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Limit cycles for discontinuous quadratic differential switching systems

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Abstract

In this paper, we study the maximum number of limit cycles by computing ε-order and ε^2-order focal values based on the methods of Yu and Han (J. Appl. Anal. Comput. 1:143-153, 2011) and Yu and Tian (Commun. Nonlinear Sci. Numer. Simul. 19:2690-2705, 2014) for discontinuous differential systems, which can bifurcate from the periodic orbits of the quadratic isochronous centers when they are perturbed inside the class of all discontinuous quadratic polynomial differential systems with the straight line of discontinuity $y = 0$. This work shows that the discontinuous systems have at least five limit cycles surrounding the origin for three different cases and four limit cycles for another case.

MSC: 34C05; 34C07

Keywords: switching systems; elementary critical point; Hopf bifurcation; limit cycle

1 Introduction

It is well known that the 16th problem of Hilbert is far from being solved even for $n = 2$; there were hundreds of references about the limit cycles of continuous planar quadratic polynomial differential systems in books [1, 2]. The classical method for Hopf bifurcation is to compute Lyapunov constants. The critical point will be a center when all Lyapunov constants are all zero. Furthermore, the center is called to be an isochronous center if all periodic constants are all zero. The quadratic polynomial differential systems having an isochronous center were classified by Loud [3]. Chicone and Jacobs proved in [4] that at most two limit cycles bifurcate from the periodic orbits of the isochronous center.

Recently, Chen and Du constructed a quadratic switching system to have nine limit cycles [5]. These examples show that there exist more limit cycles in switching systems than continuous systems, and the dynamics of these systems is more complex. A cubic switching system was constructed to show existence of 15 limit cycles in [6]. Llibre *et al.* study the maximum number of limit cycles that bifurcate from the periodic solutions of the family of isochronous cubic polynomial centers [7], and they also studied the maximum number of limit cycles which can bifurcate from the periodic orbits of the isochronous centers of discontinuous quadratic polynomial differential systems [8], namely the following systems:

$$\begin{cases} \frac{dx}{dt} = -y + x^2 + \varepsilon p_1(x,y), \\ \frac{dy}{dt} = x + xy + \varepsilon q_1(x,y) \end{cases} (y > 0), \qquad \begin{cases} \frac{dx}{dt} = -y + x^2 + \varepsilon p_2(x,y), \\ \frac{dy}{dt} = x + xy + \varepsilon q_2(x,y) \end{cases} (y < 0), \tag{1.1}$$

and

$$\begin{cases} \frac{dx}{dt} = -y + x^2 - y^2 + \varepsilon p_1(x,y), \\ \frac{dy}{dt} = x + 2xy + \varepsilon q_1(x,y) \end{cases} \quad (y > 0),$$
$$\begin{cases} \frac{dx}{dt} = -y + x^2 - y^2 + \varepsilon p_2(x,y), \\ \frac{dy}{dt} = x + 2xy + \varepsilon q_2(x,y) \end{cases} \quad (y < 0), \tag{1.2}$$

where ε is a small parameter, and

$$p_1(x,y) = a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2,$$
$$q_1(x,y) = b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2,$$
$$p_2(x,y) = c_1x + c_2y + c_3xy + c_4x^2 + c_5y^2,$$
$$q_1(x,y) = d_1x + d_2y + d_3xy + d_4x^2 + d_5y^2.$$

By the averaging theory of first order for discontinuous differential systems, they proved the following theorems.

Theorem 1.1 *For* $|\varepsilon| \neq 0$ *sufficiently small, there are discontinuous quadratic polynomial differential systems having at least five limit cycles bifurcating from the periodic orbits of the isochronous center* (1.1) *and four limit cycles bifurcating from the periodic orbits of the isochronous center* (1.2).

In this paper, we investigate the limit cycles bifurcating from the periodic orbits of the quadratic isochronous centers of system (1.3)-(1.6) by computing its ε-order and ε^2-order focal values.

$$\begin{cases} \frac{dx}{dt} = -y + x^2 + \varepsilon P_1(x,y), \\ \frac{dy}{dt} = x + xy + \varepsilon Q_1(x,y) \end{cases} \quad (y > 0), \qquad \begin{cases} \frac{dx}{dt} = -y + x^2 + \varepsilon P_2(x,y), \\ \frac{dy}{dt} = x + xy + \varepsilon Q_2(x,y) \end{cases} \quad (y < 0), \tag{1.3}$$

$$\begin{cases} \frac{dx}{dt} = -y + x^2 - y^2 + \varepsilon P_1(x,y), \\ \frac{dy}{dt} = x + 2xy + \varepsilon Q_1(x,y) \end{cases} \quad (y > 0),$$
$$\begin{cases} \frac{dx}{dt} = -y + x^2 - y^2 + \varepsilon P_2(x,y), \\ \frac{dy}{dt} = x + 2xy + \varepsilon Q_2(x,y) \end{cases} \quad (y < 0), \tag{1.4}$$

$$\begin{cases} \frac{dx}{dt} = -y - \frac{4}{3}x^2 + \varepsilon P_1(x,y), \\ \frac{dy}{dt} = x - \frac{16}{3}xy + \varepsilon Q_1(x,y) \end{cases} \quad (y > 0),$$
$$\begin{cases} \frac{dx}{dt} = -y - \frac{4}{3}x^2 + \varepsilon P_2(x,y), \\ \frac{dy}{dt} = x - \frac{16}{3}xy + \varepsilon Q_2(x,y) \end{cases} \quad (y < 0), \tag{1.5}$$

and

$$\begin{cases} \frac{dx}{dt} = -y + \frac{16}{3}x^2 - \frac{4}{3}y^2 + \varepsilon P_1(x,y), \\ \frac{dy}{dt} = x + \frac{8}{3}xy + \varepsilon Q_1(x,y) \end{cases} \quad (y > 0),$$
$$\begin{cases} \frac{dx}{dt} = -y + + \frac{16}{3}x^2 - \frac{4}{3}y^2 + \varepsilon P_2(x,y), \\ \frac{dy}{dt} = x + \frac{8}{3}xy + \varepsilon Q_2(x,y) \end{cases} \quad (y < 0), \tag{1.6}$$

where ε is a small parameter, and

$$\begin{aligned} P_1(x,y) &= \delta_1 x + a_1 x^2 + a_2 xy + a_3 y^2, \\ Q_1(x,y) &= \delta_1 y + a_4 x^2 + a_5 xy + a_6 y^2, \\ P_2(x,y) &= \delta_2 x + b_1 x^2 + b_2 xy + b_3 y^2, \\ Q_2(x,y) &= \delta_2 y + b_4 x^2 + b_5 xy + b_6 y^2. \end{aligned} \tag{1.7}$$

Our main result is the following theorems.

Theorem 1.2 *For $|\varepsilon| \neq 0$ sufficiently small, there are discontinuous quadratic polynomial differential systems having at least five limit cycles bifurcating from the periodic orbits of the isochronous center of systems* (1.3), (1.5), (1.6).

Theorem 1.3 *For $|\varepsilon| \neq 0$ sufficiently small, there are discontinuous quadratic polynomial differential systems having at least four limit cycles bifurcating from the periodic orbits of the isochronous center of system* (1.4).

In the next two sections, we shall consider the existence of small-amplitude limit cycles, based on the ε-order and ε^2-order.

2 Proof of Theorem 1.2

In this section, we complete the proof of Theorem 1.2.

Proof We consider systems (1.3), (1.5), (1.6) respectively. First of all, let us consider system (1.3). With the help of computer algebra system Mathematics, the first six ε-order Lyapunov constants at the origin are given by

$$\begin{aligned} \lambda_0 &= e^{\varepsilon\delta_1\pi} - e^{\varepsilon\delta_2\pi}, \\ \lambda_1 &= \frac{2}{3}(a_2 + a_4 + 2a_6 - b_2 - b_4 - 2b_6)\varepsilon, \\ \lambda_2 &= -\frac{\pi}{8}(4a_4 + 3a_6 - 2b_2 + 2b_4 - b_6)\varepsilon + o(\varepsilon^2), \\ \lambda_3 &= \frac{2}{15}(19a_4 + 12a_6 - 12b_2 + 17b_4)\varepsilon + o(\varepsilon^2), \\ \lambda_4 &= -\frac{\pi}{8}(a_4 + b_4)\varepsilon + o(\varepsilon^2), \\ \lambda_5 &= -\frac{8}{35}b_4\varepsilon + o(\varepsilon^2). \end{aligned} \tag{2.1}$$

For any sufficiently small $|\varepsilon| \neq 0$, $\lambda_1 = \cdots = \lambda_4 = 0$ yields that

$$\begin{aligned} a_2 &= -a_4 - 2a_6 + b_2 + b_4 + 2b_6, \\ b_6 &= 4a_4 + 3a_6 - 2b_2 + 2b_4, \\ a_6 &= \frac{1}{12}(-19a_4 + 12b_2 - 17b_4), \\ a_4 &= -b_4. \end{aligned}$$

Furthermore,

$$\begin{vmatrix} \frac{dr_1}{da_2} & \frac{dr_1}{db_6} & \frac{dr_1}{da_6} & \frac{dr_1}{da_4} \\ \frac{dr_2}{da_2} & \frac{dr_2}{db_6} & \frac{dr_2}{da_6} & \frac{dr_2}{da_4} \\ \frac{dr_3}{da_2} & \frac{dr_3}{db_6} & \frac{dr_3}{da_6} & \frac{dr_3}{da_4} \\ \frac{dr_4}{da_2} & \frac{dr_4}{db_6} & \frac{dr_4}{da_6} & \frac{dr_4}{da_4} \end{vmatrix} = -\frac{\pi^2}{60}\varepsilon^4 \neq 0,$$

so there exist five limit cycles which could be bifurcated from (1.3). The conclusion holds for system (1.3).

When all ε-order focal values are zero, we compute ε^2-order focal values of system (1.3). The ε^2-order focal values are given by

$$\begin{aligned} \mu_1 &= \frac{\pi}{8}(a_1 + 3a_3 - a_5 - b_1 - 3b_3 + b_5)b_2\varepsilon^2, \\ \mu_2 &= -\frac{2}{45}(-27a_3 + 6b_1 - 9b_3 - 6b_5)b_2\varepsilon^2 + o(\varepsilon^3), \\ \mu_3 &= \frac{\pi}{12}(b_1 - 6b_3 - b_5)b_2\varepsilon^2 + o(\varepsilon^3), \\ \mu_4 &= -\frac{48}{35}b_3b_2\varepsilon^2 + o(\varepsilon^3). \end{aligned} \tag{2.2}$$

Similarly, we could conclude that for any sufficiently small $|\varepsilon| \neq 0$, $\mu_1 = \mu_2 = \mu_3 = 0$, $\mu_4 \neq 0$ yield that

$$\begin{aligned} a_1 &= -3a_3 + a_5 + b_1 + 3b_3 - b_5, \\ a_3 &= \frac{1}{9}(2b_1 - 3b_3 - 2b_5), \\ b_1 &= 6b_3 + b_5, \qquad b_2b_3 \neq 0. \end{aligned}$$

Moreover,

$$\begin{vmatrix} \frac{d\mu_1}{da_2} & \frac{d\mu_1}{db_6} & \frac{d\mu_1}{da_6} \\ \frac{d\mu_2}{da_2} & \frac{d\mu_2}{db_6} & \frac{d\mu_2}{da_6} \\ \frac{d\mu_3}{da_2} & \frac{d\mu_3}{db_6} & \frac{d\mu_3}{da_6} \end{vmatrix} = -\frac{\pi^2}{80}b_2^3\varepsilon^6 \neq 0.$$

So when all ε-order focal values are zero, there exist four limit cycles which could be bifurcated from the origin of system (1.3).

Next, with the help of computer algebra system Mathematics, for system (1.5), the first five ε-order Lyapunov constants at the origin are given by

$$\begin{aligned} \lambda_0 &= e^{\varepsilon\delta_1\pi} - e^{\varepsilon\delta_2\pi}, \\ \lambda_1 &= \frac{2}{3}(a_2 + a_4 + 2a_6 - b_2 - b_4 - 2b_6)\varepsilon, \\ \lambda_2 &= -\frac{\pi}{6}(7a_4 + 6a_6 - 2b_2 + 5b_4 + 2b_6)\varepsilon + o(\varepsilon^2), \\ \lambda_3 &= \frac{32}{135}(59a_4 + 48a_6 - 12b_2 + 13b_4)\varepsilon + o(\varepsilon^2), \\ \lambda_4 &= -\frac{\pi}{27}(7a_4 + 12b_2 - 15b_4)\varepsilon + o(\varepsilon^2), \end{aligned} \tag{2.3}$$

$$\lambda_5 = -\frac{512}{2{,}835}(a_4 - b_4)\varepsilon + o(\varepsilon^2).$$

For any sufficiently small $|\varepsilon| \neq 0$, $\lambda_1 = \cdots = \lambda_4 = 0$ yields that

$$\begin{aligned}
a_2 &= -a_4 - 2a_6 + b_2 + b_4 + 2b_6,\\
b_6 &= \frac{1}{2}(-7a_4 - 6a_6 + 2b_2 - 5b_4),\\
a_6 &= \frac{1}{48}(-59a_4 + 12b_2 - 13b_4),\\
b_2 &= \frac{1}{12}(-7a_4 + 15b_4).
\end{aligned}$$

Direct computation yields that

$$\begin{vmatrix}
\frac{dr_1}{da_2} & \frac{dr_1}{db_6} & \frac{dr_1}{da_6} & \frac{dr_1}{da_4}\\
\frac{dr_2}{da_2} & \frac{dr_2}{db_6} & \frac{dr_2}{da_6} & \frac{dr_2}{da_4}\\
\frac{dr_3}{da_2} & \frac{dr_3}{db_6} & \frac{dr_3}{da_6} & \frac{dr_3}{da_4}\\
\frac{dr_4}{da_2} & \frac{dr_4}{db_6} & \frac{dr_4}{da_6} & \frac{dr_4}{da_4}
\end{vmatrix} = \frac{4{,}096\pi^2}{3{,}645}\varepsilon^4 \neq 0,$$

so there exist five limit cycles which could be bifurcated from (1.5).

All ε-order Lyapunov constants at the origin equal zero if and only if

$$\begin{aligned}
a_2 &= -2a_6 + b_2 + 2b_6,\\
b_6 &= (-3a_6 + b_2 - 6b_4),\\
a_6 &= \frac{1}{4}(b_2 - 6b_4),\\
b_2 &= \frac{2}{3}b_4,\\
a_4 &= b_4.
\end{aligned}$$

Then the ε^2-order Lyapunov constants at the origin of system (1.5) could be given by

$$\begin{aligned}
\mu_1 &= -\frac{\pi}{24}(4a_1 + 6a_3 - a_5 - 4b_1 - 6b_3 + b_5)a_4\varepsilon^2,\\
\mu_2 &= -\frac{16}{135}(9a_3 - 20b_1 - 21b_3 + 5b_5)a_4\varepsilon^2 + o(\varepsilon^3),\\
\mu_3 &= \frac{2\pi}{81}(20b_1 + 12b_3 - 5b_5)a_4\varepsilon^2 + o(\varepsilon^3),\\
\mu_4 &= -\frac{1{,}024}{4{,}725}b_3a_4\varepsilon^2 + o(\varepsilon^3).
\end{aligned} \tag{2.4}$$

By similar discussion, we could conclude that for any sufficiently small $|\varepsilon| \neq 0$ there exist four limit cycles which could be bifurcated from the origin of system (1.5) when all ε-order focal values are zero.

Lastly in this section, we consider system (1.6). It is easy to compute the first five ε-order Lyapunov constants at the origin with the help of computer algebra system Mathematics for system (1.6), they could be given by

$$\begin{aligned}
\lambda_0 &= e^{\varepsilon\delta_1\pi} - e^{\varepsilon\delta_2\pi},\\
\lambda_1 &= \frac{2}{3}(a_2 + a_4 + 2a_6 - b_2 - b_4 - 2b_6)\varepsilon,\\
\lambda_2 &= -\frac{\pi}{6}(13a_4 + 10a_6 - 6b_2 + 7b_4 - 2b_6)\varepsilon + o(\varepsilon^2),
\end{aligned}\tag{2.5}$$

$$\begin{aligned}
\lambda_3 &= \frac{256}{135}(3a_4 - 6b_2 + 17b_4 + 8b_6)\varepsilon + o(\varepsilon^2),\\
\lambda_4 &= \frac{400\pi}{27}(b_2 - 3b_4 - 2b_6)\varepsilon + o(\varepsilon^2),\\
\lambda_5 &= -\frac{32{,}768}{1{,}215}(b_4 - 2b_6)\varepsilon + o(\varepsilon^2).
\end{aligned}\tag{2.6}$$

For any sufficiently small $|\varepsilon| \neq 0$, $\lambda_1 = \cdots = \lambda_4 = 0$ yields that

$$\begin{aligned}
a_2 &= -a_4 - 2a_6 + b_2 + b_4 + 2b_6,\\
a_6 &= \frac{1}{10}(-13a_4 + 6b_2 - 7b_4 + 2b_6),\\
a_4 &= \frac{1}{3}(6b_2 - 17b_4 - 8b_6),\\
b_2 &= (3b_4 + 2b_6).
\end{aligned}$$

Tedious computation yields

$$\begin{vmatrix}
\frac{dr_1}{da_2} & \frac{dr_1}{da_6} & \frac{dr_1}{da_4} & \frac{dr_1}{db_2}\\
\frac{dr_2}{da_2} & \frac{dr_2}{da_6} & \frac{dr_2}{da_4} & \frac{dr_2}{db_2}\\
\frac{dr_3}{da_2} & \frac{dr_3}{da_6} & \frac{dr_3}{da_4} & \frac{dr_3}{db_2}\\
\frac{dr_4}{da_2} & \frac{dr_4}{da_6} & \frac{dr_4}{da_4} & \frac{dr_4}{db_2}
\end{vmatrix} = \frac{15{,}104\pi^2}{3{,}645}\varepsilon^4 \neq 0,$$

which implies that there exist five limit cycles which could be bifurcated from (1.6).

When all ε-order Lyapunov constants at the origin equal zero, the ε^2-order Lyapunov constants at the origin of system (1.6) could be presented by

$$\begin{aligned}
\mu_1 &= \frac{\pi}{8}(4a_1 + 10a_3 - 3a_5 - 4b_1 - 10b_3 + 3b_5)b_6\varepsilon^2,\\
\mu_2 &= \frac{9}{9}(6a_3 + 3a_5 + 8b_1 + 26b_3 - 3b_5)b_6\varepsilon^2 + o(\varepsilon^3),\\
\mu_3 &= -\frac{100\pi}{9}(b_1 + 4b_3)b_6\varepsilon^2 + o(\varepsilon^3),\\
\mu_4 &= \frac{8{,}192}{135}(2b_3 + b_5)b_6\varepsilon^2 + o(\varepsilon^3).
\end{aligned}\tag{2.7}$$

By similar discussion, the first three ε^2-order Lyapunov constants equal zero and the fourth ε^2-order Lyapunov constant does not equal zero if and only if

$$a_1 = \frac{1}{4}(-10a_3 + 3a_5 + 4b_1 + 10b_3 - 3b_5),$$

$$a_3 = \frac{1}{6}(-3a_5 - 8b_1 - 26b_3 + 3b_5),$$

$$b_1 = -4b_3, b_5 \neq -2b_3.$$

So we could conclude that for any sufficiently small $|\varepsilon| \neq 0$ there exist four limit cycles which could be bifurcated from the origin of system (1.6) when all ε-order focal values are zero because

$$\begin{vmatrix} \frac{d\mu_1}{da_1} & \frac{d\mu_1}{da_3} & \frac{d\mu_1}{db_1} \\ \frac{d\mu_2}{da_1} & \frac{d\mu_2}{da_3} & \frac{d\mu_2}{db_1} \\ \frac{d\mu_3}{da_1} & \frac{d\mu_3}{da_3} & \frac{d\mu_3}{db_1} \end{vmatrix} = \frac{\pi^2}{56} b_3^3 \varepsilon^6 \neq 0.$$

□

3 Proof of Theorem 1.3

In this section, we complete the proof of Theorem 1.3.

Proof With the help of computer algebra system Mathematics, for system (1.4), the first five ε-order Lyapunov constants at the origin are given by

$$\begin{aligned} \lambda_0 &= e^{\varepsilon\delta_1\pi} - e^{\varepsilon\delta_2\pi}, \\ \lambda_1 &= \frac{2}{3}(a_2 + a_4 + 2a_6 - b_2 - b_4 - 2b_6)\varepsilon, \\ \lambda_2 &= -\frac{\pi}{2}(a_4 + a_6 + b_4 + b_6)\varepsilon + o(\varepsilon^2), \\ \lambda_3 &= -\frac{2}{15}(3a_4 + 13b_4 + 16b_6)\varepsilon + o(\varepsilon^2), \\ \lambda_4 &= o(\varepsilon^2), \\ \lambda_5 &= -\frac{32}{105}(b_4 + b_6)\varepsilon + o(\varepsilon^2). \end{aligned} \tag{3.1}$$

For any sufficiently small $|\varepsilon| \neq 0$, $\lambda_1 = \cdots = \lambda_4 = 0$ yields that

$$a_2 = -a_4 - 2a_6 + b_2 + b_4 + 2b_6,$$

$$a_6 = -a_4 - b_4 - b_6,$$

$$a_4 = \frac{1}{3}(-13b_4 - 16b_6).$$

Further,

$$\begin{vmatrix} \frac{dr_1}{da_2} & \frac{dr_1}{da_6} & \frac{dr_1}{da_4} \\ \frac{dr_2}{da_2} & \frac{dr_2}{da_6} & \frac{dr_2}{da_4} \\ \frac{dr_3}{da_2} & \frac{dr_3}{da_6} & \frac{dr_3}{da_4} \\ \frac{dr_4}{da_2} & \frac{dr_4}{da_6} & \frac{dr_4}{da_4} \end{vmatrix} = \frac{2\pi}{15}\varepsilon^3 \neq 0,$$

so there exist four limit cycles which could be bifurcated from (1.4).

When all ε-order Lyapunov constants at the origin equal zero, the ε^2-order Lyapunov constants at the origin of system (1.4) could be given by

$$
\begin{aligned}
\mu_1 &= \frac{\pi}{8}(a_1 + a_3 - b_1 - b_3)(b_2 + 2b_6)\varepsilon^2, \\
\mu_2 &= -\frac{2}{15}(a_5 b_2 + b_2 b_5 + 2a_1 b_6 + 8a_3 b_6 + a_5 b_6 \\
&\quad + 2b_1 b_6 + 8b_3 b_6 + b_5 b_6)\varepsilon^2 + o(\varepsilon^3), \\
\mu_3 &= -\frac{\pi}{12}(a_1 + a_3 - b_1 - b_3)(b_2 - 2b_6)\varepsilon^2 + o(\varepsilon^3), \\
\nu\mu_4 &= \frac{2}{105}(6a_5 b_2 + 6b_2 b_5 - 2a_1 b_6 + 16a_3 b_6 \\
&\quad - 3a_5 b_6 - 2b_1 b_6 + 16b_3 b_6 - 3b_5 b_6)\varepsilon^2 + o(\varepsilon^3).
\end{aligned}
\tag{3.2}
$$

If $b_2 = -2b_6$, $(a_1 + a_3 - b_1 - b_3)b_6 \neq 0$, it is to testify that $\mu_3 \neq 0$. If $a_1 + a_3 - b_1 - b_3 = 0$, $b_6 \neq 0$, we have

$$\mu_5 = (b_1 + b_3)b_6\varepsilon^2 + o(\varepsilon^3).$$

By similar discussion, we could conclude that for any sufficiently small $|\varepsilon| \neq 0$ there exist at most four limit cycles which could be bifurcated from the origin of system (1.4) when all ε-order focal values are zero. □

4 Conclusion

In this paper, based on ε-order and ε^2-order focus values, we have shown that four or five limit cycles could be bifurcated from the periodic orbits of the quadratic isochronous centers for four different cases. It is unlikely to have more small-amplitude limit cycles even using higher ε^n-order focus values.

Competing interests

The author declares that they have no competing interests.

Author's contributions

In this paper, the author studies the maximum number of limit cycles by computing ε-order and ε^2-order focal values based on the methods of [9, 10] for discontinuous differential systems; the author shows that this kind of discontinuous systems have at least five limit cycles surrounding the origin for three different cases and four limit cycles for another case.

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