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A functional generalization of diamond-α integral Dresher's inequality on time scales

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Dedicated to Professor Ravi P Agarwal

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Abstract
In this paper, we establish a functional generalization of diamond-α integral Dresher's inequality on time scales. Its reverse form is also considered.
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1 Introduction

In the fifties of the previous century, Beckenbach [1] introduced a famous inequality as follows.

Let $1 \le p \le 2$ and $x_i, y_i > 0$, $i = 1, \dots, n$. Then

$$\frac{\sum_{i=1}^n (x_i + y_i)^p}{\sum_{i=1}^n (x_i + y_i)^{p-1}} \le \frac{\sum_{i=1}^n x_i^p}{\sum_{i=1}^n x_i^{p-1}} + \frac{\sum_{i=1}^n y_i^p}{\sum_{i=1}^n y_i^{p-1}}. \tag{1.1}$$

The following integral version of the above-mentioned discrete inequality is due to Dresher [2] (see also [3]):

Assume that $f(x)$ and $g(x)$ are non-negative and continuous real-valued functions on $[a, b]$, and $0 < r \le 1 \le p$, then

$$\left(\frac{\int_a^b (f(x) + g(x))^p \, dx}{\int_a^b (f(x) + g(x))^r \, dx}\right)^{1/(p-r)} \le \left(\frac{\int_a^b f^p(x) \, dx}{\int_a^b f^r(x) \, dx}\right)^{1/(p-r)} + \left(\frac{\int_a^b g^p(x) \, dx}{\int_a^b g^r(x) \, dx}\right)^{1/(p-r)}. \tag{1.2}$$

From that time, some generalizations of the Beckenbach-Dresher inequality (1.1) and (1.2) have appeared. Here, we refer to the papers of Pečarić and Beesack [4], Petree and Persson [5], Persson [6] , Varošanec [7], Anwar *et al.* [8], and Nikolova *et al.* [9], where the reader can find literature related to this inequality. Recently, Zhao [10] gave the following reverse Dresher's inequality.

Assume that $f(x)$ and $g(x)$ are non-negative and continuous real-valued functions on $[a, b]$, and $p \le 0 \le r \le 1$, then

$$\left(\frac{\int_a^b (f(x) + g(x))^p \, dx}{\int_a^b (f(x) + g(x))^r \, dx}\right)^{1/(p-r)} \ge \left(\frac{\int_a^b f^p(x) \, dx}{\int_a^b f^r(x) \, dx}\right)^{1/(p-r)} + \left(\frac{\int_a^b g^p(x) \, dx}{\int_a^b g^r(x) \, dx}\right)^{1/(p-r)}. \tag{1.3}$$

The aim of this work is to give a functional generalization of diamond-α integral Dresher's inequality for time scales. Its reverse form is also presented.

2 Main results

Let T be a time scale; that is, T is an arbitrary nonempty closed subset of real numbers. The set of the real numbers, the integers, the natural numbers, and the Cantor set are examples of time scales. But the open interval between 0 and 1, the rational numbers, the irrational numbers, and the complex numbers are not time scales. Let $a, b \in \mathrm{T}$. We now suppose that the reader is familiar with some basic facts from the theory of time scales, which can also be found in [11–22], and of delta, nabla and diamond-α dynamic derivatives.

Our main results are given in the following theorems.

Theorem 2.1 (Dresher's inequality) *Let* T *be a time scale* $a, b \in \mathrm{T}$ *with* $a < b$ *and* $0 < r \leq 1 \leq p$. *Let* $H_l(x_1, x_2, \ldots, x_l) > 0$, $F_m(x_1, x_2, \ldots, x_m)$ *and* $G_k(x_1, x_2, \ldots, x_k)$ *be three arbitrary functions of* l, m *and* k *variables, respectively. Assume that* $\{f_i(x)\}_{i=1}^m$, $\{g_i(x)\}_{i=1}^k$ *and* $\{h_i(x)\}_{i=1}^l$ *are continuous real-valued functions on* $[a, b]_{\mathrm{T}}$, *then*

$$\begin{aligned}&\left(\frac{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m) + G_k(g_1, g_2, \ldots, g_k)|^p \diamond_\alpha x}{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m) + G_k(g_1, g_2, \ldots, g_k)|^r \diamond_\alpha x}\right)^{1/(p-r)}\\&\quad\leq \left(\frac{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m)|^p \diamond_\alpha x}{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m)|^r \diamond_\alpha x}\right)^{1/(p-r)}\\&\qquad+ \left(\frac{\int_a^b H_l(h_1, h_2, \ldots, h_l)|G_k(g_1, g_2, \ldots, g_k)|^p \diamond_\alpha x}{\int_a^b H_l(h_1, h_2, \ldots, h_l)|G_k(g_1, g_2, \ldots, g_k)|^r \diamond_\alpha x}\right)^{1/(p-r)}, \qquad (2.1)\end{aligned}$$

there is equality only when the functions $|F_m(f_1, f_2, \ldots, f_m)|$ *and* $|G_k(g_1, g_2, \ldots, g_k)|$ *are effectively proportional.*

Proof First, we have

$$\begin{aligned}&\left(\int_a^b H_l(h_1, h_2, \ldots, h_l)\big|F_m(f_1, f_2, \ldots, f_m) + G_k(g_1, g_2, \ldots, g_k)\big|^p \diamond_\alpha x\right)^{1/(p-r)}\\&\quad\leq \left(\left(\int_a^b H_l|F_m|^p \diamond_\alpha x\right)^{1/p} + \left(\int_a^b H_l|G_k|^p \diamond_\alpha x\right)^{1/p}\right)^{p/(p-r)} \qquad (2.2)\end{aligned}$$

by Minkowski's inequality on time scales [18]. Next, by the right-hand side of the above inequality, we have

$$\begin{aligned}&\left(\left(\int_a^b H_l|F_m|^p \diamond_\alpha x\right)^{1/p} + \left(\int_a^b H_l|G_k|^p \diamond_\alpha x\right)^{1/p}\right)^{p/(p-r)}\\&\quad= \left(\left(\frac{\int_a^b H_l|F_m|^p \diamond_\alpha x}{\int_a^b H_l|F_m|^r \diamond_\alpha x}\right)^{1/p}\left(\int_a^b H_l|F_m|^r \diamond_\alpha x\right)^{1/p}\right.\\&\qquad\left.+ \left(\frac{\int_a^b H_l|G_k|^p \diamond_\alpha x}{\int_a^b H_l|G_k|^r \diamond_\alpha x}\right)^{1/p}\left(\int_a^b H_l|G_k|^r \diamond_\alpha x\right)^{1/p}\right)^{p/(p-r)}.\end{aligned}$$

We apply Hölder's inequality to the above equality to obtain

$$\begin{aligned}&\left(\left(\frac{\int_a^b H_l|F_m|^p\diamondsuit_\alpha x}{\int_a^b H_l|F_m|^r\diamondsuit_\alpha x}\right)^{1/p}\left(\int_a^b H_l|F_m|^r\diamondsuit_\alpha x\right)^{1/p}\right.\\&\qquad\left.+\left(\frac{\int_a^b H_l|G_k|^p\diamondsuit_\alpha x}{\int_a^b H_l|G_k|^r\diamondsuit_\alpha x}\right)^{1/p}\left(\int_a^b H_l|G_k|^r\diamondsuit_\alpha x\right)^{1/p}\right)^{p/(p-r)}\\&\quad\le\left(\left(\frac{\int_a^b H_l|F_m|^p\diamondsuit_\alpha x}{\int_a^b H_l|F_m|^r\diamondsuit_\alpha x}\right)^{1/(p-r)}+\left(\frac{\int_a^b H_l|G_k|^p\diamondsuit_\alpha x}{\int_a^b H_l|G_k|^r\diamondsuit_\alpha x}\right)^{1/(p-r)}\right)\\&\qquad\times\left(\left(\int_a^b H_l|F_m|^r\diamondsuit_\alpha x\right)^{1/r}+\left(\int_a^b H_l|G_k|^r\diamondsuit_\alpha x\right)^{1/r}\right)^{r/(p-r)}.\end{aligned}\tag{2.3}$$

By applying reverse Minkowski's inequality with $0 < r < 1$, we obtain

$$\left(\left(\int_a^b H_l|F_m|^r\diamondsuit_\alpha x\right)^{1/r}+\left(\int_a^b H_l|G_k|^r\diamondsuit_\alpha x\right)^{1/r}\right)^r\le\int_a^b H_l|F_m+G_k|^r\diamondsuit_\alpha x.\tag{2.4}$$

From (2.2), (2.3) and (2.4), we obtain the desired inequality. □

Corollary 2.1 (T = R) *Let $0 < r \le 1 \le p$. Let $H_l(x_1,x_2,\dots,x_l) > 0$, $F_m(x_1,x_2,\dots,x_m)$ and $G_k(x_1,x_2,\dots,x_k)$ be three arbitrary functions of l, m and k variables, respectively. Assume that $\{f_i(x)\}_{i=1}^m$, $\{g_i(x)\}_{i=1}^k$ and $\{h_i(x)\}_{i=1}^l$ are continuous real-valued functions on $[a,b]$, then*

$$\begin{aligned}&\left(\frac{\int_a^b H_l(h_1,h_2,\dots,h_l)|F_m(f_1,f_2,\dots,f_m)+G_k(g_1,g_2,\dots,g_k)|^p\,dx}{\int_a^b H_l(h_1,h_2,\dots,h_l)|F_m(f_1,f_2,\dots,f_m)+G_k(g_1,g_2,\dots,g_k)|^r\,dx}\right)^{1/(p-r)}\\&\quad\le\left(\frac{\int_a^b H_l(h_1,h_2,\dots,h_l)|F_m(f_1,f_2,\dots,f_m)|^p\,dx}{\int_a^b H_l(h_1,h_2,\dots,h_l)|F_m(f_1,f_2,\dots,f_m)|^r\,dx}\right)^{1/(p-r)}\\&\qquad+\left(\frac{\int_a^b H_l(h_1,h_2,\dots,h_l)|G_k(g_1,g_2,\dots,g_k)|^p\,dx}{\int_a^b H_l(h_1,h_2,\dots,h_l)|G_k(g_1,g_2,\dots,g_k)|^r\,dx}\right)^{1/(p-r)},\end{aligned}\tag{2.5}$$

there is equality only when the functions $|F_m(f_1,f_2,\dots,f_m)|$ and $|G_k(g_1,g_2,\dots,g_k)|$ are effectively proportional.

Corollary 2.2 (T = Z) *Let $0 < r \le 1 \le p$. Let $H_l(x_1,x_2,\dots,x_l) > 0$, $F_m(x_1,x_2,\dots,x_m)$ and $G_k(x_1,x_2,\dots,x_k)$ be three arbitrary functions of l, m and k variables, respectively. Assume that $\{a_{i1},a_{i2},\dots,a_{im}\}_{i=1}^n$, $\{b_{i1},b_{i2},\dots,b_{ik}\}_{i=1}^n$ and $\{c_{i1},c_{i2},\dots,c_{il}\}_{i=1}^n$ are real numbers for any $m,k,l\in\mathrm{N}$, then*

$$\begin{aligned}&\left(\frac{\sum_{i=1}^n H_l(c_{i1},c_{i2},\dots,c_{il})|F_m(a_{i1},a_{i2},\dots,a_im)+G_k(b_{i1},b_{i2},\dots,b_{ik})|^p}{\sum_{i=1}^n H_l(c_{i1},c_{i2},\dots,c_{il})|F_m(a_{i1},a_{i2},\dots,a_im)+G_k(b_{i1},b_{i2},\dots,b_{ik})|^r}\right)^{1/(p-r)}\\&\quad\le\left(\frac{\sum_{i=1}^n H_l(c_{i1},c_{i2},\dots,c_{il})|F_m(a_{i1},a_{i2},\dots,a_im)|^p}{\sum_{i=1}^n H_l(c_{i1},c_{i2},\dots,c_{il})|F_m(a_{i1},a_{i2},\dots,a_im)|^r}\right)^{1/(p-r)}\\&\qquad+\left(\frac{\sum_{i=1}^n H_l(c_{i1},c_{i2},\dots,c_{il})|G_k(b_{i1},b_{i2},\dots,b_{ik})|^p}{\sum_{i=1}^n H_l(c_{i1},c_{i2},\dots,c_{il})|G_k(b_{i1},b_{i2},\dots,b_{ik})|^r}\right)^{1/(p-r)},\end{aligned}\tag{2.6}$$

there is equality only when the functions $|F_m(a_{i1}, a_{i2}, \ldots, a_{im})|$ *and* $|G_k(b_{i1}, b_{i2}, \ldots, b_{ik})|$ *are effectively proportional.*

Theorem 2.2 (reverse Dresher's inequality) *Let* T *be a time scale,* $a, b \in \mathrm{T}$ *with* $a < b$ *and* $p \le 0 \le r \le 1$. *Let* $H_l(x_1, x_2, \ldots, x_l) > 0$, $F_m(x_1, x_2, \ldots, x_m)$ *and* $G_k(x_1, x_2, \ldots, x_k)$ *be three arbitrary functions of* l, m *and* k *variables, respectively. Assume that* $\{f_i(x)\}_{i=1}^m$, $\{g_i(x)\}_{i=1}^k$ *and* $\{h_i(x)\}_{i=1}^l$ *are continuous real-valued functions on* $[a, b]_{\mathrm{T}}$, *then*

$$\begin{aligned}
&\left(\frac{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m) + G_k(g_1, g_2, \ldots, g_k)|^p \diamond_\alpha x}{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m) + G_k(g_1, g_2, \ldots, g_k)|^r \diamond_\alpha x}\right)^{1/(p-r)} \\
&\quad \ge \left(\frac{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m)|^p \diamond_\alpha x}{\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m)|^r \diamond_\alpha x}\right)^{1/(p-r)} \\
&\qquad + \left(\frac{\int_a^b H_l(h_1, h_2, \ldots, h_l)|G_k(g_1, g_2, \ldots, g_k)|^p \diamond_\alpha x}{\int_a^b H_l(h_1, h_2, \ldots, h_l)|G_k(g_1, g_2, \ldots, g_k)|^r \diamond_\alpha x}\right)^{1/(p-r)},
\end{aligned} \tag{2.7}$$

there is equality only when the functions $|F_m(f_1, f_2, \ldots, f_m)|$ *and* $|G_k(g_1, g_2, \ldots, g_k)|$ *are effectively proportional.*

Proof Let $\alpha_1 \ge 0$, $\alpha_2 \ge 0$, $\beta_1 > 0$, and $\beta_2 > 0$, and $-1 < \lambda < 0$, applying the following Radon's inequality (see [23]):

$$\sum_{k=1}^n \frac{a_k^p}{b_k^{p-1}} < \frac{(\sum_{k=1}^n a_k)^p}{(\sum_{k=1}^n b_k)^{p-1}}, \quad x_k \ge 0, a_k > 0, 0 < p < 1,$$

we have

$$\frac{\alpha_1^{\lambda+1}}{\beta_1^\lambda} + \frac{\alpha_2^{\lambda+1}}{\beta_2^\lambda} \le \frac{(\alpha_1 + \alpha_2)^{\lambda+1}}{(\beta_1 + \beta_2)^\lambda}, \tag{2.8}$$

there is equality only when (α) and (β) are proportional. Let

$$\alpha_1 = \left(\int_a^b H_l(h_1, h_2, \ldots, h_l)\big|F_m(f_1, f_2, \ldots, f_m)\big|^p \diamond_\alpha x\right)^{1/p}, \tag{2.9}$$

$$\beta_1 = \left(\int_a^b H_l(h_1, h_2, \ldots, h_l)\big|F_m(f_1, f_2, \ldots, f_m)\big|^r \diamond_\alpha x\right)^{1/r}, \tag{2.10}$$

$$\alpha_2 = \left(\int_a^b H_l(h_1, h_2, \ldots, h_l)\big|G_k(g_1, g_2, \ldots, g_k)\big|^p \diamond_\alpha x\right)^{1/p}, \tag{2.11}$$

$$\beta_2 = \left(\int_a^b H_l(h_1, h_2, \ldots, h_l)\big|G_k(g_1, g_2, \ldots, g_k)\big|^r \diamond_\alpha x\right)^{1/r}, \tag{2.12}$$

and set $\lambda = \frac{r}{p-r}$. From (2.8)-(2.12), we have

$$\begin{aligned}
&\frac{\alpha_1^{\lambda+1}}{\beta_1^\lambda} + \frac{\alpha_2^{\lambda+1}}{\beta_2^\lambda} \\
&\quad = \frac{(\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m)|^p \diamond_\alpha x)^{(\lambda+1)/p}}{(\int_a^b H_l(h_1, h_2, \ldots, h_l)|F_m(f_1, f_2, \ldots, f_m)|^r \diamond_\alpha x)^{\lambda/r}}
\end{aligned}$$

$$\begin{aligned}
&+\frac{(\int_a^b H_l(h_1,h_2,\ldots,h_l)|G_k(g_1,g_2,\ldots,g_k)|^p\diamondsuit_\alpha x)^{(\lambda+1)/p}}{(\int_a^b H_l(h_1,h_2,\ldots,h_l)|G_k(g_1,g_2,\ldots,g_k)|^r\diamondsuit_\alpha x)^{\lambda/r}}\\
&=\left(\frac{\int_a^b H_l(h_1,h_2,\ldots,h_l)|F_m(f_1,f_2,\ldots,f_m)|^p\diamondsuit_\alpha x}{\int_a^b H_l(h_1,h_2,\ldots,h_l)|F_m(f_1,f_2,\ldots,f_m)|^r\diamondsuit_\alpha x}\right)^{1/(p-r)}\\
&\quad+\left(\frac{\int_a^b H_l(h_1,h_2,\ldots,h_l)|G_k(g_1,g_2,\ldots,g_k)|^p\diamondsuit_\alpha x}{\int_a^b H_l(h_1,h_2,\ldots,h_l)|G_k(g_1,g_2,\ldots,g_k)|^r\diamondsuit_\alpha x}\right)^{1/(p-r)}\le\frac{(\alpha_1+\alpha_2)^{\lambda+1}}{(\beta_1+\beta_2)^{\lambda}}\\
&=\frac{[(\int_a^b H_l|F_m(f_1,\ldots,f_m)|^p\diamondsuit_\alpha x)^{1/p}+(\int_a^b H_l|G_k(g_1,\ldots,g_k)|^p\diamondsuit_\alpha x)^{1/p}]^{p/(p-r)}}{[(\int_a^b H_l|F_m(f_1,\ldots,f_m)|^r\diamondsuit_\alpha x)^{1/r}+(\int_a^b H_l|G_k(g_1,\ldots,g_k)|^r\diamondsuit_\alpha x)^{1/r}]^{r/(p-r)}}.
\end{aligned}\tag{2.13}$$

Since $-1<\lambda=\frac{r}{p-r}<0$, we may assume $p<0<r$, and by Minkowski's inequality for $p<0$ and $0<r\le 1$, we obtain respectively

$$\begin{aligned}
&\left[\left(\int_a^b H_l\left|F_m(f_1,\ldots,f_m)\right|^p\diamondsuit_\alpha x\right)^{1/p}+\left(\int_a^b H_l\left|G_k(g_1,\ldots,g_k)\right|^p\diamondsuit_\alpha x\right)^{1/p}\right]^p\\
&\quad\ge\int_a^b H_l\left|F_m(f_1,\ldots,f_m)+G_k(g_1,\ldots,g_k)\right|^p\diamondsuit_\alpha x,
\end{aligned}\tag{2.14}$$

there is equality only when $|F_m(f_1,\ldots,f_m)|$ and $|G_k(g_1,\ldots,g_k)|$ are proportional, and

$$\begin{aligned}
&\left[\left(\int_a^b H_l\left|F_m(f_1,\ldots,f_m)\right|^r\diamondsuit_\alpha x\right)^{1/r}+\left(\int_a^b H_l\left|G_k(g_1,\ldots,g_k)\right|^r\diamondsuit_\alpha x\right)^{1/r}\right]^r\\
&\quad\le\int_a^b H_l\left|F_m(f_1,\ldots,f_m)+G_k(g_1,\ldots,g_k)\right|^r\diamondsuit_\alpha x
\end{aligned}\tag{2.15}$$

with equality if and only if $|F_m(f_1,\ldots,f_m)|$ and $|G_k(g_1,\ldots,g_k)|$ are proportional.

From equality conditions for (2.8), (2.14) and (2.15), it follows that the sign of equality in (2.7) holds if and only if $|F_m(f_1,\ldots,f_m)|$ and $|G_k(g_1,\ldots,g_k)|$ are proportional.

From (2.13)-(2.15), we arrive at reverse Dresher's inequality, and the theorem is completely proved. □

Corollary 2.3 (T = R) *Let $p\le 0\le r\le 1$. Let $H_l(x_1,x_2,\ldots,x_l)>0$, $F_m(x_1,x_2,\ldots,x_m)$ and $G_k(x_1,x_2,\ldots,x_k)$ be three arbitrary functions of l, m and k variables, respectively. Assume that $\{f_i(x)\}_{i=1}^m$, $\{g_i(x)\}_{i=1}^k$ and $\{h_i(x)\}_{i=1}^l$ are continuous real-valued functions on $[a,b]$, then*

$$\begin{aligned}
&\left(\frac{\int_a^b H_l(h_1,h_2,\ldots,h_l)|F_m(f_1,f_2,\ldots,f_m)+G_k(g_1,g_2,\ldots,g_k)|^p\,dx}{\int_a^b H_l(h_1,h_2,\ldots,h_l)|F_m(f_1,f_2,\ldots,f_m)+G_k(g_1,g_2,\ldots,g_k)|^r\,dx}\right)^{1/(p-r)}\\
&\quad\ge\left(\frac{\int_a^b H_l(h_1,h_2,\ldots,h_l)|F_m(f_1,f_2,\ldots,f_m)|^p\,dx}{\int_a^b H_l(h_1,h_2,\ldots,h_l)|F_m(f_1,f_2,\ldots,f_m)|^r\,dx}\right)^{1/(p-r)}\\
&\qquad+\left(\frac{\int_a^b H_l(h_1,h_2,\ldots,h_l)|G_k(g_1,g_2,\ldots,g_k)|^p\,dx}{\int_a^b H_l(h_1,h_2,\ldots,h_l)|G_k(g_1,g_2,\ldots,g_k)|^r\,dx}\right)^{1/(p-r)},
\end{aligned}\tag{2.16}$$

there is equality only when the functions $|F_m(f_1,f_2,\ldots,f_m)|$ and $|G_k(g_1,g_2,\ldots,g_k)|$ are proportional.

Corollary 2.4 (T = Z) *Let $p \leq 0 \leq r \leq 1$. Let $H_l(x_1, x_2, \ldots, x_l) > 0$, $F_m(x_1, x_2, \ldots, x_m)$ and $G_k(x_1, x_2, \ldots, x_k)$ be three arbitrary functions of l, m and k variables, respectively. Assume that $\{a_{i1}, a_{i2}, \ldots, a_{im}\}_{i=1}^n$, $\{b_{i1}, b_{i2}, \ldots, b_{ik}\}_{i=1}^n$ and $\{c_{i1}, c_{i2}, \ldots, c_{il}\}_{i=1}^n$ are real numbers for any $m, k, l \in \mathrm{N}$, then*

$$\begin{aligned}
&\left(\frac{\sum_{i=1}^n H_l(c_{i1}, c_{i2}, \ldots, c_{il})|F_m(a_{i1}, a_{i2}, \ldots, a_i m) + G_k(b_{i1}, b_{i2}, \ldots, b_{ik})|^p}{\sum_{i=1}^n H_l(c_{i1}, c_{i2}, \ldots, c_{il})|F_m(a_{i1}, a_{i2}, \ldots, a_i m) + G_k(b_{i1}, b_{i2}, \ldots, b_{ik})|^r}\right)^{1/(p-r)} \\
&\quad \geq \left(\frac{\sum_{i=1}^n H_l(c_{i1}, c_{i2}, \ldots, c_{il})|F_m(a_{i1}, a_{i2}, \ldots, a_i m)|^p}{\sum_{i=1}^n H_l(c_{i1}, c_{i2}, \ldots, c_{il})|F_m(a_{i1}, a_{i2}, \ldots, a_i m)|^r}\right)^{1/(p-r)} \\
&\qquad + \left(\frac{\sum_{i=1}^n H_l(c_{i1}, c_{i2}, \ldots, c_{il})|G_k(b_{i1}, b_{i2}, \ldots, b_{ik})|^p}{\sum_{i=1}^n H_l(c_{i1}, c_{i2}, \ldots, c_{il})|G_k(b_{i1}, b_{i2}, \ldots, b_{ik})|^r}\right)^{1/(p-r)}, \qquad (2.17)
\end{aligned}$$

there is equality only when the functions $|F_m(a_{i1}, a_{i2}, \ldots, a_{im})|$ and $|G_k(b_{i1}, b_{i2}, \ldots, b_{ik})|$ are proportional.

Obviously, Corollaries 2.2 and 2.4 are well known for the integers.

Remark 2.1 Let $\{f_i(x,y)\}_{i=1}^m$, $\{g_i(x,y)\}_{i=1}^k$ and $\{h_i(x,y)\}_{i=1}^l$ be continuous real-valued functions on $[a,b]_{\mathrm{T}} \times [a,b]_{\mathrm{T}}$, and H_l, F_m and G_k be defined as in Theorem 2.1, then by Theorems 2.1 and 2.2, we obtain functional generalizations of two-dimensional diamond-α integral Dresher's inequality and reverse Dresher's inequality on time scales.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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