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Stabilization of neutral-type indirect control systems to absolute stability state

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Abstract

This paper provides sufficient conditions for absolute stability of an indirect control Lur'e problem of neutral type. The conditions are derived using a Lyapunov-Krasovskii functional and are given in terms of a system of matrix algebraic inequalities. From these matrix inequalities a sufficient condition for linear state feedback stabilizability follows.

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1 Introduction

The problem of absolute stability is often encountered in engineering practice. One specific form of this problem is the indirect control Lur'e problem, where the system to be controlled is linear, but the control action is the output of a nonlinear scalar system that itself receives output feedback. The special case where the output of the controller is a nonlinear function of one variable whose graph lies between two lines in the first and third quadrants of the coordinate plane is usually studied. Initially only systems of ordinary differential equations were considered; see for example [1–6]. A historical overview of the absolute stability problem can be found in [7] or in the introduction of [8].

In practical control processes time delays are common and they often cause instabilities, as a result, the absolute stability problem of nonlinear control systems with delay has attracted a lot of interest [5, 6, 9–11]. Nonlinear systems of neutral type with indirect control are considered in [10–15]. Sufficient conditions for absolute stability for such systems are derived in [13, 14] by the direct Lyapunov method using Lyapunov-Krasovskii functionals, these conditions are given in Theorem 1 of this paper. The functionals are constructed by taking the sum of a quadratic form of the current coordinates, integrals over the delay of quadratic forms of the state and its derivative, and the integral of the nonlinear components of the considered system [5, 6, 9–11]. All results from [11, 13, 14] can be put into a unified form in terms of matrix algebraic inequalities. A very different approach is given in [16, 17] or [18, 19], where integral operators are used.

In this paper we also consider what to do if absolute stability of the system under investigation cannot be established using the result given in Theorem 1. There are two obvious options: either change the method of investigation or change the Lyapunov function or functional. But there is a third option: we can try to add a linear state feedback to stabilize

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the closed loop system for the previously chosen Lyapunov function or functional. There are some interesting papers devoted to the investigation of stability and stabilization tasks [20–22].

The present article is a direct extension of [22]. The remainder of this paper is organized as follows. In Section 2 the absolute stability problem of neutral type indirect nonlinear control system is formulated, some notation is defined, and a result from [13, 14] is stated. Section 3 introduces the concepts of stability and stabilization with respect to a given functional for the case of a linear control system with delay. In Section 4 the scalar case of a neutral system with nonlinear indirect control is treated. The indirect control system of neutral type in the general matrix form is considered in Section 5. Finally, some conclusion are drawn in Section 6.

2 Problem formulation and preliminaries

In this paper $\mathbb{R}_0^+ = [0,\infty)$, $\mathbb{R}^n$ is the n-dimensional vector space over the real numbers; $\mathbb{R}^{m\times n}$ will be used for the set of all $m \times n$ matrices, $\mathrm{I}_{n\times n}$ is the $n \times n$ identity matrix; $0_{m\times n}$ is an $m \times n$ matrix filled with zeros; a superscript T marks the transpose of a vector or a matrix; and $\vec{\mathrm{e}}_{k,n}$ is the unit vector along the kth coordinate direction in an n-dimensional space. Subscripts n and $n \times n$, which indicate the dimension of the space or the matrix, will be dropped whenever they are clear from the context. The Euclidean norm of a vector $a \in \mathbb{R}^n$ will be written as $|a|$, so

$$|a| = \left(\sum_{i=1}^{n} a_i^2\right)^{\frac{1}{2}}$$

and for a square matrix $A \in \mathbb{R}^{n\times n}$, $|A|$ will be the operator norm induced by the Euclidean vector norm. Recall that

$$|A| = \left(\lambda_{\max}\left(A^T A\right)\right)^{\frac{1}{2}},$$

where $\lambda_{\max}$ is the largest eigenvalue of $A^{\mathrm{T}}A$. We will write $\mathcal{C}_{n,\tau}$ for the Banach space $\mathcal{C}([-\tau,0],\mathbb{R}^n)$ of continuous functions from $[-\tau,0]$ to $\mathbb{R}^n$ with norm

$$\|x\|_\infty = \sup_{s\in[-\tau,0]} \left\{\left|x(s)\right|\right\}$$

and use $\mathcal{C}^1_{n,\tau} = \mathcal{C}^1([-\tau,0],\mathbb{R}^n)$ for the Banach space $\mathcal{C}^1([-\tau,0],\mathbb{R}^n)$ of continuous functions from $[-\tau,0]$ to $\mathbb{R}^n$ with a continuous derivative with norm

$$\|x\|_{\infty,1} = \sup_{s\in[-\tau,0]} \left\{\left|x(s)\right|, \left|\dot{x}(s)\right|\right\}.$$

We will also need the time shift operator, which operates on time dependent functions and is given by

$$\mathcal{T}_t x = s \mapsto x(s+t).$$

For a function f with domain X, the function g with domain $Y \subset X$ that coincides with f on Y will be denoted by $f|_Y$. As is usual in the literature on differential equations with delay, we will use the abbreviated notation x_t for the time shifted function x, restricted to the domain $[-\tau, 0]$, so

$$x_t = \mathcal{T}_t x|_{[-\tau,0]}.$$

In this paper we will consider a Lur'e system of neutral type with indirect control,

$$\frac{d}{dt}\big[x(t) - Dx(t-\tau)\big] = A_1 x(t) + A_2 x(t-\tau) + bf\big(\sigma(t)\big), \quad t \geq t_0, \tag{1}$$

$$\frac{d}{dt}\sigma(t) = c^{\mathrm{T}} x(t) - \rho f\big(\sigma(t)\big), \quad t \geq t_0, \tag{2}$$

$$x_{t_0} = \phi \tag{3}$$

with $\phi \in \mathcal{C}_{n,\tau}$, $A_1, A_2, D \in \mathbb{R}^{n\times n}$, $b, c \in \mathbb{R}^n$, $\rho, \tau \in \mathbb{R}$, $f \in \mathcal{C}(\mathbb{R}, \mathbb{R})$ such that $\rho > 0$, $\tau > 0$, $|D| < 1$, and

$$k_1 \sigma^2 \leq \sigma f(\sigma) \leq k_2 \sigma^2, \tag{4}$$

where $k_1, k_2 \in \mathbb{R}$ and $k_2 > k_1 > 0$. This is a special case of the more general autonomous neutral functional-differential equation

$$\frac{d}{dt}\big[x(t) - Dx(t-\tau)\big] = F(x_t), \tag{5}$$

where $D \in \mathbb{R}^{n\times n}$ and $F \in \mathcal{C}(\mathcal{C}_{n,\tau}, \mathbb{R}^n)$ with initial condition

$$x_{t_0} = \phi, \tag{6}$$

where $\phi \in \mathcal{C}^1_{n,\tau}$. If we need to refer to a specific solution of (5) with (6) then we will use the notation $x_{\langle t_0, \phi\rangle}$.

Definition 1 A pair $(x, \sigma) \in C([t_0 - \tau, \infty), \mathbb{R}^n) \times C([t_0, \infty), \mathbb{R})$ is a *solution of* (1), (2), (3) on $[t_0, \infty)$ if x satisfies (3) and the pair satisfies the system (1) and (2).

Evidently, as discussed in [23], p.169, there are obviously two families of metrics or measures for stability in this case, one based on x alone and another based on x and its derivative. A general theory of stability in two metrics or measures was first given by [24] and extended by [25]; see also [26, 27]. We use the definition of measure given in [26].

Definition 2 A function $h \in \mathcal{C}(\mathbb{R}_0^+ \times X, \mathbb{R}_0^+)$, where X is a Banach space, is called a measure in X if

$$\inf_{(t,x)\in\mathbb{R}\times X} h(t,x) = 0$$

and the set of all measures in X is denoted by $\Gamma(X)$.

Note the large difference in meaning conveyed by the subtle difference in terminology between a 'measure in X' and a 'measure on X'.

Definition 3 For given $h_0 \in \Gamma(\mathcal{C}^1_{n,\tau})$ and $h \in \Gamma(\mathcal{C}^1_{n,\tau})$ the solution $x_{\langle t_0,\phi\rangle}$ of (5) with (6) is (h_0, h) *stable* if

$$\forall\epsilon > 0 \exists\delta > 0 \forall\psi \in \mathcal{C}_{n,\tau} : h_0(t_0, \phi - \psi) \leq \delta \Rightarrow h(t, x_{\langle t_0,\phi\rangle t} - x_{\langle t_0,\psi\rangle t}) \leq \epsilon.$$

Definition 4 For given $h_0 \in \Gamma(\mathcal{C}^1_{n,\tau})$ and $h \in \Gamma(\mathcal{C}^1_{n,\tau})$ the solution $x_{\langle t_0,\phi\rangle}$ of (5) with (6) is (h_0, h) *asymptotically stable* if it is (h_0, h) *stable* and

$$\exists\delta > 0 \forall\epsilon > 0 \exists T > t_0 \forall t \geq T \forall\psi \in \mathcal{C}_{n,\tau} :$$
$$h_0(t_0, \phi - \psi) \leq \delta \Rightarrow h(t, x_{\langle t_0,\phi\rangle t} - x_{\langle t_0,\psi\rangle t}) \leq \epsilon$$

or equivalently if it is (h_0, h) *stable* and

$$\exists\delta > 0 \forall\psi \in \mathcal{C}_{n,\tau} : h_0(t_0, \phi - \psi) \leq \delta \Rightarrow \lim_{t\to\infty} h(t, x_{\langle t_0,\phi\rangle t} - x_{\langle t_0,\psi\rangle t}) \to 0.$$

Definition 5 For given $h_0 \in \Gamma(\mathcal{C}^1_{n,\tau})$ and $h \in \Gamma(\mathcal{C}^1_{n,\tau})$ the solution $x_{\langle t_0,\phi\rangle}$ of (5) with (6) is (h_0, h) *exponentially stable* (after for instance [27, 28]) if

$$\exists\rho > 0 \exists K > 0 \exists\lambda > 0 \forall t \geq T \forall\psi \in \mathcal{C}_{n,\tau} :$$
$$h_0(t_0, \phi - \psi) \leq \rho \Rightarrow h(t, x_{\langle t_0,\phi\rangle t} - x_{\langle t_0,\psi\rangle t}) \leq K h_0(t_0, \phi - \psi) e^{-\lambda(t-t_0)}.$$

Definition 6 For given $h_0 \in \Gamma(\mathcal{C}^1_{n,\tau})$ and $h \in \Gamma(\mathcal{C}^1_{n,\tau})$ the solution $x_{\langle t_0,\phi\rangle}$ of (5) with (6) is (h_0, h) *globally asymptotically stable* if

$$\forall\epsilon > 0 \forall\psi \in \mathcal{C}_{n,\tau} \exists T > t_0 \forall t \geq T : h(t, x_{\langle t_0,\phi\rangle t} - x_{\langle t_0,\psi\rangle t}) \leq \epsilon.$$

Definition 7 We call the zero solution $x : t \mapsto 0_{n\times1}$, $\sigma : t \mapsto 0$ of (1), (2) *stable* if it is (h_0, h) stable for $h_0(t, \phi) = \|\phi\|_\infty$ and $h(t, \langle x_t, \sigma_t\rangle) = \sqrt{|x_t(0)|^2 + |\sigma_t(0)|^2}$.

Definition 8 We call the zero solution $x : t \mapsto 0_{n\times1}$, $\sigma : t \mapsto 0$ of (1), (2) *asymptotically stable* if it is (h_0, h) asymptotically stable for $h_0(t, \phi) = \|\phi\|_\infty$ and $h(t, \langle x_t, \sigma_t\rangle) = \sqrt{|x_t(0)|^2 + |\sigma_t(0)|^2}$.

Definition 9 We call the zero solution $x : t \mapsto 0_{n\times1}$, $\sigma : t \mapsto 0$ of (1), (2) *globally asymptotically stable* if it is (h_0, h) globally asymptotically stable for $h_0(t, \phi) = \|\phi\|_\infty$ and $h(t, \langle x_t, \sigma_t\rangle) = \sqrt{|x_t(0)|^2 + |\sigma_t(0)|^2}$.

Definition 10 We call the zero solution $x : t \mapsto 0_{n\times1}$, $\sigma : t \mapsto 0$ of (1), (2) *globally asymptotically stable in metric* $\mathcal{C}^1$ if it is (h_0, h) globally asymptotically stable for $h_0(t, \phi) = \|\phi\|_\infty$ and

$$h\bigl(t, \langle x_t, \sigma_t\rangle\bigr) = \max\Bigl(\sqrt{\bigl|x_t(0)\bigr|^2 + \bigl|\sigma_t(0)\bigr|^2}, \sqrt{\bigl|\dot{x}_t(0)\bigr|^2 + \bigl|\dot{\sigma}_t(0)\bigr|^2}\Bigr).$$

Definition 11 The system (1), (2) is called *absolutely stable* if the zero solution of the system (1), (2) is globally asymptotically stable for an arbitrary function $f(\sigma)$ that satisfies (4).

To investigate the system (1), (2) we use a Lyapunov-Krasovskii functional of the form

$$\begin{aligned} V[x,\sigma,t] &= x^T(t)Hx(t) \\ &\quad + \int_{s=t-\tau}^{t} e^{-\zeta(t-s)}\left\{x^T(s)G_1x(s) + \dot{x}^T(s)G_2\dot{x}(s)\right\} ds \\ &\quad + \beta \int_{w=0}^{\sigma(t)} f(w)\,dw, \end{aligned} \tag{7}$$

where $H, G_1, G_2 \in \mathbb{R}^{n\times n}$ and $\beta, \gamma \in \mathbb{R}$, $\beta > 0$, $\zeta > 0$.

We define the matrix

$$S[A_1, A_2, b, c, \rho, \tau, H, G_1, G_2, \beta, \zeta] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12}^{\mathrm{T}} & S_{22} & S_{23} & S_{24} \\ S_{34}^{\mathrm{T}} & S_{34}^{\mathrm{T}} & S_{33} & S_{34} \\ S_{14}^{\mathrm{T}} & S_{24}^{\mathrm{T}} & S_{34}^{\mathrm{T}} & S_{44} \end{bmatrix}, \tag{8}$$

where

$$\begin{aligned} &S_{11} = -A_1^T H - HA_1 - G_1 - A_1^T G_2 A_1, \qquad S_{12} = -HA_2 - A_1^T G_2 A_2, \\ &S_{13} = -HD - A_1^T G_2 D, \qquad S_{14} = -Hb - A_1^T G_2 b - \frac{1}{2}\beta c, \\ &S_{22} = e^{-\zeta\tau} G_1 - A_2^T G_2 A_2, \qquad S_{23} = A_2 G_2 D, \qquad S_{24} = -A_2^T G_2 b, \\ &S_{33} = e^{-\zeta\tau} G_2 - D^T G_2 D, \qquad S_{34} = -D^T G_2 b, \qquad S_{44} = \beta\rho - b^T G_2 b. \end{aligned} \tag{9}$$

In [13, 14] a general theorem was proved, that provided sufficient conditions for absolute stability and estimates of the exponential decay for the solutions of the system (1), (2), when the elements of the matrices A_1 and A_2 were only known to lie in given intervals. When A_1 and A_2 are known exactly the following theorem follows immediately.

Theorem 1 *Let* $|D| < 1$, $\rho, \tau > 0$ *and suppose that there exist positive definite matrices* G_1, G_2, H, *and constants* $\zeta > 0$, $\beta > 0$ *such that the matrix* $S[A_1, A_2, b, c, \rho, \tau, H, G_1, G_2, \beta, \zeta]$ *is positive definite. In that case the system* (1), (2) *is absolutely stable in metric* with respect to the metric defined earlier for $\mathcal{C}^1$.

Corollary 1 *Let* $|D| < 1$, $\rho, \tau > 0$ *and suppose that there exist positive definite matrices* G_1, G_2, H, *and constants* $0 < \lambda < 1$, $\beta > 0$ *such that the matrix* $\tilde{S}[A_1, A_2, b, c, \rho, \tau, H, G_1, G_2, \beta, \lambda]$ *given by* S_{ij} *for* $(i,j) \notin \{(2,2),(3,3)\}$ *and* $\tilde{S}_{22} = \lambda G_1 - A_2^T G_2 A_2$, $\tilde{S}_{33} = \lambda G_2 - D^T G_2 D$ *is positive definite. In that case the system* (1), (2) *is absolutely stable in metric* in metric $\mathcal{C}^1$ *for all finite delays* τ.

Proof For each τ this follows from Theorem 1 by taking $\zeta = \tau^{-1}\log\lambda$. □

Note 1 In this corollary there are no conditions on the delay other than $\tau > 0$.

In analogy with the definition of exponential stability in terms of two measures we can use the existence of a Lyapunov-Krasovkii functional with specific properties to define a new type of stability. The definition is based on the inequality

$$\frac{d}{dt}V[x,t] \leq -\gamma V[x,t]. \tag{10}$$

Definition 12 A system is *stable with respect to the functional V with exponent $\gamma > 0$* if inequality (10) holds for the total derivative of the functional $V[x,t]$ along any solution of $x : t \mapsto x(t)$ of the system.

For some systems it can be profitable to examine the possibility of stabilizing the system by allowing a specific type of linear state feedback.

Definition 13 A system is *stabilizable with respect to functional V and state feedback of a given type* if the adding state feedback of that type results in a system that is stable with respect to the functional V with exponent $\gamma > 0$.

To illustrate the use of these definitions in the next two sections we will apply these definitions first in the case of a linear system with delay and then in the case of a scalar nonlinear neutral system with indirect control.

3 A Lyapunov-Krasovkii functional approach to a linear problem with delay

Before considering the general problem of stabilization of nonlinear control systems, an example of a linear control system with delay is used to introduce the concept of stability and stabilization with respect to a given functional and to demonstrate the methodology. Let us consider the control system

$$\dot{x}(t) = A_1x(t) + A_2x(t-\tau) + bu(t) \tag{11}$$

with $A_1, A_2 \in \mathbb{R}^{n\times n}$, $b, c \in \mathbb{R}^n$, and $u(t)$ is a scalar function and $\tau > 0$ is constant. To investigate the system (7) we use a Lyapunov-Krasovskii functional of the form

$$V\bigl[x(t)\bigr] = x^T(t)Hx(t) + \int_{s=-\tau}^{0} e^{\gamma s}x^T(t+s)Gx(t+s)\,ds, \tag{12}$$

where $H, G \in \mathbb{R}^{n\times n}$ and $\gamma \in \mathbb{R}$, $\zeta > 0$. We will consider controls of the form

$$u(t) = c^T x(t) + d^T x(t-\tau), \tag{13}$$

where $c, d \in \mathbb{R}^n$. First, let us consider stability with respect to the functional (12).

Theorem 2 *Consider* (11) *for $b = 0$ and with given A_1, A_2. Let there be positive definite matrices G and H and a constant $\gamma > 0$ such that the matrix*

$$\begin{aligned} &M[A_1, A_2, G, H, \gamma, \tau] \\ &\quad= \begin{bmatrix} -(A_1^{\mathrm{T}}H + HA_1 + G + \gamma H) & -HA_2 \\ -A_2^{\mathrm{T}}H & e^{-\gamma\tau}G \end{bmatrix} \end{aligned} \tag{14}$$

is positive definite. In that case the system (11) *is stable with respect to functional* (12) *with matrices G, H, and exponent γ.*

Proof Let $x(t)$ be a solution of (11). We introduce the vector

$$y(t) = \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}.$$

We can now write

$$x(t) = [\mathrm{I} \quad 0]y(t), \qquad x(t-\tau) = [0 \quad \mathrm{I}]y(t), \qquad \dot{x}(t) = [A_1 \quad A_2]y(t). \tag{15}$$

To show that the system (11) is stable with respect to functional (12) we need to show that (10) holds. For this we need to take the derivative of $V[x(t)]$:

$$\begin{aligned} \frac{d}{dt}V\bigl[x(t)\bigr] &= \dot{x}^{\mathrm{T}}(t)Hx(t) + x^{\mathrm{T}}(t)H\dot{x}(t) \\ &\quad + \frac{d}{dt}\biggl(\int_{\xi=t-\tau}^{t} e^{-\gamma(t-\xi)}x^{\mathrm{T}}(\xi)Gx(\xi)\,d\xi\biggr). \end{aligned} \tag{16}$$

The terms containing H in (16) can be rewritten in terms of $y(t)$ by using (15)

$$\dot{x}^{\mathrm{T}}(t)Hx(t) + x^{\mathrm{T}}(t)H\dot{x}(t) = y^{\mathrm{T}}(t)\begin{bmatrix} (A_1^{\mathrm{T}}H + HA_1) & HA_2 \\ A_2^{T}H & 0 \end{bmatrix} y(t). \tag{17}$$

To rewrite the terms in (16) containing the integral we will use

$$V\bigl[x(t)\bigr] - x^{\mathrm{T}}(t)Hx(t) = \int_{s=t-\tau}^{t} e^{\gamma(s-t)}x^{\mathrm{T}}(s)Gx(s)\,ds$$

and

$$\frac{d}{dt}\int_{s=t-\tau}^{t} e^{\gamma(s-t)}g(s)\,ds = -\gamma\int_{s=t-\tau}^{t} e^{\gamma(s-t)}g(s)\,ds + g(t) - e^{-\gamma\tau}g(t-\tau).$$

If we insert $g(t) = x^{\mathrm{T}}(s)Gx(s)$ then this results in

$$\begin{aligned} &\frac{d}{dt}\bigl(V\bigl[x(t)\bigr] - x^{\mathrm{T}}(t)Hx(t)\bigr) \\ &\quad = (-\gamma)\bigl(V\bigl[x(t)\bigr] - x^{\mathrm{T}}(t)Hx(t) + x^{\mathrm{T}}(t)Gx(t) + e^{-\gamma\tau}\bigl(-x^{\mathrm{T}}(t-\tau)Gx(t-\tau)\bigr), \end{aligned}$$

which with the aid of (15) can be put into matrix form

$$\begin{aligned} &\frac{d}{dt}\bigl(V\bigl[x(t)\bigr] - x^{\mathrm{T}}(t)Hx(t)\bigr) \\ &\quad = (-\gamma)\bigl(V\bigl[x(t)\bigr] - x^{\mathrm{T}}(t)Hx(t)\bigr) + y^{T}(t)\begin{bmatrix} G & 0 \\ 0 & -e^{-\gamma\tau}G \end{bmatrix} y(t). \end{aligned}$$

If we combine these results, then we get

$$\frac{d}{dt}V\bigl[x(t)\bigr] = -y^{T}(t)My(t) - \gamma V\bigl[x(t)\bigr]$$

and by positive definiteness of M we have (10). □

Example 1 If in Theorem 2 we take

$$A_1 = \begin{bmatrix} -1 & \frac{2}{5} \\ \frac{1}{5} & -1 \end{bmatrix}; \qquad A_2 = \frac{1}{1{,}000}\begin{bmatrix} -10 & 1 \\ 3 & -10 \end{bmatrix}; \qquad \tau = 1,$$

then the conditions of the theorem are satisfied when we take

$$G = \begin{bmatrix} 18 & -3 \\ -3 & 18 \end{bmatrix}, \qquad H = \begin{bmatrix} 100 & -40 \\ -40 & 100 \end{bmatrix}, \qquad \gamma = 1.$$

Corollary 2 *Let there be positive definite matrices H and G, vectors c and d, and a constant $\gamma > 0$ such that the matrix*

$$M\big[A_1 + bc^T, A_2 + bd^T, G, H\big]$$

is also positive definite. In that case the system (11) *is stabilizable with respect to functional* (12) *with state feedback of type* (13), *matrices G, H, and exponent γ.*

Proof This follows immediately from Theorem 2. □

Corollary 3 *If the pair (A, b) is controllable and*

$$R = \begin{bmatrix} b & A_1 b & A_1^2 b & \cdots & A_1^{n-2} b & A_1^{n-1} b \end{bmatrix}$$

and $\det(\lambda I - A_1) = \lambda^n + p_1\lambda^{n-1} + \cdots + p_n$ *and we define*

$$\tilde{A}_1 = \left[\begin{array}{c|c} \begin{matrix} 0_{(n-1)\times 1} \\ \mathrm{I}_{(n-1)\times(n-1)} \end{matrix} & -p \end{array}\right] = R^{-1}A_1R,$$

$$\tilde{A}_2 = R^{-1}A_1R, \qquad \tilde{H} = R^THR, \qquad \tilde{G} = R^TGR,$$

$$\tilde{b} = R^{-1}b = \vec{e}_n, \qquad \tilde{c} = R^Tc, \qquad \tilde{d} = R^Td,$$

and the matrix

$$\tilde{M} = \begin{bmatrix} -(\tilde{A}_1 + \frac{1}{2}\gamma I)^T\tilde{H} - \tilde{H}(\tilde{A}_1 + \frac{1}{2}\gamma I) - G & -\tilde{H}A_2 \\ -A_2^TH & e^{-\gamma\tau}G \end{bmatrix}$$

is positive definite, then the system (11) *is stabilizable with respect to functional* (12) *with state feedback of type* (13), *matrices G, H, and exponent γ.*

Proof If we apply the change of basis $y(t) = R^{-1}x(t)$, then this corollary follows immediately from the previous corollary. □

4 A scalar Lur'e system of neutral type with indirect control

Let us consider an indirect control system of neutral type described by a two scalar equations

$$\frac{d}{dt}\big[x(t) - dx(t-\tau)\big] = a_1x(t) + a_2x(t-\tau) + bf\big(\sigma(t)\big), \tag{18}$$

$$\frac{d}{dt}\sigma(t) = cx(t) - \rho f\big(\sigma(t)\big), \tag{19}$$

where $t \geq t_0 \geq 0$, x is the state function, σ is the control defined on $[t_0, \infty)$, a_1, a_2, b, c, $-1 < d < 1$, $\rho > 0$, $\tau > 0$ are constants, and f is a continuous function on $\mathbb{R}$ that satisfies the sector condition (4).

For this case the Lyapunov-Krasovskii functional (12) can be written as

$$\begin{aligned} V\big[x(t), \sigma(t), t\big] = {} & h \cdot \big(x(t)\big)^2 \\ & + \int_{s=t-\tau}^{t} e^{-\zeta(t-s)} \big\{ g_1\big(x(t)\big)^2 + g_2\big(\dot{x}(s)\big)^2 \big\}\, ds \\ & + \int_{w=0}^{\sigma(t)} f(w)\, dw, \end{aligned}$$

where $h > 0$, $g_1 > 0$, $g_2 > 0$, $\zeta > 0$ are constants, (x, σ) is a solution of (18), (19), and $t \geq t_0 \geq 0$. We define

$$\begin{aligned} & s_{11} = -2a_1 h - g_1 - a_1^2 g_2, \qquad s_{12} = -a_2 h - a_1 a_2 g_2, \qquad s_{13} = -hd - a_1\, dg_2, \\ & s_{14} = -hb - a_1 g_2, \qquad s_{22} = e^{-\zeta\tau} g_1 - a_2^2 g_2, \qquad s_{23} = -a_2 g_2 d, \qquad s_{24} = -a_2 g_2 b, \\ & s_{33} = \big(e^{-\zeta\tau} - d^2\big) g_2, \qquad s_{34} = -dg_2 b, \qquad s_{44} = \beta\rho - b^2 g_2, \end{aligned}$$

and the symmetric matrix

$$S[a_1, a_2, b, c, \rho, h, g_1, g_2, \beta, \zeta] = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{13} & s_{23} & s_{33} & s_{34} \\ s_{14} & s_{24} & s_{34} & s_{44} \end{bmatrix},$$

where $s_{ij} = s_{ji}$. Our first result is a theorem on the absolute stability for the system (18), (19).

Theorem 3 *If there exist constants* $h > 0$, $g_1 > 0$, $g_2 > 0$, $\beta > 0$, $\zeta > 0$ *such that the matrix* $S[a_1, a_2, b, c, \rho, h, g_1, g_2, \beta, \zeta]$ *is positive definite, then the system* (18), (19) *is absolutely stable.*

Proof The proof of this theorem follows directly from Theorem 1. □

Example 2 If in Theorem 3 we take

$$a_1 = -1, \qquad a_2 = \frac{1}{2}, \qquad d = -\frac{1}{10}, \qquad b = \frac{1}{10}, \qquad c = \frac{1}{10}, \qquad \rho = 1, \qquad \tau = 1,$$

then the conditions of the theorem are satisfied for

$$g_1 = \frac{3}{5}, \qquad g_2 = \frac{1}{2}, \qquad h = \frac{9}{10}, \qquad \beta = \frac{1}{2}, \qquad \zeta = \frac{4}{5}.$$

From Sylvester's criterion [29], Theorem 7.2.5, it follows that a necessary and sufficient condition for positive definiteness of the matrix S is that all of the leading principal minors are positive, that is,

$$s_{11} > 0, \tag{20}$$

$$s_{11}s_{22} - (s_{12})^2 > 0, \tag{21}$$

$$\det \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{pmatrix} > 0, \tag{22}$$

$$\det(S) > 0. \tag{23}$$

From inequalities (20) to (23) we can determine whether or not the matrix S is positive definite. If it is then the system (18), (19) is absolutely stable. Another approach is based on the lemma on the properties of block matrices given below.

Lemma 1 *Let A be a regular $n \times n$ matrix, B be an $n \times q$ matrix, and C be a regular $q \times q$ matrix. Let a Hermitian matrix S be represented as*

$$S = \begin{bmatrix} A & B \\ B^* & C \end{bmatrix}.$$

This matrix S is positive definite if and only if the matrices A and

$$C - B^* A^{-1} B$$

are positive definite. Here B^* denotes the Hermitian adjoint.

Proof See [30], Theorem 1.12. □

Now we can use this to formulate another set of stability conditions.

Theorem 4 *For $S = S[a_1, a_2, b, c, \rho, h, g_1, g_2, \beta, \zeta]$ let*

$$W_{11} = [\mathrm{I}_{2\times 2} \quad 0_{2\times 2}] S \begin{bmatrix} \mathrm{I}_{2\times 2} \\ 0_{2\times 2} \end{bmatrix},$$

$$W_{22} = [0_{2\times 2} \quad \mathrm{I}_{2\times 2}] S \begin{bmatrix} 0_{2\times 2} \\ \mathrm{I}_{2\times 2} \end{bmatrix},$$

$$W_{12} = [\mathrm{I}_{2\times 2} \quad 0_{2\times 2}] S \begin{bmatrix} 0_{2\times 2} \\ \mathrm{I}_{2\times 2} \end{bmatrix},$$

and suppose there exist constants $h > 0$, $g_1 > 0$, $g_2 > 0$, $\beta > 0$, $\zeta > 0$ such that the inequalities (20), (21) *hold and the matrix*

$$W_{22} - W_{12}^T W_{11}^{-1} W_{12} \tag{24}$$

is positive definite. In that case the system (18), (19) *is absolutely stable.*

Proof According to Lemma 1, S is positive definite if and only if W_{11} and $W_{22} - W_{12}^T W_{11}^{-1} W_{12}$ are positive definite. This completes the proof. □

The crucial assumption in Theorem 4 is the assumption of positive definiteness of the matrix $S[a_1, a_2, b, c, \rho, h, g_1, g_2, \beta, \zeta]$. If we cannot find suitable constants $c > 0$, $h > 0$, $g_1 > 0$, $g_2 > 0$, $\beta > 0$, $\zeta > 0$ to ensure positive definiteness, then we cannot apply Theorem 4. If that is the case, then we can consider modification of the control function in (18), (19) by adding a linear combination of the state at t and at $t - \tau$

$$\frac{d}{dt}\big[x(t) - dx(t-\tau)\big] = a_1 x(t) + a_2 x(t-\tau) + bf\big(\sigma(t)\big) + u(t), \tag{25}$$

$$\frac{d}{dt}\sigma(t) = cx(t) - \rho f\big(\sigma(t)\big) + \nu(t), \tag{26}$$

where

$$u(t) = c_1 x(t) + c_2 x(t-\tau),$$
$$\nu(t) = c_3 x(t),$$

and c_1, c_2, and c_3 are suitable constants. Inserting the definitions of u and ν in system (25), (26) results in

$$\frac{d}{dt}\big[x(t) - dx(t-\tau)\big] = (a_1 + c_1)x(t) + (a_2 + c_2)x(t-\tau) + bf\big(\sigma(t)\big), \tag{27}$$

$$\frac{d}{dt}\sigma(t) = (c + c_3)x(t) - \rho f\big(\sigma(t)\big). \tag{28}$$

In this case the matrix of the total derivative takes of the functional along the solution will be of the form

$$S[a_1 + c_1, a_2 + c_2, b, c + c_3, \rho, h, g_1, g_2, \beta, \zeta].$$

To stabilize the system we need to find c_1, c_2, and c_3 such that

$$S[a_1 + c_1, a_2 + c_2, b, c + c_3, \rho, h, g_1, g_2, \beta, \zeta]$$

is positive definite. We can now either use the Sylvester criterion [31] and look for c_1, c_2, and c_3 such that the leading principal minors of S_3 are positive or use Lemma 1 by defining

$$W_{11} = [\mathrm{I}_{2\times 2} \quad 0_{2\times 2}] S_3 \begin{bmatrix} \mathrm{I}_{2\times 2} \\ 0_{2\times 2} \end{bmatrix},$$

$$W_{22} = [0_{2\times 2} \quad \mathrm{I}_{2\times 2}] S_3 \begin{bmatrix} 0_{2\times 2} \\ \mathrm{I}_{2\times 2} \end{bmatrix},$$

$$W_{12} = [\mathrm{I}_{2\times 2} \quad 0_{2\times 2}] S_3 \begin{bmatrix} 0_{2\times 2} \\ \mathrm{I}_{2\times 2} \end{bmatrix},$$

where

$$S_3 = S[a_1 + c_1, a_2 + c_2, b, c + c_3, \rho, h, g_1, g_2, \beta, \zeta]$$

and look for c_1, c_2, and c_3 such that the matrices S_{11} and $S_{22} - S_{12}^T S_{11}^{-1} S_{12}$ are positive definite.

5 Stabilization

Let us return to our original system (1), (2). According to Theorem 1 for absolute stability of the system (1), (2) we need the matrix

$$S[A_1, A_2, b, c, \rho, \tau, H, G_1, G_2, \beta, \zeta]$$

to be positive definite. From the Sylvester criterion [31] it follows that we can verify that the matrix is positive definite by calculating its leading principal minors, that is, by verifying the positivity of $3n + 1$ determinants. Using the results of Lemma 1 we will give another set of absolute stability conditions. To do so we give names to specific blocks in matrix (8) as follows:

$$W_{11} = [\mathrm{I}_{2n\times 2n} \quad 0_{(n+1)\times(n+1)}] S_3 \begin{bmatrix} \mathrm{I}_{2n\times 2n} \\ 0_{(n+1)\times(n+1)} \end{bmatrix},$$

$$W_{12} = [\mathrm{I}_{2n\times 2n} \quad 0_{(n+1)\times(n+1)}] S_3 \begin{bmatrix} 0_{2n\times 2n} \\ \mathrm{I}_{(n+1)\times(n+1)} \end{bmatrix},$$

$$W_{22} = [0_{2n\times 2n} \quad \mathrm{I}_{(n+1)\times(n+1)}] S_3 \begin{bmatrix} 0_{2n\times 2n} \\ \mathrm{I}_{(n+1)\times(n+1)} \end{bmatrix},$$

where

$$S_3 = S[A_1, A_2, b, c, \rho, \tau, H, G_1, G_2, \beta, \zeta].$$

Theorem 5 *The sufficient conditions of absolute stability of neutral-type indirect control system* (1), (2) *are the existence of the positive definite matrices* W_{11} *and* $W_{22} - (W_{12})^T (W_{11})^{-1} W_{12}$.

Proof According to Lemma 1 the condition imposed on the matrices W_{11} and $W_{22} - (W_{12})^T (W_{11})^{-1} W_{12}$ implies that S_3 is positive definite. Theorem 1 now implies that the system is stable. □

Therefore, the absolute stability investigation problem is reduced to the task of checking of positive definiteness for two matrices, one of which is $2n$-dimensional and the other is $n + 1$-dimensional. Note that we can use Lemma 1 to reduce the proof of positive definiteness of the $2n$-dimensional case to positive definiteness of two n-dimensional matrices.

Example 3 When the matrices have special properties, Theorem 5 can be quite useful. For example suppose we have

$$A_1 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & \sqrt{10} \end{bmatrix}, \qquad A_2 = \frac{1}{4} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad D = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Now A_1 is negative definite and

$$A_2^{\mathrm{T}} D = 0, \qquad D^{\mathrm{T}} b = 0, \qquad A_2^{\mathrm{T}} b = 0,$$

or in other words b is in the intersection of the null spaces of D^{T} and A_2^{T} and the image of D is in the null space A_2^{T}.

If we could take $H = -hA_1$, $G_1 = g_1 I$, and $G_2 = g_2 I$, then our matrix S would be of the form

$$\begin{bmatrix} (2h-g_2)A_1^{\mathrm{T}}A_1 - g_1 I & (h-g_2)A_1^{\mathrm{T}}A_2 & (h-g_2)A_1^{\mathrm{T}}D & (h-g_2)A_1^{\mathrm{T}}b - \frac{1}{2}\beta c \\ (h-g_2)A_2^{\mathrm{T}}A_1 & g_1 e^{-\zeta\tau} I - g_2 A_2^{\mathrm{T}}A_2 & 0 & 0 \\ (h-g_2)D^{\mathrm{T}}A_1 & 0 & g_2(e^{-\zeta\tau} I - D^{\mathrm{T}}D) & 0 \\ (h-g_2)b^{\mathrm{T}}A_1 - \frac{1}{2}\beta c^{\mathrm{T}} & 0 & 0 & \beta\rho - g_2 b^{\mathrm{T}} b \end{bmatrix}.$$

It is interesting to examine under what conditions we could actually do this and still prove positive definiteness of the matrix. To apply Theorem 5 to this matrix we need the following matrices to be positive definite:

$$W_{11} = \begin{bmatrix} (2h-g_2)A_1^{\mathrm{T}}A_1 - g_1 I & (h-g_2)A_1^{\mathrm{T}}A_2 \\ (h-g_2)A_2^{\mathrm{T}}A_1 & g_1 e^{-\zeta\tau} I - g_2 A_2^{\mathrm{T}}A_2 \end{bmatrix}$$

and

$$W_{22} - W_{12}^{\mathrm{T}} W_{11}^{-1} W_{12},$$

where

$$W_{22} = \begin{bmatrix} g_2(e^{-\zeta\tau} I - D^{\mathrm{T}}D) & 0 \\ 0 & \beta\rho - g_2 b^{\mathrm{T}} b \end{bmatrix}$$

and

$$W_{12} = \begin{bmatrix} (h-g_2)A_1^{\mathrm{T}}D & (h-g_2)A_1^{\mathrm{T}}b - \frac{1}{2}\beta c \\ 0 & 0 \end{bmatrix}.$$

Note that we can apply Lemma 1 to W_{11}, so the proof of positive definiteness of W_{11} reduces to the proofs that

$$(2h-g_2)A_1^{\mathrm{T}}A_1 - g_1 I$$

and

$$g_1 e^{-\zeta\tau} I - g_2 A_2^{\mathrm{T}}A_2 - (h-g_2)^2 A_2^{\mathrm{T}}A_1 S_{11}^{-1} A_1^{\mathrm{T}}A_2$$

are positive definite.

A tempting further simplification would be $h = g_2$, which would simplify W_{11} and W_{12} to

$$W_{11} = \begin{bmatrix} hA_1^{\mathrm{T}}A_1 - g_1 I & 0 \\ 0 & g_1 e^{-\zeta\tau} I - g_2 A_2^{\mathrm{T}} A_2 \end{bmatrix}$$

and

$$W_{12} = \begin{bmatrix} 0 & -\frac{1}{2}\beta c \\ 0 & 0 \end{bmatrix},$$

while for $W_{22} - W_{12}^{\mathrm{T}} W_{11}^{-1} W_{12}$ we would get

$$\begin{bmatrix} g_2(e^{-\zeta\tau} I - D^{\mathrm{T}} D) & 0 \\ 0 & \beta\rho - g_2 b^{\mathrm{T}} b \end{bmatrix} - \frac{1}{4}\beta^2 \begin{bmatrix} 0 & 0 \\ 0 & c^{\mathrm{T}}(hA_1^{\mathrm{T}}A_1 - g_1 I)^{-1} c \end{bmatrix}$$

to get a positive definite S. Under these assumptions we would need the following matrices to be positive definite:

$$S_{11} = hA_1^{\mathrm{T}} A_1 - g_1 I, \tag{29}$$

$$S_{22} = g_1 e^{-\zeta\tau} I - g_2 A_2^{\mathrm{T}} A_2, \tag{30}$$

$$S_{33} = g_2 \left(e^{-\zeta\tau} I - D^{\mathrm{T}} D\right), \tag{31}$$

and we would need $r(\rho, b, c, \beta, g_1, h)$ defined by

$$r(\rho, b, c, \beta, g_1, h) = \beta\rho - g_2 b^{\mathrm{T}} b - \frac{1}{4}\beta^2 c^{\mathrm{T}} \left(hA_1^{\mathrm{T}} A_1 - g_1 I\right)^{-1} c \tag{32}$$

to be positive.

For (31) we need $g_2 > 0$ and $\exp(-\zeta\tau) > \|D^{\mathrm{T}} D\|$ which can be realized by taking $\zeta > -\log \|D^{\mathrm{T}} D\|$. This is possible because $\|D\| < 1$. For (30) is possible only if $g_1 e^{-\zeta\tau} > h\|A_2^{\mathrm{T}} A_2\|$ and for (29) we need $h\|A_1^{\mathrm{T}} A_1\| > g_1$. For (32) to hold we need

$$\frac{1}{4}\beta^2 c^{\mathrm{T}} \left(hA_1^{\mathrm{T}} A_1 - g_1 I\right)^{-1} c - \beta\rho + g_2 b^{\mathrm{T}} b < 0,$$

which is solvable if and only if

$$\rho^2 > g_2 \beta^2 \left(c^{\mathrm{T}} \left(hA_1^{\mathrm{T}} A_1 - g_1 I\right)^{-1} c\right) b^{\mathrm{T}} b.$$

For our example we find

$$W_{11} = h \begin{bmatrix} \frac{5}{4} & -\frac{3}{4} & 0 \\ -\frac{3}{4} & \frac{5}{4} & 0 \\ 0 & 0 & \frac{5}{4} \end{bmatrix} - g_1 I, \tag{33}$$

$$S_{22} = g_1 e^{-\zeta\tau} I - h \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{16} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{34}$$

$$S_{33} = h\left(e^{-\zeta\tau}I - \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \end{bmatrix}\right), \tag{35}$$

and

$$r = \beta\rho - \frac{1}{16}h - \frac{1}{4}\beta^2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{\mathrm{T}} \left(h\begin{bmatrix} \frac{5}{4} & -\frac{3}{4} & 0 \\ -\frac{3}{4} & \frac{5}{4} & 0 \\ 0 & 0 & \frac{5}{4} \end{bmatrix} - g_1 I\right)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} > 0. \tag{36}$$

We see that for $g_1 = h/4$ and $0 < \zeta < (\log 4)/\tau$ the matrices

$$h\begin{bmatrix} 1 & -\frac{3}{4} & 0 \\ -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{37}$$

$$h\left(\frac{1}{4}e^{-\zeta\tau}I - \frac{1}{16}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right), \tag{38}$$

$$h\left(e^{-\zeta\tau}I - \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) \tag{39}$$

are positive definite and

$$r = \beta\rho - \frac{1}{16}h - \frac{1}{4h}\beta^2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} 1 & -\frac{3}{4} & 0 \\ -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} > 0, \tag{40}$$

which, after insertion of the inverse matrix,

$$r = \beta\rho - \frac{1}{16}h - \frac{1}{4h}\beta^2 h^{-2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \frac{16}{7} & \frac{12}{7} & 0 \\ \frac{12}{7} & \frac{16}{7} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} > 0, \tag{41}$$

reduces to

$$\frac{9}{4}\beta^2 h - \beta\rho + \frac{1}{16}h < 0. \tag{42}$$

This holds for

$$\frac{4\rho - \sqrt{16\rho^2 - 9h^2}}{18h} < \beta < \frac{4\rho + \sqrt{16\rho^2 - 9h^2}}{18h},$$

which is solvable as long as $16\rho^2 > 9h^2$.

If we cannot find suitable matrices G_1, G_2, H, and constants $\beta > 0$, $\zeta > 0$ to ensure positive definiteness, or such matrices and constants do not exist, then Theorem 5 is not applicable. In such a case we can try to construct a feedback control u, v, such that the modified system

$$\frac{d}{dt}\big[x(t) - Dx(t-\tau)\big] = A_1x(t) + A_2x(t-\tau) + bf\big(\sigma(t)\big) + u(t), \tag{43}$$

$$\frac{d}{dt}\sigma(t) = c^Tx(t) - \rho f\big(\sigma(t)\big) + v(t), \tag{44}$$

$$u(t) = C_1x(t) + C_2x(t-\tau), \tag{45}$$

$$v(t) = C_3x(t) \tag{46}$$

will be absolutely stable, where C_1 and C_2 are $n \times n$ matrices and C_3 is a $1 \times n$ matrix.

Define

$$S_4 = S\big[A_1 + C_1, A_2 + C_2, b, c + C_3^T, \rho, \tau, H, G_1, G_2, \beta, \zeta\big].$$

We give a generalization of the two previous options of finding of the stabilization conditions to the case of the system (1), (2).

Theorem 6 *Suppose that there are matrices C_1, C_2, and C_3, such that the matrix S_4 is positive definite. In that case the system* (1), (2) *is stabilizable with respect to the state feedback shown in* (43), (44), *and the functional* (7).

Proof The proof follows immediately from Theorem 1. □

Using the results of Lemma 1, we can replace verification of positive definiteness of matrix S_4 by verification of positive definiteness of two matrices of lower dimensionality.

Theorem 7 *Define*

$$\tilde{S}_{11} = [\mathrm{I}_{2n\times 2n} \quad 0_{(n+1)\times(n+1)}]S_4 \begin{bmatrix} \mathrm{I}_{2n\times 2n} \\ 0_{(n+1)\times(n+1)} \end{bmatrix},$$

$$\tilde{S}_{12} = [\mathrm{I}_{2n\times 2n} \quad 0_{(n+1)\times(n+1)}]S_4 \begin{bmatrix} 0_{2n\times 2n} \\ \mathrm{I}_{(n+1)\times(n+1)} \end{bmatrix},$$

$$\tilde{S}_{22} = [0_{2n\times 2n} \quad \mathrm{I}_{(n+1)\times(n+1)}]S_4 \begin{bmatrix} 0_{2n\times 2n} \\ \mathrm{I}_{(n+1)\times(n+1)} \end{bmatrix}.$$

Suppose that there are matrices C_1, C_2, and C_3, such that the matrices $\tilde{S}_{11}$ and $\tilde{S}_{11} - (\tilde{S}_{12})^T(\tilde{S}_{11})^{-1}\tilde{S}_{12}$ are positive definite. In that case the system (1), (2) *is stabilizable with respect to the state feedback shown in* (43), (44), *and the functional* (7).

6 Conclusions

We discussed the stabilization problem for an indirect control Lur'e system of neutral type. Based on the direct Lyapunov method (Lyapunov-Krasovskii approach) several stabilization criteria were given in terms of a set of matrix algebraic inequalities. A sufficient condition for absolutely stability of the closed loop system was presented.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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