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Robust stability analysis for discrete and distributed time-delays Markovian jumping reaction-diffusion integro-differential equations with uncertain parameters

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Abstract

In this paper, the authors employ Lyapunov stability theory, and the M -matrix, H -matrix, and linear matrix inequality (LMI) techniques and variational methods to obtain the LMI-based stochastically exponential robust stability criterion for discrete and distributed time-delays Markovian jumping reaction-diffusion integro-differential equations with uncertain parameters, whose background of physics and engineering is bidirectional associative memory (BAM) neural networks. It is worth mentioning that an LMI-based stability criterion can easily be computed by the Matlab toolbox which has high efficiency and other advantages in large-scale engineering calculations. Since using the M -matrix and H -matrix methods is not easy in obtaining the LMI criterion conditions, the methods employed in this paper improve those of previous related literature to some extent. Moreover, a numerical example is presented to illustrate the effectiveness of the proposed methods.

Keywords: exponential stability; Laplace diffusion; distributed time-delays; M -matrix; Lyapunov functional

1 Introduction and preliminaries

The time stability analysis of reaction-diffusion time-delays neural networks has attracted more and more interests ([1–5] and the references therein) due to the practical importance and successful applications of neural networks in many areas, such as image processing, combinatorial optimization, signal processing, pattern recognition. Generally, an important precondition of the above applications is that the equilibrium of the neural networks should be stable. Hence, the stability analysis of neural networks has always been an important research topic. Many methods have been proposed to reduce the conservatism of the stability criteria, such as the model transformation method, the free-weighting-matrix approach, constructing novel Lyapunov functionals method, the delay decomposition technique, and the weighting-matrix decomposition method. For example, by using a Lyapunov functional, a modified stability condition for neural networks with discrete and distributed delays has been obtained in [6]; by constructing a general Lyapunov functional and convex combination approach, the stability criterion for neural networks with mixed delays has been obtained in [7]; by partitioning the time delay and using the Jensen integral

inequalities, the stability condition on delayed neural networks with both discrete and distributed delays has been obtained in [8], by employing homomorphic mapping theory and M -matrix theory, the stability criterion for neural networks with discrete and distributed time-delays has been derived in [9]. However, the above methods seem to become less effective in studying the stability of reaction-diffusion integro-differential equations. Some variational methods should be added (see, e.g., [1, 3, 4]). In the actual operations, diffusion effects cannot be avoided in neural networks when electrons are moving in asymmetric electromagnetic fields. Strictly speaking, all the neural networks models should have been reaction-diffusion partial differential equations. Recently, Lyapunov functionals method and the Poincaré inequality were employed to investigate the stability of discrete and distributed time-delays reaction-diffusion integro-differential equations in [1]. But it should be noted that parameter uncertainties cannot be inevitable as usual. Besides, the neural networks are often disturbed by environmental noise in the real world. The noise may influence the stability of the equilibrium and vary some structure parameters, which usually correspond to the Markov process. Systems with Markovian jumping parameters are frequently dominated by discrete-state homogeneous Markov processes, and each state of the parameters represents a mode of the system. During the recent decade, bidirectional associative memory (BAM) neural networks, neural networks with Markovian jumping parameters, have been extensively studied ([10–14] and the references therein) due to the fact that systems with Markovian jumping parameters are useful in modeling abrupt phenomena, such as random failures, changing in the interconnections of subsystems, and operating in different points of a nonlinear plant. So, in this paper, we are to employ Lyapunov stability theory and the M -matrix, H -matrix, and linear matrix equality (LMI) techniques and variational methods to investigate the stochastically robust exponential stability of a class of discrete and distributed time-delays Markovian jumping reaction-diffusion uncertain parameter integro-differential equations (BAM neural networks) as follows:

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial t} = \nabla \cdot (\Theta \circ \nabla u) - A(r(t))u + (C(r(t)) + \Delta C(r(t)))f(v) + (E(r(t)) \\ \quad + \Delta E(r(t)))f(v(t - \tau(t), x)) + (L(r(t)) + \Delta L(r(t))) \int_{t-\varrho}^t f(v(s, x)) ds, \\ \quad t \geq 0, x \in \Omega, \\ \frac{\partial v}{\partial t} = \nabla \cdot (\Lambda \circ \nabla v) - B(r(t))v + (D(r(t)) + \Delta D(r(t)))g(u) + (H(r(t)) \\ \quad + \Delta H(r(t)))g(u(t - \gamma(t), x)) + (W(r(t)) + \Delta W(r(t))) \int_{t-\rho}^t g(u(s, x)) ds, \\ \quad t \geq 0, x \in \Omega, \\ u(s, x) = \phi(s, x), \quad v(s, x) = \psi(s, x), \quad (s, x) \in [-\tau, 0] \times \Omega, \\ u(t, x) = v(t, x) = 0 \in R^n, \quad (t, x) \in [0, +\infty) \times \partial\Omega, \end{array} \right. \quad (1.1)$$

where $x \in \Omega$, and Ω is a bounded domain in R^m with a smooth boundary $\partial\Omega$ of class C^2 by Ω (see, e.g., [15]). $u = u(t, x) = (u_1(t, x), u_2(t, x), \dots, u_n(t, x))^T$, $v = v(t, x) = (v_1(t, x), v_2(t, x), \dots, v_n(t, x))^T \in R^n$, and $u_i(t, x)$ and $v_j(t, x)$ are state variables of the i th neuron and the j th neuron at time t and in space variable x . $f(v) = f(v(t, x)) = (f_1(v_1(t, x)), \dots, f_n(v_n(t, x)))^T$, $g(u) = g(u(t, x)) = (g_1(u_1(t, x)), \dots, g_n(u_n(t, x)))^T$, and $f_j(u_j(t, x))$, $g_j(u_j(t, x))$ are neuron activation functions of the j th unit at time t and in space variable x . $f(v(t - \tau(t), x)) = (f_1(v_1(t - \tau_1(t), x)), \dots, f_j(v_j(t - \tau_j(t), x)), \dots, f_n(v_n(t - \tau_n(t), x)))^T$, $g(u(t - \gamma(t), x)) = (g_1(u_1(t - \gamma_1(t), x)), \dots, g_j(u_j(t - \gamma_j(t), x)), \dots, g_n(u_n(t - \gamma_n(t), x)))^T$, $\tau_j(t)$ and $\gamma_j(t)$ are discrete delays, and ϱ and ρ are distributed time-delays. τ is the constant, satisfying $0 \leq \max\{\tau_j(t), \gamma_j(t), \varrho, \rho\} \leq \tau$ for all $j = 1, 2, \dots, n$. $\Theta = \Theta(t, x, u)$, $\Lambda = \Lambda(t, x, v)$, and $\Theta \circ \nabla u = (\Theta_{ik} \frac{\partial u_i}{\partial x_k})_{n \times m}$, $\Lambda \circ \nabla v =$

$(\Lambda_{ik} \frac{\partial u_i}{\partial x_k})_{n \times m}$ are the Hadamard products of matrix $\Theta = (\Theta_{ik})_{n \times m}$ and ∇u , and of matrix $\Lambda = (\Lambda_{ik})_{n \times m}$ and ∇u , respectively (see [16]). The smooth functions $\Theta_{ik} = \Theta_{ik}(t, x, u) \geq 0$ and $\Lambda_{ik} = \Lambda_{ik}(t, x, v) \geq 0$ are diffusion operators. $(\Omega_*, \Upsilon, \mathbb{P})$ is the given probability space where Ω_* is the sample space, Υ is the σ -algebra of the subset of the sample space, and \mathbb{P} is the probability measure defined on Υ . Let $S = \{1, 2, \dots, N\}$ and the random form process $\{r(t) : [0, +\infty) \rightarrow S\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories with generator $\Pi = (\pi_{ij})_{N \times N}$ and transition probability from mode i at time t to mode j at time $t + \Delta t$, $i, j \in S$,

$$\mathbb{P}(r(t + \delta) = j \mid r(t) = i) = \begin{cases} \pi_{ij}\delta + o(\delta), & j \neq i, \\ 1 + \pi_{ij}\delta + o(\delta), & j = i, \end{cases}$$

where $\pi_{ij} \geq 0$ is the transition probability rate from i to j ($j \neq i$) and $\pi_{ii} = -\sum_{j=1, j \neq i}^N \pi_{ij}$, $\delta > 0$, and $\lim_{\delta \rightarrow 0} o(\delta)/\delta = 0$.

Define the norm $\| \cdot \|$ for vector $u \in R^n$ and matrix $C = (c_{ij})_{n \times n}$ as follows:

$$\|u\| = \sqrt{\sum_{i=1}^n u_i^2}, \quad \|C\| = \sqrt{\lambda_{\max}(CC^T)}.$$

Denote $|u| = (|u_1|, |u_2|, \dots, |u_n|)^T$ for the vector $u = (u_1, \dots, u_n)^T \in R^n$, and $|C| = (|c_{ij}|)_{n \times n}$ for the matrix $C = (c_{ij})_{n \times n}$.

Definition 1.1 For symmetric matrices A, B , we denote $A < B$ or $B > A$ if matrix $B - A$ is a positive definite matrix. Particularly, $A > 0$ if a symmetric matrix A is a positive definite matrix.

In this paper, we denote by $\lambda_{\max} C$ and $\lambda_{\min} C$ the maximum and minimum eigenvalue of matrix C . For any mode $r(t) = r \in S$, we denote $A_r = A(r(t))$ for convenience. So do $B_r, C_r, D_r, E_r, H_r, L_r$, and W_r . In the real world, the parameter uncertainties are inevitable. We assume that

$$\begin{aligned} \Delta C_r &\in [-C_r^*, C_r^*], & \Delta D_r &\in [-D_r^*, D_r^*], & \Delta E_r &\in [-E_r^*, E_r^*], \\ \Delta H_r &\in [-H_r^*, H_r^*], & \Delta L_r &\in [-L_r^*, L_r^*], & \Delta W_r &\in [-W_r^*, W_r^*], \end{aligned} \tag{1.2}$$

where $C_r^*, D_r^*, E_r^*, H_r^*, L_r^*, W_r^*$ are nonnegative matrices. A_r, B_r are diagonal matrices.

There are positive definite diagonal matrices $\underline{A}_r, \underline{B}_r, \bar{A}_r$, and \bar{B}_r such that

$$0 < \underline{A}_r \leq A_r \leq \bar{A}_r, \quad 0 < \underline{B}_r \leq B_r \leq \bar{B}_r, \quad r \in S. \tag{1.3}$$

There exist positive definite diagonal matrices F and G such that

$$|f(u) - f(v)| \leq F|u - v|, \quad |g(u) - g(v)| \leq G|u - v|, \quad \forall u, v \in R^n. \tag{1.4}$$

Throughout this paper, we assume that $f(0) = g(0) = 0 \in R^n$. Then $u = 0, v = 0$ is the null solution for the system (1.1).

Let $u(t, x; \phi, \psi, i_0), v(t, x; \phi, \psi, i_0)$ denote the state trajectory from the initial condition $r(0) = i_0, u(t_0 + \theta, x; \phi, \psi, i_0) = \phi(\theta, x), v(t_0 + \theta, x; \phi, \psi, i_0) = \psi(\theta, x)$ on $-\tau \leq \theta \leq 0$ in $L^2_{\mathcal{F}_0}([-\tau, 0] \times \Omega; R^n)$. (Remark: $t_0 = 0$.) Here, $L^2_{\mathcal{F}_0}([-\tau, 0] \times \Omega; R^n)$ denotes the family of all \mathcal{F}_0 -measurable $C([-\tau, 0] \times \Omega; R^n)$ -value random variables $\xi = \{\xi(\theta, x) : -\tau \leq \theta \leq 0, x \in \Omega\}$ such that $\sup_{-\tau \leq \theta \leq 0} \mathbb{E}\|\xi(\theta)\|_2^2 < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure \mathbb{P} .

Definition 1.2 The system (1.1) is said to be globally stochastically exponentially robust stability if for every initial condition $\phi, \psi \in L^2_{\mathcal{F}_0}([-\tau, 0] \times \Omega; R^n), r(0) = i_0$, there exist scalars $c_1 > 0, c_2 > 0, \beta > 0$, and $\gamma > 0$ such that for any solution $u(t, x; \phi, \psi, i_0), v(t, x; \phi, \psi, i_0)$,

$$\begin{aligned} & \mathbb{E}(\|u(t, x; \phi, \psi, i_0)\|_2^2) + \mathbb{E}(\|v(t, x; \phi, \psi, i_0)\|_2^2) \\ & \leq \gamma e^{-\beta t} \left[c_1 \sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\phi(\theta, x)\|_2^2) + c_2 \sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\psi(\theta, x)\|_2^2) \right], \end{aligned}$$

$t \geq t_0$, for all admissible uncertainties satisfying (1.1), where the norm $\|u\|_2 = (\sum_{i=1}^n \int_{\Omega} u_i^2(t, x) dx)^{\frac{1}{2}}$ for $u = (u_1(t, x), \dots, u_n(t, x))^T \in R^n$.

Definition 1.3 [17] A matrix $A = (a_{ij})_{n \times n}$ is an H -matrix if its comparison matrix $M(A) = (m_{ij})_{n \times n}$ is an M -matrix, where

$$m_{ij} = \begin{cases} |a_{ii}|, & i = j, \\ -|a_{ij}|, & i \neq j. \end{cases}$$

Lemma 1.4 [17] $A, B \in z_n \triangleq \{A = (a_{ij})_{n \times n} \in R^{n \times n} : a_{ij} \leq 0, i \neq j\}$, if A is a M -matrix, A, B satisfy $a_{ij} \leq b_{ij}, i, j = 1, 2, \dots, n$, then B is a M -matrix.

Lemma 1.5 [18] For given constants k_1 and k_2 satisfying $k_1 > k_2 > 0, V(t)$ is a nonnegative continuous function on $[t_0 - \tau, t_0]$, if the following inequality holds:

$$D^+ V(t) \leq k_1 V(t) + k_2 \bar{V}(t),$$

where

$$D^+ V(t) = \limsup_{\Delta t \rightarrow 0^+} \frac{V(t + \Delta t) - V(t)}{\Delta t},$$

$$\bar{V}(t) = \limsup_{t-\tau \leq s \leq t} V(s),$$

$\tau > 0$ is a constant. Then, as $t \geq t_0$, we have

$$V(t) \leq \bar{V}(t_0) e^{-\lambda(t-t_0)},$$

where λ is the unique positive solution of the following equation:

$$\lambda = k_1 - k_2 e^{\lambda \tau}.$$

Lemma 1.6 (Schur complement [19]) *Given matrices $\mathcal{Q}(t), \mathcal{S}(t),$ and $\mathcal{R}(t)$ with appropriate dimensions, where $\mathcal{Q}(t) = \mathcal{Q}(t)^T, \mathcal{R}(t) = \mathcal{R}(t)^T,$ then*

$$\begin{pmatrix} \mathcal{Q}(t) & \mathcal{S}(t) \\ \mathcal{S}^T(t) & \mathcal{R}(t) \end{pmatrix} > 0,$$

if and only if

$$\mathcal{R}(t) > 0, \quad \mathcal{Q}(t) - \mathcal{S}(t)\mathcal{R}^{-1}(t)\mathcal{S}^T(t) > 0;$$

or

$$\mathcal{Q}(t) > 0, \quad \mathcal{R}(t) - \mathcal{S}^T(t)\mathcal{Q}^{-1}(t)\mathcal{S}(t) > 0,$$

where $\mathcal{Q}(t), \mathcal{S}(t),$ and $\mathcal{R}(t)$ are dependent on $t.$

2 Main result

Denote by λ_1 the first eigenvalue of $-\Delta$ in Sobolev space $W_0^{1,2}(\Omega),$ where

$$\lambda_1 = \min \left\{ \int_{\Omega} |\nabla \eta(x)|^2 dx : \eta(x) \in W_0^{1,2}(\Omega), \int_{\Omega} |\eta(x)|^2 dx = 1 \right\}$$

(see [20, 21] for details).

Lemma 2.1 *Let $P_r = \text{diag}(p_{r1}, p_{r2}, \dots, p_{rn})$ be a positive definite matrix, $\alpha_r > 0$ with $\alpha_r I \leq P_r,$ and u, v be a solution of system (1.1). Then we have*

$$\begin{cases} \int_{\Omega} u^T P_r \nabla \cdot (\Theta \circ \nabla u) dx \leq -\lambda_1 \alpha_r \Theta_* \|u\|_2^2, \\ \int_{\Omega} v^T P_r \nabla \cdot (\Lambda \circ \nabla v) dx \leq -\lambda_1 \alpha_r \Lambda_* \|v\|_2^2, \end{cases}$$

where I denotes the identity matrix, $\Theta_* = \min_{ik} \inf_{t,x,u} \Theta(t, x, u)_{ik} \geq 0, \Lambda_* = \min_{ik} \inf_{t,x,v} \Lambda(t, x, v)_{ik} \geq 0.$

Proof Since u, v is a solution of system (1.1), it follows by the Gauss formula and the Dirichlet boundary condition that

$$\begin{aligned} & \int_{\Omega} u^T P_r (\nabla \cdot (\Theta \circ \nabla u)) dx \\ &= \int_{\Omega} u^T P_r \left(\sum_{k=1}^m \frac{\partial}{\partial x_k} \left(\Theta_{1k} \frac{\partial u_1}{\partial x_k} \right), \dots, \sum_{k=1}^m \frac{\partial}{\partial x_k} \left(\Theta_{nk} \frac{\partial u_n}{\partial x_k} \right) \right)^T dx \\ &= - \sum_{k=1}^m \sum_{j=1}^n \int_{\Omega} p_{rj} \Theta_{jk} \left(\frac{\partial u_j}{\partial x_k} \right)^2 dx \leq -\lambda_1 \alpha_r \Theta_* \|u\|_2^2. \end{aligned}$$

Similarly, we can prove that $\int_{\Omega} v^T P_r \nabla \cdot (\Lambda \circ \nabla v) dx \leq -\lambda_1 \alpha_r \Lambda_* \|v\|_2^2.$ Thus, the proof is completed. □

Throughout this paper, we define Φ , Ψ , $\tilde{\Phi}$, and $\tilde{\Psi}$ as follows:

$$\begin{aligned} \Phi &= \frac{1}{\alpha} \left[2\lambda_1 \alpha_r \Theta_* I + 2P_r \underline{A}_r - \beta \|C_r\| (\lambda_{\max} F) I - P_r \Delta C_r \Delta C_r^T P_r - G^2 \right. \\ &\quad - \beta \|D_r\| (\lambda_{\max} G) I - \beta \| |E_r| \|^2 I \\ &\quad \left. - P_r \Delta E_r \Delta E_r^T P_r - \varrho \beta \| |L_r| \|^2 I - \varrho P_r \Delta L_r \Delta L_r^T P_r - \sum_{j=1}^N \pi_{rj} P_j \right], \\ \Psi &= \frac{1}{\alpha} \left[2\lambda_1 \alpha_r \Lambda_* I + 2P_r \underline{B}_r - F^2 - \beta \|C_r\| (\lambda_{\max} F) I \right. \\ &\quad - \beta \|D_r\| (\lambda_{\max} G) I - P_r \Delta D_r \Delta D_r^T P_r - \beta \| |H_r| \|^2 I \\ &\quad \left. - P_r \Delta H_r \Delta H_r^T P_r - \rho \beta \| |W_r| \|^2 I - \rho P_r \Delta W_r \Delta W_r^T P_r - \sum_{j=1}^N \pi_{rj} P_j \right], \\ \tilde{\Phi} &= \frac{1}{\alpha} \left[2\lambda_1 \alpha_r \Theta_* I + 2P_r \underline{A}_r - \beta \|C_r\| (\lambda_{\max} F) I - P_r C_r^* C_r^{*T} P_r - G^2 - \beta \|D_r\| (\lambda_{\max} G) I \right. \\ &\quad \left. - \beta \| |E_r| \|^2 I - P_r E_r^* E_r^{*T} P_r - \varrho \beta \| |L_r| \|^2 I - \varrho P_r L_r^* L_r^{*T} P_r - \sum_{j=1}^N \pi_{rj} P_j \right], \\ \tilde{\Psi} &= \frac{1}{\alpha} \left[2\lambda_1 \alpha_r \Lambda_* I + 2P_r \underline{B}_r - F^2 - \beta \|C_r\| (\lambda_{\max} F) I \right. \\ &\quad - \beta \|D_r\| (\lambda_{\max} G) I - P_r D_r^* D_r^{*T} P_r - \beta \| |H_r| \|^2 I \\ &\quad \left. - P_r H_r^* H_r^{*T} P_r - \rho \beta \| |W_r| \|^2 I - \rho P_r W_r^* W_r^{*T} P_r - \sum_{j=1}^N \pi_{rj} P_j \right]. \end{aligned}$$

Lemma 2.2 *If the following two inequalities hold:*

$$\tilde{\Phi} - \frac{(1 + \beta)(\tau + 1) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I > 0, \quad \forall r \in S, \tag{2.1}$$

and

$$\tilde{\Psi} - \frac{(1 + \beta)(\tau + 1) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I > 0, \quad \forall r \in S, \tag{2.2}$$

we have

$$\Phi - \frac{(1 + \beta)(\tau + 1) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I > 0, \quad \forall r \in S, \tag{2.3}$$

and

$$\Psi - \frac{(1 + \beta)(\tau + 1) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I > 0, \quad \forall r \in S, \tag{2.4}$$

where I denotes the identity matrix, and $\alpha_r, \beta, \alpha > 0$.

Proof Let

$$U = \frac{1}{\alpha} (P_r C_r^* C_r^{*T} P_r + P_r E_r^* E_r^{*T} P_r + \varrho P_r L_r^* L_r^{*T} P_r). \tag{2.5}$$

Since C_r^*, E_r^*, L_r^* all are nonnegative matrices, U is also a nonnegative matrix.

Then $(\tilde{\Phi} + U - \frac{(1+\beta)(\tau+1)\max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I)$ is a positive diagonal matrix, and $(\tilde{\Phi} - \frac{(1+\beta)(\tau+1)\max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I)$ is a M -matrix.

Let

$$Q = \frac{1}{\alpha} [P_r \Delta C_r \Delta C_r^T P_r + P_r \Delta E_r \Delta E_r^T P_r + \varrho P_r \Delta L_r \Delta L_r^T P_r]. \tag{2.6}$$

It follows by (1.2) that $|Q_{ij}| \leq U_{ij}$ for all $i, j = 1, 2, \dots, n$. Here, we denote matrices $Q = (Q_{ij})_{n \times n}$ and $U = (U_{ij})_{n \times n}$. For convenience, we denote

$$\begin{aligned} \hat{\Phi} &= \tilde{\Phi} - \frac{(1+\beta)(\tau+1)\max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I, \\ \underline{\Phi} &= \Phi - \frac{(1+\beta)(\tau+1)\max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I. \end{aligned}$$

Denote, in addition, $\tilde{Q} = \frac{1}{\alpha} [2\lambda_1 \alpha_r \Theta_* I + 2P_r \underline{A}_r - \beta \|C_r\| (\lambda_{\max} F) I - G^2 - \beta \|D_r\| (\lambda_{\max} G) I - \beta \|E_r\|^2 I - \varrho \beta \|L_r\|^2 I - \sum_{j=1}^N \pi_{rj} P_j] - \frac{(1+\beta)(\tau+1)\max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I$, and $\tilde{Q} = \text{diag}(\tilde{Q}_1, \tilde{Q}_2, \dots, \tilde{Q}_n)$ is a diagonal matrix. So the comparison matrix $M(\underline{\Phi}) = (\mu_{ij})$ of $\underline{\Phi}$ is

$$\mu_{ij} = \begin{cases} -|Q_{ij}|, & i \neq j, \\ \tilde{Q}_i - Q_{ii}, & i = j. \end{cases}$$

It is obvious that $\mu_{ij} \geq \hat{\Phi}_{ij}$ for all $i, j = 1, 2, \dots, n$. From Lemma 1.4, we know that $M(\underline{\Phi})$ is an M -matrix. According to Definition 1.3, $\underline{\Phi}$ is a H -matrix with positive diagonal elements, it is to say $\underline{\Phi} > 0$. Hence we have proved (2.3) by way of (2.1). Similarly, we can also derive (2.4) from the condition (2.2). The proof is completed. \square

Theorem 2.3 *Assume that (1.2)-(1.4) hold. If, in addition, there exist a sequence of positive definite diagonal matrices $P_r (r \in S)$ and positive scalars $\beta, \alpha_r (r \in S)$ such that the following LMIs conditions hold:*

$$\begin{pmatrix} Q_r & \frac{1}{\sqrt{\alpha}} P_r C_r^* & \frac{1}{\sqrt{\alpha}} P_r E_r^* & \sqrt{\frac{\varrho}{\alpha}} P_r L_r^* \\ \frac{1}{\sqrt{\alpha}} (C_r^*)^T P_r & I & 0 & 0 \\ \frac{1}{\sqrt{\alpha}} (E_r^*)^T P_r & 0 & I & 0 \\ \sqrt{\frac{\varrho}{\alpha}} (L_r^*)^T P_r & 0 & 0 & I \end{pmatrix} > 0, \quad r \in S, \tag{2.7}$$

$$\begin{pmatrix} \Omega_r & \frac{1}{\sqrt{\alpha}} P_r D_r^* & \frac{1}{\sqrt{\alpha}} P_r H_r^* & \sqrt{\frac{\varrho}{\alpha}} P_r W_r^* \\ \frac{1}{\sqrt{\alpha}} (D_r^*)^T P_r & I & 0 & 0 \\ \frac{1}{\sqrt{\alpha}} (H_r^*)^T P_r & 0 & I & 0 \\ \sqrt{\frac{\varrho}{\alpha}} (W_r^*)^T P_r & 0 & 0 & I \end{pmatrix} > 0, \quad r \in S, \tag{2.8}$$

$$\alpha_r I \leq P_r, \quad r \in S, \tag{2.9}$$

$$P_r \leq \beta I, \quad r \in S, \tag{2.10}$$

then the system (1.1) is stochastically global exponential robust stability, where

$$\begin{aligned} Q_r &\triangleq \frac{1}{\alpha} \left[2\lambda_1 \alpha_r \Theta_* I + 2P_r \underline{A}_r - \beta \|C_r\| (\lambda_{\max} F) I - G^2 - \beta \|D_r\| (\lambda_{\max} G) I \right. \\ &\quad \left. - \beta \| |E_r| \|^2 I - \varrho \beta \| |L_r| \|^2 I - \sum_{j=1}^N \pi_{rj} P_j \right] \\ &\quad - \frac{(1 + \beta)(\tau + 1) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I, \\ \Omega_r &\triangleq \frac{1}{\alpha} \left[2\lambda_1 \alpha_r \Lambda_* I + 2P_r \underline{B}_r - \beta \|C_r\| (\lambda_{\max} F) I - F^2 - \beta \|D_r\| (\lambda_{\max} G) I - \beta \| |H_r| \|^2 I \right. \\ &\quad \left. - \varrho \beta \| |W_r| \|^2 I - \sum_{j=1}^N \pi_{rj} P_j \right] \\ &\quad - \frac{(1 + \beta)(\tau + 1) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha} I, \end{aligned}$$

and α is a positive scalar with $\alpha < \alpha_r$ for all $r \in S$.

Proof Let $P_r = \text{diag}(p_{r1}, \dots, p_{rn})$ be a positive definite matrix for mode $r \in S$, we consider the following Lyapunov-Krasovskii functional:

$$V(t) = \int_{\Omega} (u^T(t, x) P_r u(t, x) + v^T(t, x) P_r v(t, x)) \, dx, \quad r \in S.$$

It follows from Lemma 2.1 that

$$\begin{aligned} \int_{\Omega} 2u^T P_r \nabla \cdot (\Theta \circ \nabla u) \, dx &\leq -2\lambda_1 \int_{\Omega} u^T \alpha_r \Theta_* I u \, dx, \quad \text{and} \\ \int_{\Omega} 2v^T P_r \nabla \cdot (\Lambda \circ \nabla v) \, dx &\leq -2\lambda_1 \int_{\Omega} v^T \alpha_r \Lambda_* I v \, dx. \end{aligned} \tag{2.11}$$

Obviously, we can get by (1.3)

$$\begin{aligned} \int_{\Omega} -2u^T P_r A_r u \, dx &\leq \int_{\Omega} -2u^T P_r \underline{A}_r u \, dx, \quad \text{and} \\ \int_{\Omega} -2v^T P_r B_r v \, dx &\leq \int_{\Omega} -2v^T P_r \underline{B}_r v \, dx. \end{aligned} \tag{2.12}$$

Besides, we can derive the following inequalities by the assumptions on f, g, β , and $C_r, D_r, E_r, H_r, L_r, W_r$:

$$\begin{aligned} &\int_{\Omega} 2u^T P_r (C_r + \Delta C_r) f(v) \, dx \\ &\leq 2 \int_{\Omega} (\|u\| \cdot \|P_r\| \cdot \|C_r\| \cdot \|f(v)\| + u^T P_r \Delta C_r f(v)) \, dx \end{aligned}$$

$$\begin{aligned}
 &\leq \int_{\Omega} (2\|u\| \cdot \beta \|C_r\| \cdot \|f(v)\| + 2u^T P_r \Delta C_r f(v)) \, dx \\
 &\leq \int_{\Omega} [u^T (\beta \|C_r\| (\lambda_{\max} F) I + P_r \Delta C_r \Delta C_r^T P_r) u \\
 &\quad + v^T (F^2 + \beta \|C_r\| (\lambda_{\max} F) I) v] \, dx,
 \end{aligned} \tag{2.13}$$

$$\begin{aligned}
 &\int_{\Omega} 2v^T P_r (D_r + \Delta D_r) g(u) \, dx \\
 &\leq 2 \int_{\Omega} (\|v\| \cdot \beta \|D_r\| \cdot \|g(u)\| + v^T P_r \Delta D_r g(u)) \, dx \\
 &\leq \int_{\Omega} [u^T (G^2 + \beta \|D_r\| (\lambda_{\max} G) I) u \\
 &\quad + v^T (\beta \|D_r\| (\lambda_{\max} G) I + P_r \Delta D_r \Delta D_r^T P_r) v] \, dx,
 \end{aligned} \tag{2.14}$$

$$\begin{aligned}
 &\int_{\Omega} 2u^T P_r (E_r + \Delta E_r) f(v(t - \tau(t), x)) \, dx \\
 &= \int_{\Omega} (2u^T P_r E_r f(v(t - \tau(t), x)) + 2u^T P_r \Delta E_r f(v(t - \tau(t), x))) \, dx \\
 &\leq \int_{\Omega} (2|u|^T \beta |E_r| |f(v(t - \tau(t), x))| + 2u^T P_r \Delta E_r f(v(t - \tau(t), x))) \, dx \\
 &\leq \int_{\Omega} (\beta (|u|^T |E_r| |E_r^T| |u| + f^T(v(t - \tau(t), x)) f(v(t - \tau(t), x))) \\
 &\quad + u^T P_r \Delta E_r \Delta E_r^T P_r u + f^T(v(t - \tau(t), x)) f(v(t - \tau(t), x))) \, dx \\
 &\leq \int_{\Omega} [u^T (\beta \| |E_r| \|^2 I + P_r \Delta E_r \Delta E_r^T P_r) u \\
 &\quad + v^T (t - \tau(t), x) (1 + \beta) F^2 v(t - \tau(t), x)] \, dx,
 \end{aligned} \tag{2.15}$$

$$\begin{aligned}
 &\int_{\Omega} 2v^T P_r (H_r + \Delta H_r) g(u(t - \gamma(t), x)) \, dx \\
 &= \int_{\Omega} (2v^T P_r H_r g(u(t - \gamma(t), x)) + 2v^T P_r \Delta H_r g(u(t - \gamma(t), x))) \, dx \\
 &\leq \int_{\Omega} (\beta (|v|^T |H_r| |H_r^T| |v| + g^T(u(t - \gamma(t), x)) g(u(t - \gamma(t), x))) \\
 &\quad + v^T P_r \Delta H_r \Delta H_r^T P_r v + g^T(u(t - \gamma(t), x)) g(u(t - \gamma(t), x))) \, dx \\
 &\leq \int_{\Omega} [u^T (t - \gamma(t), x) (1 + \beta) G^2 u(t - \gamma(t), x) \\
 &\quad + v^T (\beta \| |H_r| \|^2 I + P_r \Delta H_r \Delta H_r^T P_r) v] \, dx,
 \end{aligned} \tag{2.16}$$

$$\begin{aligned}
 &\int_{\Omega} \left(2u^T P_r (L_r + \Delta L_r) \int_{t-\varrho}^t f(v(s, x)) \, ds \right) \, dx \\
 &= \int_{\Omega} \left(\int_{t-\varrho}^t 2u^T P_r L_r f(v(s, x)) \, ds + \int_{t-\varrho}^t 2u^T P_r \Delta L_r f(v(s, x)) \, ds \right) \, dx \\
 &\leq \int_{\Omega} \left[\int_{t-\varrho}^t (\beta (|u|^T |L_r| |L_r^T| |u| + f^T(v(s, x)) f(v(s, x)))) \, ds \right. \\
 &\quad \left. + \int_{t-\varrho}^t (u^T P_r \Delta L_r \Delta L_r^T P_r u + f^T(v(s, x)) f(v(s, x))) \, ds \right] \, dx
 \end{aligned}$$

$$\begin{aligned}
 &\leq \int_{\Omega} \left[u^T (\varrho\beta \| |L_r| \|^2 I + \varrho P_r \Delta L_r \Delta L_r^T P_r) u \right. \\
 &\quad \left. + \int_{t-\varrho}^t v^T(s, x) (1 + \beta) F^2 v(s, x) ds \right] dx, \tag{2.17} \\
 &\int_{\Omega} \left(2v^T P_r (W_r + \Delta W_r) \int_{t-\rho}^t g(u(s, x)) ds \right) dx \\
 &= \int_{\Omega} \left(\int_{t-\rho}^t 2v^T P_r W_r g(u(s, x)) ds + \int_{t-\rho}^t 2v^T P_r \Delta W_r g(u(s, x)) ds \right) dx \\
 &\leq \int_{\Omega} \left[\int_{t-\rho}^t \beta (|v|^T |W_r| |W_r|^T |v| + g^T(u(s, x)) g(u(s, x))) ds \right. \\
 &\quad \left. + \int_{t-\rho}^t (v^T P_r \Delta W_r \Delta W_r^T P_r v + g^T(u(s, x)) g(u(s, x))) ds \right] dx \\
 &\leq \int_{\Omega} \left[v^T (\rho\beta \| |W_r| \|^2 I + \rho P_r \Delta W_r \Delta W_r^T P_r) v \right. \\
 &\quad \left. + \int_{t-\rho}^t u^T(s, x) (1 + \beta) G^2 u(s, x) ds \right] dx. \tag{2.18}
 \end{aligned}$$

Moreover, we can conclude from the restrictions of α, β that

$$\begin{aligned}
 &\int_{\Omega} \left(\int_{t-\rho}^t u^T(s, x) (1 + \beta) G^2 u(s, x) ds + \int_{t-\varrho}^t v^T(s, x) (1 + \beta) F^2 v(s, x) ds \right) dx \\
 &\leq (1 + \beta) \int_{\Omega} \left[\int_{t-\tau}^t (u^T(s, x) G^2 u(s, x) + v^T(s, x) F^2 v(s, x) ds) \right] dx \\
 &\leq \frac{(1 + \beta) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha_r} \\
 &\quad \times \int_{\Omega} \left[\int_{t-\tau}^t (u^T(s, x) P_r u(s, x) + v^T(s, x) P_r v(s, x) ds) \right] dx \\
 &\leq \frac{(1 + \beta) \tau \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha_r} \sup_{t-\tau \leq s \leq t} V(s), \tag{2.19}
 \end{aligned}$$

and

$$\begin{aligned}
 &\int_{\Omega} \left[u^T(t - \gamma(t), x) (1 + \beta) G^2 u(t - \gamma(t), x) + v^T(t - \tau(t), x) (1 + \beta) F^2 v(t - \tau(t), x) \right] dx \\
 &\leq \frac{(1 + \beta) \max\{(\lambda_{\max} F)^2, (\lambda_{\max} G)^2\}}{\alpha_r} \sup_{t-\tau \leq s \leq t} V(s). \tag{2.20}
 \end{aligned}$$

Then we can conclude from (2.9)-(2.20) and the weak infinitesimal operator \mathcal{L} that

$$\begin{aligned}
 \mathcal{L}V(t) = &\lim_{\delta \rightarrow 0^+} \frac{1}{\delta} \left\{ \mathbb{E} \left[\int_{\Omega} (u^T(t + \delta, x) P(r(t + \delta))) u(t + \delta, x) \right. \right. \\
 &\quad \left. \left. + v^T(t + \delta, x) P(r(t + \delta)) v(t + \delta, x) \right) dx \Big| r(t) = r \right] \\
 &\quad \left. - \int_{\Omega} (u^T(t, x) P_r u(t, x) + v^T(t, x) P_r v(t, x)) dx \right\}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{\delta \rightarrow 0^+} \frac{\mathbb{E}[\int_{\Omega} u^T(t + \delta, x)(P_r + \sum_{j=1}^N \pi_{rj}P_j)u(t + \delta, x) dx - \int_{\Omega} u^T(t, x)P_r u(t, x) dx]}{\delta} \\
 &\quad + \lim_{\delta \rightarrow 0^+} \frac{\mathbb{E}[\int_{\Omega} v^T(t + \delta, x)(P_r + \sum_{j=1}^N \pi_{rj}P_j)v(t + \delta, x) dx - \int_{\Omega} v^T(t, x)P_r v(t, x) dx]}{\delta} \\
 &\leq -\mathbb{E} \int_{\Omega} \left[u^T \left(2\lambda_1 \alpha_r \Theta_* I + 2P_r \underline{A}_r - \beta \|C_r\|(\lambda_{\max} F)I - P_r \Delta C_r \Delta C_r^T P_r - G^2 \right. \right. \\
 &\quad \left. \left. - \beta \|D_r\|(\lambda_{\max} G)I - \beta \| |E_r| \|^2 I - P_r \Delta E_r \Delta E_r^T P_r - \varrho \beta \| |L_r| \|^2 I \right. \right. \\
 &\quad \left. \left. - \varrho P_r \Delta L_r \Delta L_r^T P_r - \sum_{j=1}^N \pi_{rj} P_j \right) u \right. \\
 &\quad \left. + v^T \left(2\lambda_1 \alpha_r \Lambda_* I + 2P_r \underline{B}_r - F^2 - \beta \|C_r\|(\lambda_{\max} F)I \right. \right. \\
 &\quad \left. \left. - \beta \|D_r\|(\lambda_{\max} G)I - P_r \Delta D_r \Delta D_r^T P_r - \beta \| |H_r| \|^2 I \right. \right. \\
 &\quad \left. \left. - P_r \Delta H_r \Delta H_r^T P_r - \rho \beta \| |W_r| \|^2 I - \rho P_r \Delta W_r \Delta W_r^T P_r - \sum_{j=1}^N \pi_{rj} P_j \right) v \right. \\
 &\quad \left. + (1 + \beta) \left(u^T(t - \gamma(t), x) G^2 u(t - \gamma(t), x) + v^T(t - \tau(t), x) F^2 v(t - \tau(t), x) \right. \right. \\
 &\quad \left. \left. + \int_{t-\rho}^t u^T(s, x) G^2 u(s, x) ds + \int_{t-\varrho}^t v^T(s, x) F^2 v(s, x) ds \right) \right] dx \\
 &\leq -\min_{r \in S} \{ \lambda_{\min} \Phi, \lambda_{\min} \Psi \} \mathbb{E}V(t) \\
 &\quad + \frac{(1 + \beta)(\tau + 1) \max \{ (\lambda_{\max} F)^2, (\lambda_{\max} G)^2 \}}{\alpha} \sup_{t-\tau \leq s \leq t} \mathbb{E}V(s). \tag{2.21}
 \end{aligned}$$

Let $\Delta t > 0$ be small enough, we can integrate (2.21) from t to $t + \Delta t$ as follows:

$$\begin{aligned}
 &\mathbb{E}V(t + \Delta t) - \mathbb{E}V(t) \\
 &\leq \int_t^{t+\Delta t} \left(-\min_{r \in S} \{ \lambda_{\min} \Phi, \lambda_{\min} \Psi \} \mathbb{E}V(\theta) \right. \\
 &\quad \left. + \frac{(1 + \beta)(\tau + 1) \max \{ (\lambda_{\max} F)^2, (\lambda_{\max} G)^2 \}}{\alpha} \sup_{\theta-\tau \leq s \leq \theta} \mathbb{E}V(s) \right) d\theta. \tag{2.22}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 D^+ \mathbb{E}V(t) &\leq -\min_{r \in S} \{ \lambda_{\min} \Phi, \lambda_{\min} \Psi \} \mathbb{E}V(t) \\
 &\quad + \frac{(1 + \beta)(\tau + 1) \max \{ (\lambda_{\max} F)^2, (\lambda_{\max} G)^2 \}}{\alpha} \sup_{t-\tau \leq s \leq t} \mathbb{E}V(s). \tag{2.23}
 \end{aligned}$$

Let $k_1 = \min_{r \in S} \{ \lambda_{\min} \Phi, \lambda_{\min} \Psi \}$, and $k_2 = \frac{(1+\beta)(\tau+1) \max \{ (\lambda_{\max} F)^2, (\lambda_{\max} G)^2 \}}{\alpha}$. Moreover, the conditions (2.1) and (2.2) are satisfied owing to (2.7)-(2.10) and Schur complement theorem (Lemma 1.6), and we can conclude from Lemma 2.2 that k_1 and k_2 satisfy all the conditions in Lemma 1.5. Since k_1 and k_2 are independent of mode $r \in S$, it follows by Lemma 1.5

that

$$\mathbb{E}V(t) \leq \mathbb{E}\bar{V}(0)e^{-\lambda t}, \tag{2.24}$$

where λ is the unique positive solution of the equation $\lambda = k_1 - k_2e^{\lambda\tau}$, and

$$\mathbb{E}\bar{V}(0) = \limsup_{-\tau \leq s \leq 0} \mathbb{E}V(s) \leq \beta \left[\sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\phi(\theta, x)\|_2^2) + \sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\psi(\theta, x)\|_2^2) \right], \tag{2.25}$$

where $\beta = \max_{r \in S}(\lambda_{\max} P_r)$.

Moreover, (2.24) and (2.25) yield

$$\begin{aligned} \left(\min_{r \in S} \alpha_r\right) \mathbb{E} \int_{\Omega} (u^T u + v^T v) dx &\leq \mathbb{E}V(t) \\ &\leq \beta \left[\sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\phi(\theta, x)\|_2^2) + \sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\psi(\theta, x)\|_2^2) \right] e^{-\lambda t}, \end{aligned}$$

from which one derives

$$\begin{aligned} \mathbb{E}\|u\|_2^2 + \mathbb{E}\|v\|_2^2 &\leq \frac{\beta}{\min_{r \in S} \alpha_r} e^{-\lambda t} \left[\sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\phi(\theta, x)\|_2^2) + \sup_{-\tau \leq \theta \leq 0} \mathbb{E}(\|\psi(\theta, x)\|_2^2) \right], \\ t &\geq 0. \end{aligned}$$

According to Definition 1.2, we know that the system (1.1) is stochastically global exponential robust stability. The proof is completed. \square

Remark 2.1 In 2015, [9], Theorem 3.3 offered a stability criterion for the following neural networks ([9], (22)) with discrete and distributed time-delays by way of M -matrix and H -matrix methods:

$$\dot{z}(t) = -Cz(t) + Ag(z(t)) + Bg(z(t - \tau(t))) + D \int_{t-\sigma}^t g(z(s)) ds.$$

However, using the M -matrix and H -matrix methods is not easy in obtaining the LMI criterion conditions. Motivated by some methods of [9], we employ the Schur complement technique to derive the LMI-based stability criterion - Theorem 2.3.

3 Example

Consider two modes for the Markovian jumping system (1.1).

For Mode 1,

$$\begin{aligned} \underline{A}_1 &= \begin{pmatrix} 2.9 & 0 \\ 0 & 2.1 \end{pmatrix}, & \underline{B}_1 &= \begin{pmatrix} 2.7 & 0 \\ 0 & 1.9 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} 0.5 & 0.01 \\ 0.01 & 0.6 \end{pmatrix} = D_1 = E_1 = H_1 = L_1 = W_1, \\ C_1^* = E_1^* = L_1^* &= \begin{pmatrix} 0.05 & 0.01 \\ 0.01 & 0.06 \end{pmatrix}, & D_1^* = H_1^* = W_1^* &= \begin{pmatrix} 0.07 & 0.03 \\ 0.03 & 0.06 \end{pmatrix}. \end{aligned}$$

For Mode 2,

$$\begin{aligned} \underline{A}_2 &= \begin{pmatrix} 2.8 & 0 \\ 0 & 2 \end{pmatrix}, & \underline{B}_2 &= \begin{pmatrix} 2.1 & 0 \\ 0 & 2.8 \end{pmatrix}, \\ C_2 &= \begin{pmatrix} 0.7 & 0.01 \\ 0.01 & 0.5 \end{pmatrix} = D_2 = E_2 = H_2 = L_2 = W_2, \\ C_2^* = E_2^* = L_2^* &= \begin{pmatrix} 0.06 & 0.02 \\ 0.02 & 0.05 \end{pmatrix}, & D_2^* = H_2^* = W_2^* &= \begin{pmatrix} 0.07 & 0.03 \\ 0.03 & 0.06 \end{pmatrix}. \end{aligned}$$

Let $u = (u_1, u_2)^T, v = (v_1, v_2)^T \in R^2, x = (x_1, x_2) \in (0, 10) \times (0, 10) \subset R^2$, and then $\lambda_1 = 0.02\pi^2 = 0.1974$ (see [22], Remark 2.5). Assume that $\varrho = 0.8, \rho = 0.7, \tau = 1.2$, and

$$\begin{aligned} F &= \begin{pmatrix} 0.3 & 0 \\ 0 & 0.2 \end{pmatrix}, & G &= \begin{pmatrix} 0.2 & 0 \\ 0 & 0.3 \end{pmatrix}, \\ \Theta &= \begin{pmatrix} 0.005 & 0.009 \\ 0.006 & 0.008 \end{pmatrix}, & \Lambda &= \begin{pmatrix} 0.007 & 0.009 \\ 0.006 & 0.004 \end{pmatrix}. \end{aligned}$$

So $\Lambda_* = 0.004, \Theta_* = 0.005$. Let $\pi_{11} = -0.6, \pi_{12} = 0.6; \pi_{21} = 0.2, \pi_{22} = -0.2$. Let $\alpha = 5$, then we use the Matlab LMI toolbox to solve the linear matrices inequalities (2.7)-(2.10), and then the feasibility data follows:

$$P_1 = \begin{pmatrix} 5.5763 & 0 \\ 0 & 5.7127 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 5.9111 & 0 \\ 0 & 5.8758 \end{pmatrix},$$

and $\alpha_1 = 5.2048, \alpha_2 = 5.3765, \beta = 6.1317$. Hence, $\alpha = 5 < \alpha_1$ and $\alpha = 5 < \alpha_2$. According to Theorem 2.3, we know that the system (1.1) is stochastically global exponential robust stability (see Figures 1-4).

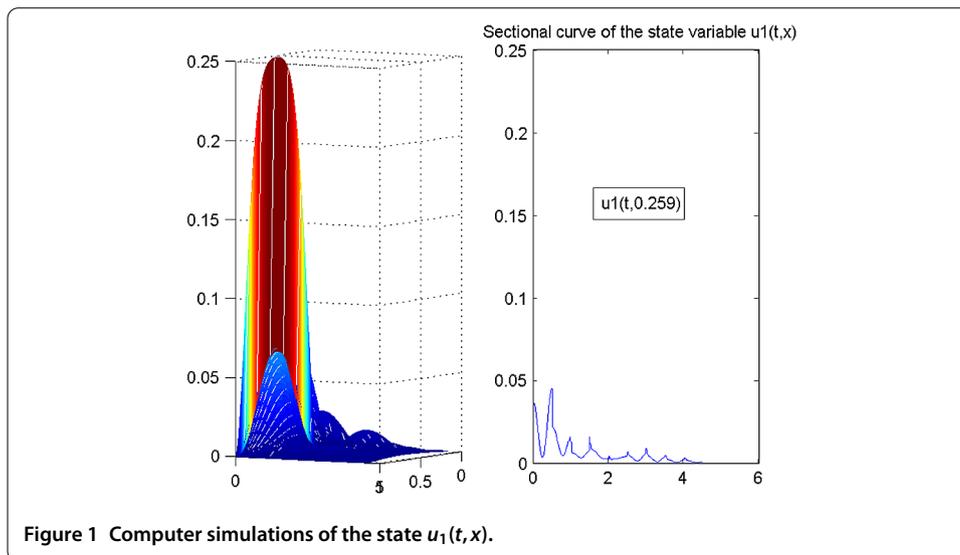
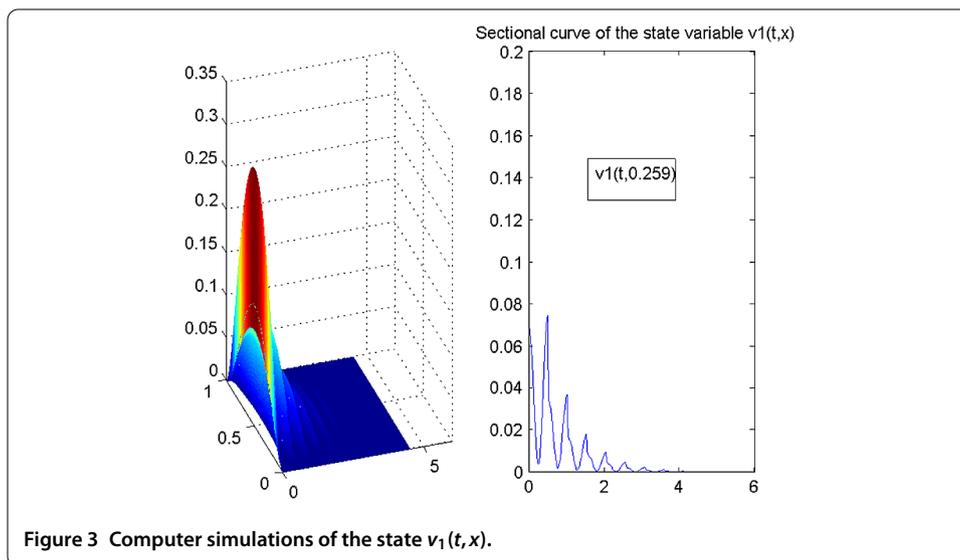
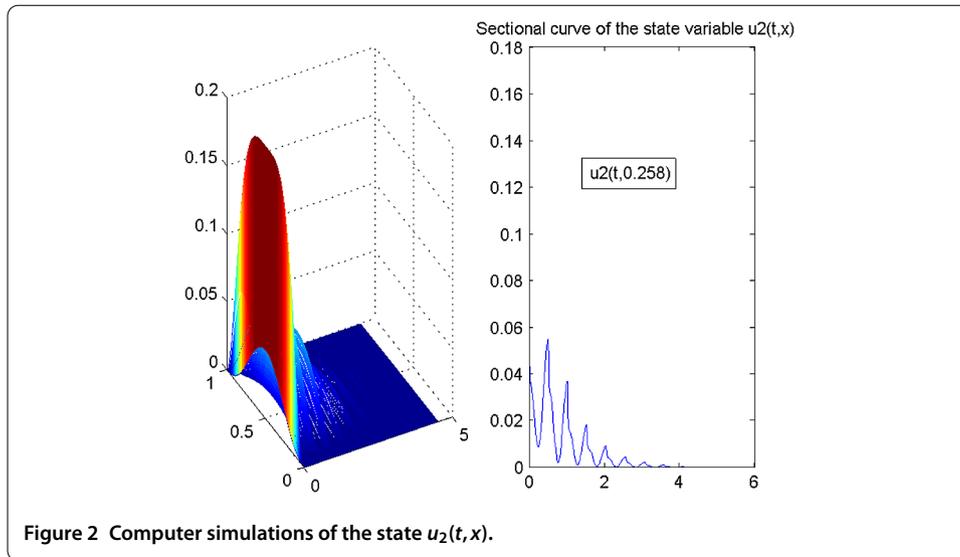
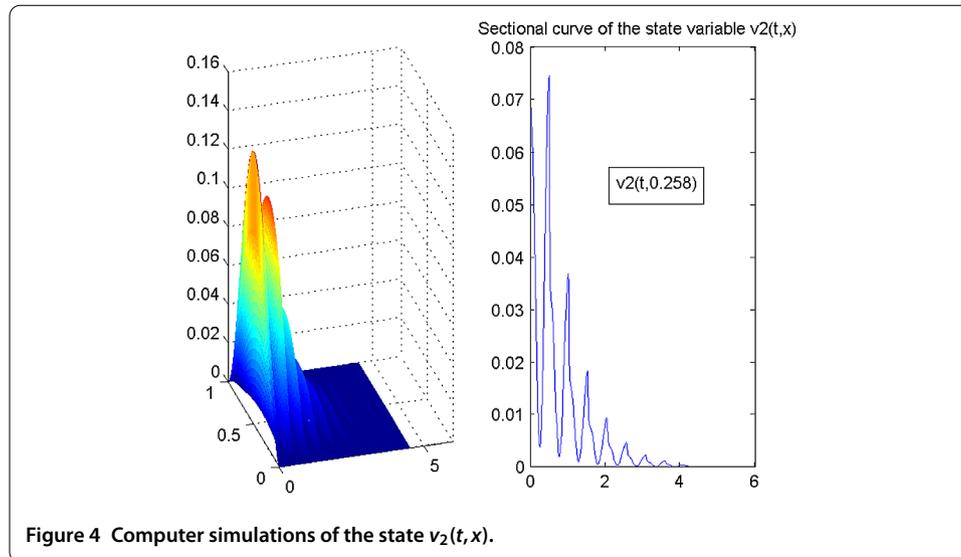


Figure 1 Computer simulations of the state $u_1(t, x)$.



4 Conclusions

LMI-based stability criterion can easily be computed by the Matlab toolbox which has a high efficiency and other advantages in large-scale engineering calculations. In this paper, we employ Lyapunov stability theory, the M -matrix, H -matrix and linear matrix equality (LMI) techniques and variational methods to obtain the LMI-based stochastically exponential robust stability criterion for discrete and distributed time-delays Markovian jumping reaction-diffusion integro-differential equations with uncertain parameters, whose background of physics and engineering is BAM neural networks. Since using the M -matrix and H -matrix methods is not easy in obtaining the LMI criterion conditions, the methods employed in this paper improve those of previous related literature to some extent. Moreover, a numerical example in Section 3 is presented to illustrate the effectiveness of the proposed methods.



Competing interests

The authors declare that they have no competing interests.

Authors' contributions

XY wrote the original manuscript, carrying out the main part of this work. SZ examined the original manuscript in detail before submission. XW and RR are in charge of the correspondence. All authors typed, read, and approved the final manuscript.

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