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The fractional modified Zakharov system for plasmas with a quantum correction

Lang Li†, Lingyu Jin†, Changping Xie† and Shaomei Fang*

*Correspondence:
dz90@scau.edu.cn
Department of Applied Mathematics, South China Agricultural University, Wushan Road, Guangzhou, 510642, China
†Equal contributors

Abstract

In this paper, we consider the fractional modified Zakharov system with a quantum correction. This system can be regarded as a generalization of the Garcia model to the fractional order. By the properties of the fractional Sobolev spaces and *a priori* estimates, we overcome the mathematical difficulty arising in the fractional model and establish the global existence and uniqueness of the solution.

Keywords: fractional modified Zakharov system; Galerkin method; quantum correction

1 Introduction

In this paper, we consider the following fractional modified Zakharov system

$$i\partial_t E + \partial_{xx} E - H^2(-\partial_{xx})^\alpha E = nE, \tag{1}$$

$$\partial_{tt} n - \partial_{xx} n + H^2(-\partial_{xx})^\beta n = \partial_{xx}\big(|E|^2\big), \tag{2}$$

$$E(x,0) = E_0(x), \qquad n(x,0) = n_0(x), \qquad \partial_t n(x,0) = n_1(x), \tag{3}$$

$$E(x+2\pi,t) = E(x,t), \qquad n(x+2\pi,t) = n(x,t), \qquad \partial_t n(x+2\pi,t) = \partial_t n(x,t), \tag{4}$$

where $1 < \alpha \le \beta < 2$, $E : \mathbb{R}^{1+1} \to \mathbb{C}$ represents the slowly varying envelope of the high-frequency electric field, $n : \mathbb{R}^{1+1} \to \mathbb{R}$ denotes the low-frequency variation of the density of the ions and H is the dimensionless quantum parameter given by the ratio of the ion plasmon and electron thermal energies.

When $H = 0$, (1)-(2) reduce to the classical Zakharov system. It is well known that the classical Zakharov system [1], which describes the nonlinear interaction of Langmuir waves and ion-acoustic waves, is one of the most important models in plasma physics. Physically, many authors have paid much attention to investigate the nonlinear properties of this system, such as the existence of solitons, chaos, hyperchaos, Hopf bifurcation. For more details, see [2–4]. Mathematically, this system was also broadly studied concerning its existence, uniqueness and regularity of the solutions, see [5–8].

When $H > 0$, $\alpha = 2$, $\beta = 2$, it was exactly the Garcia model [9] which obtained by using a quantum fluid approach. Since this system took into account the quantum corrections, it has became more important on the investigation of the quantum counterpart of some of the plasma physics phenomena [10, 11]. In this sense, (1)-(2) can be regarded as a fractional generalization of the Garcia model.

The interest in the study of differential equations of fractional order lies in the fact that fractional derivatives provide a powerful and useful tool for the description of memory and hereditary properties of various material and process. With this advantage, particularly in some interdisciplinary fields, the fractional order models become more realistic and practical than the classical integer-order models [12]. Recently, the fractional differential equations have been attracting great attention and widely been used in the areas of physics, engineering, chemistry, biology, economics, control theory, signal and image processing, biophysics, aerodynamics, *etc.* [13–15]. For example, the authors [16] studied the chaotic behavior of a generalization of the Liu system with fractional order. In [17], the existence and uniqueness of the solution for fractional neutral differential equations with infinite delay was obtained. Other new results concerning the numerical investigation on the dynamics and properties of the system for fractional differential equations can be found in [18–25], *etc.*

Concerning the mathematical issues for (1)-(2), You *et al.* [25] established the global existence of the solution to (1)-(2) with initial boundary conditions via *a priori* estimates and Galerkin method when $H > 0$, $\alpha = 2$, $\beta = 2$. Jin *et al.* obtained the existence of weak solutions and the global attractor for modified Zakharov equations with a quantum correction [26, 27]. To our knowledge, there is no general existence theory that has been developed for (1)-(4).

In this paper, we are interested in studying the global existence and uniqueness of the weak solution. Motivated by [24], we combine the energy method with *a priori* estimates to establish the existence of the weak solution in some fractional Sobolev spaces. To overcome the mathematical difficulty arises in the fractional model, we use the properties of fractional Sobolev spaces and the linear interpolation to deal with the nonlinear terms.

This paper is organized as follows. In Section 2, we obtain *a priori* estimates. In Section 3, we prove the existence and uniqueness of global solution to the problem (1)-(4). In Section 4, we give a conclusion for our results.

Now we give some notations.

Let $\Omega = [0, 2\pi]$. We denote by $L^p(\Omega)$ the space of all the pth integrable functions f normed by

$$\|f\|_{L^p(\Omega)} = \left(\int_\Omega \left|f(x)\right|^p dx\right)^{\frac{1}{p}}, \qquad \|f\|_{L^\infty(\Omega)} = \operatorname*{ess\,sup}_{\Omega} \left|f(x)\right|.$$

When no confusion arises, we set $L^p := L^p(\Omega)$ for $1 \le p \le \infty$.

The space $L^p(0, T; X)$ consists of all measurable functions $f : [0, T] \to X$ with

$$\|f\|_{L^p(0,T;X)} = \left(\int_0^T \|f\|_X^p \, dt\right)^{\frac{1}{p}} < \infty,$$

for $1 \le p < \infty$, and

$$\|f\|_{L^\infty(0,T;X)} = \operatorname*{ess\,sup}_{0 \le t \le T} \|f\|_X < \infty.$$

If u is a periodic function, we can express it by Fourier series and write

$$u = \sum_{j \in \mathbb{Z}} a_j e^{ij\cdot x}.$$

$\Lambda^\alpha u$ can be defined by

$$\Lambda^\alpha u = \sum_{j\in\mathbb{Z}} |j|^\alpha a_j e^{ij\cdot x},$$

where $\Lambda := (-\Delta)^{\frac{1}{2}}$ is the so-called Zygmund operator.

Define

$$A = \left\{ u \;\middle|\; u = \sum_{j\in\mathbb{Z}} a_j e^{ij\cdot x}, \sum_{j\in\mathbb{Z}} a_j^2 < \infty, \sum_{j\in\mathbb{Z}} |j|^{2\alpha} a_j^2 < \infty \right\}.$$

H^α is a complete space of A with the induced norm

$$\|u\|_{H^\alpha} = \left(\sum_{j\in\mathbb{Z}} a_j^2\right)^{\frac{1}{2}} + \left(\sum_{j\in\mathbb{Z}} |j|^{2\alpha} a_j^2\right)^{\frac{1}{2}}.$$

Then H^α is a Banach space. It is easy to show that H^α is a Hilbert space with the inner product

$$(u,v)_{H^\alpha} = \left(\Lambda^\alpha u, \Lambda^\alpha v\right) = \sum_{j\in\mathbb{Z}} |j|^{2\alpha} a_j b_j,$$

where $\Lambda^\alpha v = \sum_{j\in\mathbb{Z}} |j|^\alpha b_j e^{ij\cdot x}$.

We denote by C a positive constant which may change from one line to the next line.

2 *A priori* estimates

To study the solution of the fractional system, we bring in ϕ, and transform (1)-(4) into the following form

$$i\partial_t E + \partial_{xx} E - H^2 \Lambda^{2\alpha} E = nE, \tag{5}$$

$$\partial_t n - \partial_{xx}\phi = 0, \tag{6}$$

$$\partial_t \phi - n - H^2 \Lambda^{2(\beta-1)} n - |E|^2 = 0, \tag{7}$$

$$E(x,0) = E_0(x), \qquad n(x,0) = n_0(x), \qquad \phi(x,0) = \phi_0(x), \tag{8}$$

$$E(x+2\pi,t) = E(x,t), \qquad n(x+2\pi,t) = n(x,t), \qquad \phi(x+2\pi,t) = \phi(x,t). \tag{9}$$

Lemma 1 *Assume that* $E_0(x) \in L^2(\Omega)$, *then we have*

$$\|E(x,t)\|_{L^2}^2 = \|E_0(x)\|_{L^2}^2.$$

Proof Multiplying (5) by $\overline{E}$, integrating over Ω, and then taking the imaginary part, we have

$$\frac{1}{2}\frac{d}{dt}\|E\|_{L^2}^2 = 0.$$

This completes the proof. □

Lemma 2 *Assume that $E_0(x) \in H^\alpha(\Omega)$, $n_0(x) \in H^{\beta-1}(\Omega)$, $\phi_0(x) \in H^1(\Omega)$, then we have*

$$\sup_{0\le t\le T}\left(\|\partial_x E\|_{L^2}^2 + \|\partial_x\phi\|_{L^2}^2 + \left\|\Lambda^\alpha E\right\|_{L^2}^2 + \|n\|_{L^2}^2 + \left\|\Lambda^{\beta-1} n\right\|_{L^2}^2\right) \le C,$$

where C is a positive constant depending on $\|E_0\|_{H^\alpha}$, $\|n_0\|_{H^{\beta-1}}$, $\|\phi_0\|_{H^1}$, and T.

Proof Multiplying (6) by n, and integrating over Ω, we have

$$\frac{1}{2}\frac{d}{dt}\|n\|_{L^2}^2 - \int_\Omega \partial_{xx}\phi n\,dx = 0.$$

Since

$$\begin{aligned}\int_\Omega \partial_{xx}\phi n\,dx &= \int_\Omega \partial_{xx}\phi\left(\partial_t\phi - H^2\Lambda^{2(\beta-1)}n - |E|^2\right)dx \\ &= -\frac{1}{2}\frac{d}{dt}\|\partial_x\phi\|_{L^2}^2 - \frac{H^2}{2}\frac{d}{dt}\left\|\Lambda^{(\beta-1)}n\right\|_{L^2}^2 - \int_\Omega \partial_t n|E|^2\,dx,\end{aligned}$$

it follows that

$$\frac{1}{2}\frac{d}{dt}\left(\|n\|_{L^2}^2 + \|\partial_x\phi\|_{L^2}^2 + H^2\left\|\Lambda^{(\beta-1)}n\right\|_{L^2}^2\right) + \int_\Omega \partial_t n|E|^2\,dx = 0. \tag{10}$$

Multiplying (5) by $\overline{\partial_t E}$, integrating over Ω, and taking the real part, we have

$$\frac{1}{2}\frac{d}{dt}\|\partial_x E\|_{L^2}^2 + \frac{H^2}{2}\frac{d}{dt}\left\|\Lambda^\alpha E\right\|_{L^2}^2 + \frac{1}{2}\int_\Omega n\partial_t\left(|E|^2\right)dx = 0. \tag{11}$$

Since

$$\frac{d}{dt}\int_\Omega n|E|^2\,dx = \int_\Omega \partial_t n|E|^2\,dx + \int_\Omega n\partial_t\left(|E|^2\right)dx,$$

from (10) and (11), we can deduce that

$$\begin{aligned}&\frac{d}{dt}\left(\frac{1}{2}\|n\|_{L^2}^2 + \frac{1}{2}\|\partial_x\phi\|_{L^2}^2 + \frac{H^2}{2}\left\|\Lambda^{(\beta-1)}n\right\|_{L^2}^2 + \|\partial_x E\|_{L^2}^2\right. \\ &\quad\left. + H^2\left\|\Lambda^\alpha E\right\|_{L^2}^2 + \int_\Omega n\left(|E|^2\right)dx\right) = 0.\end{aligned}$$

Set

$$\begin{aligned}\Psi(t) &= \frac{1}{2}\|n\|_{L^2}^2 + \frac{1}{2}\|\partial_x\phi\|_{L^2}^2 + \frac{H^2}{2}\left\|\Lambda^{(\beta-1)}n\right\|_{L^2}^2 + \|\partial_x E\|_{L^2}^2 \\ &\quad + H^2\left\|\Lambda^\alpha E\right\|_{L^2}^2 + \int_\Omega n\left(|E|^2\right)dx,\end{aligned}$$

we have

$$\Psi(t) = \Psi(0).$$

By using the Hölder inequality, the Gagliardo-Nirenberg inequality, and Lemma 1, we have

$$\begin{aligned}
&\frac{1}{2}\|n\|_{L^2}^2+\frac{1}{2}\|\partial_x\phi\|_{L^2}^2+\frac{H^2}{2}\|\Lambda^{(\beta-1)}n\|_{L^2}^2+\|\partial_xE\|_{L^2}^2+H^2\|\Lambda^{\alpha}E\|_{L^2}^2\\
&\quad\le\Psi(0)+\left|\int_\Omega n|E|^2\,dx\right|\\
&\quad\le\Psi(0)+\frac{1}{4}\|n\|_{L^2}^2+\|E\|_{L^4}^4\\
&\quad\le\Psi(0)+\frac{1}{4}\|n\|_{L^2}^2+C\|E\|_{L^2}^3\|\partial_xE\|_{L^2}\\
&\quad\le\Psi(0)+\frac{1}{4}\|n\|_{L^2}^2+\frac{1}{4}\|\partial_xE\|_{L^2}^2+C.
\end{aligned}$$

Owing to the inequality

$$\begin{aligned}
\Psi(0)&\le C\big(\|n_0\|_{L^2}^2+\|\partial_x\phi_0\|_{L^2}^2+\|\Lambda^{(\beta-1)}n_0\|_{L^2}^2+\|\partial_xE_0\|_{L^2}^2+\|\Lambda^{\alpha}E_0\|_{L^2}^2\big)\\
&\le C\big(\|E_0\|_{H^\alpha}+\|n_0\|_{H^{\beta-1}}+\|\phi_0\|_{H^1}^2\big),
\end{aligned}$$

we obtain the desired result. □

Lemma 3 *Assume that $E_0(x)\in H^{2\alpha}(\Omega)$, $n_0(x)\in H^{\beta}(\Omega)$, $\phi_0(x)\in H^2(\Omega)$, then we have*

$$\sup_{0\le t\le T}\big(\|\partial_tE\|_{L^2}^2+\|\partial_tn\|_{L^2}^2+\|\partial_t\phi\|_{L^2}^2+\|\Lambda^{2\alpha}E\|_{L^2}^2+\|\Lambda^{\beta}n\|_{L^2}^2+\|\partial_{xx}\phi\|_{L^2}^2\big)\le C,$$

where C is a positive constant depending on $\|E_0\|_{H^{2\alpha}}$, $\|n_0\|_{H^\beta}$, $\|\phi_0\|_{H^2}$ and T.

Proof Differentiating (5) with respect to t, then multiplying it by $\overline{E}_t$, integrating over Ω, and taking the imaginary part, we have

$$\frac{1}{2}\frac{d}{dt}\|\partial_tE\|_{L^2}^2=\operatorname{Im}\int_\Omega\partial_t(nE)\overline{E}_t\,dx.$$

Therefore

$$\begin{aligned}
\frac{d}{dt}\|\partial_tE\|_{L^2}^2&=2\operatorname{Im}\int_\Omega\partial_t(nE)\overline{E}_t\,dx\\
&=2\operatorname{Im}\int_\Omega\partial_tnE\overline{E}_t\,dx\\
&\le2\|E\|_{L^\infty}\|\partial_tn\|_{L^2}\|\partial_tE\|_{L^2}\\
&\le C\big(\|\partial_tn\|_{L^2}^2+\|\partial_tE\|_{L^2}^2\big),
\end{aligned}\tag{12}$$

where we have used the fact $\|E\|_{L^\infty}\le C\|\partial_xE\|_{L^2}$ and Lemma 2.

Differentiating (6) with respect to t, then multiplying it by ∂_tn, and integrating over Ω, we have

$$\frac{1}{2}\frac{d}{dt}\|\partial_tn\|_{L^2}^2+\frac{1}{2}\frac{d}{dt}\|\partial_xn\|_{L^2}^2+\frac{H^2}{2}\frac{d}{dt}\|\Lambda^{\beta}n\|_{L^2}^2=\int_\Omega\partial_tn\partial_{xx}\big(|E|^2\big)\,dx.$$

By using the Hölder inequality, the Gagliardo-Nirenberg inequality, and Lemma 2, we have

$$\begin{aligned}\int_\Omega \partial_t n \partial_{xx}\big(|E|^2\big)\,dx &\le C\big(\|\partial_t n\|_{L^2}^2 + \big\|\partial_{xx}\big(|E|^2\big)\big\|_{L^2}^2\big)\\ &\le C\big(\|\partial_t n\|_{L^2}^2 + \|E\|_{L^\infty}^2\|\partial_{xx}E\|_{L^2}^2 + \|\partial_x E\|_{L^4}^4\big)\\ &\le C\big(\|\partial_t n\|_{L^2}^2 + \|\partial_{xx}E\|_{L^2}^2 + 1\big)\\ &\le C\big(\|\partial_t n\|_{L^2}^2 + \big\|\Lambda^{2\alpha}E\big\|_{L^2}^2 + 1\big).\end{aligned}$$

From (5), it is easy to see that

$$\big\|\Lambda^{2\alpha}E\big\|_{L^2}^2 \le C\big(\|\partial_t E\|_{L^2}^2 + \|\partial_{xx}E\|_{L^2}^2 + \|nE\|_{L^2}^2\big).$$

By using the Gagliardo-Nirenberg inequality, the ϵ-Young inequality and Lemma 2, we have

$$\begin{aligned}\big\|\Lambda^{2\alpha}E\big\|_{L^2}^2 &\le C\big(\|\partial_t E\|_{L^2}^2 + \big\|\Lambda^{2\alpha}E\big\|_{L^2}^{\frac{2}{\alpha}}\|E\|_{L^2}^{2-\frac{2}{\alpha}} + \|E\|_{L^\infty}^2\|n\|_{L^2}^2\big)\\ &\le \frac{1}{2}\big\|\Lambda^{2\alpha}E\big\|_{L^2}^2 + C\big(\|\partial_t E\|_{L^2}^2 + 1\big).\end{aligned}$$

Thus

$$\big\|\Lambda^{2\alpha}E\big\|_{L^2}^2 \le C\big(\|\partial_t E\|_{L^2}^2 + 1\big). \tag{13}$$

Consequently

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|\partial_t n\|_{L^2}^2 + \frac{1}{2}\frac{d}{dt}\|\partial_x n\|_{L^2}^2 + \frac{H^2}{2}\frac{d}{dt}\big\|\Lambda^\beta n\big\|_{L^2}^2\\ &\quad\le C\big(\|\partial_t n\|_{L^2}^2 + \|\partial_t E\|_{L^2}^2 + 1\big).\end{aligned} \tag{14}$$

From (12) and (14), we have

$$\begin{aligned}&\frac{d}{dt}\big(\|\partial_t E\|_{L^2}^2 + \|\partial_t n\|_{L^2}^2 + \|\partial_x n\|_{L^2}^2 + H^2\big\|\Lambda^\beta n\big\|_{L^2}^2\big)\\ &\quad\le C\big(\|\partial_t n\|_{L^2}^2 + \|\partial_t E\|_{L^2}^2 + 1\big).\end{aligned}$$

Thus the Gronwall inequality yields the estimate

$$\sup_{0\le t\le T}\big(\|\partial_t E\|_{L^2}^2 + \|\partial_t n\|_{L^2}^2 + \|\partial_x n\|_{L^2}^2 + \big\|\Lambda^\beta n\big\|_{L^2}^2\big) \le C,$$

where C is a positive constant depending on $\|E_0\|_{H^{2\alpha}}$, $\|n_0\|_{H^\beta}$, $\|\phi_0\|_{H^2}$, and T.

Then (6) and (13) imply

$$\|\partial_{xx}\phi\|_{L^2}^2 + \big\|\Lambda^{2\alpha}E\big\|_{L^2}^2 \le C.$$

From (7), we can also obtain

$$\|\partial_t\phi\|_{L^2}^2 \le C,$$

where we have used the fact $\beta - 1 < \frac{\beta}{2}$. Thus we complete the proof. □

Lemma 4 *Assume that* $E_0(x) \in H^{4\alpha}(\Omega)$, $n_0(x) \in H^{2\beta}(\Omega)$, $\phi_0(x) \in H^{\beta+2}(\Omega)$, *then we have*

$$\sup_{0\le t\le T}\left(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \|\partial_{tt}\phi\|_{L^2}^2 + \left\|\Lambda^{2\alpha}\partial_t E\right\|_{L^2}^2 + \left\|\Lambda^{\beta}\partial_t n\right\|_{L^2}^2 + \|\partial_{xx}\partial_t\phi\|_{L^2}^2\right) \le C,$$

where C is a positive constant depending on $\|E_0\|_{H^{4\alpha}}$, $\|n_0\|_{H^{2\beta}}$, $\|\phi_0\|_{H^{\beta+2}}$, *and T*.

Proof Differentiating (6) with respect to t twice, multiplying it by $\partial_{tt}n$, and integrating over Ω, we have

$$\frac{d}{dt}\left(\|\partial_{tt}n\|_{L^2}^2 + \|\partial_x\partial_t n\|_{L^2}^2 + H^2\left\|\Lambda^{\beta}\partial_t n\right\|_{L^2}^2\right) - 2\int_\Omega \partial_t\partial_{xx}\left(|E|^2\right)\partial_{tt}n\,dx = 0.$$

By using the Hölder inequality, the Gagliardo-Nirenberg inequality, and Lemma 3, we have

$$\begin{aligned}
&\left|\int_\Omega \partial_t\partial_{xx}\left(|E|^2\right)\partial_{tt}n\,dx\right| \\
&\quad\le C\left(\|\partial_{tt}n\|_{L^2}^2 + \left\|\partial_t\partial_{xx}\left(|E|^2\right)\right\|_{L^2}^2\right) \\
&\quad\le C\left(\|\partial_{tt}n\|_{L^2}^2 + \|\partial_t\partial_{xx}E\|_{L^2}^2\|E\|_{L^\infty}^2 + \|\partial_t\partial_x E\|_{L^2}^2\|\partial_x E\|_{L^\infty}^2\right) \\
&\quad\le C\left(\|\partial_{tt}n\|_{L^2}^2 + \left\|\Lambda^{2\alpha}\partial_t E\right\|_{L^2}^2 + 1\right).
\end{aligned} \tag{15}$$

Since

$$i\partial_{tt}E + \partial_{xx}\partial_t E - H^2\Lambda^{2\alpha}\partial_t E - \partial_t(nE) = 0,$$

we have

$$\begin{aligned}
\left\|\Lambda^{2\alpha}\partial_t E\right\|_{L^2}^2 &\le C\left(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{xx}\partial_t E\|_{L^2}^2 + \left\|\partial_t(nE)\right\|_{L^2}^2\right) \\
&\le C\left(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{xx}\partial_t E\|_{L^2}^2 + \|n\|_{L^\infty}^2\|\partial_t E\|_{L^2}^2 + \|E\|_{L^\infty}^2\|\partial_t n\|_{L^2}^2\right) \\
&\le C\left(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{xx}\partial_t E\|_{L^2}^2 + 1\right).
\end{aligned}$$

By using the ϵ-Young inequality and the Gagliardo-Nirenberg inequality, we have

$$\left\|\Lambda^{2\alpha}\partial_t E\right\|_{L^2}^2 \le C\|\partial_{tt}E\|_{L^2}^2 + \frac{1}{2}\left\|\Lambda^{2\alpha}\partial_t E\right\|_{L^2}^2 + C.$$

Thus

$$\left\|\Lambda^{2\alpha}\partial_t E\right\|_{L^2}^2 \le C\left(\|\partial_{tt}E\|_{L^2}^2 + 1\right). \tag{16}$$

Combining (15) and (16) leads us to the estimate

$$\begin{aligned}
&\frac{d}{dt}\left(\|\partial_{tt}n\|_{L^2}^2 + \|\partial_x\partial_t n\|_{L^2}^2 + H^2\left\|\Lambda^{\beta}\partial_t n\right\|_{L^2}^2\right) \\
&\quad\le C\left(\|\partial_{tt}n\|_{L^2}^2 + \|\partial_{tt}E\|_{L^2}^2 + 1\right).
\end{aligned} \tag{17}$$

Differentiating (5) with respect to t twice, multiplying it by $\overline{\partial_{tt}E}$, integrating over Ω, and then taking the imaginary part, we have

$$\frac{1}{2}\frac{d}{dt}\|\partial_{tt}E\|_{L^2}^2 - \operatorname{Im}\int_\Omega \partial_{tt}(nE)\overline{\partial_{tt}E}\,dx = 0.$$

Therefore

$$\begin{aligned}\frac{d}{dt}\|\partial_{tt}E\|_{L^2}^2 &\le C\big(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \|\partial_t E\partial_t n\|_{L^2}^2\big) \\ &\le C\big(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \|\partial_t E\|_{L^2}^2\|\partial_t n\|_{L^\infty}^2\big) \\ &\le C\big(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \|\partial_x\partial_t n\|_{L^2}^2\big).\end{aligned} \tag{18}$$

From (17) and (18), we have

$$\begin{aligned}&\frac{d}{dt}\big(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \|\partial_x\partial_t n\|_{L^2}^2 + H^2\big\|\Lambda^\beta\partial_t n\big\|_{L^2}^2\big) \\ &\quad\le C\big(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \|\partial_x\partial_t n\|_{L^2}^2 + 1\big).\end{aligned}$$

Using the Gronwall inequality, we have

$$\sup_{0\le t\le T}\big(\|\partial_{tt}E\|_{L^2}^2 + \|\partial_{tt}n\|_{L^2}^2 + \big\|\Lambda^\beta\partial_t n\big\|_{L^2}^2\big) \le C.$$

Therefore, from (16), we can obtain

$$\sup_{0\le t\le T}\big\|\Lambda^{2\alpha}\partial_t E\big\|_{L^2}^2 \le C.$$

From (5)-(7), we can also get the following estimates easily:

$$\sup_{0\le t\le T}\big(\|\partial_{tt}\phi\|_{L^2}^2 + \big\|\Lambda^{4\alpha}E\big\|_{L^2}^2 + \big\|\Lambda^{2\beta}n\big\|_{L^2}^2 + \big\|\Lambda^{\beta+2}\phi\big\|_{L^2}^2\big) \le C.$$

Thus we complete the proof. □

Lemma 5 *Assume that $E_0(x)\in H^{2k\alpha}(\Omega)$, $n_0(x)\in H^{k\beta}(\Omega)$, $\phi_0(x)\in H^{2+(k-1)\beta}(\Omega)$, $k\ge 2$. Then*

$$\sup_{0\le t\le T}\big(\big\|(\partial_t)^k E\big\|_{L^2}^2 + \big\|(\partial_t)^k n\big\|_{L^2}^2 + \big\|(\partial_t)^k\phi\big\|_{L^2}^2\big) \le C,$$

and

$$\sup_{0\le t\le T}\big(\big\|\Lambda^{2\alpha}(\partial_t)^{k-1}E\big\|_{L^2}^2 + \big\|\Lambda^{\beta}(\partial_t)^{k-1}n\big\|_{L^2}^2 + \big\|\partial_{xx}(\partial_t)^{k-1}\phi\big\|_{L^2}^2\big) \le C,$$

where C is a positive constant depending on the initial data and T.

Proof The proof is by induction on k, the case $k = 2$ being Lemma 4 above.

Assume now the lemma is valid for some integer $k\ge 2$, and suppose then

$$E_0(x)\in H^{2m\alpha}(\Omega),\qquad n_0(x)\in H^{m\beta}(\Omega),\qquad \phi_0(x)\in H^{2+(m-1)\beta}(\Omega),\quad m = k+1.$$

Differentiating (5) $(k+1)$-times with respect to t, then multiplying it by $\overline{(\partial_t)^{k+1}E}$, integrating over Ω, and taking the imaginary part, we have

$$\frac{1}{2}\frac{d}{dt}\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 = \operatorname{Im}\int_\Omega (\partial_t)^{k+1}(nE)\overline{(\partial_t)^{k+1}E}\,dx.$$

Therefore

$$\begin{aligned}\frac{d}{dt}\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 &= 2\operatorname{Im}\int_\Omega (\partial_t)^{k+1}(nE)\overline{(\partial_t)^{k+1}E}\,dx\\ &\le C\bigl(\left\|(\partial_t)^{k+1}n\right\|_{L^2}^2 + \left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 + 1\bigr).\end{aligned} \tag{19}$$

Differentiating (6) $(k+1)$-times with respect to t, then multiplying it by $(\partial_t)^{k+1}n$, and integrating over Ω, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\bigl(\left\|(\partial_t)^{k+1}n\right\|_{L^2}^2 + \left\|\partial_x(\partial_t)^k n\right\|_{L^2}^2 + H^2\left\|\Lambda^\beta(\partial_t)^k n\right\|_{L^2}^2\bigr)\\ &\quad= \int_\Omega \partial_{xx}(\partial_t)^k\bigl(|E|^2\bigr)(\partial_t)^{k+1}n\,dx.\end{aligned}$$

Therefore

$$\begin{aligned}&\frac{d}{dt}\bigl(\left\|(\partial_t)^{k+1}n\right\|_{L^2}^2 + \left\|\partial_x(\partial_t)^k n\right\|_{L^2}^2 + H^2\left\|\Lambda^\beta(\partial_t)^k n\right\|_{L^2}^2\bigr)\\ &\quad\le C\bigl(\left\|(\partial_t)^{k+1}n\right\|_{L^2}^2 + \left\|\partial_{xx}(\partial_t)^k\bigl(|E|^2\bigr)\right\|_{L^2}^2\bigr).\end{aligned} \tag{20}$$

Since

$$i(\partial_t)^{k+1}E + \partial_{xx}(\partial_t)^k E - H^2\Lambda^{2\alpha}(\partial_t)^k E = (\partial_t)^k(nE),$$

it follows that

$$\begin{aligned}&\left\|\Lambda^{2\alpha}(\partial_t)^k E\right\|_{L^2}^2\\ &\quad\le C\bigl(\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 + \left\|\partial_{xx}(\partial_t)^k E\right\|_{L^2}^2 + \left\|(\partial_t)^k(nE)\right\|_{L^2}^2\bigr)\\ &\quad\le C\Biggl(\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 + \left\|(\partial_{xx})^\alpha(\partial_t)^k E\right\|_{L^2}^{\frac{2}{\alpha}}\left\|(\partial_t)^k E\right\|_{L^2}^{2-\frac{2}{\alpha}}\\ &\qquad + \left\|(\partial_t)^k nE\right\|_{L^2}^2 + \sum_{i+j=k,0<i<k,0<j<k}\left\|(\partial_t)^i n(\partial_t)^j E\right\|_{L^2}^2 + \left\|n(\partial_t)^k E\right\|_{L^2}^2\Biggr)\\ &\quad\le C\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 + \frac{1}{2}\left\|(\partial_{xx})^\alpha(\partial_t)^k E\right\|_{L^2}^2 + C + \left\|(\partial_t)^k n\right\|_{L^2}^2\|E\|_{L^\infty}^2\\ &\qquad + \sum_{i+j=k,0<i<k,0<j<k}\left\|(\partial_t)^i n\right\|_{L^\infty}^2\left\|(\partial_t)^j E\right\|_{L^2}^2 + \|n\|_{L^\infty}^2\left\|(\partial_t)^k E\right\|_{L^2}^2\\ &\quad\le C\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 + \frac{1}{2}\left\|(\partial_{xx})^\alpha(\partial_t)^k E\right\|_{L^2}^2 + \sum_{i+j=k,0<i<k,0<j<k}\left\|\Lambda^\beta(\partial_t)^i n\right\|_{L^2}^2\left\|(\partial_t)^j E\right\|_{L^2}^2 + C\\ &\quad\le C\left\|(\partial_t)^{k+1}E\right\|_{L^2}^2 + \frac{1}{2}\left\|(\partial_{xx})^\alpha(\partial_t)^k E\right\|_{L^2}^2 + C,\end{aligned}$$

i.e.

$$\|\Lambda^{2\alpha}(\partial_t)^k E\|_{L^2}^2 \le C\|(\partial_t)^{k+1}E\|_{L^2}^2 + C.$$

Therefore

$$\begin{aligned}\|\partial_{xx}(\partial_t)^k E\|_{L^2}^2 &\le C\|\Lambda^{2\alpha}(\partial_t)^k E\|_{L^2}^{\frac{2}{\alpha}}\|(\partial_t)^k E\|_{L^2}^{2-\frac{2}{\alpha}} \le C\big(\|\Lambda^{2\alpha}(\partial_t)^k E\|_{L^2}^2 + 1\big)\\ &\le C\big(\|(\partial_t)^{k+1}E\|_{L^2}^2 + 1\big).\end{aligned}$$

Substituting it into (20), we have

$$\begin{aligned}&\frac{d}{dt}\big(\|(\partial_t)^{k+1}n\|_{L^2}^2 + \|\partial_x(\partial_t)^k n\|_{L^2}^2 + H^2\|\Lambda^\beta(\partial_t)^k n\|_{L^2}^2\big)\\ &\quad\le C\big(\|(\partial_t)^{k+1}n\|_{L^2}^2 + \|(\partial_t)^{k+1}E\|_{L^2}^2 + 1\big).\end{aligned}\tag{21}$$

From (19) and (21), we have

$$\begin{aligned}&\frac{d}{dt}\big(\|(\partial_t)^{k+1}E\|_{L^2}^2 + \|(\partial_t)^{k+1}n\|_{L^2}^2 + \|\partial_x(\partial_t)^k n\|_{L^2}^2 + H^2\|\Lambda^\beta(\partial_t)^k n\|_{L^2}^2\big)\\ &\quad\le C\big(\|(\partial_t)^{k+1}n\|_{L^2}^2 + \|(\partial_t)^{k+1}E\|_{L^2}^2 + 1\big).\end{aligned}$$

Thus the Gronwall inequality yields

$$\|(\partial_t)^{k+1}E\|_{L^2}^2 + \|(\partial_t)^{k+1}n\|_{L^2}^2 + \|\partial_x(\partial_t)^k n\|_{L^2}^2 + H^2\|\Lambda^\beta(\partial_t)^k n\|_{L^2}^2 \le C.$$

Then from (7), we can deduce that

$$\|\partial_{xx}(\partial_t)^k\phi\|_{L^2}^2 \le C.$$

Since

$$(\partial_t)^{k+1}\phi - (\partial_t)^k n - H^2\Lambda^{2(\beta-1)}(\partial_t)^k n - (\partial_t)^k\big(|E|^2\big) = 0,$$

we have

$$\|(\partial_t)^{k+1}\phi\|_{L^2}^2 \le C\big(\|(\partial_t)^k n\|_{L^2}^2 + \|\Lambda^{2(\beta-1)}(\partial_t)^k n\|_{L^2}^2 + \|(\partial_t)^k\big(|E|^2\big)\|_{L^2}^2\big) \le C,$$

where we have used the fact $\beta - 1 < \frac{\beta}{2}$. The proof is complete. □

3 The existence result of the solution

Now we state our main result as follows.

Theorem 1 *Assume that* $1 < \alpha \le \beta < 2$, *and* $E_0(x) \in H^{2\alpha}(\Omega)$, $n_0(x) \in H^\beta(\Omega)$, $\phi_0(x) \in H^2(\Omega)$. *Then there exists a unique global solution to problem* (5)-(9),

$$\begin{aligned}&E \in L^\infty\big(0,T;H^{2\alpha}\big), \qquad n \in L^\infty\big(0,T;H^\beta\big), \qquad \phi \in L^\infty\big(0,T;H^2\big),\\ &E_t \in L^\infty\big(0,T;L^2\big), \qquad n_t \in L^\infty\big(0,T;L^2\big), \qquad \phi_t \in L^\infty\big(0,T;L^2\big).\end{aligned}$$

Proof By using the Galerkin method, we find the approximate solution as follows

$$E_m(t) = \sum_{j=0}^{m} g_{jm}(t)\omega_j, \qquad n_m(t) = \sum_{j=0}^{m} h_{jm}(t)\omega_j, \qquad \phi_m(t) = \sum_{j=0}^{m} l_{jm}(t)\omega_j,$$

where $\omega_j = e^{ijx}$, $0 \le j \le m$. According to the Galerkin method, these undetermined coefficients $g_{jm}(t)$, $h_{jm}(t)$, $l_{jm}(t)$ need to satisfy the following initial boundary value problem of the system of ordinary differential equations

$$\bigl(i\partial_t E_m + \partial_{xx}E_m - H^2\Lambda^{2\alpha}E_m - n_m E_m, \omega_j\bigr) = 0, \tag{22}$$

$$(\partial_t n_m - \partial_{xx}\phi_m, \omega_j) = 0, \tag{23}$$

$$\bigl(\partial_t \phi_m - n_m + H^2(-\partial_{xx})^{\beta-1}n_m - |E_m|^2, \omega_j\bigr) = 0, \tag{24}$$

$$\begin{aligned} &E_m(x+2\pi,t) = E_m(x,t), \qquad n_m(x+2\pi,t) = n_m(x,t), \\ &\phi_m(x+2\pi,t) = \phi_m(x,t), \end{aligned} \tag{25}$$

$$E_m(x,0) = E_{0m}(x) \in \operatorname{span}\{\omega_j, 0 \le j \le m\}, \tag{26}$$

$$n_m(x,0) = n_{0m}(x) \in \operatorname{span}\{\omega_j, 0 \le j \le m\}, \tag{27}$$

$$\phi_m(x,0) = \phi_{0m}(x) \in \operatorname{span}\{\omega_j, 0 \le j \le m\}, \tag{28}$$

where $E_{0m}(x) \stackrel{H^{2\alpha}}{\longrightarrow} E_0(x)$, $n_{0m}(x) \stackrel{H^{\beta}}{\longrightarrow} n_0(x)$, and $\phi_{0m}(x) \stackrel{H^{2}}{\longrightarrow} \phi_0(x)$ as $m \to \infty$. Similar to the proof of Lemma 3, we can deduce that the sequence $\{E_m\}_{m=1}^{\infty}$ is bounded in $L^\infty(0,T;H^{2\alpha}(\Omega))$, $\{n_m\}_{m=1}^{\infty}$ is bounded in $L^\infty(0,T;H^{\beta}(\Omega))$, $\{\phi_m\}_{m=1}^{\infty}$ is bounded in $L^\infty(0,T;H^{2}(\Omega))$, $\{E_{mt}\}_{m=1}^{\infty}$ is bounded in $L^\infty(0,T;L^{2}(\Omega))$, $\{n_{mt}\}_{m=1}^{\infty}$ is bounded in $L^\infty(0,T;L^{2}(\Omega))$, $\{\phi_{mt}\}_{m=1}^{\infty}$ is bounded in $L^\infty(0,T;L^{2}(\Omega))$.

By a compactness argument, we can choose subsequences, still denoted by $E_m(x,t)$, $n_m(x,t)$, $\phi_m(x,t)$, such that

$$\begin{aligned}
&E_m(x,t) \to E(x,t) \quad \text{in } L^\infty\bigl(0,T;H^{2\alpha}(\Omega)\bigr) \text{ star weakly}, \\
&n_m(x,t) \to n(x,t) \quad \text{in } L^\infty\bigl(0,T;H^{\beta}(\Omega)\bigr) \text{ star weakly}, \\
&\phi_m(x,t) \to \phi(x,t) \quad \text{in } L^\infty\bigl(0,T;H^{2}(\Omega)\bigr) \text{ star weakly}, \\
&E_{mt}(x,t) \to E_t(x,t) \quad \text{in } L^\infty\bigl(0,T;L^{2}(\Omega)\bigr) \text{ star weakly}, \\
&n_{mt}(x,t) \to n_t(x,t) \quad \text{in } L^\infty\bigl(0,T;L^{2}(\Omega)\bigr) \text{ star weakly}, \\
&\phi_{mt}(x,t) \to \phi_t(x,t) \quad \text{in } L^\infty\bigl(0,T;L^{2}(\Omega)\bigr) \text{ star weakly}, \\
&E_m(x,t) \to E(x,t) \quad \text{in } L^2\bigl(0,T;L^{2}(\Omega)\bigr) \text{ strongly and a.e.}, \\
&n_m(x,t) \to n(x,t) \quad \text{in } L^2\bigl(0,T;L^{2}(\Omega)\bigr) \text{ strongly and a.e.}, \\
&n_m(x,t)E_m(x,t) \to n(x,t)E(x,t) \quad \text{in } L^\infty\bigl(0,T;L^{2}(\Omega)\bigr) \text{ star weakly}, \\
&\bigl|E_m(x,t)\bigr|^2 \to |E(x,t)|^2 \quad \text{in } L^\infty\bigl(0,T;L^{2}(\Omega)\bigr) \text{ star weakly}.
\end{aligned}$$

By using the density of ω_j in L^2, we obtain the existence of a local solution for the problem (5)-(9). By the continuous extension principle, from the conditions of the theorem and *a priori* estimates in Section 2, we can get the existence of the global solution for the problem (5)-(9).

Next, we will show the uniqueness of the solution.

Suppose that there are two solutions (E_1, n_1, ϕ_1) and (E_2, n_2, ϕ_2). Let

$$\widetilde{E} = E_1 - E_2, \qquad \widetilde{n} = n_1 - n_2, \qquad \widetilde{\phi} = \phi_1 - \phi_2,$$

then we have

$$i\partial_t \widetilde{E} + \partial_{xx}\widetilde{E} - H^2\Lambda^{2\alpha}\widetilde{E} = n_1\widetilde{E} + \widetilde{n}E_2, \tag{29}$$

$$\partial_t \widetilde{n} - \partial_{xx}\widetilde{\phi} = 0, \tag{30}$$

$$\partial_t \widetilde{\phi} - \widetilde{n} + H^2\Lambda^{2\beta-2}\widetilde{n} = |E_1|^2 - |E_2|^2, \tag{31}$$

with initial data

$$\widetilde{E}|_{t=0} = 0, \qquad \widetilde{n}|_{t=0} = 0, \qquad \widetilde{\phi}|_{t=0} = 0,$$

and periodic boundary conditions

$$\widetilde{E}(x + 2\pi, t) = \widetilde{E}(x, t), \qquad \widetilde{n}(x + 2\pi, t) = \widetilde{n}(x, t), \qquad \widetilde{\phi}(x + 2\pi, t) = \widetilde{\phi}(x, t).$$

Multiplying (29) by $\overline{\widetilde{E}}$, integrating over Ω, and then taking the imaginary part, we have

$$\begin{aligned}
\frac{d}{dt}\|\widetilde{E}\|_{L^2}^2 &= 2\,\mathrm{Im}\int_\Omega \widetilde{n}E_2\overline{\widetilde{E}}\,dx \\
&\le C\|E_2\|_{L^\infty}\|\widetilde{n}\|_{L^2}\|\widetilde{E}\|_{L^2} \\
&\le C\bigl(\|\widetilde{n}\|_{L^2}^2 + \|\widetilde{E}\|_{L^2}^2\bigr).
\end{aligned} \tag{32}$$

Differentiating (30) with respect to t, then multiplying it by $\partial_t\widetilde{n}$, integrating over Ω, we have

$$\frac{1}{2}\frac{d}{dt}\bigl(\|\partial_t\widetilde{n}\|_{L^2}^2 + \|\partial_x\widetilde{n}\|_{L^2}^2 + H^2\bigl\|\Lambda^\beta\widetilde{n}\bigr\|_{L^2}^2\bigr) = \int_\Omega (\partial_{xx})(E_1\overline{\widetilde{E}} + \widetilde{E}\overline{E_2})\partial_t\widetilde{n}\,dx.$$

Therefore

$$\begin{aligned}
&\frac{d}{dt}\bigl(\|\partial_t\widetilde{n}\|_{L^2}^2 + \|\partial_x\widetilde{n}\|_{L^2}^2 + H^2\bigl\|\Lambda^\beta\widetilde{n}\bigr\|_{L^2}^2\bigr) \\
&\quad\le C\bigl(\|\widetilde{E}\|_{L^2} + \|\partial_{xx}\widetilde{E}\|_{L^2}\bigr)\|\partial_t\widetilde{n}\|_{L^2} \\
&\quad\le C\bigl(\|\widetilde{E}\|_{L^2}^2 + \|\partial_{xx}\widetilde{E}\|_{L^2}^2 + \|\partial_t\widetilde{n}\|_{L^2}^2\bigr).
\end{aligned} \tag{33}$$

Since

$$\begin{aligned}
\bigl\|\Lambda^{2\alpha}\widetilde{E}\bigr\|_{L^2} &\le \|\partial_t\widetilde{E}\|_{L^2} + \|\partial_{xx}\widetilde{E}\|_{L^2} + \|\widetilde{n}\|_{L^2}\|E_2\|_{L^\infty} + \|n_1\|_{L^\infty}\|\widetilde{E}\|_{L^2} \\
&\le \|\partial_t\widetilde{E}\|_{L^2} + \frac{1}{2}\bigl\|\Lambda^{2\alpha}\widetilde{E}\bigr\|_{L^2} + C\|\widetilde{n}\|_{L^2} + C\|\widetilde{E}\|_{L^2},
\end{aligned}$$

we have

$$\|\Lambda^{2\alpha}\widetilde{E}\|_{L^2} \le C\big(\|\partial_t\widetilde{E}\|_{L^2} + \|\widetilde{E}\|_{L^2} + \|\widetilde{n}\|_{L^2}\big).$$

Furthermore,

$$\begin{aligned}\|\partial_{xx}\widetilde{E}\|_{L^2}^2 &\le C\big(\|\Lambda^{2\alpha}\widetilde{E}\|_{L^2}^2 + \|\widetilde{E}\|_{L^2}^2\big)\\ &\le C\big(\|\partial_t\widetilde{E}\|_{L^2}^2 + \|\widetilde{E}\|_{L^2}^2 + \|\widetilde{n}\|_{L^2}^2\big).\end{aligned}$$

Substituting it into (33), we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\big(\|\partial_t\widetilde{n}\|_{L^2}^2 + \|\partial_x\widetilde{n}\|_{L^2}^2 + H^2\|\Lambda^{\beta}\widetilde{n}\|_{L^2}^2\big)\\ &\quad\le C\big(\|\partial_t\widetilde{E}\|_{L^2}^2 + \|\widetilde{E}\|_{L^2}^2 + \|\widetilde{n}\|_{L^2}^2 + \|\partial_t\widetilde{n}\|_{L^2}^2\big).\end{aligned}\tag{34}$$

Differentiating (29) with respect to t, we have

$$i\partial_{tt}\widetilde{E} + \partial_{xx}\partial_t\widetilde{E} - H^2\Lambda^{2\alpha}\partial_t\widetilde{E} = \partial_t n_1\widetilde{E} + n_1\partial_t\widetilde{E} + \partial_t\widetilde{n}E_2 + \widetilde{n}\partial_t E_2.\tag{35}$$

Multiplying (35) by $\partial_t\overline{\widetilde{E}}$, integrating over Ω, and taking the imaginary part, we have

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|\partial_t\widetilde{E}\|_{L^2}^2 &\le \|\partial_t n_1\|_{L^\infty}\|\widetilde{E}\|_{L^2}\|\partial_t\widetilde{E}\|_{L^2}\\ &\quad + \|\partial_t\widetilde{n}\|_{L^2}\|E_2\|_{L^\infty}\|\partial_t\widetilde{E}\|_{L^2} + \|\widetilde{n}\|_{L^2}\|\partial_t E_2\|_{L^\infty}\|\|\partial_t\widetilde{E}\|_{L^2}\\ &\le C\big(\|\widetilde{E}\|_{L^2}^2 + \|\partial_t\widetilde{E}\|_{L^2}^2 + \|\widetilde{n}\|_{L^2}^2 + \|\partial_t\widetilde{n}\|_{L^2}^2\big).\end{aligned}\tag{36}$$

Recall the inequality

$$\frac{1}{2}\frac{d}{dt}\|\widetilde{n}\|_{L^2}^2 \le \|\widetilde{n}\|_{L^2}\|\partial_t\widetilde{n}\|_{L^2} \le C\big(\|\widetilde{n}\|_{L^2}^2 + \|\partial_t\widetilde{n}\|_{L^2}^2\big).\tag{37}$$

Combining (32), (34), (36) and (37), we have

$$\begin{aligned}&\frac{d}{dt}\big(\|\widetilde{E}\|_{L^2}^2 + \|\partial_t\widetilde{E}\|_{L^2}^2 + \|\widetilde{n}\|_{L^2}^2 + \|\partial_t\widetilde{n}\|_{L^2}^2 + \|\partial_x\widetilde{n}\|_{L^2}^2 + H^2\|\Lambda^{\beta}\widetilde{n}\|_{L^2}^2\big)\\ &\quad\le C\big(\|\widetilde{E}\|_{L^2}^2 + \|\partial_t\widetilde{E}\|_{L^2}^2 + \|\widetilde{n}\|_{L^2}^2 + \|\partial_t\widetilde{n}\|_{L^2}^2 + \|\partial_x\widetilde{n}\|_{L^2}^2 + H^2\|\Lambda^{\beta}\widetilde{n}\|_{L^2}^2\big).\end{aligned}$$

Using the Gronwall inequality and initial data, we deduce

$$\widetilde{n} = 0, \qquad \widetilde{E} = 0, \qquad \widetilde{\phi} = 0.$$ □

Theorem 2 *Assume that* $1 < \alpha \le \beta < 2$, *and* $E_0(x) \in H^{2k\alpha}(\Omega)$, $n_0(x) \in H^{k\beta}(\Omega)$, $\phi_0(x) \in H^{2+(k-1)\beta}(\Omega)$, $k \ge 2$. *Then there exists a unique global solution to* (5)-(9).

Proof Using Lemma 5 and the embedding theory of Sobolev spaces, we can prove the theorem above. Since the proof is similar to the proof of Theorem 1, we omit it. □

4 Conclusions

In the past decades, quantum plasmas have been attracting considerable attention, both from the physical and mathematical viewpoints. In this paper, we have considered the global existence of the smooth solutions to the fractional modified Zakharov system, when the quantum correction was taken into account. For the other mathematical property of the fractional modified Zakharov system, it is a subject which is still very open to new developments. We will consider the numerical solutions to the fractional modified Zakharov system in a coming study.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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