

RESEARCH

Open Access



# On a new eco-epidemiological model for migratory birds with modified Leslie-Gower functional schemes

Kuangang Fan<sup>1</sup>, Yan Zhang<sup>2,3\*</sup> and Shujing Gao<sup>3</sup>

\*Correspondence:

zhyang8401@163.com

<sup>2</sup>School of Mathematics and Statistics, Wuhan University, Wuhan, 430000, P.R. China

<sup>3</sup>School of Mathematics and Computer Science, Gannan Normal University, Ganzhou, 341000, P.R. China

Full list of author information is available at the end of the article

## Abstract

Migratory birds are critical to the prevalence of many epidemic diseases. In this paper, a new two species eco-epidemiological model with disease in the migratory prey is formulated. A modified Leslie-Gower functional scheme, with saturated incidence and recovery rate are considered in this new model. Through theoretical analysis, a series of conditions are established to ensure the extinction, permanence of the disease, and to keep the system globally attractive. It was observed that if the lower threshold value  $R_* > 1$ , the infective population of the periodic system is permanent, whereas if the upper threshold value  $R^* \leq 1$ , then the disease will go to extinction. Our results also show that predation could be a good choice to control disease and enhance permanence.

**Keywords:** migratory birds; Leslie-Gower functional response; saturated incidence and recovery rate

## 1 Introduction

Nowadays, an important issue in applied mathematics is to study the influence of epidemiological parameters on ecological systems. Since Kermac-Mckendric (1927) first proposed the SIR systems, many attentions have been paid to this field. In 1989, Haderler and Freedman described a model for predator and prey with parasitic infection [1]. From then on, more and more predator-prey models were proposed and discussed under the frame work of eco-epidemiology; see [2–7] and references therein. The biological significance of these works is that we can see how epidemic diseases affect the interactions of prey and predators and how predators act as biological control to disease transmissions. In nature, migratory birds are responsible for the prevalence of many epidemic diseases, such as WNV, which was introduced in the Middle East by migrating white storks [8], HPAI that broke in Mexico in 1994 and was introduced by some wild migrating birds [9, 10], and so on. However, there are few papers analyzing the role of migratory birds, especially by mathematical models and analysis, except the works of Chatterjee *et al.* [10–13], Gao *et al.* [14] and Zhang *et al.* [15].

In [10], Chatterjee and Chattopadhyay assumed the prey population migrated with disease and proposed a one-season eco-epidemiological predator-prey model for migratory birds. In [11], Chatterjee *et al.* modified and analyzed their model in [10] by taking time lags into consideration. Their analysis showed that we could control the outbreak of the disease

by making use of the time lag factor suitably. In [12], the author introduced standard incidence into the model and obtained the stability of equilibrium point in the presence or absence of environmental fluctuations. Chatterjee (in [13]) discussed an eco-epidemiological model with a nonautonomous recruitment rate and a general functional response. They showed that the contact rate, the predation, and the recovery rate were central to the extinction of the disease. In [14], Gao *et al.* considered a competitive model for migratory birds and economical birds population. They analyzed the model and discussed dynamics of the model. Zhang *et al.* in [15] proposed a time-dependent model for migratory birds with saturated incidence rate. They also analyzed the dynamics of the system, such as permanence, extinction, and global attractivity of the model. In [15], for simplicity, only the bilinear predation rate was considered for migratory birds and the diversity of the functional responses was not referred to.

As we all know, the functional response is a critical factor in the research of the population dynamics for predator-prey models. The mutual interference between predator and prey can influence the relationship between them. In the past decades, more and more different forms of ratio-dependent functional responses were proposed, such as of the Crowley-Martin type, the Beddington-DeAngelis type, the Leslie-Gower type, the Hassell-Varley type, and so on [16]. In this paper, we consider a modified Leslie-Gower functional response, in which the Leslie-Gower term is  $\frac{P(t)}{k_i(t)+S(t)+I(t)}$ ,  $i = 1, 2$ , to describe the dynamics between migratory preys and their predators.

To construct the model for migratory birds, we suppose that the prey population, including the susceptible population,  $S$ , and the infected population,  $I$ , migrate into the system. The incidence rate and recovery rate are assumed to take saturated forms which are more realistic as many researchers suggested, that is,  $\frac{\beta(t)SI}{1+\gamma(t)S}$  and  $\frac{f(t)}{1+\alpha(t)I}$ . Then without predation, the  $SI$  model can be expressed as follows:

$$\begin{cases} \dot{S}(t) = \Lambda(t) - \frac{\beta(t)S(t)I(t)}{1+\gamma(t)S(t)} - d(t)S(t) + \frac{f(t)I(t)}{1+\alpha(t)I(t)}, \\ \dot{I}(t) = \frac{\beta(t)S(t)I(t)}{1+\gamma(t)S(t)} - e(t)I(t) - \frac{f(t)I(t)}{1+\alpha(t)I(t)}, \end{cases} \tag{1.1}$$

where  $\Lambda(t)$ ,  $\beta(t)$  denote the instantaneous recruitment rate of the prey population and the force of the infective (contact rate) at time  $t$ .  $f(t)$  represents the recovery rate of the infected prey from the disease.  $d(t)$ ,  $e(t)$  denote the natural death rate and the mortality rate including the natural death rate and the diseased death rate for susceptible and infective prey population at time  $t$ , respectively. Obviously,  $d(t) \leq e(t)$  for all  $t \geq 0$ .  $\gamma(t) > 0$  and  $\alpha(t) > 0$  measure the force of the inhibition effect at time  $t$ .

We assume that the predator population  $P$  eat both the susceptible and the infective prey in a form of modified Leslie-Gower scheme. Inspired by the above factors, we propose a nonautonomous differential equation for migratory birds,

$$\begin{cases} \dot{S}(t) = \Lambda(t) - \frac{\beta(t)S(t)I(t)}{1+\gamma(t)S(t)} - d(t)S(t) + \frac{f(t)I(t)}{1+\alpha(t)I(t)} - \frac{c_1(t)S(t)P(t)}{w_1(t)+S(t)+I(t)}, \\ \dot{I}(t) = \frac{\beta(t)S(t)I(t)}{1+\gamma(t)S(t)} - e(t)I(t) - \frac{f(t)I(t)}{1+\alpha(t)I(t)} - \frac{c_2(t)I(t)P(t)}{w_1(t)+S(t)+I(t)}, \\ \dot{P}(t) = P(t) \left[ r(t) - \frac{c_3(t)P(t)}{w_2(t)+\sigma(t)S(t)+I(t)} \right], \end{cases} \tag{1.2}$$

and the initial conditions are

$$S(0) > 0, \quad I(0) > 0, \quad P(0) > 0. \tag{1.3}$$

Denote the set  $\Theta = \{(S, I, P) \in R^3 : S > 0, I > 0, P > 0\}$ , then we can prove that it is a positively invariant set of system (1.2). In fact, letting the right equations of system (1.2) by  $F_i(S, I, P)$  ( $i = 1, 2, 3$ ) and  $X = (S, I, P)^T \in R^3$ , then system (1.2) can be rewritten in a vector form as  $F(X) = [F_1(X), F_2(X), F_3(X)]^T$ , where  $F \in C^\infty(R^3)$ . Thus, system (1.2) becomes

$$\dot{X} = F(X) \tag{1.4}$$

with  $X(0) = X_0 \in R_+^3$ . Therefore, for any  $X(0) \in R_+^3$  satisfying  $X_i = 0$ , then  $F_i(X)|_{X_i=0} \geq 0$  ( $i = 1, 2, 3$ ). Thus, the set  $\Theta$  is positively invariant. (For more details, please see [17].)

The other parameters for model (1.2) are defined as shown below:

- $r(t)$  is the growth rate of the predator population.
- $c_1(t)$  ( $c_2(t)$ ) is the maximum value of per capita rate of  $S$  (respectively,  $I$ ) due to  $P$  at time  $t$ . Because the predators catch the infected prey more easily than the healthy ones, we have  $c_1(t) \leq c_2(t)$ .
- $c_3(t)$  is the maximum value of the per capita rate of  $P$  due to  $S$  and  $I$  at time  $t$  ([16]).
- $w_1(t)$  denotes the level of environment protection to prey at time  $t$  and  $w_2(t)$  has a similar meaning to  $w_1(t)$ .
- $\sigma(t)$  denotes the effects on the predator by absorbing the susceptible prey and  $\sigma(t) \leq 1$  for all  $t \geq 0$ .

The rest of this paper is organized as follows. In Section 2, we analyze the nonautonomous differential equations for migratory birds and establish a set of sufficient conditions to discuss the extinction, the permanence of the disease, and keep the system globally attractive. In Section 3, some results are presented for the periodic system. In Section 4, we verify our theoretical results and outline a discussion by making comparison among the new model (1.2), the  $SI$  model (1.1) and the model in [15] with the help of numerical simulation. Finally, some conclusions are given in Section 5.

## 2 The analysis of the model

To proceed, we give some appropriate definitions and notations and list them in the following.

For convenience, we denote

$$f^u = \sup_{t \geq 0} f(t), \quad f^v = \inf_{t \geq 0} f(t), \quad \bar{f} = \frac{1}{t} \int_0^t f(s) ds,$$

where  $f(t)$  is a continuous and bounded function that defined on  $R_+ = [0, +\infty)$ . Moreover, we make some assumptions as below:

- (B1)  $\Lambda(t), \beta(t), \gamma(t), \alpha(t), d(t), e(t), f(t), \sigma(t), r(t), w_i(t)$  ( $i = 1, 2$ ) and  $c_i(t)$  ( $i = 1, 2, 3$ ) are all nonnegative, continuous functions and bounded on  $R_+$ ;
- (B2) there are constants  $\omega_i > 0$  ( $i = 1, 2, 3, 4, 5, 6$ ) satisfying

$$\begin{aligned} \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_1} \Lambda(\theta) d\theta &> 0, & \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_2} d(\theta) d\theta &> 0, \\ \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_3} r(\theta) d\theta &> 0, & \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_4} e(\theta) d\theta &> 0, \\ \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_5} \frac{c_1(\theta)}{w_1(\theta)} d\theta &> 0, & \liminf_{t \rightarrow +\infty} \int_t^{t+\omega_6} \frac{c_3(\theta)}{w_2(\theta)} d\theta &> 0; \end{aligned}$$

- (B3)  $d^m > 0, w_1^m > 0, w_2^m > 0$ .

**Theorem 2.1** *Under assumptions (B1)-(B3), if there is a constant  $\omega_7 > 0$  satisfying*

$$\liminf_{t \rightarrow +\infty} \int_t^{t+\omega_7} \frac{c_3(\theta)}{w_2(\theta) + M_1} d\theta > 0, \tag{2.1}$$

where the constant  $M_1 = \max\{(\frac{\Lambda}{d})^\nu, 1\}$ , then both the prey population and the predator population are permanent.

*Proof* First of all, suppose that  $(S, I, P)$  is an arbitrary positive solution of model (1.2) with initial conditions (1.3). By the first two equations of (1.2), we have

$$\dot{S}(t) + \dot{I}(t) \leq \Lambda(t) - d(t)(S(t) + I(t)),$$

for all  $t \geq 0$ . Then applying the conclusion of Lemma 2.1 in [18] and the comparison theorem, there are constants  $M_1 = \max\{(\frac{\Lambda}{d})^\nu, 1\}$  and  $T_1 > 0$  satisfying

$$S(t) + I(t) \leq M_1, \quad \text{for all } t \geq T_1. \tag{2.2}$$

Applying (2.2) to system (1.2), we obtain

$$\dot{P}(t) \leq P(t) \left[ r(t) - \frac{c_3(t)}{w_2(t) + M_1} P(t) \right],$$

for all  $t \geq T_1$ . Using the condition (2.1), Lemma 1 in [19], and the comparison theorem, there are constants  $M_2 = \max\{(\frac{w_2 r + M_1 r}{c_3})^\nu, 1\}$  and  $T_2 (\geq T_1)$  satisfying

$$P(t) \leq M_2, \quad \text{for all } t \geq T_2. \tag{2.3}$$

Second, from inequality (2.3) and system (1.2), we have

$$\dot{S}(t) + \dot{I}(t) \geq \Lambda(t) - \left[ e(t) + \frac{c_1(t) + c_2(t)}{w_1(t)} M_2 \right] (S(t) + I(t)), \quad \text{for all } t \geq T_2.$$

Applying Lemma 2.1 in [18] again, there are constants  $m_1 = \min\{(\frac{\Lambda}{e + M_2(c_1 + c_2)/w_1})^\nu, 1\}$  and  $T_3 > T_2$  satisfying

$$S(t) + I(t) \geq m_1, \quad \text{for all } t \geq T_3. \tag{2.4}$$

Next, considering the last equation of model (1.2)

$$\dot{P}(t) \geq P(t) \left[ r(t) - \frac{c_3(t)}{w_2(t)} P(t) \right].$$

Applying Lemma 1 in [19] and the comparison theorem, we see that there exist constants  $m_2 = \min\{(\frac{w_2 r}{c_3})^\nu, 1\}$  and  $T_4 > T_3$  satisfying

$$P(t) \geq m_2, \quad \text{for all } t \geq T_4. \tag{2.5}$$

Thus, by (2.2)-(2.5), we have the following results:

$$m_1 \leq \liminf_{t \rightarrow +\infty} (S(t) + I(t)) \leq \limsup_{t \rightarrow +\infty} (S(t) + I(t)) \leq M_1,$$

$$m_2 \leq \liminf_{t \rightarrow +\infty} P(t) \leq \limsup_{t \rightarrow +\infty} P(t) \leq M_2.$$

It completes the proof. □

Now we give the results about the permanence of the infective prey. Suppose  $S_0(t), p_0(t)$  are an arbitrary fixed solution of the system

$$\dot{S}(t) = \Lambda(t) - \frac{c_1(t)}{w_1(t)} M_0^2 - d(t)S$$

and

$$\dot{p}(t) = p \left( r(t) + \frac{c_3(t)}{w_2^2(t)} \sigma(t) M_0^2 - \frac{c_3(t)}{w_2(t)} p \right),$$

respectively, where  $M_0 = \frac{1}{m_2} + M_1 + M_2$ , then we can obtain the theorem about the permanence of the infective prey population as follows.

**Theorem 2.2** *Under assumptions (B1), (B2), (B3), if there exist constants  $\lambda > 0$  and  $\omega_8 > 0$  satisfying*

$$\liminf_{t \rightarrow +\infty} \frac{1}{\lambda} \int_t^{t+\lambda} \left( \frac{\beta(\theta)S_0(\theta)}{1 + \gamma(\theta)S_0(\theta)} - e(\theta) - f(\theta) - \frac{c_2(\theta)p_0(\theta)}{w_1(\theta) + S_0(\theta)} \right) d\theta > 0$$

and

$$\liminf_{t \rightarrow +\infty} \int_t^{t+\omega_8} \left( \Lambda(\theta) - \frac{c_1(\theta)}{w_1(\theta)} M_0^2 \right) d\theta > 0, \tag{2.6}$$

then the infected prey population  $I$  is permanent.

*Proof* Our proof is motivated by the work of Zhang and Teng [18] and Niu [20]. Choose an arbitrary solution of system (1.2) and denote it by  $(S(t), I(t), P(t))$ . Then, by (2.2)-(2.6), there are constants  $0 < \varepsilon_1, \varepsilon_2 < 1$ , and  $t_1 > 0$  satisfying

$$\int_t^{t+\lambda} \left( \frac{\beta(\theta)(S_0(\theta) - \varepsilon_1)}{1 + \gamma(\theta)(S_0(\theta) - \varepsilon_1)} - e(\theta) - f(\theta) - \frac{c_2(\theta)(p_0(\theta) + \varepsilon_1)}{w_1(\theta) + S_0(\theta) - \varepsilon_1} \right) d\theta > \varepsilon_2 \tag{2.7}$$

and

$$S(t) \leq M_0, \quad I(t) \leq M_0, \quad M_0^{-1} \leq P(t) \leq M_0, \tag{2.8}$$

for all  $t \geq t_1$ .

First of all, we prove that there is a constant  $\alpha > 0$ , being independent of any positive solution of system (1.2) and satisfying

$$\limsup_{t \rightarrow \infty} I(t) > \alpha. \tag{2.9}$$

Consider the auxiliary equation

$$\dot{x}(t) = \Lambda(t) - d(t)x(t) - \left( \beta(t)M_0\alpha + \frac{c_1(t)}{w_1(t)}M_0^2 \right). \tag{2.10}$$

Applying Lemma 4 in [21], we see that for the given constants  $\varepsilon_1 > 0$  and  $M_0 > 0$ , there are positive constants  $\delta_1 = \delta_1(\varepsilon_1) > 0$ ,  $G_1 = G_1(\varepsilon_1, M_0) > 0$ , satisfying for any  $t_0 \in R_+$  and  $x_0 \in [0, M_0]$ , if  $\beta(t)M_0\alpha < \delta_1$  for all  $t \geq t_0$ ,

$$|x(t, t_0, x_0) - S_0(t)| < \varepsilon_1, \quad \text{for all } t \geq t_0 + G_1. \tag{2.11}$$

Here,  $x(t, t_0, x_0)$  is the solution of equation (2.10) with initial value  $x(t_0) = x_0$ .

In addition, we consider the equation

$$\dot{v}(t) = v \left( r(t) - \frac{c_3(t)}{w_2(t)}v + \frac{c_3(t)}{w_2^2(t)}(\sigma(t)M_0^2 + M_0\alpha) \right). \tag{2.12}$$

Based on Lemma 2 in [20], for the given constants  $\varepsilon_1 > 0$  and  $M_0 > 0$ , there are positive constants  $\delta_2 = \delta_2(\varepsilon_1) > 0$ ,  $G_2 = G_2(\varepsilon_1, M_0) > 0$ , satisfying that, for any  $t_0 \in R_+$  and  $M_0^{-1} \leq v_0 \leq M_0$ , if  $\frac{c_3(t)}{w_2^2(t)}M_0\alpha < \delta_2$  for all  $t \geq t_0$ , we have

$$|v(t, t_0, v_0) - p_0(t)| < \varepsilon_1, \quad \text{for all } t \geq t_0 + G_2, \tag{2.13}$$

and here,  $v(t, t_0, v_0)$  is the solution of equation (2.12) with initial value  $v(t_0) = v_0$ .

Choose a constant  $\alpha_0 = \frac{1}{2} \left\{ \frac{\delta_1}{\beta^\mu M_0 + 1}, \frac{\delta_2}{(c_3/w_2^2)^\mu M_0 + 1} \right\}$  and suppose (2.9) is not true, then for the positive solution  $(S(t), I(t), P(t))$  of system (1.2), there exists a  $Z \in R_3^+$  satisfying initial condition  $(S(0), I(0), P(0)) = Z$  and

$$\limsup_{t \rightarrow \infty} I(t) < \alpha_0.$$

Thus, from the definition of a superior limit, we see that there is a constant  $t_2 (> t_1)$  such that

$$I(t) < \alpha_0, \tag{2.14}$$

for all  $t \geq t_2$ . Hence, from model (1.2), we obtain

$$\dot{S}(t) \geq \Lambda(t) - d(t)S(t) - \frac{c_1(t)}{w_1(t)}M_0^2 - \beta(t)\alpha_0M_0.$$

Let  $x(t), v(t)$  be the solution of equations (2.10), (2.12), which satisfy the conditions  $x(t_2) = S(t_2)$  and  $v(t_2) = P(t_2)$ , respectively. Applying the comparison theorem, we have

$$S(t) \geq x(t), \quad P(t) \leq v(t),$$

for all  $t \geq t_2$ . So by (2.11), (2.13) we get

$$S(t) \geq S_0(t) - \varepsilon_1, \quad \text{for all } t \geq t_2 + G_1, \tag{2.15}$$

and

$$P(t) \leq p_0(t) + \varepsilon_1, \quad \text{for all } t \geq t_2 + G_2. \tag{2.16}$$

Then, from the equation for  $I(t)$  in system (1.2), we further have

$$\dot{I}(t) \geq I(t) \left[ \frac{\beta(t)(S_0(t) - \varepsilon_1)}{1 + \gamma(t)(S_0(t) - \varepsilon_1)} - (e(t) + f(t)) - \frac{c_2(t)(p_0(t) + \varepsilon_1)}{w_1(t) + S_0(t) - \varepsilon_1} \right], \quad \text{for all } t \geq T^*,$$

where  $T^* = t_2 + G_1 + G_2$ , thus

$$I(t) \geq I(T^*) \exp \left( \int_{T^*}^t \left[ \frac{\beta(\theta)(S_0(\theta) - \varepsilon_1)}{1 + \gamma(\theta)(S_0(\theta) - \varepsilon_1)} - (e(\theta) + f(\theta)) - \frac{c_2(\theta)(p_0(\theta) + \varepsilon_1)}{w_1(\theta) + S_0(\theta) - \varepsilon_1} \right] d\theta \right).$$

Therefore, from (2.7), we have  $I(t) \rightarrow +\infty$ , as  $t \rightarrow +\infty$ , which contradicts (2.14). Hence, (2.9) is true.

Second, we claim that it is impossible that  $I(t) \leq \alpha_0$ , for all  $t \geq t_0$ . From this claim, we have two cases. In the first case, there exists a  $T \geq T^*$ , such that  $I(t) \geq \alpha_0$  for all  $t \geq T$  and in the second case,  $I(t)$  oscillates about  $\alpha_0$  for all large  $t$ .

Obviously, we merely have to take the second case into consideration. Now, we are in a position to prove  $I(t) \geq \alpha_0 \exp(-h_1 H + h_2 \lambda) \triangleq m$  for sufficiently large  $t$ , where

$$h_1 = \limsup_{t \geq 0} \left[ e(t) + f(t) + \frac{c_2(t)}{w_1(t)} M_0 \right],$$

$$h_2 = \limsup_{t \geq 0} \left[ \beta(t) S_0(t) + e(t) + f(t) + \frac{c_2(t)(y_0(t) + \varepsilon_1)}{w_1(t) + S_0(t)} \right],$$

and

$$H = \max\{B_1, B_2\}.$$

Let  $t_1^*, t_2^*$  be sufficiently large such that

$$I(t_1^*) = I(t_2^*) = \alpha_0; \quad I(t) < \alpha_0, \quad \text{for all } t \in (t_1^*, t_2^*).$$

If  $t_2^* - t_1^* \leq H$ , then considering the second equation of model (1.2) and integrating it from  $t_1^*$  to  $t$ , we have

$$\begin{aligned} I(t) &= I(t_1^*) \exp \left( \int_{t_1^*}^t \left[ \frac{\beta(\theta)S(\theta)}{1 + \gamma(\theta)S(\theta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)I(\theta)} - \frac{c_2(\theta)P(\theta)}{w_1(\theta) + S(\theta) + I(\theta)} \right] d\theta \right) \\ &\geq I(t_1^*) \exp \left( \int_{t_1^*}^t \left[ -e(\theta) - f(\theta) - \frac{c_2(\theta)}{w_1(\theta)} M_0 \right] d\theta \right) \\ &\geq \alpha_0 \exp(-h_1 H), \quad \text{for all } t \in [t_1^*, t_2^*]. \end{aligned} \tag{2.17}$$

If  $t_2^* - t_1^* > H$ , taking a similar proof as that in (2.15), (2.16), we obtain

$$S(t) \geq S_0(t) - \varepsilon_1, \quad P(t) \leq p_0(t) + \varepsilon_1, \quad \text{for all } t \in [t_1^* + H, t_2^*]. \tag{2.18}$$

Then for any  $t \in [t_1^*, t_2^*]$ , when  $t \leq t_1^* + H$ , we have

$$I(t) \geq \alpha_0 \exp(-h_1 H).$$

When  $t > t_1^* + H$ , we choose a nonnegative integer  $q$  such that  $t \in [t_1^* + H + q\lambda, t_1^* + H + (q + 1)\lambda)$ , then by (2.7), (2.17), and (2.18) we have

$$\begin{aligned} I(t) &= I(t_1^* + H) \\ &\times \exp\left(\int_{t_1^* + H}^t \left[ \frac{\beta(\theta)S(\theta)}{1 + \gamma(\theta)S(\theta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)I(\theta)} - \frac{c_2(\theta)P(\theta)}{w_1(\theta) + S(\theta) + I(\theta)} \right] d\theta\right) \\ &\geq \alpha_0 \exp(-h_1 H) \\ &\times \exp\left(\int_{t_1^* + H}^t \left[ \frac{\beta(\theta)(S_0(\theta) - \varepsilon_1)}{1 + \gamma(\theta)(S_0(\theta) - \varepsilon_1)} - e(\theta) - f(\theta) - \frac{c_2(\theta)(p_0(\theta) + \varepsilon_1)}{w_1(\theta) + S_0(\theta) - \varepsilon_1} \right] d\theta\right) \\ &= \alpha_0 \exp(-h_1 H) \exp\left\{ \left[ \int_{t_1^* + H}^{t_1^* + H + q\lambda} + \int_{t_1^* + H + q\lambda}^t \right] \right. \\ &\quad \left. \times \left( \frac{\beta(\theta)(S_0(\theta) - \varepsilon_1)}{1 + \gamma(\theta)(S_0(\theta) - \varepsilon_1)} - e(\theta) - f(\theta) - \frac{c_2(\theta)(p_0(\theta) + \varepsilon_1)}{w_1(\theta) + S_0(\theta) - \varepsilon_1} \right) d\theta \right\} \\ &\geq \alpha_0 \exp(-h_1 H) \\ &\quad \times \exp\left(\int_{t_1^* + H + q\lambda}^t \left[ \frac{\beta(\theta)(S_0(\theta) - \varepsilon_1)}{1 + \gamma(\theta)(S_0(\theta) - \varepsilon_1)} - e(\theta) - f(\theta) - \frac{c_2(\theta)(p_0(\theta) + \varepsilon_1)}{w_1(\theta) + S_0(\theta) - \varepsilon_1} \right] d\theta\right) \\ &\geq \alpha_0 \exp(-(h_1 H + h_2 \lambda)) \\ &\triangleq m. \end{aligned}$$

Thus, we finally obtain

$$I(t) \geq m, \quad \text{for all } t \in [t_1^*, t_2^*].$$

This completes the proof. □

Next we turn to a discussion of how to control the disease and have the following result.

**Theorem 2.3** *Under assumptions (B1), (B2), (B3), if there are constants  $\xi, \lambda^* > 0$  satisfying*

$$(B4) \quad \liminf_{t \rightarrow \infty} \int_t^{t+\xi} \beta(\theta) d\theta > 0,$$

$$(B5) \quad \limsup_{t \rightarrow +\infty} \frac{1}{\lambda^*} \int_t^{t+\lambda^*} \left( \frac{\beta(\theta)S_0^*(\theta)}{1 + \gamma(\theta)S_0^*(\theta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)S_0^*(\theta)} - \frac{c_2(\theta)p_0^*(\theta)}{w_1(\theta) + S_0^*(\theta)} \right) d\theta \leq 0,$$

where  $S_0^*(t), p_0^*(t)$  are fixed solutions of the following equations:

$$\dot{S}(t) = \Lambda(t) - d(t)S$$

and

$$\dot{p}(t) = p \left( r(t) - \frac{c_3(t)}{w_2(t)} p \right),$$

respectively, then the infected prey  $I$  will go to extinction.

*Proof* First of all, we prove that there is a constant  $t_1 \geq T$  satisfying  $I(t_1) < \sigma$ , where  $\sigma$  is a sufficiently small positive constant.

By assumption (B4), there are constants  $\eta > 0$  and  $T_0 > 0$  satisfying

$$\int_t^{t+\xi} \beta(\theta) d\theta \geq \eta, \quad \text{for all } t \geq T_0.$$

For any sufficiently small  $0 < \sigma < 1$ , let  $\sigma_0 = \min\{\frac{\lambda^* \eta \sigma}{2\xi}, \frac{1}{2} \eta \sigma\}$ . If (B5) holds, we can see that there exist  $\delta > 0$  and  $T_1 \geq T_0$  satisfying

$$\int_t^{t+\lambda^*} \left( \frac{\beta(\theta)(S_0^*(\theta) + \delta)}{1 + \gamma(\theta)(S_0^*(\theta) + \delta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)(S_0^*(\theta) + \delta)} - \frac{c_2(\theta)(p_0^*(\theta) - \delta)}{w_1(\theta) + S_0^*(\theta) + \delta} \right) d\theta \leq \sigma_0,$$

for all  $t \geq T_1$ . Let  $n_0$  be an integer such that  $\frac{2\xi}{\lambda^*} \leq n_0 \leq \frac{2\xi}{\lambda^*} + 1$  and  $\lambda_0 = n_0 \lambda^*$ , then

$$\begin{aligned} & \int_t^{t+\lambda_0} \left( \frac{\beta(\theta)(S_0^*(\theta) + \delta)}{1 + \gamma(\theta)(S_0^*(\theta) + \delta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)(S_0^*(\theta) + \delta)} \right. \\ & \quad \left. - \frac{c_2(\theta)(p_0^*(\theta) - \delta)}{w_1(\theta) + S_0^*(\theta) + \delta} - \beta(\theta)\sigma \right) d\theta \\ & \leq \int_t^{t+n_0\lambda^*} \left( \frac{\beta(\theta)(S_0^*(\theta) + \delta)}{1 + \gamma(\theta)(S_0^*(\theta) + \delta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)(S_0^*(\theta) + \delta)} \right. \\ & \quad \left. - \frac{c_2(\theta)(p_0^*(\theta) - \delta)}{w_1(\theta) + S_0^*(\theta) + \delta} \right) d\theta \\ & \quad - \int_t^{t+2\xi} \beta(\theta)\sigma d\theta \\ & \leq n_0\sigma_0 - 2\eta\sigma \\ & \leq -\frac{1}{2}\eta\sigma. \end{aligned} \tag{2.19}$$

By the first two equations of system (1.2), Lemma 2.1 in [18], and applying the comparison theorem, we see that there is a constant  $T_2 \geq T_1$  satisfying

$$S(t) + I(t) \leq S_0^*(t) + \delta, \quad \text{for all } t \geq T_2.$$

Moreover, from model (1.2), we also see that there is a  $T_3 \geq T_2$  satisfying

$$P(t) \geq p_0^*(t) - \delta, \quad \text{for all } t \geq T_3.$$

Denote  $T = \max\{T_2, T_3\}$ ,  $h = \sup_{t \geq T} \{\beta(t)(S_0^*(t) + \delta) + e(t) + f(t) + \frac{c_2(t)p_0^*(t)}{w_1(t) + S_0^*(t)} + \beta(t)\}$ , so for all  $t \geq T$ , it yields

$$\dot{I}(t) \leq I(t) \left[ \frac{\beta(t)(S_0^*(t) + \delta - I(t))}{1 + \gamma(t)(S_0^*(t) + \delta - I(t))} - e(t) - \frac{f(t)}{1 + \alpha(t)(S_0^*(t) + \delta)} - \frac{c_2(t)(p_0^*(t) - \delta)}{w_1(t) + S_0^*(t) + \delta} \right]. \tag{2.20}$$

Suppose that  $I(t) \geq \sigma$  for all  $t \geq T$ , then let  $q^* \geq 0$  be an integer satisfying  $t \in [T + q^*\lambda_0, T + (q^* + 1)\lambda_0)$ , and we integrate (2.20) from  $T$  to  $t$ , yielding

$$\begin{aligned} I(t) &\leq I(T) \exp\left(\int_T^t \left[ \frac{\beta(\theta)(S_0^*(\theta) + \delta)}{1 + \gamma(\theta)(S_0^*(\theta) + \delta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)(S_0^*(\theta) + \delta)} \right. \right. \\ &\quad \left. \left. - \frac{c_2(\theta)(p_0^*(\theta) - \delta)}{w_1(\theta) + S_0^*(\theta) + \delta} - \beta(\theta)\sigma \right] d\theta\right) \\ &= I(T) \exp\left\{ \left[ \int_T^{T+q^*\lambda_0} + \int_{T+q^*\lambda_0}^t \right] \right. \\ &\quad \times \left( \frac{\beta(\theta)(S_0^*(\theta) + \delta)}{1 + \gamma(\theta)(S_0^*(\theta) + \delta)} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)(S_0^*(\theta) + \delta)} \right. \\ &\quad \left. \left. - \frac{c_2(\theta)(p_0^*(\theta) - \delta)}{w_1(\theta) + S_0^*(\theta) + \delta} - \beta(\theta)\sigma \right) d\theta \right\} \\ &\leq I(T) \exp\left(-\frac{1}{2}\eta\sigma q^*\right) \exp(\lambda_0 h). \end{aligned}$$

Thus,  $I(t) \rightarrow 0$  as  $t \rightarrow +\infty$ , which contradicts  $I(t) \geq \sigma$ , and we can see that there must be a  $t_1 \geq T$  such that  $I(t_1) < \sigma$ .

Next, we prove that

$$I(t) \leq \sigma \exp(h\lambda_0) \tag{2.21}$$

for all  $t \geq t_1$ . If the above inequality is not true, then there is a  $t_2 > t_1$  satisfying  $I(t_2) > \sigma \exp(h\lambda_0)$ . Therefore, there must be a constant  $t_3 \in (t_1, t_2)$  satisfying  $I(t_3) = \sigma$  and  $I(t) > \sigma$  for all  $t \in (t_3, t_2)$ . Then we can choose an integer  $l_1 \geq 0$  such that  $t_2 \in [t_3 + l_1\lambda_0, t_3 + (l_1 + 1)\lambda_0)$  and integrate (2.20) from  $t_3$  to  $t_2$ , and we have

$$\begin{aligned} \sigma \exp(h\lambda_0) &< I(t_2) \\ &\leq I(t_3) \exp\left(\int_{t_3}^{t_2} \left[ \frac{\beta(t)(S_0^*(t) + \delta - I(t))}{1 + \gamma(t)(S_0^*(t) + \delta - I(t))} - e(t) - \frac{f(t)}{1 + \alpha(t)(S_0^*(t) + \delta)} \right. \right. \\ &\quad \left. \left. - \frac{c_2(t)(p_0^*(t) - \delta)}{w_1(t) + S_0^*(t) + \delta} \right] dt\right) \\ &\leq I(t_3) \exp\left\{ \left[ \int_{t_3}^{t_3+l_1\lambda_0} + \int_{t_3+l_1\lambda_0}^{t_2} \right] \right. \\ &\quad \times \left( \frac{\beta(t)(S_0^*(t) + \delta)}{1 + \gamma(t)(S_0^*(t) + \delta)} - e(t) - \frac{f(t)}{1 + \alpha(t)(S_0^*(t) + \delta)} \right. \\ &\quad \left. \left. - \frac{c_2(t)(p_0^*(t) - \delta)}{w_1(t) + S_0^*(t) + \delta} - \beta(t)\sigma \right) dt \right\} \\ &\leq \sigma \exp\left(-\frac{1}{2}\eta\sigma l_1\right) \exp(\lambda_0 h). \end{aligned}$$

It is a contradiction. Therefore, (2.21) holds.

Finally, as  $\sigma$  is an arbitrarily small constant, we can obtain  $I(t) \rightarrow 0$ , as  $t \rightarrow +\infty$ .

This completes the proof. □

Next, the global attractivity of the model will be discussed. First, the definition will be given below.

**Definition 2.1** ([20]) The system (1.2) is said to be globally attractive if any two solutions  $(S_1(t), I_1(t), P_1(t))$  and  $(S_2(t), I_2(t), P_2(t))$  of system (1.2) with initial conditions (1.3) satisfy

$$\lim_{t \rightarrow +\infty} |S_1(t) - S_2(t)| = 0, \quad \lim_{t \rightarrow +\infty} |I_1(t) - I_2(t)| = 0, \quad \lim_{t \rightarrow +\infty} |P_1(t) - P_2(t)| = 0.$$

**Theorem 2.4** *Under assumptions (B1), (B2), (B3), if there exist constants  $\mu_i > 0$  ( $i = 1, 2, 3$ ) satisfying  $\liminf_{t \rightarrow \infty} A_i(t) > 0$ , where*

$$\begin{aligned} A_1(t) &= \mu_1 d(t) - \mu_2 \beta(t) - \mu_1 \left[ \frac{c_1(t)}{w_1(t)} M_2 + \frac{(c_2(t) - c_1(t))}{w_1^2(t)} M_1 M_2 \right] \\ &\quad - \mu_2 \frac{c_2(t) M_2}{w_1^2(t)} - \mu_3 \frac{c_3(t) \sigma(t)}{w_2^2(t)} M_2, \\ A_2(t) &= \mu_2 \frac{\beta(t)}{(1 + \gamma(t) M_1)^2} + \mu_1 [d(t) - e(t)] - \mu_1 M_2 \frac{(c_2(t) - c_1(t))(w_1(t) + M_1)}{w_1^2(t)} \\ &\quad - \mu_2 \alpha(t) f(t) - \mu_3 M_2 \frac{c_3(t)(\sigma(t) + 1)}{w_2^2(t)}, \\ A_3(t) &= -\mu_1 \frac{c_2(t) M_1}{w_1^2(t)} (w_1(t) + M_1) - \mu_2 \frac{c_2(t)(w_1(t) + M_1)}{w_1^2(t)} + \mu_3 \frac{c_3(t)[w_2(t) + \sigma(t) m_1]}{[w_2(t) + (\sigma(t) + 1) M_1]^2}, \end{aligned} \tag{2.22}$$

then system (1.2) is globally attractive.

*Proof* Let  $x = S + I$ , then model (1.2) can be rewritten as follows:

$$\begin{aligned} \dot{x}(t) &= \Lambda(t) - d(t)x - [e(t) - d(t)]I - \frac{c_1(t)}{w_1(t) + x} P(x - I) - \frac{c_2(t)}{w_1(t) + x} PI, \\ \dot{I}(t) &= I \left[ \frac{\beta(t)(x - I)}{1 + \gamma(t)(x - I)} - e(t) - \frac{c_2(t)P}{w_1(t) + x} - \frac{f(t)}{1 + \alpha(t)I} \right], \\ \dot{P}(t) &= P \left[ r(t) - \frac{c_3(t)}{w_2(t) + \sigma(t)x + (1 - \sigma(t))I} P \right]. \end{aligned} \tag{2.23}$$

Suppose that  $(x_1(t), I_1(t), P_1(t))$ ,  $(x_2(t), I_2(t), P_2(t))$  are two arbitrary solutions of model (2.23). By (2.2), (2.3), we obtain

$$m_1 \leq x_k(t) \leq M_1, \quad I_k(t) \leq M_1, \quad P_k(t) \leq M_2, \quad \text{for all } t \geq 0 \text{ and } k = 1, 2. \tag{2.24}$$

Define a Liapunov function

$$V(t) = \mu_1 |x_1(t) - x_2(t)| + \mu_2 |\ln I_1(t) - \ln I_2(t)| + \mu_3 |\ln P_1(t) - \ln P_2(t)|.$$

Then we have

$$\begin{aligned} D^+(V(t)) &= \mu_1 \operatorname{sgn}(x_1 - x_2) \left\{ -d(t)(x_1 - x_2) - (e(t) - d(t))(I_1 - I_2) \right. \\ &\quad \left. - c_1(t) \left[ \frac{P_1 x_1}{w_1(t) + x_1} - \frac{P_2 x_2}{w_1(t) + x_2} \right] + (c_1(t) - c_2(t)) \left[ \frac{P_1 I_1}{w_1(t) + x_1} - \frac{P_2 I_2}{w_1 + x_2} \right] \right\} \end{aligned}$$

$$\begin{aligned}
 & + \mu_2 \operatorname{sgn}(I_1 - I_2) \left\{ \frac{\beta(t)(x_1 - I_1)}{1 + \gamma(t)(x_1 - I_1)} - \frac{\beta(t)(x_2 - I_2)}{1 + \gamma(t)(x_2 - I_2)} - \frac{c_2(t)P_1}{w_1(t) + x_1} \right. \\
 & \left. + \frac{c_2(t)P_2}{w_1(t) + x_2} - \frac{f(t)}{1 + \alpha(t)I_1} + \frac{f(t)}{1 + \alpha(t)I_2} \right\} \\
 & + \mu_3 \operatorname{sgn}(P_1 - P_2) \left\{ -\frac{c_3(t)P_1}{w_2(t) + \sigma(t)x_1 + (1 - \sigma(t))I_1} \right. \\
 & \left. + \frac{c_3(t)P_2}{w_2(t) + \sigma(t)x_2 + (1 - \sigma(t))I_2} \right\} \\
 \leq & \mu_1 \left\{ -d(t)|x_1 - x_2| - (d(t) - e(t))|I_1 - I_2| + \frac{c_1(t)}{w_1(t)}M_2|x_1 - x_2| \right. \\
 & \left. + \left[ \frac{c_1(t)M_1}{w_1(t)} + \frac{(c_2(t) - c_1(t))M_1}{w_1(t)} + \frac{c_2(t)M_1^2}{w_1^2(t)} \right] |P_1 - P_2| \right. \\
 & + (c_2(t) - c_1(t)) \frac{M_2}{w_1(t)} |I_1 - I_2| \\
 & \left. + (c_2(t) - c_1(t)) \frac{M_1M_2}{w_1^2(t)} |I_1 - I_2| + (c_2(t) - c_1(t)) \frac{M_1M_2}{w_1^2(t)} |x_1 - x_2| \right\} \\
 & + \mu_2 \left\{ \beta(t)|x_1 - x_2| - \frac{\beta(t)}{(1 + \gamma(t)M_1)^2} |I_1 - I_2| + f(t)\alpha(t)|I_1 - I_2| \right. \\
 & \left. + c_2(t) \frac{(M_1 + w_1(t))}{w_1^2(t)} |P_1 - P_2| + c_2(t) \frac{M_2}{w_1^2(t)} |x_1 - x_2| \right\} \\
 & + \mu_3 \left\{ -c_3(t) \frac{(w_2(t) + m_1\sigma(t))}{[w_2(t) + (1 + \sigma(t))M_1]^2} |P_1 - P_2| \right. \\
 & \left. + \frac{c_3(t)\sigma(t)M_2}{w_2^2(t)} |x_1 - x_2| + \frac{(1 + \sigma(t))M_2}{w_2^2(t)} |I_1 - I_2| \right\} \\
 = & \left\{ -\mu_1 d(t) + \mu_2 \beta(t) + \mu_1 \left[ \frac{c_1(t)}{w_1(t)}M_2 + \frac{(c_2(t) - c_1(t))}{w_1^2(t)}M_1M_2 \right] \right. \\
 & \left. + \mu_2 \frac{c_2(t)M_2}{w_1^2(t)} \right\} |x_1 - x_2| + \mu_3 \frac{c_3(t)\sigma(t)M_2}{w_2^2(t)} |x_1 - x_2| \\
 & + \left\{ -\mu_2 \frac{\beta(t)}{(1 + \gamma(t)M_1)^2} - \mu_1 [d(t) - e(t)] \right\} |I_1 - I_2| \\
 & + \left\{ \mu_1 M_2 \frac{(c_2(t) - c_1(t))(w_1(t) + M_1)}{w_1^2(t)} + \mu_2 \alpha(t)f(t) \right. \\
 & \left. + \mu_3 M_2 c_3(t) \frac{(1 + \sigma(t))}{w_2^2(t)} \right\} |I_1 - I_2| \\
 & + \left\{ \mu_1 \frac{c_2(t)M_1}{w_1^2(t)} (w_1(t) + M_1) + \mu_2 \frac{c_2(t)(w_1(t) + M_1)}{w_1(t)^2} \right. \\
 & \left. - \mu_3 \frac{c_3(t)[w_2(t) + m_1\sigma(t)]}{[w_2(t) + (1 + \sigma(t))M_1]^2} \right\} |P_1 - P_2|.
 \end{aligned}$$

Applying the conditions  $\liminf_{t \rightarrow \infty} A_i(t) > 0$  ( $i = 1, 2, 3$ ) and the definition of the inferior limit, we see that there are constants  $\bar{\alpha} > 0$  and  $T^\circ > 0$  such that  $A_i(t) \geq \bar{\alpha}$  ( $i = 1, 2, 3$ ) for all  $t \geq T^\circ$ . Thus we obtain

$$D^+(V(t)) \leq -\bar{\alpha} (|x_1 - x_2| + |I_1 - I_2| + |P_1 - P_2|), \tag{2.25}$$

for all  $t \geq T^*$ . Integrating (2.25) from  $T^*$  to  $t$ , we obtain

$$V(t) - V(T^*) \leq -\bar{\alpha} \int_{T_0}^t (|x_1(s) - x_2(s)| + |I_1(s) - I_2(s)| + |P_1(s) - P_2(s)|) ds,$$

therefore,

$$\bar{\alpha} \int_{T_0}^t (|x_1(s) - x_2(s)| + |I_1(s) - I_2(s)| + |P_1(s) - P_2(s)|) ds \leq V(T^*) < +\infty. \tag{2.26}$$

At the same time, by (2.23), (2.24), it can be seen that  $\frac{d}{dt}(x_1 - x_2)$ ,  $\frac{d}{dt}(I_1 - I_2)$ ,  $\frac{d}{dt}(P_1 - P_2)$  are all bounded on  $[0, \infty)$ . By (2.26), we see that

$$\lim_{t \rightarrow \infty} |x_1(t) - x_2(t)| = 0, \quad \lim_{t \rightarrow \infty} |I_1(t) - I_2(t)| = 0, \quad \lim_{t \rightarrow \infty} |P_1(t) - P_2(t)| = 0.$$

The proof is completed. □

**Remark 1** For model (1.1), we can also give the condition of the global attractivity for this *SI* model without predation as that for model (1.2) in Theorem 2.4, that is, if

$$\begin{aligned} &\liminf_{t \rightarrow \infty} [\mu_1 d(t) - \mu_2 \beta(t)] > 0, \\ &\liminf_{t \rightarrow \infty} \left[ \mu_2 \frac{\beta(t)}{(1 + \gamma(t)M_1)^2} + \mu_1 (d(t) - e(t)) - \mu_2 \alpha(t)f(t) \right] > 0, \end{aligned}$$

then system (1.1) is globally attractive.

### 3 Some results for the periodic system

If model (1.2) is an  $\omega$ -periodic system, then assumptions (B1), (B2), (B4) can degenerate into the following forms:

- (A1) Parameters  $\Lambda(t)$ ,  $\beta(t)$ ,  $\gamma(t)$ ,  $\alpha(t)$ ,  $d(t)$ ,  $e(t)$ ,  $f(t)$ ,  $r(t)$ ,  $\sigma(t)$ ,  $w_i(t)$  ( $i = 1, 2$ ), and  $c_i(t)$  ( $i = 1, 2, 3$ ) are all nonnegative, continuous periodic functions which have a period  $\omega > 0$ ,
- (A2)  $\bar{\Lambda} > 0$ ,  $\bar{d} > 0$ ,  $\bar{r} > 0$ ,  $\bar{e} > 0$ ,  $\overline{c_1/w_1} > 0$ ,  $\overline{c_3/w_2} > 0$ ,
- (A4)  $\bar{\beta} > 0$ .

Then we have some results for the periodic system as shown below.

**Corollary 3.1** *Under assumptions (A1), (A2), (B3), if*

$$R_* = \frac{\overline{\beta S_0 / (1 + \gamma S_0)}}{(\bar{e} + \bar{f} + \frac{c_2 p_0^*}{w_1 + S_0})} > 1,$$

*then the infective prey population  $I$  of model (1.2) is permanent.*

**Corollary 3.2** *Under assumptions (A1), (A2), (B3), (A4), if*

$$R^* = \frac{\overline{\beta S_0^* / (1 + \gamma S_0^*)}}{(\bar{e} + \frac{\bar{f}}{1 + \alpha S_0^*} + \frac{c_2 p_0^*}{w_1 + S_0^*})} \leq 1,$$

*then the infective prey population  $I$  of model (1.2) goes to extinction.*

**Corollary 3.3** Under assumptions (B1), (B2), if  $d^v > 0$ ,  $w_1^v > 0$ ,  $(\frac{c_3}{w_2})^v > 0$ , and there exists a constant  $\lambda > 0$  satisfying

$$\liminf_{t \rightarrow \infty} \frac{1}{\lambda} \int_t^{t+\lambda} \left( \frac{\beta(\theta)(\frac{\Lambda}{d} - \frac{c_1 M_0^2}{w_1 d})^v}{1 + \gamma(\theta)(\frac{\Lambda}{d} - \frac{c_1 M_0^2}{w_1 d})^v} - e(\theta) - f(\theta) - \frac{c_2(\theta)}{w_1(\theta) + (\frac{\Lambda}{d} - \frac{c_1 M_0^2}{w_1 d})^v} \left( \frac{r w_2}{c_3} + \frac{\sigma M_0^2}{w_2} \right)^u \right) d\theta > 0,$$

then the infective prey population  $I$  of model (1.2) is permanent.

**Corollary 3.4** Under assumptions (B1), (B2), if  $d^v > 0$ ,  $w_1^v > 0$ ,  $(\frac{c_3}{w_2})^v > 0$ , and there exist constants  $\lambda^* > 0$ ,  $\lambda > 0$  satisfying

$$\liminf_{t \rightarrow \infty} \int_t^{t+\gamma} \beta(\theta) d\theta > 0,$$

$$\limsup_{t \rightarrow \infty} \frac{1}{\lambda^*} \int_t^{t+\lambda^*} \left( \frac{\beta(\theta)(\frac{\Lambda}{d})^u}{1 + \gamma(\theta)(\frac{\Lambda}{d})^u} - e(\theta) - \frac{f(\theta)}{1 + \alpha(\theta)(\frac{\Lambda}{d})^u} - \frac{c_2(\theta)(\frac{r w_2}{c_3})^v}{w_1(\theta) + (\frac{\Lambda}{d})^u} \right) d\theta \leq 0,$$

then the infective prey population  $I$  of model (1.2) goes to extinction.

**Remark 2** For model (1.1) without predation, assumptions (B1), (B2), (B4) are equivalent to the following forms:

- (D1) Parameters  $\Lambda(t)$ ,  $\beta(t)$ ,  $\alpha(t)$ ,  $\gamma(t)$ ,  $d(t)$ ,  $e(t)$ ,  $f(t)$  are all nonnegative, continuous periodic functions which have a period  $\omega > 0$ ,
- (D2)  $\bar{\Lambda} > 0$ ,  $\bar{d} > 0$ ,  $\bar{e} > 0$ ,
- (D4)  $\bar{\beta} > 0$ .

If assumptions (D1), (D2), (B3), (D4) hold, then from Corollaries 3.1 and 3.2, we can obtain the threshold value between extinction and permanence of the infective population in system (1.1), that is,

- (1) If  $\widehat{R} = \frac{\beta S_0^* / (1 + \gamma S_0^*)}{(\bar{e} + f / (1 + \alpha S_0^*))} \leq 1$ , then the infective prey population of model (1.1) goes to extinction;
- (2) If  $\widehat{R} = \frac{\beta S_0^* / (1 + \gamma S_0^*)}{(\bar{e} + f / (1 + \alpha S_0^*))} > 1$ , then the infective prey population of model (1.1) is permanent.

#### 4 Numerical simulation and discussion

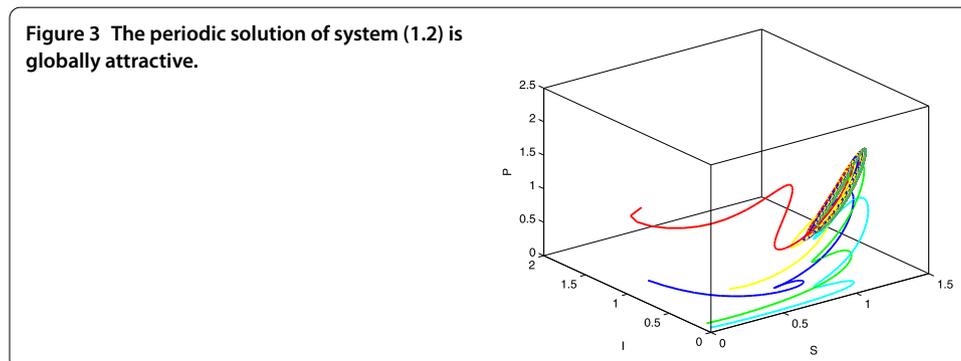
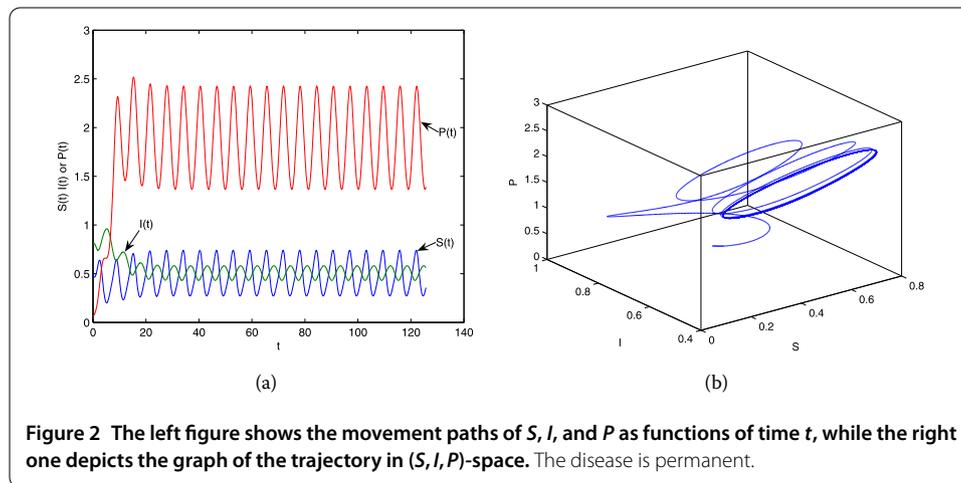
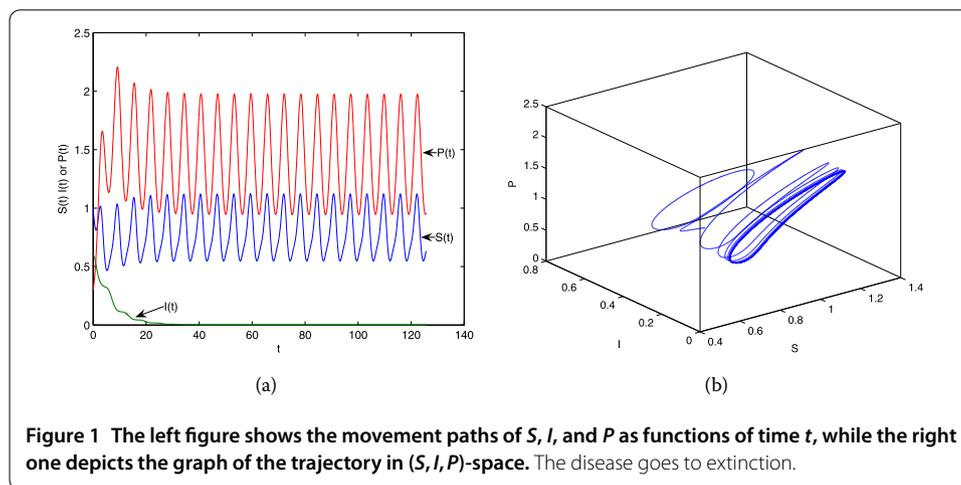
In this section, a set of numerical simulations are carried out to confirm and visualize our theoretical results. The role of predation on the system dynamics is discussed by comparing system (1.2) with the  $SI$  model (1.1). Moreover, the effects of the functional response in controlling disease is compared between system (1.2) and the model in [15].

First, for model (1.2), we choose the parameters  $\Lambda(t) = 0.5 + 0.3 \sin t$ ,  $d(t) = 0.6 + 0.2 \sin(2t)$ ,  $e(t) = 0.3 + 0.2 \sin t$ ,  $f(t) = 0.05 + 0.04 \sin t$ ,  $r(t) = 0.5 + 0.4 \sin t$ ,  $\alpha(t) = 0.2 + 0.01 \sin t$ ,  $\gamma(t) = 0.05 + 0.01 \sin t$ ,  $w_1(t) = 8 + 0.5 \cos t$ ,  $w_2(t) = 0.2 + 0.08 \sin t$ ,  $\sigma(t) = 0.8 + 0.1 \sin t$ ,  $c_1(t) = 0.2 + 0.1 \sin t$ ,  $c_2(t) = 0.4 + 0.1 \sin t$ ,  $c_3(t) = 0.3 + 0.1 \sin t$ . Then assumptions (C1), (C2), and (B3) hold. Let  $\beta(t) = 0.36 + 0.1 \sin t$ , by calculation we see that the upper threshold value  $R^* = 0.9062 < 1$ , which satisfies the conditions in Corollary 3.2. Thus, the infected prey population will go to extinction (see Figure 1). Then, let the infective rate

increase to  $\beta(t) = 0.9 + 0.1 \sin t$ , being similar to the above calculation, we can obtain the lower threshold value  $R_* = 2.2271 > 1$  and see that model (1.2) is permanent from Figure 2, which verifies the conclusion of Corollary 3.1.

Second, let  $d(t) = 0.5 + 0.1 \sin t$ ,  $e(t) = 1.2 + 0.2 \sin t$ ,  $\gamma(t) = 0.3 + 0.01 \sin t$ ,  $\beta(t) = 1.8 + 0.1 \sin t$ , and the other parameters are the same as in Figure 1. Considering system (1.2) with initial conditions  $(0.5, 1.7, 0.6)$ ,  $(0.01, 0.07, 0.09)$ ,  $(0.2, 1.1, 0.03)$ ,  $(0.03, 0.06, 0.02)$ ,  $(0.3, 0.3, 0.3)$ . From Figure 3, we can see that system (1.2) is globally attractive.

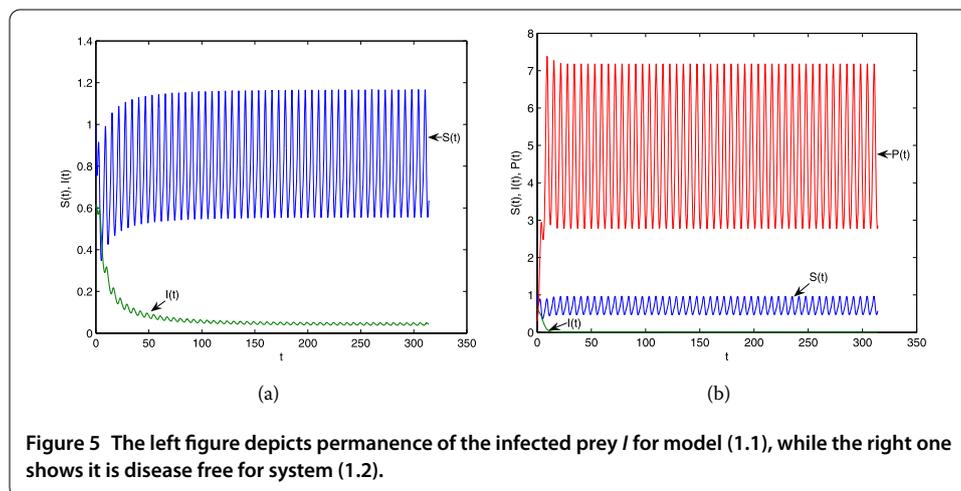
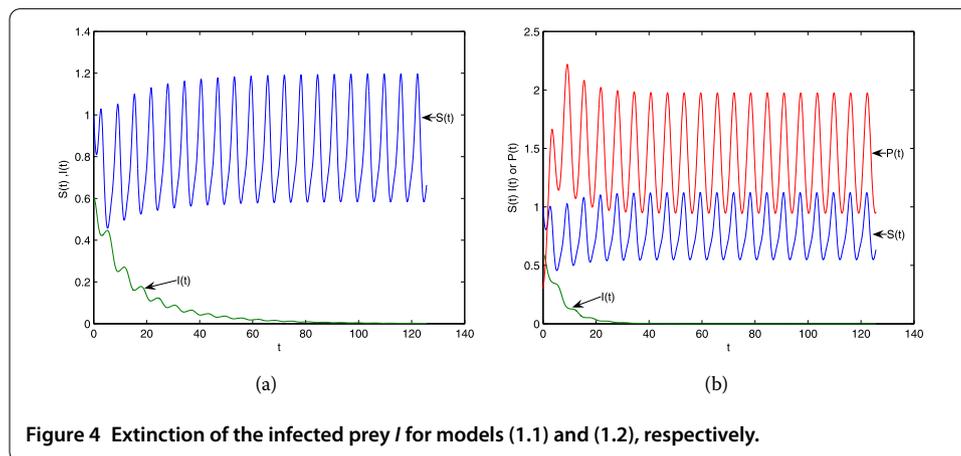
Third, we will study the role of predation on system dynamics through making a comparison between model (1.2) and (1.1).

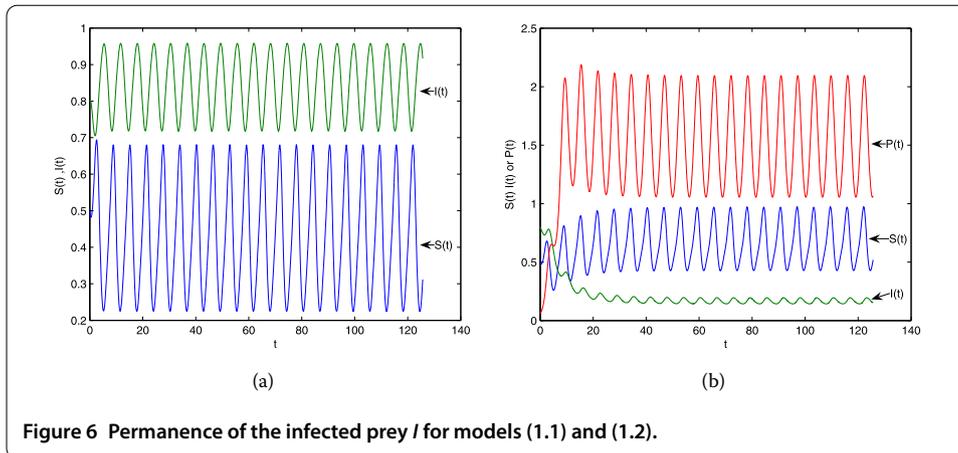


Now, let  $\beta(t) = 0.38 + 0.1 \sin t$  and retain the other parameter values as in Figure 1, then we can see that the two upper threshold values are  $\widehat{R}_* = 0.9987 < 1$ ,  $R_* = 0.9528 < 1$  for models (1.1) and (1.2), respectively, which shows that the infected prey  $I$  goes to extinction for both models (see Figure 4). Observing that  $\widehat{R}_* > R_*$ , which means we have predation, the infected prey  $I$  in model (1.2) will be extinct more early and easily than in system (1.1). The results can also be observed from Figure 5, in which we choose  $\beta(t) = 0.58 + 0.01 \sin t$ ,  $c_3(t) = 0.08 + 0.01 \sin t$ ,  $e(t) = 0.4 + 0.02 \sin t$ , and we can obtain the threshold values,  $\widehat{R}_* = 1.1019$ ,  $R_* = 0.9567$ , for models (1.1) and (2.2). The figure shows that model (1.2) is disease free, while the infected prey population for model (1.1) without predation is permanent. Then we conclude that the predator can be used as a bio-controller to keep the model disease free.

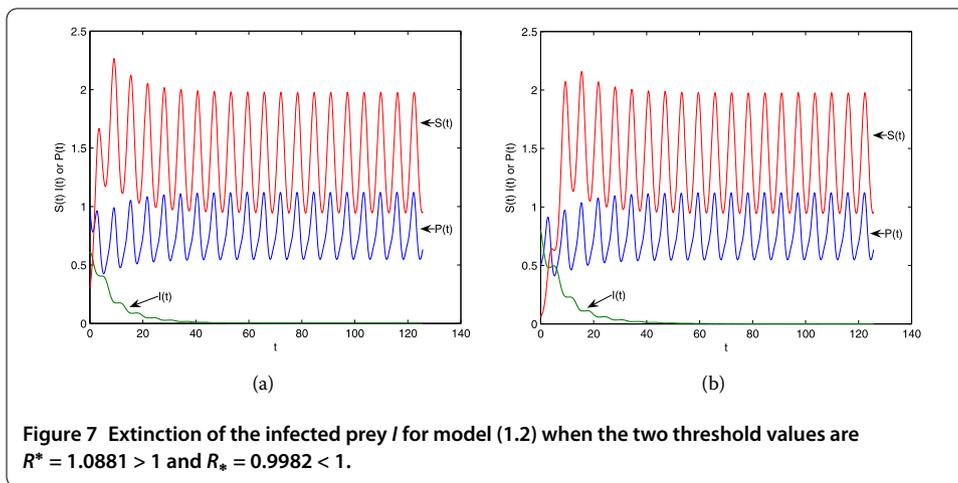
Next, let the infection rate increase to  $\beta(t) = 0.8 + 0.1 \sin t$ , we can easily get  $\widehat{R}_* = 2.0255 > 1$ ,  $R_* = 1.9611 > 1$  and  $\widehat{R}_* > R_*$  for models (1.1) and (1.2), respectively. Then we can observe that all the species of system (1.1) and (1.2) enter into a steady state from Figure 6. Therefore, we could conclude that predation is benefit for controlling disease and enhancing permanence in a predator-prey model.

Fourth, some discussions are given for the intermediate case where  $R_* > 1$  while  $R_* \leq 1$ . Choose the infection rate  $\beta(t) = 0.438 + 0.1 \sin t$  and retain the other parameter values as in Figure 1, then we can see that the two threshold values are  $R_* = 1.0881 > 1$  and  $R_* =$





**Figure 6** Permanence of the infected prey  $I$  for models (1.1) and (1.2).

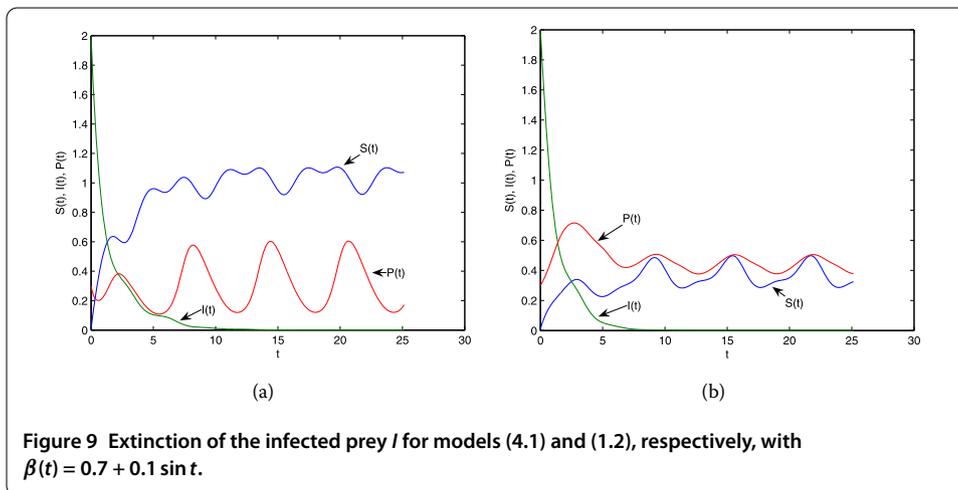
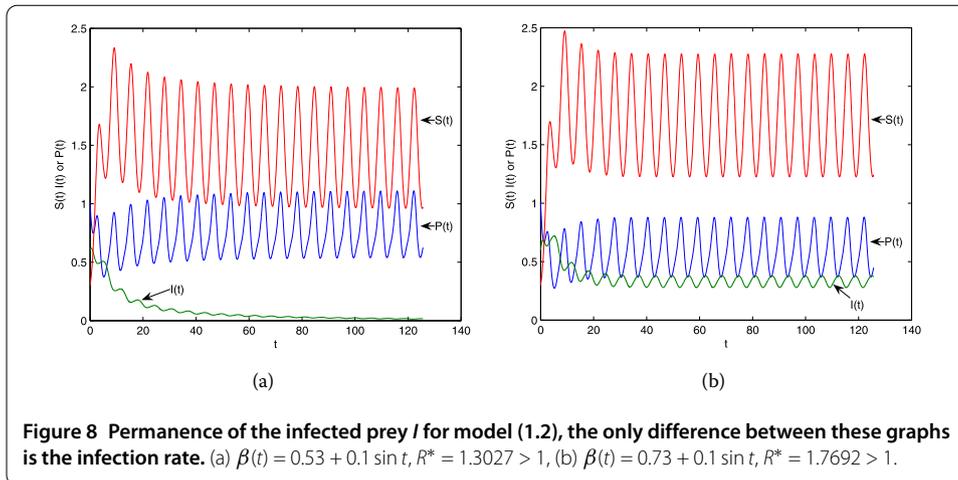


**Figure 7** Extinction of the infected prey  $I$  for model (1.2) when the two threshold values are  $R^* = 1.0881 > 1$  and  $R_* = 0.9982 < 1$ .

$0.9982 < 1$  for model (1.2). From Figure 7, it can be shown that the infected prey  $I$  goes to extinction. Changing the infection rate from  $\beta(t) = 0.53 + 0.1 \sin t$  to  $\beta(t) = 0.73 + 0.1 \sin t$ , then we have the upper threshold values  $R^* = 1.3027$  and  $R^* = 1.7692$ , respectively, which are also greater than 1, however, by Figure 8(a)-(b), it can be seen that the infected prey population is permanent. From Figure 7, it could be concluded that if the lower threshold value  $R^* \leq 1$ , the infected prey could go to extinction. In addition, comparing Figure 7(a) and Figure 8, it could be shown that the condition we obtained for the extinction of the infected prey is only a sufficient condition. What is the sufficient necessary condition? This will be left as our future consideration.

Last but not least, we turn to the role of the functional response in controlling disease. In [15], we considered a predator-prey model with a linear predation rate for migratory birds, that is,

$$\begin{aligned}
 \dot{S}(t) &= \Lambda(t) - \frac{\beta(t)S(t)I(t)}{1 + \alpha(t)S(t)} - d(t)S(t) + f(t)I(t) - k_1(t)S(t)P(t), \\
 \dot{I}(t) &= \frac{\beta(t)S(t)I(t)}{1 + \alpha(t)S(t)} - (e(t) + f(t))I(t) - k_2(t)I(t)P(t), \\
 \dot{P}(t) &= r(t)P(t) \left[ 1 - \frac{P(t)}{K(t)} \right] + k_1'(t)S(t)P(t) - k_2'(t)I(t)P(t),
 \end{aligned} \tag{4.1}$$



by theoretical analysis, we showed that if  $R^* = \frac{\overline{\beta S_0 / (1 + \alpha S_0)}}{(\overline{e} + f + k_2 \gamma_0)} \leq 1$ , then the infective prey population of system (4.1) goes to extinction. However, in this paper, we assume that the predator eat both the susceptible and the infected prey population with modified Leslie-Gower schemes and we obtain the upper threshold value  $R^*$  to determine the extinction of the infection. Now we give a numerical simulation to study the effects of different predation rates in controlling the disease. Let  $\Lambda(t) = 0.2 + 0.1 \sin t, d(t) = 0.5 + 0.2 \sin(2t), e(t) = 0.6 + 0.2 \sin(2t), f(t) = 0.05 + 0.045 \sin(2t), r(t) = 0.5 + 0.4 \sin t, \beta(t) = 0.7 + 0.1 \sin t, K(t) = 0.1 + 0.08 \sin t, w_1(t) = 2 + 0.5 \cos t, w_2(t) = 0.1 + 0.08 \sin t, c_1(t) = k_1(t) = 0.2 + 0.08 \sin t, c_2(t) = k_2(t) = 1 + 0.5 \cos t, c_3(t) = r(t) = 0.5 + 0.4 \sin t, \gamma(t) = 0.2 + 0.1 \sin t, k'_1(t) = 1.1 + \sin t, k'_2(t) = 0.2 + 0.08 \cos t, \sigma(t) = 0.8 + 0.1 \sin t$ , and  $\alpha(t) = 0.2 + 0.1 \sin t$  for system (4.1) while  $\alpha(t) = 0$  for system (1.2). Then we can obtain the upper threshold values  $R^* = 0.4226 < 1$  for model (1.2) and  $R^* = 0.8836 < 1$  for model (4.1), from which we see that the infected prey  $I$  goes to extinction for both models, and Figure 9 confirms it. Moreover, obviously,  $R^* < R^{l*}$  and we can conclude that the modified Leslie-Gower functional predation rate may be a good choice that can be used to control the disease more easily and effectively.

### 5 Conclusion

In this paper, a new nonautonomous predator-prey model for migratory birds has been considered. The main results for permanence, extinction of the disease, and global attrac-

tivity of the system are obtained in Theorems 2.1-2.4. Theorem 2.1 shows that the predator and prey in the model are permanent if the condition (2.1), which is the inferior limit of the minimum loss of the predator on interval  $[t, t + \omega_7]$  for some constant  $\omega_7 > 0$ , is established.

In Theorem 2.2,  $s_0(t)$  is the density of the susceptible prey without infected prey at time  $t$ , satisfying  $\dot{S}(t) = \Lambda(t) - \frac{c_1(t)}{k_1(t)} M_0^2 - d(t)S$ . It is shown that  $s_0(t)$  is a globally attractive state of the susceptible prey. In addition,  $p_0(t)$  is the density of the predator without any infected prey at time  $t$ , satisfying  $\dot{p}(t) = p(r(t) + \frac{c_3(t)}{k_2(t)} \sigma(t) M_0^2 - \frac{c_3(t)}{k_2(t)} p)$ . From Lemma 1 of [19], it can be shown that  $p_0(t)$  is also a globally attractive state of the predator. Then  $\beta(t)S_0(t) - e(t) - f(t) - \frac{c_2(t)p_0(t)}{k_1(t) + S_0(t)}$  is the available minimum growth rate of the infected prey at time  $t$ . Thus, the left hand of inequality (2.6) implies an inferior limit of the available minimum growth rate of the infected prey in the mean on the interval  $[t, t + \lambda]$ . By Theorem 2.2, the infected prey will be permanent when the inferior limit is positive.

Theorem 2.3 implies that the infected prey will be extinct when the superior limit of the available maximum growth rate of the infected prey in the mean on interval  $[t, t + \lambda^*]$  for some constant  $\lambda^* > 0$  is non-positive.

In Theorem 2.4, through constructing a Liapunov function, a diagonal dominance condition for the global attractivity of system (1.2) is presented.

#### Competing interests

The authors declare that they have no competing of interests regarding the publication of this paper.

#### Authors' contributions

All authors, KF, YZ, and SG, contributed substantially to this paper, participated in drafting and checking the manuscript, and have approved the version to be published.

#### Author details

<sup>1</sup>School of Mechanical and Electrical Engineering, Jiangxi University of Science and Technology, Ganzhou, 341000, P.R. China. <sup>2</sup>School of Mathematics and Statistics, Wuhan University, Wuhan, 430000, P.R. China. <sup>3</sup>School of Mathematics and Computer Science, Gannan Normal University, Ganzhou, 341000, P.R. China.

#### Acknowledgements

The research have been supported by The Natural Science Foundation of China (11261004), the bidding project of Gannan Normal university (15zb01), The Foundation of Education Committee of Jiangxi (GJJ150674) and the key projects of the Natural Science Foundation of Jiangxi University of Science and Technology (NSFJ2015-K09).

Received: 29 November 2015 Accepted: 30 March 2016 Published online: 07 April 2016

#### References

- Hadeler, KP, Freedman, HI: Predator-prey populations with parasitic infection. *J. Math. Biol.* **27**, 609-631 (1989)
- Muhammadhaji, A, Teng, ZD, Rehim, M: On a two species stochastic Lotka-Volterra competition system. *J. Dyn. Control Syst.* **21**, 495-511 (2015)
- Misraa, OP, Sinhab, P, Singh, C: Stability and bifurcation analysis of a prey-predator model with age based predation. *Appl. Math. Model.* **37**, 6519-6529 (2013)
- Zu, L, Jiang, DQ, O'Regan, D, Ge, B: Periodic solution for a non-autonomous Lotka-Volterra predator-prey model with random perturbation. *J. Math. Anal. Appl.* **430**, 428-437 (2015)
- Zhao, TT, Xiao, YN: Plant disease models with nonlinear impulsive cultural control strategies for vegetatively propagated plants. *Math. Comput. Simul.* **107**, 61-91 (2015)
- Zhang, TL, Kang, RN, Wang, K, Liu, JL: Global dynamics of an SEIR epidemic model with discontinuous treatment. *Adv. Differ. Equ.* **2015**, 361 (2015). doi:10.1186/s13662-015-0695-0
- Hu, ZY, Teng, ZD, Jia, CJ, et al.: Dynamical analysis and chaos control of a discrete SIS epidemic model. *Adv. Differ. Equ.* **2014**, 58 (2014). doi:10.1186/1687-1847-2014-58
- Malkinson, M, Banet, C, Weisman, Y, Pokamunski, S, King, R: Introduction of West Nile Virus in the Middle East by migrating white storks. *Emerg. Infect. Dis.* **8**, 392-397 (2002)
- Earn, DJD, Dushoff, J, Levin, SA: Ecology and evolution of the flu. *Trends Ecol. Evol.* **17**, 334-340 (2002)
- Chatterjee, S, Pal, S, Chattopadhyay, J: Role of migratory birds under environmental fluctuation-a mathematical study. *J. Biol. Syst.* **16**, 81-106 (2008)
- Chatterjee, S, Chattopadhyay, J: Role of migratory bird population in a simple eco-epidemiological model. *Math. Comput. Model. Dyn. Syst.* **13**, 99-114 (2007)
- Chatterjee, S: Alternative prey source coupled with prey recovery enhance stability between migratory prey and their predator in the presence of disease. *Nonlinear Anal., Real World Appl.* **11**, 4415-4430 (2010)

13. Chatterjee, S, Das, K, Chattopadhyay, J: Time delay factor can be used as a key factor for preventing the outbreak of a disease—results drawn from a mathematical study of a one season eco-epidemiological model. *Nonlinear Anal., Real World Appl.* **8**, 1472-1493 (2007)
14. Gao, S, Zhang, F, He, Y: The effects of migratory bird population in a nonautonomous eco-epidemiological model. *Appl. Math. Model.* **37**, 3903-3916 (2013)
15. Zhang, Y, Gao, SJ, Liu, YJ: Analysis of a nonautonomous model for migratory birds with saturation incidence rate. *Commun. Nonlinear Sci. Numer. Simul.* **17**, 1659-1672 (2012)
16. Zhou, XY, Cui, JA, Shi, XY, et al.: A modified Leslie-Gower predator-prey model with prey infection. *J. Appl. Math. Comput.* **33**, 471-487 (2010)
17. Das, KP, Chatterjee, S, Chattopadhyay, J: Disease in prey population and body size of intermediate predator reduce the prevalence of chaos—conclusion drawn from Hastings-Powell model. *Ecol. Complex.* **6**, 363-374 (2009)
18. Zhang, TL, Teng, ZD: On a nonautonomous SEIRS model in epidemiology. *Bull. Math. Biol.* **69**, 2537-2559 (2007)
19. Teng, ZD, Li, Z: Permanence and asymptotic behavior of the  $N$ -species nonautonomous Lotka-Volterra competitive systems. *Comput. Math. Appl.* **39**, 107-116 (2000)
20. Niu, X, Zhang, L, Teng, ZD: The asymptotic behavior of a nonautonomous eco-epidemic model with disease in the prey. *Appl. Math. Model.* **35**, 457-470 (2011)
21. Feng, X, Teng, ZD, Zhang, L: Permanence for nonautonomous  $N$ -species Lotka-Volterra competitive systems with feedback controls. *Rocky Mt. J. Math.* **38**, 1355-1376 (2008). doi:10.1216/RMJ-2008-38-5-1355

Submit your manuscript to a SpringerOpen<sup>®</sup> journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

---

Submit your next manuscript at ► [springeropen.com](http://springeropen.com)

---