, 0)$:

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dy_{11}(t)}{dt} \\ \frac{dy_2(t)}{dt} \end{pmatrix} = \begin{pmatrix} a_1 - \beta_1 \widehat{y_1(t)} & 0 & 0 \\ k_1 \beta_1 \widehat{y_1(t)} & -d_1 & 0 \\ 0 & 0 & a_2 \end{pmatrix} \begin{pmatrix} x_1(t) \\ y_{11}(t) \\ y_2(t) \end{pmatrix}.$$

It is easy to obtain the fundamental solution matrix

$$\Phi(t) = \begin{pmatrix} \exp\left[\int_0^t (a_1 - \beta_1 \widehat{y_1(t)}) ds\right] & 0 & 0 \\ \exp\left[\int_0^t k_1 \beta_1 \widehat{y_1(t)} ds\right] & \exp(-d_1 t) & 0 \\ 0 & 0 & \exp(a_2 t) \end{pmatrix}.$$

The linearization of the fourth, fifth, and sixth equations of (2.1) is

$$\begin{pmatrix} x_1((n+l)\tau^+) \\ y_{11}((n+l)\tau^+) \\ y_2((n+l)\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & D \\ 0 & 0 & 1-D \end{pmatrix} \begin{pmatrix} x_1((n+l)\tau) \\ y_{11}((n+l)\tau) \\ y_2((n+l)\tau) \end{pmatrix}.$$

The linearization of the seventh, eighth, and ninth equations of (2.1) is

$$\begin{pmatrix} x_1((n+1)\tau^+) \\ y_{11}((n+1)\tau^+) \\ y_2((n+1)\tau^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1((n+1)\tau) \\ y_{11}((n+1)\tau) \\ y_2((n+1)\tau) \end{pmatrix}.$$

The stability of the periodic solution $(0, \widehat{y_1(t)}, 0)$ is determined by the eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & D \\ 0 & 0 & 1-D \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(\tau),$$

where

$$\lambda_1 = \exp\left[\int_0^\tau (a_1 - \beta_1 \widehat{y_1(t)}) ds\right],$$

$$\lambda_2 = e^{-d_1 \tau} < 1,$$

and

$$\lambda_3 = (1-D)e^{a_2 \tau}.$$

According to condition (4.1), (4.2), and the Floquet theory [21], *i.e.*

$$\exp\left[\int_0^\tau (a_1 - \beta_1 \widehat{y_1(s)}) ds\right] < 1$$

and

$$(1-D)e^{a_2 \tau} < 1,$$

we have

$$\lambda_1 < 1$$

and

$$\lambda_3 < 1,$$

the prey-extinction boundary periodic solution $(0, \widehat{y_1(t)}, 0)$ of (2.1) is locally stable.

In the following, we will prove the global attraction. From condition (4.2), we can choose an $\varepsilon > 0$ such that

$$\rho = \exp \left[\int_0^\tau (a_1 - \beta_1(\widehat{y_1(s)} - \varepsilon)) ds \right] < 1.$$

From the second equation of (2.1), we notice that $\frac{dy_1(t)}{dt} \geq -d_1 y_1(t)$. Then we consider the following impulsive comparative differential equation:

$$\begin{cases} \frac{dy_{21}(t)}{dt} = -d_1 y_{21}(t), \\ \frac{dy_{22}(t)}{dt} = y_{22}(t)(a_2 - b_2 y_{22}(t)), \\ \Delta y_{21}(t) = D y_{22}(t), \\ \Delta y_{22}(t) = -D y_{22}(t), \\ \Delta y_{21}(t) = \mu, \\ \Delta y_{22}(t) = 0, \end{cases} \quad \begin{matrix} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ t = (n+l)\tau, \\ t = (n+1)\tau. \end{matrix} \quad (4.3)$$

From Lemma 3.6 and the comparison theorem of impulsive equations (see Theorem 3.1.1 in [29]), we have $y_1(t) \geq y_{21}(t)$, $y_2(t) \geq y_{22}(t)$, and $y_{21}(t) \rightarrow \widehat{y_1(s)}$, $y_{22}(t) \rightarrow 0$, as $t \rightarrow \infty$. Then

$$\begin{cases} y_1(t) \geq y_{21}(t) \geq \widehat{y_1(s)} - \varepsilon, \\ y_2(t) \geq y_{22}(t) \geq -\varepsilon, \end{cases} \quad (4.4)$$

for all t large enough. For convenience, we may assume (4.4) holds for all $t \geq 0$. From (2.1) and (4.4), we get

$$\frac{dx_1(t)}{dt} \leq [a_1 - \beta_1(\widehat{y_1(s)} - \varepsilon)] x_1(t). \quad (4.5)$$

So $x_1((n+1)\tau) \leq x_1(n\tau^+) \exp[\int_{n\tau}^{(n+1)\tau} (a_1 - \beta_1(\widehat{y_1(s)} - \varepsilon)) ds]$. Hence, $x_1(n\tau) \leq x_1(0^+) \rho^n$ and $x_1(n\tau) \rightarrow 0$ as $n \rightarrow \infty$, therefore $x_1(t) \rightarrow 0$ ($i = 1, 2$) as $t \rightarrow \infty$.

Next, we will prove that $y_1(t) \rightarrow \widehat{y_1(t)}$ and $y_2(t) \rightarrow 0$ as $t \rightarrow \infty$. For $0 < \varepsilon_1 < \frac{d_1}{k_1 \beta_1}$ small enough, there must exist a $t_0 > 0$ such that $0 < x_1(t) < \varepsilon_1$ for all $t \geq t_0$. Without loss of generality, we may assume that $0 < x_1(t) < \varepsilon_1$ for all $t \geq 0$. For System (2.1) we have

$$-d_1 y_1(t) \leq \frac{dy_1(t)}{dt} \leq -(d_1 - k_1 \beta_1 \varepsilon_1) y_1(t), \quad (4.6)$$

then we have $y_{21}(t) \leq y_1(t) \leq y_{31}(t)$, $y_{22}(t) \leq y_2(t) \leq y_{32}(t)$, and $y_{21}(t) \rightarrow \widehat{y_1(t)}$, $y_{22}(t) \rightarrow 0$, $y_{31}(t) \rightarrow \widehat{y_{31}(t)}$, $y_{32}(t) \rightarrow 0$, as $t \rightarrow \infty$. Meanwhile $(y_{21}(t), y_{22}(t))$ and $(y_{31}(t), y_{32}(t))$ are the

solutions of (4.3) and

$$\left\{ \begin{array}{l} \frac{dy_{31}(t)}{dt} = -(d_1 - k_1\beta_1\varepsilon_1)y_{31}(t), \\ \frac{dy_{32}(t)}{dt} = y_{32}(t)(a_2 - b_2y_{32}(t)), \end{array} \right\} \quad t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left\{ \begin{array}{l} \Delta y_{31}(t) = Dy_{32}(t), \\ \Delta y_{32}(t) = -Dy_{32}(t), \end{array} \right\} \quad t = (n+l)\tau, \\ \left\{ \begin{array}{l} \Delta y_{31}(t) = \mu, \\ \Delta y_{32}(t) = 0, \end{array} \right\} \quad t = (n+1)\tau,$$
(4.7)

respectively. We have

$$\widehat{y_{31}}(t) = \begin{cases} y_{31}^* e^{-(d_1 - k_1\beta_1\varepsilon_1)(t - n\tau)}, & t \in [n\tau, (n+l)\tau), \\ y_{31}^{****} e^{-(d_1 - k_1\beta_1\varepsilon_1)(t - (n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \quad (4.8)$$

where $y_{31}^{****} = e^{-(d_1 - k_1\beta_1\varepsilon_1)\tau} y_{31}^*$, and y_{31}^* is determined as

$$y_{31}^* = \frac{\mu}{1 - e^{-(d_1 - k_1\beta_1\varepsilon_1)\tau}}.$$

For any $\varepsilon_2 > 0$, there exists a t_1 , $t > t_1$ such that

$$\widehat{y_{21}}(t) - \varepsilon_2 < y_1(t) < \widehat{y_{31}}(t) + \varepsilon_2$$

and

$$\widehat{y_{22}}(t) - \varepsilon_2 < y_2(t) < \widehat{y_{32}}(t) + \varepsilon_2.$$

Let $\varepsilon_1 \rightarrow 0$, so we have

$$\widehat{y_1}(t) - \varepsilon_2 < y_1(t) < \widehat{y_1}(t) + \varepsilon_2$$

and

$$-\varepsilon_2 < y_2(t) < +\varepsilon_2,$$

for t large enough, which implies $y_1(t) \rightarrow \widehat{y_1}(t)$ and $y_2(t) \rightarrow 0$ as $t \rightarrow \infty$. This completes the proof. \square

We can easily prove Theorem 4.2 similar to Theorem 4.1.

Theorem 4.2 *If*

$$1 - e^{-a_2\tau + 2[\ln \frac{a_2 + b_2 y_2^{**}(e^{a_2 l\tau} - 1)}{a_2} + \ln \frac{a_2 + b_2 y_2^{****}(e^{a_2 \tau} - 1)}{a_2 + b_2 y_2^{****}(e^{a_2 l\tau} - 1)}]} < D < 1 - e^{-a_2\tau} \quad (4.9)$$

and

$$a_1\tau < \frac{\beta_1}{d_1} [y_1^{**}(1 - e^{-d_1 l\tau}) + y_1^{****}(e^{-d_1 l\tau} - e^{-d_1 \tau})] \quad (4.10)$$

hold, the prey-extinction boundary periodic solution $(0, \widetilde{y_1(t)}, \widetilde{y_2(t)})$ of (2.1) is globally asymptotically stable, where y_1^{**} , y_1^{***} , y_2^{**} , and y_2^{***} are defined as (3.8) and (3.14).

The next task is to investigate the permanence of System (2.1).

Definition 4.3 System (2.1) is said to be permanent if there are constants $m, M > 0$ (independent of the initial value) and a finite time T_0 such that for all solutions $(x_1(t), y_1(t), y_2(t))$ with all initial values $x_1(0^+) > 0$, $y_1(0^+) > 0$, $y_2(0^+) > 0$, $m \leq x_1(t) \leq M$, $m \leq y_1(t) \leq M$, $m \leq y_2(t) \leq M$ hold for all $t \geq T_0$. Here T_0 may depend on the initial values $(x_1(0^+), y_1(0^+), y_2(0^+))$.

Theorem 4.4 If

$$0 < D < 1 - e^{-a_2 \tau} \quad (4.11)$$

and

$$a_1 \tau > \frac{\beta_1}{d_1} [y_1^{**} (1 - e^{-d_1 l \tau}) + y_1^{***} (e^{-d_1 l \tau} - e^{-d_1 \tau})] \quad (4.12)$$

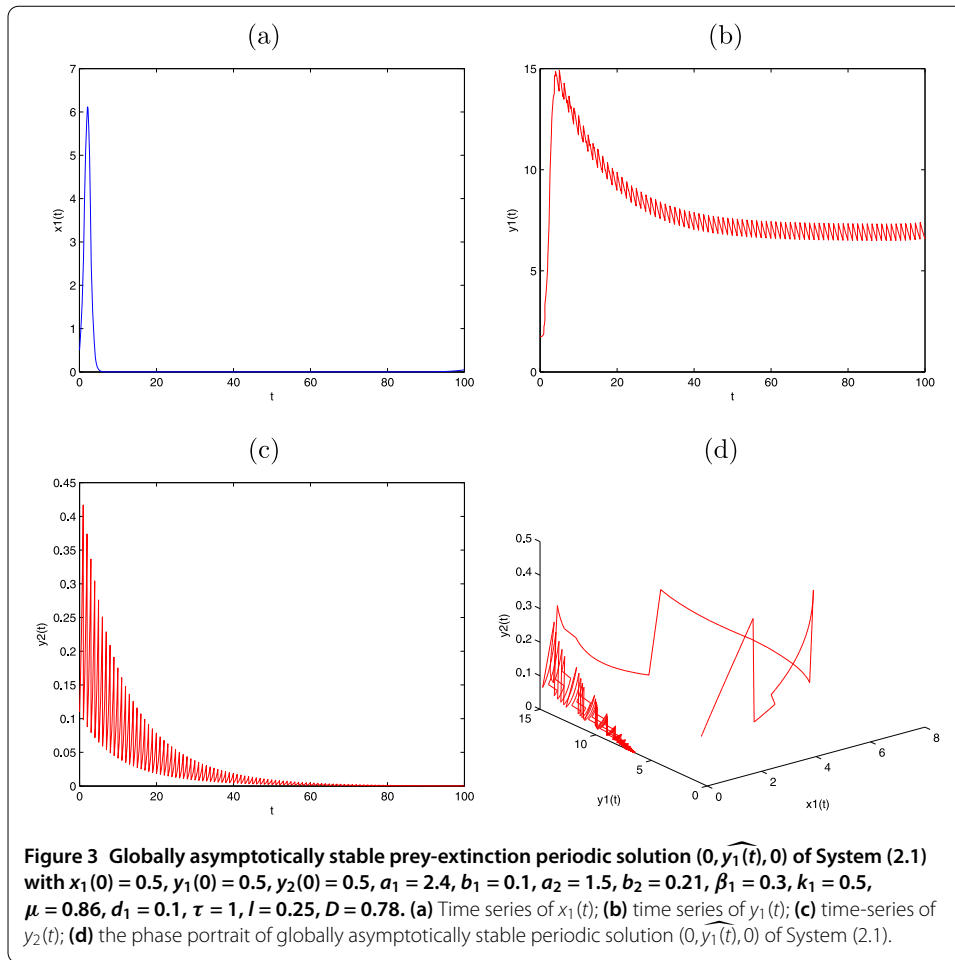
hold, System (2.1) is permanent, where y_1^{**} and y_1^{***} are defined as (3.8) and (3.14).

Proof Suppose $(x_1(t), y_1(t), y_2(t))$ is a solution of (2.1) with $x_1(0) > 0$, $y_1(0) > 0$, $y_2(0) > 0$. By Lemma 3.4, we have proved there exists a constant $M > 0$ such that $x_1(t) \leq M$, $y_1(t) \leq M$, $y_2(t) \leq M$, for t large enough. From (2.1), Theorem 4.1, and condition (4.11), we have

$$\begin{cases} y_1(t) > \widetilde{y_1(t)} - \varepsilon_2 \\ > y_1^{**} e^{-d_1 l \tau} + y_1^{***} e^{-d_1 (1-l) \tau} - \varepsilon_2 \triangleq m_1, \\ y_2(t) > \widetilde{y_2(t)} - \varepsilon_2 \\ > \frac{a_2 y_1^{**} e^{a_2 l \tau}}{a_2 + b_2 y_1^{**} (e^{a_2 l \tau} - 1)} + \frac{a_2 y_1^{***} e^{a_2 (1-l) \tau}}{a_2 + b_2 y_1^{***} (e^{a_2 (1-l) \tau} - 1)} - \varepsilon_2 \triangleq m_2, \end{cases} \quad (4.13)$$

for ε_2 small enough. So we only need to find $m_3 > 0$ and ε_3 such that $x_1(t) > m_3$ for t large enough. Otherwise, we can select $m_4 > 0$ small enough satisfying $m_4 < \frac{d_1}{k_1 \beta_1}$, and we prove $x_1(t) < m_4$ cannot hold for $t \geq 0$. By condition (4.12) and choosing ε_3 small enough, we can obtain

$$\begin{aligned} \sigma_i &= a_1 \tau \\ &- \frac{\beta_1}{(d_1 - k_1 \beta_1 m_4)} [y_{41}^{**} (1 - e^{-(d_1 - k_1 \beta_1 m_4) l \tau}) + y_{41}^{***} (e^{-(d_1 - k_1 \beta_1 m_4) l \tau} - e^{-(d_1 - k_1 \beta_1 m_4) \tau})] \\ &- \beta_1 \varepsilon_3 \tau \\ &> 0, \end{aligned}$$

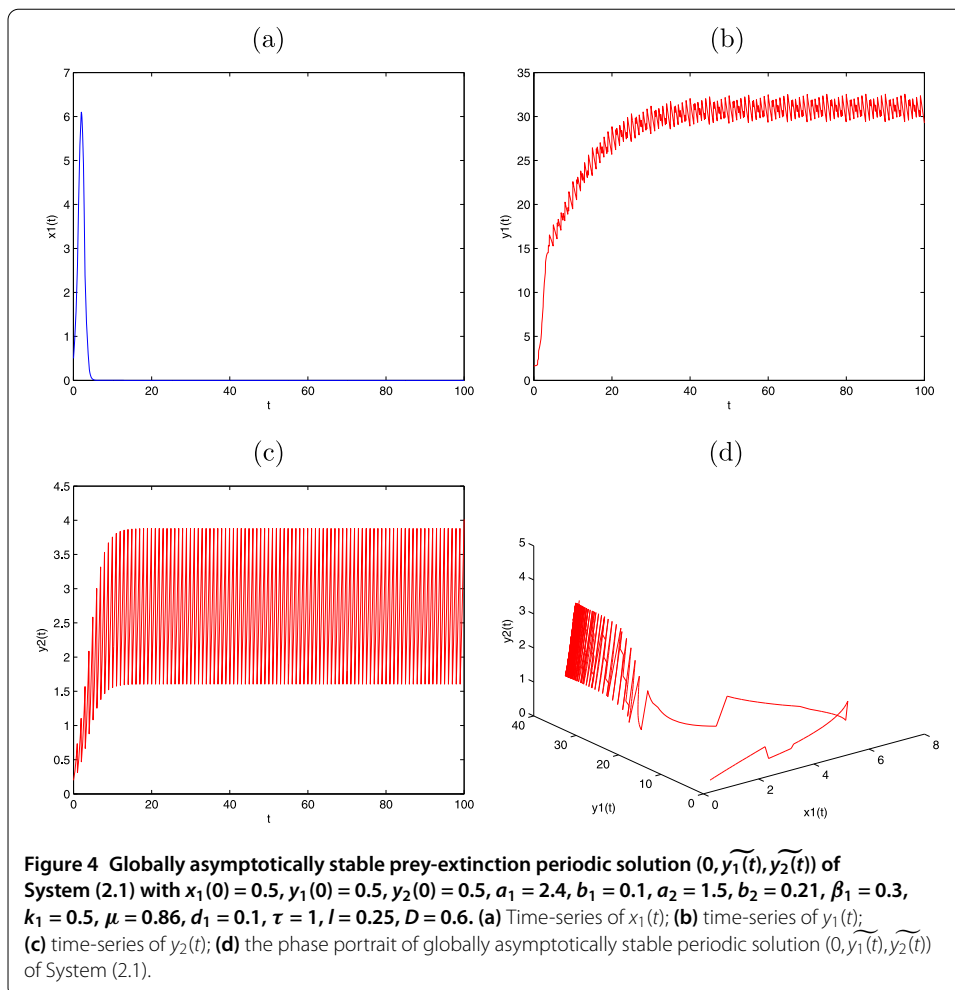


with y_{41}^{**} and y_{41}^{***} defined as (4.17) and (4.18). Then

$$\left\{ \begin{array}{l} \frac{dy_1(t)}{dt} < -(d_1 - k_1\beta_1 m_4)y_1(t), \\ \frac{dy_2(t)}{dt} = y_2(t)(a_2 - b_2 y_2(t)), \\ \Delta y_1(t) = D y_2(t), \\ \Delta y_2(t) = -D y_2(t), \\ \Delta y_1(t) = \mu, \\ \Delta y_2(t) = 0, \end{array} \right\} \quad \begin{array}{l} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ t = (n+l)\tau, \\ t = (n+1)\tau. \end{array} \quad (4.14)$$

By Lemma 3.6, we have $y_1(t) \leq y_{41}(t)$, $y_2(t) \leq y_{42}(t)$ and $y_{41}(t) \rightarrow \overline{y_{41}(t)}$, $y_{42}(t) \rightarrow \overline{y_{42}(t)}$, $t \rightarrow \infty$, where $(y_{41}(t), y_{42}(t))$ is the solution of

$$\left\{ \begin{array}{l} \frac{dy_{41}(t)}{dt} = -(d_1 - k_1\beta_1 m_4)y_{41}(t), \\ \frac{dy_{42}(t)}{dt} = y_{42}(t)(a_2 - b_2 y_{42}(t)), \\ \Delta y_{41}(t) = D y_{42}(t), \\ \Delta y_{42}(t) = -D y_{42}(t), \\ \Delta y_{41}(t) = \mu, \\ \Delta y_{42}(t) = 0, \end{array} \right\} \quad \begin{array}{l} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ t = (n+l)\tau, \\ t = (n+1)\tau. \end{array} \quad (4.15)$$



with

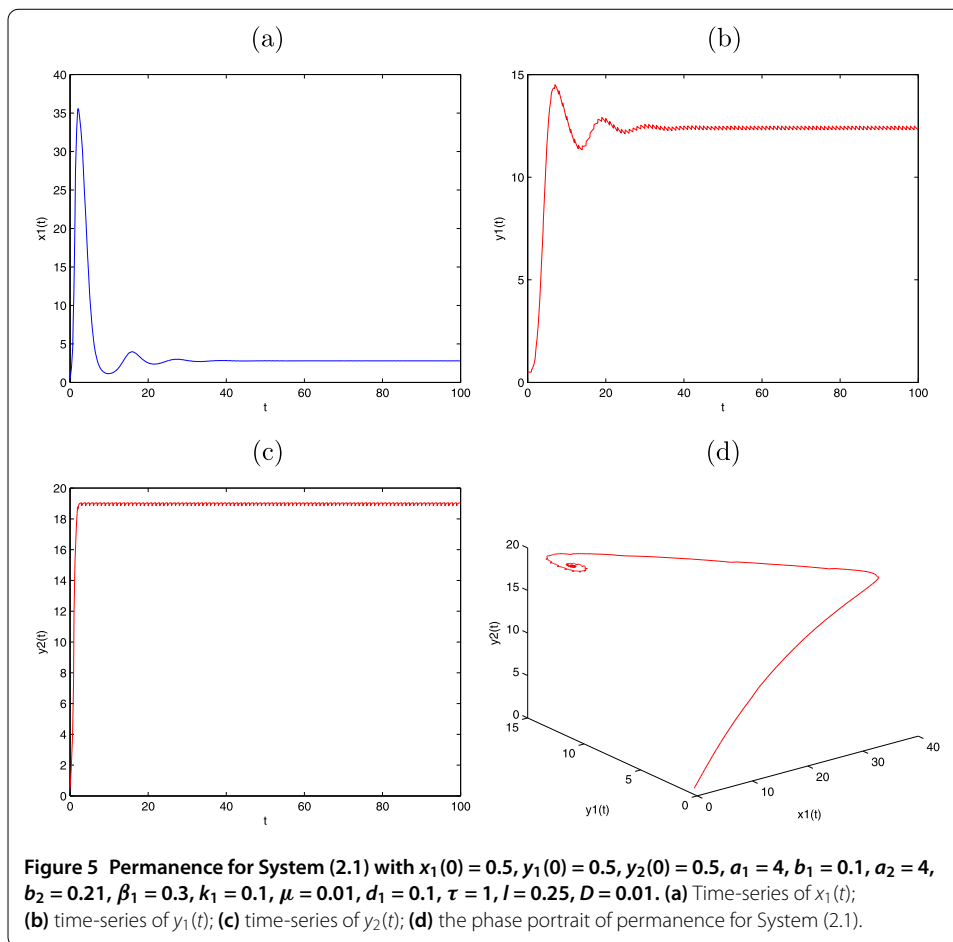
$$\begin{cases} \widetilde{y_{41}(t)} = \begin{cases} y_{41}^{**} e^{-(d_1 - k_1 \beta_1 m_4)(t - n\tau)}, & t \in [n\tau, (n+l)\tau), \\ y_{41}^{***} e^{-(d_1 - k_1 \beta_1 m_4)(t - (n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \\ \widetilde{y_{42}(t)} = \begin{cases} \frac{a_2 y_{42}^{**} e^{a_2(t - n\tau)}}{a_2 + b_2 y_{42}^{**} (e^{a_2(t - n\tau)} - 1)}, & t \in [n\tau, (n+l)\tau), \\ \frac{a_2 y_{42}^{***} e^{a_2(t - (n+l)\tau)}}{a_2 + b_2 y_{42}^{***} (e^{a_2(t - (n+l)\tau)} - 1)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \end{cases} \quad (4.16)$$

where y_{41}^{**} and y_{42}^{**} are determined as

$$\begin{cases} y_{41}^{**} = \frac{B_{41}(B_{42} - a_2)}{(1 - A_4)[a_2 C_{42} + C_{41}(B_{42} - a_2)]} + \frac{\mu}{1 - A_4} > 0, & 0 < D < 1 - e^{-a_2 \tau}, \\ y_{42}^{**} = \frac{B_{42} - a_2}{C_{42}} > 0, & 0 < D < 1 - e^{-a_2 \tau}, \end{cases} \quad (4.17)$$

with $A_4 = e^{-(d_1 - k_1 \beta_1 m_4)\tau} < 1, B_{41} = Da_2 e^{a_2 l \tau} > 0, B_{42} = (1 - D)a_2 e^{a_2 \tau} > 0, C_{41} = b_2(e^{a_2 \tau} - 1) > 0, C_{42} = b_2(e^{a_2 \tau} - 1)[1 + (1 - D)e^{a_2 l \tau}] > 0$, and y_{41}^{***} and y_{42}^{***} are defined as

$$\begin{cases} y_{41}^{***} = e^{-(d_1 - k_1 \beta_1 m_4)l\tau} y_{41}^{**} + D \times \frac{a_2 y_{42}^{**} e^{a_2 l \tau}}{a_2 + b_2 y_{42}^{**} (e^{a_2 l \tau} - 1)}, \\ y_{42}^{***} = (1 - D) \times \frac{a_2 y_{42}^{**} e^{a_2 l \tau}}{a_2 + b_2 y_{42}^{**} (e^{a_2 l \tau} - 1)}. \end{cases} \quad (4.18)$$



Therefore, there exist $T_1 > 0$ and $\varepsilon_3 > 0$ such that

$$y_1(t) \leq y_{41}(t) \leq \overline{y_{41}(t)} - \varepsilon_3$$

and

$$y_2(t) \leq y_{42}(t) \leq \overline{y_{42}(t)} - \varepsilon_3.$$

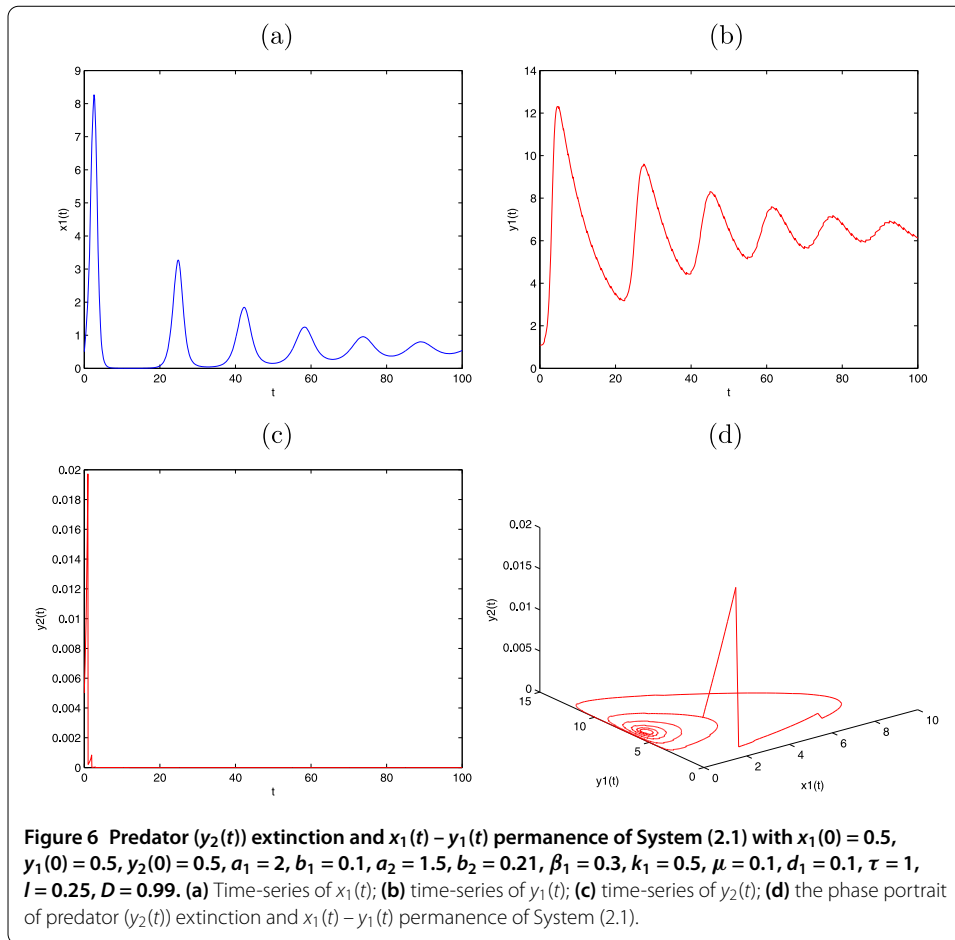
Then

$$\frac{dx_1(t)}{dt} \geq [a_1 - \beta_1(\overline{y_1(t)} - \varepsilon_3)]x_1(t), \quad (4.19)$$

for $t \geq T_1$, let $N_1 \in \mathbb{N}$ and $N_1\tau > T_1$. Integrating (4.19) on $(n\tau, (n+1)\tau)$, $n \geq N_1$, we have

$$x_1((n+1)\tau) \geq x_1(n\tau^+) \exp\left(\int_{n\tau}^{(n+1)\tau} [a_1 - \beta_1(\overline{y_1(t)} - \varepsilon_3)] dt\right) = x_1(n\tau)e^\sigma,$$

then $x_1((N_1 + k)\tau) \geq x_1(N_1\tau^+)e^{k\sigma} \rightarrow \infty$, as $k \rightarrow \infty$, which is a contradiction to the boundedness of $x_1(t)$. Hence, there exists a $t_1 > 0$ such that $x_1(t) \geq m_3$. This completes the proof. \square



5 Discussion

In this paper, we establish a predator-prey model with impulsive diffusion and releasing on predator population. This predator-prey model for two regions, which are connected by diffusion of the predator population, portrays the evolution of the population. We prove that all solutions of the investigated system are uniformly ultimately bounded. From Theorem 4.1, the prey-extinction periodic solution $(0, \widetilde{y_1}(t), 0)$ of System (2.1) is globally asymptotically stable. From Theorem 4.2, the prey-extinction periodic solution $(0, \widetilde{y_1}(t), \widetilde{y_2}(t))$ of System (2.1) is globally asymptotically stable. From Theorem 4.4, System (2.1) is permanent. It is assumed that $x_1(0) = 0.5$, $y_1(0) = 0.5$, $y_2(0) = 0.5$, $a_1 = 2.4$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.21$, $\beta_1 = 0.3$, $k_1 = 0.5$, $\mu = 0.86$, $d_1 = 0.1$, $\tau = 1$, $l = 0.25$, $D = 0.78$; the conditions (4.1) and (4.2) are obviously satisfied, then the prey-extinction periodic solution $(0, \widetilde{y_1}(t), 0)$ of System (2.1) is globally asymptotically stable (see Figure 3). It is assumed that $x_1(0) = 0.5$, $y_1(0) = 0.5$, $y_2(0) = 0.5$, $a_1 = 2.4$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.21$, $\beta_1 = 0.3$, $k_1 = 0.5$, $\mu = 0.86$, $d_1 = 0.1$, $\tau = 1$, $l = 0.25$, $D = 0.6$; the conditions (4.9) and (4.10) are obviously satisfied, then the prey-extinction periodic solution $(0, \widetilde{y_1}(t), \widetilde{y_2}(t))$ of System (2.1) is globally asymptotically stable (see Figure 4). It is also assumed that $x_1(0) = 0.5$, $y_1(0) = 0.5$, $y_2(0) = 0.5$, $a_1 = 4$, $b_1 = 0.1$, $a_2 = 4$, $b_2 = 0.21$, $\beta_1 = 0.3$, $k_1 = 0.1$, $\mu = 0.01$, $d_1 = 0.1$, $\tau = 1$, $l = 0.25$, $D = 0.01$; the conditions (4.11) and (4.12) are obviously satisfied, then System (2.1) is permanent (see Figure 5). It is also assumed that $x_1(0) = 0.5$, $y_1(0) = 0.5$, $y_2(0) = 0.5$, $a_1 = 2$, $b_1 = 0.1$, $a_2 = 1.5$,

$b_2 = 0.21$, $\beta_1 = 0.3$, $k_1 = 0.5$, $\mu = 0.1$, $d_1 = 0.1$, $\tau = 1$, $l = 0.25$, $D = 0.99$; predator $(y_2(t))$ will go into extinction and $x_1(t) - y_1(t)$ will be permanent in System (2.1) (see Figure 6).

From the simulations, we can guess that there exist three controlling thresholds with D . It is always assumed that $0 < D^* < D^{**} < D^{***} < 1$. If $0 < D < D^*$ holds, System (2.1) is permanent. If $D^* < D < D^{**}$ holds, the prey-extinction periodic solution $(0, \widetilde{y_1(t)}, \widetilde{y_2(t)})$ of System (2.1) is globally asymptotically stable. If $D^{**} < D < D^{***}$ holds, the prey-extinction periodic solution $(0, \widetilde{y_1(t)}, 0)$ of System (2.1) is globally asymptotically stable. If $D^{***} < D < 1$ holds, the predator $y_2(t)$ will go into extinction, prey $x_1(t)$ and predator $y_1(t)$ will be permanent. We can discuss parameter μ similar to parameter D . We discover that the diffusive rate of the predator population plays an important role in pest management. We conclude that the impulsive diffusion and the released predator provide reliable tactic bases for pest management.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

JJ carried out the main part of this article, LS corrected the manuscript, SH, LM, and YJ brought forward some suggestions on this article. All authors have read and approved the final manuscript.

Author details

¹School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang, 550004, P.R. China.

²School of Continuous Education, Guizhou University of Finance and Economics, Guiyang, 550004, P.R. China. ³College of Mathematics and Information Science, Anshan Normal University, Anshan, 114007, P.R. China.

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