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Global robust exponential synchronization of BAM recurrent FNNs with infinite distributed delays and diffusion terms on time scales

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Abstract
In this article, the global robust exponential synchronization of reaction-diffusion BAM recurrent fuzzy neural networks (FNNs) with infinite distributed delays on time scales is investigated. Applied Lyapunov functional and inequality skills, some sufficient criteria are established to guarantee the global robust exponential synchronization of reaction-diffusion BAM recurrent FNNs with infinite distributed delays on time scales. One example is given to illustrate the effectiveness of our results.

Keywords: globally robust exponential synchronization; reaction-diffusion BAM recurrent FNNs; infinite distributed delays; Lyapunov functional; time scales

1 Introduction

The study on the artificial neural networks has attracted much attention because of their potential applications such as signal processing, image processing, pattern classification, quadratic optimization, associative memory, moving object speed detection, *etc*. Many kinds of models of neural networks have been proposed by some famous scholars. One of these important neural network models is the bidirectional associative memory (BAM) neural network models, which were first introduced by Kosko [1–3]. It is a special class of recurrent neural networks that can store bipolar vector pairs. The BAM neural network is composed of neurons arranged in two layers, the X-layer and the Y-layer. The neurons in one layer are fully interconnected to the neurons in the other layer. Through iterations of forward and backward information flows between the two layers, it performs a two-way associative search for stored bipolar vector pairs and generalize the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks possesses good application prospects in some fields such as pattern recognition, signal and image process, artificial intelligence [4]. In general, artificial neural networks have complex dynamical behaviors such as stability, synchronization, periodic or almost periodic solutions, invariant sets and attractors, and so forth. We can refer to [5–27] and the references cited therein. Therefore, the analysis of dynamical behaviors for neural networks is a necessary step for practical design of neural networks. As one of the famous neural network models, it has attracted many attention in the past two decades [28–48] since the BAM model was proposed by Kosko. The dynamical behaviors such as uniqueness, global asymptotic stability, exponential stability and invariant sets and attractors of the equilibrium point or periodic solutions were investigated for BAM neural networks with different types of time delays (see [28–44, 48]).

Synchronization has attracted much attention after it was proposed by Carrol *et al.* [49, 50]. The principle of drive-response synchronization is this: the driver system sends a signal through a channel to the responder system, which uses this signal to synchronize itself with the driver. Namely, the response system is influenced by the behavior of the drive system, but the drive system is independent of the response one. In recent years, many results concerning a synchronization problem of time lag neural networks have been investigated in the literature [5, 6, 8–15, 27, 36, 49, 50].

As is well known, both in biological and man-made neural networks, strictly speaking, diffusion effects cannot be avoided when electrons are moving in asymmetric electromagnetic fields, so we must consider that the activations vary in space as well as in time. Many researchers have studied the dynamical properties of continuous time reaction-diffusion neural networks (see, for example, [8, 11, 17, 18, 24, 25, 27, 32, 48]).

However, in mathematical modeling of real world problems, we will encounter some other inconveniences such as complexity and uncertainty or vagueness. Fuzzy theory is considered as a more suitable setting for the sake of taking vagueness into consideration. Based on traditional cellular neural networks (CNNs), T Yang and LB Yang proposed the fuzzy CNNs (FCNNs) [23] which integrate fuzzy logic into the structure of traditional CNNs and maintain local connectedness among cells. Unlike previous CNNs structures, FCNNs have fuzzy logic between their template input and/or output besides the sum of product operation. FCNNs are very a useful paradigm for image processing problems, which is a cornerstone in image processing and pattern recognition. Therefore, it is necessary to consider both the fuzzy logic and delay effect on dynamical behaviors of neural networks. To the best of our knowledge, few authors have considered the synchronization of reaction-diffusion recurrent fuzzy neural networks with delays and Dirichlet boundary conditions on time scales which is a challenging and important problem in theory and application. Therefore, in this paper, we will investigate the global robust exponential synchronization of delayed reaction-diffusion BAM recurrent fuzzy neural networks (FNNs) on time scales as follows:

$$\begin{cases} u_i^{\Delta}(t,x) = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(a_{ik}\frac{\partial u_i}{\partial x_k}) - b_i u_i(t,x) + \sum_{j=1}^{m} c_{ij} f_j(v_j(t-\tau,x)) + I_i \\ \qquad + \bigwedge_{j=1}^{n} p_{ij} F_j(u_j(t-\tau,x)) + \bigwedge_{j=1}^{n} r_{ij} \int_0^{+\infty} k_{ij}(s) F_j(u_j(t-s,x))\Delta s \\ \qquad + \bigvee_{j=1}^{n} q_{ij} F_j(u_j(t-\tau,x)) + \bigvee_{j=1}^{n} w_{ij} \int_0^{+\infty} k_{ij}(s) F_j(u_j(t-s,x))\Delta s \\ \qquad + \sum_{j=1}^{n} d_{ij}\mu_j + \bigwedge_{j=1}^{n} S_{ij}\mu_j + \bigvee_{j=1}^{n} T_{ij}\mu_j, \\ v_j^{\Delta}(t,x) = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(\xi_{jk}\frac{\partial v_j}{\partial x_k}) - \eta_j v_j(t,x) + \sum_{i=1}^{n} \zeta_{ji} g_i(u_i(t-\tau,x)) + J_j \\ \qquad + \bigwedge_{i=1}^{m} \lambda_{ji} G_i(v_i(t-\tau,x)) + \bigwedge_{i=1}^{m} \rho_{ji} \int_0^{+\infty} \kappa_{ji}(s) G_i(v_i(t-s,x))\Delta s \\ \qquad + \bigvee_{i=1}^{m} \pi_{ji} G_i(v_i(t-\tau,x)) + \bigvee_{i=1}^{m} \sigma_{ji} \int_0^{+\infty} \kappa_{ji}(s) G_i(v_i(t-s,x))\Delta s \\ \qquad + \sum_{i=1}^{m} h_{ji}\nu_i + \bigwedge_{i=1}^{m} M_{ji}\nu_i + \bigvee_{i=1}^{m} N_{ji}\nu_i, \end{cases} \tag{1.1}$$

subject to the following initial conditions

$$\begin{cases} u_i(s,x) = \phi_i(s,x), & (s,x) \in [-\tau,0]_{\mathbb{T}} \times \Omega, \\ v_j(s,x) = \varphi_j(s,x), & (s,x) \in [-\tau,0]_{\mathbb{T}} \times \Omega, \end{cases} \tag{1.2}$$

and Dirichlet boundary conditions

$$\begin{cases} u_i(t,x) = 0, & (t,x) \in [0,\infty)_{\mathbb{T}} \times \partial\Omega, \\ v_j(t,x) = 0, & (t,x) \in [0,\infty)_{\mathbb{T}} \times \partial\Omega, \end{cases} \tag{1.3}$$

where $i = 1,2,\dots,n; j = 1,2,\dots,m$. $\mathbb{T} \subset \mathbb{R}$ is a time scale and $\mathbb{T} \cap [0,+\infty) \triangleq [0,+\infty)_{\mathbb{T}}$ is unbounded and $\mathbb{T} \cap [-\tau,0] \triangleq [\tau,0]_{\mathbb{T}} \neq \phi$. $\tau > 0$ is constant time delay. $x = (x_1,x_2,\dots,x_l)^T \in \Omega \subset \mathbb{R}^l$ and $\Omega = \{x = (x_1,x_2,\dots,x_l)^T : |x_i| < l_i, i = 1,2,\dots,l\}$ is a bounded compact set with smooth boundary $\partial\Omega$ in space $\mathbb{R}^l$. $u = (u_1,u_2,\dots,u_n)^T \in \mathbb{R}^n$, $v = (v_1,v_2,\dots,v_m)^T \in \mathbb{R}^m$. $u_i(t,x)$ and $v_j(t,x)$ are the state of the ith neurons and the jth neurons at time t and in space x, respectively. $I = (I_1,I_2,\dots,I_n)^T \in \mathbb{R}^n$ and $J = (J_1,J_2,\dots,J_m)^T \in \mathbb{R}^m$ are constant input vectors. The smooth functions $a_{ik} > 0$ and $\xi_{jk} > 0$ correspond to the transmission diffusion operators along with the ith neurons and the jth neurons, respectively. $b_i > 0$, $\eta_j > 0$, μ_j, ν_i, c_{ij}, p_{ij}, $r_{ij,}$ q_{ij}, w_{ij}, d_{ij}, S_{ij}, T_{ij}, ζ_{ji}, λ_{ji}, ρ_{ji}, π_{ji}, σ_{ji}, h_{ji}, M_{ji}, M_{ji} are constants. b_i and η_j denote the rate with which the ith neurons and jth neurons will reset their potential to the resting state in isolation when disconnected from the network and external inputs, respectively. c_{ij}, p_{ij}, $r_{ij,}$ q_{ij}, w_{ij}, d_{ij}, S_{ij}, T_{ij}, ζ_{ji}, λ_{ji}, ρ_{ji}, π_{ji}, σ_{ji}, h_{ji}, M_{ji}, M_{ji} denote the connection weights. $f_j(\cdot)$ ($j = 1,2,\dots,m$) and $g_i(\cdot)$ ($i = 1,2,\dots,n$) denote the activation function of the jth neurons of Y-layer on the ith neurons of X-layer and the ith neurons of X-layer on the jth neurons of Y-layer at time t and in space x, respectively. $F_j(\cdot)$ ($j = 1,2,\dots,n$) denotes the fuzzy activation function of the jth neurons on the ith neurons inside of X-layer. $G_i(\cdot)$ ($i = 1,2,\dots,m$) denotes the fuzzy activation function of the ith neurons on the jth neurons inside of Y-layer. μ_j ($j = 1,2,\dots,n$) denotes the bias of the jth neurons on the ith neurons inside of X-layer. ν_i ($i = 1,2,\dots,m$) denotes the bias of the ith neurons on the jth neurons inside of Y-layer. $\bigwedge$, $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively. $\phi(t,x) = (\phi_1(t,x),\phi_2(t,x),\dots,\phi_n(t,x))^T : [-\tau,0]_{\mathbb{T}} \times \Omega \to \mathbb{R}^n$, $\varphi(t,x) = (\varphi_1(t,x),\varphi_2(t,x),\dots,\varphi_m(t,x))^T : [-\tau,0]_{\mathbb{T}} \times \Omega \to \mathbb{R}^m$ are rd-continuous with respect to $t \in [-\tau,0]_{\mathbb{T}}$ and continuous with respect to $x \in \Omega$.

In order to investigate the global robust exponential synchronization for system (1.1)-(1.3), the quantities b_i, a_{ik}, c_{ij}, p_{ij}, $r_{ij,}$ q_{ij}, w_{ij}, η_j, ξ_{jk}, ζ_{ji}, λ_{ji}, ρ_{ji}, π_{ji} and σ_{ji} may be considered as intervals as follows: $0 < \underline{b}_i \le b_i < \infty$, $\underline{a}_{ik} \le a_{ik} \le \overline{a}_{ik}$, $|\underline{c}_{ij}| \le |c_{ij}| \le |\overline{c}_{ij}|$, $|\underline{p}_{ij}| \le |p_{ij}| \le |\overline{p}_{ij}|$, $|\underline{r}_{ij}| \le |r_{ij}| \le |\overline{r}_{ij}|$, $|\underline{q}_{ij}| \le |q_{ij}| \le |\overline{q}_{ij}|$, $|\underline{w}_{ij}| \le |w_{ij}| \le |\overline{w}_{ij}|$, $0 < \underline{\eta}_j \le \eta_j < \infty$, $\underline{\xi}_{jk} \le \xi_{jk} \le \overline{\xi}_{jk}$, $|\underline{\zeta}_{ji}| \le |\zeta_{ji}| \le |\overline{\zeta}_{ji}|$, $|\underline{\lambda}_{ji}| \le |\lambda_{ji}| \le |\overline{\lambda}_{ji}|$, $|\underline{\rho}_{ji}| \le |\rho_{ji}| \le |\overline{\rho}_{ji}|$, $|\underline{\pi}_{ji}| \le |\pi_{ji}| \le |\overline{\pi}_{ji}|$, $|\underline{\sigma}_{ji}| \le |\sigma_{ji}| \le |\overline{\sigma}_{ji}|$.

Take the time scale $\mathbb{T} = \mathbb{R}$ (real number set), then system (1.1)-(1.3) can be changed into the following continuous case (1.4)-(1.6):

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(a_{ik}\frac{\partial u_i}{\partial x_k}) - b_i u_i(t,x) + \sum_{j=1}^{m} c_{ij} f_j(v_j(t-\tau,x)) + I_i \\ \qquad + \bigwedge_{j=1}^{n} p_{ij} F_j(u_j(t-\tau,x)) + \bigwedge_{j=1}^{n} r_{ij} \int_0^{+\infty} k_{ij}(s) F_j(u_j(t-s,x))\,\mathrm{d}s \\ \qquad + \bigvee_{j=1}^{n} q_{ij} F_j(u_j(t-\tau,x)) + \bigvee_{j=1}^{n} w_{ij} \int_0^{+\infty} k_{ij}(s) F_j(u_j(t-s,x))\,\mathrm{d}s \\ \qquad + \sum_{j=1}^{n} d_{ij}\mu_j + \bigwedge_{j=1}^{n} S_{ij}\mu_j + \bigvee_{j=1}^{n} T_{ij}\mu_j, \\ \frac{\partial v_j(t,x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(\xi_{jk}\frac{\partial v_j}{\partial x_k}) - \eta_j v_j(t,x) + \sum_{i=1}^{n} \zeta_{ji} g_i(u_i(t-\tau,x)) + J_j \\ \qquad + \bigwedge_{i=1}^{m} \lambda_{ji} G_i(v_i(t-\tau,x)) + \bigwedge_{i=1}^{m} \rho_{ji} \int_0^{+\infty} \kappa_{ji}(s) G_i(v_i(t-s,x))\,\mathrm{d}s \\ \qquad + \bigvee_{i=1}^{m} \pi_{ji} G_i(v_i(t-\tau,x)) + \bigvee_{i=1}^{m} \sigma_{ji} \int_0^{+\infty} \kappa_{ji}(s) G_i(v_i(t-s,x))\,\mathrm{d}s \\ \qquad + \sum_{i=1}^{m} h_{ji}\nu_i + \bigwedge_{i=1}^{m} M_{ji}\nu_i + \bigvee_{i=1}^{m} N_{ji}\nu_i, \end{cases} \tag{1.4}$$

subject to the following initial conditions

$$\begin{cases} u_i(s,x) = \phi_i(s,x), & (s,x) \in [-\tau,0] \times \Omega, \\ v_j(s,x) = \varphi_j(s,x), & (s,x) \in [-\tau,0] \times \Omega, \end{cases} \tag{1.5}$$

and Dirichlet boundary conditions

$$\begin{cases} u_i(t,x) = 0, & (t,x) \in [0,\infty) \times \partial\Omega, \\ v_j(t,x) = 0, & (t,x) \in [0,\infty) \times \partial\Omega. \end{cases} \tag{1.6}$$

Take the time scale $\mathbb{T} = \mathbb{Z}$ (integer number set), then system (1.1)-(1.3) can be changed into the following discrete case (1.7)-(1.9):

$$\begin{cases} \Delta_t u_i(t,x) = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(a_{ik}\frac{\partial u_i}{\partial x_k}) - b_i u_i(t,x) + \sum_{j=1}^{m} c_{ij} f_j(v_j(t-\tau,x)) + I_i \\ \qquad + \bigwedge_{j=1}^{n} p_{ij} F_j(u_j(t-\tau,x)) + \bigwedge_{j=1}^{n} r_{ij} \sum_{s=0}^{\infty} k_{ij}(s) F_j(u_j(t-s,x)) \\ \qquad + \bigvee_{j=1}^{n} q_{ij} F_j(u_j(t-\tau,x)) + \bigvee_{j=1}^{n} w_{ij} \sum_{s=0}^{\infty} k_{ij}(s) F_j(u_j(t-s,x)) \\ \qquad + \sum_{j=1}^{n} d_{ij}\mu_j + \bigwedge_{j=1}^{n} S_{ij}\mu_j + \bigvee_{j=1}^{n} T_{ij}\mu_j, \\ \Delta_t v_j(t,x) = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(\xi_{jk}\frac{\partial v_j}{\partial x_k}) - \eta_j v_j(t,x) + \sum_{i=1}^{n} \zeta_{ji} g_i(u_i(t-\tau,x)) + J_j \\ \qquad + \bigwedge_{i=1}^{m} \lambda_{ji} G_i(v_i(t-\tau,x)) + \bigwedge_{i=1}^{m} \rho_{ji} \sum_{s=0}^{\infty} \kappa_{ji}(s) G_i(v_i(t-s,x)) \\ \qquad + \bigvee_{i=1}^{m} \pi_{ji} G_i(v_i(t-\tau,x)) + \bigvee_{i=1}^{m} \sigma_{ji} \sum_{s=0}^{\infty} \kappa_{ji}(s) G_i(v_i(t-s,x)) \\ \qquad + \sum_{i=1}^{m} h_{ji}\nu_i + \bigwedge_{i=1}^{m} M_{ji}\nu_i + \bigvee_{i=1}^{m} N_{ji}\nu_i, \end{cases} \tag{1.7}$$

subject to the following initial conditions

$$\begin{cases} u_i(s,x) = \phi_i(s,x), & (s,x) \in \{-\tau,-\tau+1,\dots,-2,-1,0\} \times \Omega, \\ v_j(s,x) = \varphi_j(s,x), & (s,x) \in \{-\tau,-\tau+1,\dots,-2,-1,0\} \times \Omega, \end{cases} \tag{1.8}$$

and Dirichlet boundary conditions

$$\begin{cases} u_i(t,x) = 0, & (t,x) \in \mathbb{Z}^+ \times \partial\Omega, \\ v_j(t,x) = 0, & (t,x) \in \mathbb{Z}^+ \times \partial\Omega, \end{cases} \tag{1.9}$$

where $t \in \mathbb{Z}$, τ is a positive integer, $\mathbb{Z}^+ = \{0,1,2,\dots\}$, $\Delta_t u_i(t,x) \triangleq u_i(t+1,x) - u_i(t,x)$, $\Delta_t v_j(t,x) \triangleq v_j(t+1,x) - v_j(t,x)$.

If we choose $\mathbb{T} = \mathbb{R}$, then $\sigma(t) = t$, $\mu(t) = 0$. In this case, system (1.1)-(1.3) is the continuous reaction-diffusion BAM recurrent FNNs (1.4)-(1.6). If $\mathbb{T} = \mathbb{Z}$, then $\mu(t) = 1$, system (1.1)-(1.3) is the discrete difference reaction-diffusion BAM recurrent FNNs (1.7)-(1.9). In this paper, we study the global robust exponential synchronization of reaction-diffusion BAM recurrent FNNs (1.1)-(1.3), which unify both the continuous case and the discrete difference case. What is more, system (1.1)-(1.3) is a good model for handling many problems such as predator-prey forecast or optimizing of goods output.

The rest of this paper is organized as follows. In Section 2, some notations and basic theorems or lemmas on time scales are given. In Section 3, the main results of global robust exponential synchronization are obtained by constructing the appropriate Lyapunov functional and applying inequality skills. In Section 4, one example is given to illustrate the effectiveness of our results.

2 Preliminaries

In this section, we first recall some basic definitions and lemmas on time scales which are used in what follows.

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\rho, \sigma : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu :\to \mathbb{R}^+$ are defined, respectively, by

$$\begin{aligned}
&\sigma(t) = \inf\{s \in \mathbb{T} : s > t\},\\
&\rho(t) = \sup\{s \in \mathbb{T} : s < t\},\\
&\mu(t) = \sigma(t) - t.
\end{aligned}$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$, otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$, otherwise $\mathbb{T}_k = \mathbb{T}$.

Definition 2.1 ([51]) A function $f : \mathbb{T} \to \mathbb{R}$ is called regulated provided its right-hand side limits exist (finite) at all right-hand side points in $\mathbb{T}$ and its left-hand side limits exist (finite) at all left-hand side points in $\mathbb{T}$.

Definition 2.2 ([51]) A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-hand side limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous function $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{\mathrm{rd}} = C_{\mathrm{rd}}(\mathbb{T}) = C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$.

Definition 2.3 ([51]) Assume $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$. Then we define $f^\Delta(t)$ to be the number (if it exists) with the property that given any $\epsilon > 0$ there exists a neighborhood U of t (*i.e.*, $U = (t - \Delta, t + \Delta) \cap \mathbb{T}$ for some $\Delta > 0$) such that

$$\bigl|\bigl[f\bigl(\sigma(t)\bigr) - f(s)\bigr] - f^\Delta(t)\bigl[\sigma(t) - s\bigr]\bigr| < \epsilon\bigl|\sigma(t) - s\bigr|$$

for all $s \in U$. We call $f^\Delta(t)$ the delta (or Hilger) derivative of f at t. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that is a differentiable and whose derivative is rd-continuous is denoted by $C^1_{\mathrm{rd}} = C^1_{\mathrm{rd}}(\mathbb{T}) = C^1_{\mathrm{rd}}(\mathbb{R}, \mathbb{T})$.

If f is continuous, then f is rd-continuous. If f is rd-continuous, then f is regulated. If f is delta differentiable at t, then f is continuous at t.

Lemma 2.1 ([51]) *Let f be regulated, then there exists a function F which is delta differentiable with region of differentiation D such that $F^\Delta(t) = f(t)$ for all $t \in D$.*

Definition 2.4 ([51]) Assume that $f : \mathbb{T} \to \mathbb{R}$ is a regulated function. Any function F as in Lemma 2.1 is called a Δ-antiderivative of f. We define the indefinite integral of a regulated function f by

$$\int f(t)\Delta t = F(t) + C,$$

where C is an arbitrary constant and F is a Δ-antiderivative of f. We define the Cauchy integral by $\int_a^b f(s)\Delta s = F(b) - F(a)$ for all $a, b \in \mathbb{T}$.

A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^\Delta(t) = f(t)$ for all $t \in \mathbb{T}^k$.

Lemma 2.2 ([51]) *If $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C(\mathbb{T}, \mathbb{R})$, then*

(i) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$,

(ii) *if* $f(t) \geq 0$ *for all* $a \leq t \leq b$, *then* $\int_a^b f(t)\Delta t \geq 0$,

(iii) *if* $|f(t)| \leq g(t)$ *on* $[a, b) \triangleq \{t \in \mathbb{T} : a \leq t \leq b\}$, *then* $|\int_a^b f(t)\Delta t| \leq \int_a^b g(t)\Delta t$.

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive if $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T}, \mathbb{R})$. We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0 \text{ for all } t \in \mathbb{T}\}$. If p is a regressive function, then the generalized exponential function $e_p(t, s)$ is defined by $e_p(t, s) = \exp\{\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\}$ for all $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q = p + q + \mu pq,$$
$$\ominus p = -\frac{p}{1 + \mu p},$$
$$p \ominus q = p \oplus p(\ominus q).$$

If $p \in \mathcal{R}^+$, then $\ominus p \in \mathcal{R}^+$.

The generalized exponential function has the following properties.

Lemma 2.3 ([51]) *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then*

(i) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;

(ii) $1/e_p(t, s) = e_{\ominus p}(t, s)$;

(iii) $e_p(t, s) = 1/e_p(s, t) = e_{\ominus p}(s, t)$;

(iv) $e_p(t, s)e_p(s, r) = e_p(t, r)$;

(v) $[e_p(t, s)]^\Delta = p(t)e_p(t, s)$;

(vi) $[e_p(c, \cdot)]^\Delta = -p[e_p(c, \cdot)]^\sigma$ *for all* $c \in \mathbb{T}$;

(vii) $(\mathrm{d}/\mathrm{d}z)[e_z(t, s)] = [\int_s^t 1/(1 + \mu(\tau)z)\Delta\tau]e_z(t, s)$.

Lemma 2.4 ([51]) *Assume that $f, g : \mathbb{T} \to \mathbb{R}$ are delta differentiable at $t \in \mathbb{T}^k$. Then*

$$(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = g^\Delta(t)f(t) + g(\sigma(t))f^\Delta(t).$$

Lemma 2.5 ([52]) *For each $t \in \mathbb{T}$, let N be a neighborhood of t. Then, for $V \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R}^+)$, define $D^+V^\Delta(t)$ to mean that, given $\epsilon > 0$, there exists a right neighborhood $N_\epsilon \cap N$ of t such* *that*

$$\frac{1}{u(t)}\big[V\big(\sigma(t)\big) - V(t) - \mu(t)f(t)\big] < D^+V^\Delta(t) + \epsilon \quad \textit{for each } s \in N_\epsilon, s > t,$$

where $\mu(t) = \sigma(t) - s$. *If* t *is right-scattered and* $V(t)$ *is continuous at* t, *this reduces to* $D^+V^\Delta(t) = \frac{V(\sigma(t))-V(t)}{\sigma(t)-t}$.

Next, we introduce the Banach space which is suitable for system (1.1)-(1.3).

Let $\Omega = \{x = (x_1, x_2, \ldots, x_l)^T : |x_i| < l_i, i = 1, 2, \ldots, l\}$ be an open bounded domain in $\mathbb{R}^l$ with smooth boundary $\partial\Omega$. Let $C_{\mathrm{rd}}(\mathbb{T}\times\Omega, \mathbb{R}^{n+m})$ be the set consisting of all the vector function $y(t,x) = (y_1(t,x), y_2(t,x), \ldots, y_{n+m}(t,x))^T$ which is rd-continuous with respect to $t \in \mathbb{T}$ and continuous with respect to $x \in \Omega$. For every $t \in \mathbb{T}$ and $x \in \Omega$, we define the set $C^t_{\mathbb{T}} = \{y(t,\cdot) : y \in C(\Omega, \mathbb{R}^{n+m})\}$. Then $C^t_{\mathbb{T}}$ is a Banach space with the norm $\|y(t,\cdot)\| = (\sum_{i=1}^{n+m}\|y_i(t,\cdot)\|_2^2)^{1/2}$, where $\|y_i(t,\cdot)\|_2 = (\int_\Omega |y_i(t,x)|^2\,\mathrm{d}x)^{1/2}$. Let $C_{\mathrm{rd}}([-\tau,0]_{\mathbb{T}}\times\Omega, \mathbb{R}^{n+m})$ consist of all functions $f(t,x)$ which map $[-\tau,0]_{\mathbb{T}}\times\Omega$ into $\mathbb{R}^{n+m}$ and $f(t,x)$ is rd-continuous with respect to $t \in [-\tau,0]_{\mathbb{T}}$ and continuous with respect to $x \in \Omega$. For every $t \in [-\tau,0]_{\mathbb{T}}$ and $x \in \Omega$, we define the set $C^t_{[-\tau,0]_{\mathbb{T}}} = \{u(t,\cdot) : u \in C(\Omega, \mathbb{R}^{n+m})\}$. Then $C^t_{[-\tau,0]_{\mathbb{T}}}$ is a Banach space equipped with the norm $\|\psi\|_0 = (\sum_{i=1}^{n+m}\|\phi_i\|_1^2)^{1/2}$, where $\psi(t,x) = (\psi_1(t,x), \psi_2(t,x), \ldots, \psi_{n+m}(t,x))^T \in C^t_{[-\tau,0]_{\mathbb{T}}}$, $\|\psi_i(t,\cdot)\|_1 = (\int_\Omega |\psi_i(\cdot,x)|_\tau^2\,\mathrm{d}x)^{1/2}$, $|\psi_i(\cdot,x)|_\tau = \sup_{s\in[-\tau,0]_{\mathbb{T}}}|\psi_i(s,x)|$.

In order to achieve the global robust exponential synchronization, the following system (2.1)-(2.3) is the controlled slave system corresponding to the master system (1.1)-(1.3):

$$\begin{cases} \tilde{u}_i^\Delta(t,x) = \sum_{k=1}^l \frac{\partial}{\partial x_k}(a_{ik}\frac{\partial \tilde{u}_i}{\partial x_k}) - b_i\tilde{u}_i(t,x) + \sum_{j=1}^m c_{ij}f_j(\tilde{v}_j(t-\tau,x)) + I_i \\ \qquad + \bigwedge_{j=1}^n p_{ij}F_j(\tilde{u}_j(t-\tau,x)) + \bigwedge_{j=1}^n r_{ij}\int_0^{+\infty} k_{ij}(s)F_j(\tilde{u}_j(t-s,x))\Delta s \\ \qquad + \bigvee_{j=1}^n q_{ij}F_j(\tilde{u}_j(t-\tau,x)) + \bigvee_{j=1}^n w_{ij}\int_0^{+\infty} k_{ij}(s)F_j(\tilde{u}_j(t-s,x))\Delta s \\ \qquad + \sum_{j=1}^n d_{ij}\mu_j + \bigwedge_{j=1}^n S_{ij}\mu_j + \bigvee_{j=1}^n T_{ij}\mu_j + m_iE_i(t,x), \\ \tilde{v}_j^\Delta(t,x) = \sum_{k=1}^l \frac{\partial}{\partial x_k}(\xi_{jk}\frac{\partial \tilde{v}_j}{\partial x_k}) - \eta_j\tilde{v}_j(t,x) + \sum_{i=1}^n \zeta_{ji}g_i(\tilde{u}_i(t-\tau,x)) + J_j \\ \qquad + \bigwedge_{i=1}^m \lambda_{ji}G_i(\tilde{v}_i(t-\tau,x)) + \bigwedge_{i=1}^m \rho_{ji}\int_0^{+\infty} \kappa_{ji}(s)G_i(\tilde{v}_i(t-s,x))\Delta s \\ \qquad + \bigvee_{i=1}^m \pi_{ji}G_i(\tilde{v}_i(t-\tau,x)) + \bigvee_{i=1}^m \sigma_{ji}\int_0^{+\infty} \kappa_{ji}(s)G_i(\tilde{v}_i(t-s,x))\Delta s \\ \qquad + \sum_{i=1}^m h_{ji}\nu_i + \bigwedge_{i=1}^m M_{ji}\nu_i + \bigvee_{i=1}^m N_{ji}\nu_i + m_{n+j}E_{n+j}(t,x), \end{cases} \tag{2.1}$$

subject to the following initial conditions

$$\begin{cases} \tilde{u}_i(s,x) = \tilde{\phi}_i(s,x), & (s,x) \in [-\tau,0]_{\mathbb{T}}\times\Omega, \\ \tilde{v}_j(s,x) = \tilde{\varphi}_j(s,x), & (s,x) \in [-\tau,0]_{\mathbb{T}}\times\Omega, \end{cases} \tag{2.2}$$

and Dirichlet boundary conditions

$$\begin{cases} \tilde{u}_i(t,x) = 0, & (t,x) \in [0,\infty)_{\mathbb{T}}\times\partial\Omega, \\ \tilde{v}_j(t,x) = 0, & (t,x) \in [0,\infty)_{\mathbb{T}}\times\partial\Omega, \end{cases} \tag{2.3}$$

where $E_i(t,x) = \tilde{u}_i(t,x) - u_i(t,x)$ $(i = 1,2,\ldots,n)$ and $E_{n+j}(t,x) = \tilde{v}_{n+j}(t,x) - v_{n+j}(t,x)$ $(j = 1,2,\ldots,m)$ are error functions. $m_k > 0$ $(k = 1,2,\ldots,n+m)$ is a constant error weighting coefficient. $\tilde{u}(t,x) = (\tilde{u}_1(t,x), \tilde{u}_2(t,x), \ldots, \tilde{u}_n(t,x))^T \in C_{\mathrm{rd}}(\mathbb{T}\times\Omega, \mathbb{R}^n)$, $\tilde{v}(t,x) = (\tilde{v}_1(t,x), \tilde{v}_2(t,x), \ldots, \tilde{v}_m(t,x))^T \in C_{\mathrm{rd}}(\mathbb{T}\times\Omega, \mathbb{R}^m)$, $\tilde{\phi}(t,x) = (\tilde{\phi}_1(t,x), \tilde{\phi}_2(t,x), \ldots, \tilde{\phi}_n(t,x))^T \in C([-\tau,0]\times\Omega, \mathbb{R}^n)$, $\tilde{\varphi}(t,x) = (\tilde{\varphi}_1(t,x), \tilde{\varphi}_2(t,x), \ldots, \tilde{\varphi}_m(t,x))^T \in C([-\tau,0]\times\Omega, \mathbb{R}^m)$.

From (1.1)-(1.3) and (2.1)-(2.3), we obtain the error system (2.4)-(2.6) as follows:

$$\begin{cases} E_i^{\Delta}(t,x) = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(a_{ik}\frac{\partial E_i}{\partial x_k}) + (m_i - b_i)E_i(t,x) \\ \qquad + \sum_{j=1}^{m} c_{ij}[f_j(\tilde{v}_j(t-\tau,x)) - f_j(v_j(t-\tau,x))] \\ \qquad + \bigwedge_{j=1}^{n} p_{ij}[F_j(\tilde{u}_j(t-\tau,x)) - F_j(u_j(t-\tau,x))] \\ \qquad + \bigwedge_{j=1}^{n} r_{ij}\int_0^{+\infty} k_{ij}(s)[F_j(\tilde{u}_j(t-s,x)) - F_j(u_j(t-s,x))]\Delta s \\ \qquad + \bigvee_{j=1}^{n} q_{ij}[F_j(\tilde{u}_j(t-\tau,x)) - F_j(u_j(t-\tau,x))] \\ \qquad + \bigvee_{j=1}^{n} w_{ij}\int_0^{+\infty} k_{ij}(s)[F_j(\tilde{u}_j(t-s,x)) - F_j(u_j(t-s,x))]\Delta s, \\ E_{n+j}^{\Delta}(t,x) = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(\xi_{jk}\frac{\partial E_{n+j}}{\partial x_k}) + (m_{n+j} - \eta_j)E_{n+j}(t,x) \\ \qquad + \sum_{i=1}^{n} \zeta_{ji}[g_i(\tilde{u}_i(t-\tau,x)) - g_i(u_i(t-\tau,x))] \\ \qquad + \bigwedge_{i=1}^{m} \lambda_{ji}[G_i(\tilde{v}_i(t-\tau,x)) - G_i(v_i(t-\tau,x))] \\ \qquad + \bigwedge_{i=1}^{m} \rho_{ji}\int_0^{+\infty} \kappa_{ji}(s)[G_i(\tilde{v}_i(t-s,x)) - G_i(v_i(t-s,x))]\Delta s \\ \qquad + \bigvee_{i=1}^{m} \pi_{ji}[G_i(\tilde{v}_i(t-\tau,x)) - G_i(v_i(t-\tau,x))] \\ \qquad + \bigvee_{i=1}^{m} \sigma_{ji}\int_0^{+\infty} \kappa_{ji}(s)[G_i(\tilde{v}_i(t-s,x)) - G_i(v_i(t-s,x))]\Delta s, \end{cases} \tag{2.4}$$

subject to the following initial conditions

$$\begin{cases} E_i(s,x) = \tilde{\phi}_i(s,x) - \phi_i(s,x), & (s,x) \in [-\tau,0]_{\mathbb{T}} \times \Omega, \\ E_{n+j}(s,x) = \tilde{\varphi}_j(s,x) - \varphi_j(s,x), & (s,x) \in [-\tau,0]_{\mathbb{T}} \times \Omega, \end{cases} \tag{2.5}$$

and Dirichlet boundary conditions

$$\begin{cases} E_i(t,x) = 0, & (t,x) \in [0,\infty)_{\mathbb{T}} \times \partial\Omega, \\ E_{n+j}(t,x) = 0, & (t,x) \in [0,\infty)_{\mathbb{T}} \times \partial\Omega. \end{cases} \tag{2.6}$$

The following definition is significant to study the global robust exponential synchronization of coupled neural networks (1.1)-(1.3) and (2.1)-(2.3).

Definition 2.5 Let $y(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x), v_1(t,x), v_2(t,x), \dots, v_m(t,x))^T \in \mathbb{R}^{n+m}$ and $\tilde{y}(t,x) = (\tilde{u}_1(t,x), \tilde{u}_2(t,x), \dots, \tilde{u}_n(t,x), \tilde{v}_1(t,x), \tilde{v}_2(t,x), \dots, \tilde{v}_m(t,x))^T \in \mathbb{R}^{n+m}$ be the solution vectors of system (1.1)-(1.3) and its controlled slave system (2.1)-(2.3), respectively. $E(t,x) = (E_1(t,x), E_2(t,x), \dots, E_{n+m}(t,x))^T \in \mathbb{R}^{n+m}$ is the error vector. Then the coupled systems (1.1)-(1.3) and (2.1)-(2.3) are said to be globally exponentially synchronized if there exists a controlled input vector $z(t,x) = (m_1E_1(t,x), m_2E_2(t,x), \dots, m_{n+m}E_{n+m}(t,x))^T$ and a positive constant $\alpha \in \mathcal{R}^+$ and $M \geq 1$ such that

$$\|E(t,\cdot)\| = \|\tilde{y}(t,\cdot) - y(t,\cdot)\| \leq Me_{\ominus\alpha}(t,0), \quad t \in [0,\infty)_{\mathbb{T}},$$

where α is called the degree of exponential synchronization on time scales.

3 Main results

In this section, we will consider the global robust exponential synchronization of coupled systems (1.1)-(1.3) and (2.1)-(2.3). At first, we need to introduce some useful lemmas.

Lemma 3.1 ([53]) *Let Ω be a cube $|x_i| < l_i$ $(i = 1, 2, \dots, l)$ and assume that $h(x)$ is a real-valued function belonging to $C^1(\Omega)$ which vanishes on the boundary $\partial\Omega$ of Ω, i.e.,*

$h(x)|_{\partial\Omega} = 0$. *Then*

$$\int_\Omega h^2(x)\,\mathrm{d}x \le l_i^2 \int_\Omega \left|\frac{\partial h}{\partial x_i}\right|^2 \mathrm{d}x.$$

Lemma 3.2 ([23]) *Suppose that* $y = (y_1, y_2, \dots, y_{n+m})^T$ *and* $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_{n+m})^T$ *are the solutions to systems* (1.1)-(1.3) *and* (2.1)-(2.3), *respectively, then*

$$\left|\bigwedge_{j=1}^m p_{ij} f_j(\tilde{y}_j) - \bigwedge_{j=1}^m p_{ij} f_j(y_j)\right| \le \sum_{j=1}^m |p_{ij}| \left|f_j(\tilde{y}_j) - f_j(y_j)\right|,$$

$$\left|\bigvee_{j=1}^m q_{ij} f_j(\tilde{y}_j) - \bigvee_{j=1}^m q_{ij} f_j(y_j)\right| \le \sum_{j=1}^m |q_{ij}| \left|f_j(\tilde{y}_j) - f_j(y_j)\right|,$$

$$\left|\bigwedge_{j=1}^n p_{ij} g_j(\tilde{y}_j) - \bigwedge_{j=1}^n p_{ij} g_j(y_j)\right| \le \sum_{j=1}^n |p_{ij}| \left|g_j(\tilde{y}_j) - g_j(y_j)\right|,$$

$$\left|\bigvee_{j=1}^n q_{ij} g_j(\tilde{y}_j) - \bigvee_{j=1}^n q_{ij} g_j(y_j)\right| \le \sum_{j=1}^n |q_{ij}| \left|g_j(\tilde{y}_j) - g_j(y_j)\right|,$$

$$\left|\bigwedge_{j=1}^n p_{ij} F_j(\tilde{y}_j) - \bigwedge_{j=1}^n p_{ij} F_j(y_j)\right| \le \sum_{j=1}^n |p_{ij}| \left|F_j(\tilde{y}_j) - F_j(y_j)\right|,$$

$$\left|\bigvee_{j=1}^n q_{ij} F_j(\tilde{y}_j) - \bigvee_{j=1}^n q_{ij} F_j(y_j)\right| \le \sum_{j=1}^n |q_{ij}| \left|F_j(\tilde{y}_j) - F_j(y_j)\right|,$$

$$\left|\bigwedge_{j=1}^m p_{ij} G_j(\tilde{y}_j) - \bigwedge_{j=1}^m p_{ij} G_j(y_j)\right| \le \sum_{j=1}^m |p_{ij}| \left|f_j(\tilde{y}_j) - G_j(y_j)\right|,$$

$$\left|\bigvee_{j=1}^m q_{ij} G_j(\tilde{y}_j) - \bigvee_{j=1}^m q_{ij} G_j(y_j)\right| \le \sum_{j=1}^m |q_{ij}| \left|G_j(\tilde{y}_j) - G_j(y_j)\right|.$$

Throughout this paper, we always assume that:

(H_1) The neurons activation f_j, F_i, g_i and G_j are Lipschitz continuous, that is, there exist positive constants α_j, β_i, γ_i and δ_j such that $|f_j(\xi) - f_j(\eta)| \le \alpha_j|\xi - \eta|$, $|F_i(\xi) - F_i(\eta)| \le \beta_i|\xi - \eta|$, $|g_i(\xi) - g_i(\eta)| \le \gamma_i|\xi - \eta|$, $|G_j(\xi) - G_j(\eta)| \le \delta_j|\xi - \eta|$ for any $\xi, \eta \in \mathbb{R}$, $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$.

(H_2) The delay kernels $k_{ij}, \kappa_{ji} : [0, +\infty)_{\mathbb{T}} \to [0, +\infty)$ ($i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$) are real-valued non-negative rd-continuous functions and satisfy the following conditions:

$$\int_0^\infty k_{ij}(s)\Delta s = 1, \qquad \int_0^\infty s k_{ij}(s)\Delta s < \infty,$$
$$\int_0^\infty \kappa_{ji}(s)\Delta s = 1, \qquad \int_0^\infty s\kappa_{ji}(s)\Delta s < \infty,$$

and there exist constants $\omega_1 > 0$, $\omega_2 > 0$ such that

$$\int_0^\infty k_{ij}(s) e_{\omega_1}(s, 0)\Delta s < \infty, \qquad \int_0^\infty \kappa_{ji}(s) e_{\omega_2}(s, 0)\Delta s < \infty.$$

(H_3) The following conditions are always satisfied:

$$\begin{aligned}
&-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|+\sum_{\nu=1}^{n}\beta_\nu\bigl(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\bigr)\\
&\quad+\sum_{\nu=1}^{n}\beta_i\bigl(|\overline{p}_{\nu i}|+|\overline{q}_{\nu i}|\bigr)e_{1\oplus 1}(\tau,0)+\sum_{\nu=1}^{n}\beta_i\bigl(|\overline{r}_{\nu i}|+|\overline{w}_{\nu i}|\bigr)\int_0^{+\infty}k_{\nu i}(s)e_{1\oplus 1}(s,0)\Delta s\\
&\quad+\sum_{j=1}^{m}\gamma_i|\overline{\zeta}_{ji}|e_{1\oplus 1}(\tau,0)<0,\quad i=1,2,\ldots,n;\\
&-\sum_{k=1}^{l}\frac{2\underline{\xi}_{jk}}{l_k^2}+2(m_{n+j}-\underline{\eta}_j)+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|+\sum_{\varrho=1}^{m}\delta_\varrho\bigl(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|+|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\bigr)\\
&\quad+\sum_{\varrho=1}^{m}\delta_j\bigl(|\overline{\lambda}_{\varrho j}|+|\overline{\pi}_{\varrho j}|\bigr)e_{1\oplus 1}(\tau,0)+\sum_{\varrho=1}^{m}\delta_j\bigl(|\overline{\rho}_{\varrho j}|+|\overline{\sigma}_{\varrho j}|\bigr)\int_0^{+\infty}\kappa_{\varrho j}(s)e_{1\oplus 1}(s,0)\Delta s\\
&\quad+\sum_{i=1}^{n}\alpha_j|\overline{c}_{ij}|e_{1\oplus 1}(\tau,0)<0,\quad j=1,2,\ldots,m.
\end{aligned}$$

Theorem 3.1 *Assume that* (H_1)-(H_3) *hold. Then the controlled slave system* (2.1)-(2.3) *is globally robustly exponentially synchronous with the master system* (1.1)-(1.3).

Proof Calculating the delta derivation of $\|E_i(t,\cdot)\|_2^2$ ($i=1,2,\ldots,n$) and $\|E_{n+j}(t,\cdot)\|_2^2$ ($j=1,2,\ldots,m$) along the solution of (2.1), we can obtain

$$\begin{aligned}
&\bigl(\|E_i(t,\cdot)\|_2^2\bigr)^\Delta\\
&\quad=\int_\Omega\bigl(\bigl(E_i(t,x)\bigr)^2\bigr)^\Delta\,\mathrm{d}x\\
&\quad=\int_\Omega\bigl(E_i(t,x)+E_i\bigl(\sigma(t),x\bigr)\bigr)\bigl(E_i(t,x)\bigr)^\Delta\,\mathrm{d}x\\
&\quad=\int_\Omega\bigl(2E_i(t,x)+\mu(t)\bigl(E_i(t,x)\bigr)^\Delta\bigr)\bigl(E_i(t,x)\bigr)^\Delta\,\mathrm{d}x\\
&\quad=2\int_\Omega E_i(t,x)\bigl(E_i(t,x)\bigr)^\Delta\,\mathrm{d}x+\mu(t)\int_\Omega\bigl(\bigl(E_i(t,x)\bigr)^\Delta\bigr)^2\,\mathrm{d}x\\
&\quad=2\sum_{k=1}^{l}\int_\Omega E_i(t,x)\frac{\partial}{\partial x_k}\left(a_{ik}\frac{\partial E_i}{\partial x_k}\right)\mathrm{d}x+2\int_\Omega(m_i-b_i)\bigl(E_i(t,x)\bigr)^2\,\mathrm{d}x\\
&\qquad+2\int_\Omega E_i(t,x)\sum_{j=1}^{m}c_{ij}\bigl[f_j\bigl(\tilde{v}_j(t-\tau,x)\bigr)-f_j\bigl(v_j(t-\tau,x)\bigr)\bigr]\,\mathrm{d}x\\
&\qquad+2\int_\Omega E_i(t,x)\bigwedge_{j=1}^{n}p_{ij}\bigl[F_j\bigl(\tilde{u}_j(t-\tau,x)\bigr)-F_j\bigl(u_j(t-\tau,x)\bigr)\bigr]\,\mathrm{d}x\\
&\qquad+2\int_\Omega E_i(t,x)\bigvee_{j=1}^{n}q_{ij}\bigl[F_j\bigl(\tilde{u}_j(t-\tau,x)\bigr)-F_j\bigl(u_j(t-\tau,x)\bigr)\bigr]\,\mathrm{d}x\\
&\qquad+2\int_\Omega E_i(t,x)\left[\bigwedge_{j=1}^{n}r_{ij}\int_0^{+\infty}k_{ij}(s)\bigl[F_j\bigl(\tilde{u}_j(t-s,x)\bigr)-F_j\bigl(u_j(t-s,x)\bigr)\bigr]\Delta s\right]\mathrm{d}x
\end{aligned}$$

$$+ 2\int_\Omega E_i(t,x)\left[\bigvee_{j=1}^{n} w_{ij}\int_0^{+\infty} k_{ij}(s)\left[F_j\left(\tilde{u}_j(t-s,x)\right) - F_j\left(u_j(t-s,x)\right)\right]\Delta s\right]\mathrm{d}x$$
$$+ \mu(t)\left\|\left(E_i(t,\cdot)\right)^\Delta\right\|_2^2 \tag{3.1}$$

and

$$\begin{aligned}
&\left(\left\|E_{n+j}(t,\cdot)\right\|_2^2\right)^\Delta \\
&\quad= \int_\Omega \left(\left(E_{n+j}(t,x)\right)^2\right)^\Delta \mathrm{d}x \\
&\quad= \int_\Omega \left(E_{n+j}(t,x) + E_{n+j}\left(\sigma(t),x\right)\right)\left(E_{n+j}(t,x)\right)^\Delta \mathrm{d}x \\
&\quad= \int_\Omega \left(2E_{n+j}(t,x) + \mu(t)\left(E_{n+j}(t,x)\right)^\Delta\right)\left(E_{n+j}(t,x)\right)^\Delta \mathrm{d}x \\
&\quad= 2\int_\Omega E_{n+j}(t,x)\left(E_i(t,x)\right)^\Delta \mathrm{d}x + \mu(t)\int_\Omega \left(\left(E_{n+j}(t,x)\right)^\Delta\right)^2 \mathrm{d}x \\
&\quad= 2\sum_{k=1}^{l}\int_\Omega E_{n+j}(t,x)\frac{\partial}{\partial x_k}\left(\xi_{jk}\frac{\partial E_{n+j}}{\partial x_k}\right)\mathrm{d}x + 2\int_\Omega (m_{n+j}-\eta_j)\left(E_{n+j}(t,x)\right)^2 \mathrm{d}x \\
&\qquad+ 2\int_\Omega E_{n+j}(t,x)\sum_{i=1}^{n}\zeta_{ji}\left[g_i\left(\tilde{u}_i(t-\tau,x)\right) - g_i\left(u_i(t-\tau,x)\right)\right]\mathrm{d}x \\
&\qquad+ 2\int_\Omega E_{n+j}(t,x)\bigwedge_{i=1}^{m}\lambda_{ji}\left[G_i\left(\tilde{v}_i(t-\tau,x)\right) - G_i\left(v_i(t-\tau,x)\right)\right]\mathrm{d}x \\
&\qquad+ 2\int_\Omega E_{n+j}(t,x)\bigvee_{i=1}^{m}\pi_{ji}\left[G_i\left(\tilde{v}_i(t-\tau,x)\right) - G_i\left(v_i(t-\tau,x)\right)\right]\mathrm{d}x \\
&\qquad+ 2\int_\Omega E_{n+j}(t,x)\left[\bigwedge_{i=1}^{m}\rho_{ji}\int_0^{+\infty}\kappa_{ji}(s)\left[G_i\left(\tilde{v}_i(t-s,x)\right) - G_i\left(v_i(t-s,x)\right)\right]\Delta s\right]\mathrm{d}x \\
&\qquad+ 2\int_\Omega E_{n+j}(t,x)\left[\bigvee_{i=1}^{m}\sigma_{ji}\int_0^{+\infty}\kappa_{ji}(s)\left[G_i\left(\tilde{v}_i(t-s,x)\right) - G_i\left(v_i(t-s,x)\right)\right]\Delta s\right]\mathrm{d}x \\
&\qquad+ \mu(t)\left\|\left(E_{n+j}(t,\cdot)\right)^\Delta\right\|_2^2.
\end{aligned} \tag{3.2}$$

Employing Green's formula [17], Dirichlet boundary condition (2.6) and Lemma 3.1, we have

$$\begin{aligned}
&\sum_{k=1}^{l}\int_\Omega E_i(t,x)\frac{\partial}{\partial x_k}\left(a_{ik}\frac{\partial E_i}{\partial x_k}\right)\mathrm{d}x \\
&\quad= \sum_{k=1}^{l}\int_{\partial\Omega} a_{ik}E_i(t,x)\frac{\partial E_i(t,x)}{\partial n_k}\mathrm{d}S - \sum_{k=1}^{l}\int_\Omega a_{ik}\left(\frac{\partial E_i(t,x)}{\partial x_k}\right)^2 \mathrm{d}x \\
&\quad= -\sum_{k=1}^{l}\int_\Omega a_{ik}\left(\frac{\partial E_i(t,x)}{\partial x_k}\right)^2 \mathrm{d}x \\
&\quad\le -\sum_{k=1}^{l}\int_\Omega \frac{a_{ik}}{l_k^2}\left(E_i(t,x)\right)^2 \mathrm{d}x
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
&\sum_{k=1}^{l}\int_{\Omega}E_{n+j}(t,x)\frac{\partial}{\partial x_k}\left(\xi_{jk}\frac{\partial E_{n+j}}{\partial x_k}\right)\mathrm{d}x \\
&\quad=\sum_{k=1}^{l}\int_{\partial\Omega}\xi_{jk}E_{n+j}(t,x)\frac{\partial E_{n+j}(t,x)}{\partial n_k}\,\mathrm{d}S-\sum_{k=1}^{l}\int_{\Omega}\xi_{jk}\left(\frac{\partial E_{n+j}(t,x)}{\partial x_k}\right)^2\mathrm{d}x \\
&\quad=-\sum_{k=1}^{l}\int_{\Omega}\xi_{jk}\left(\frac{\partial E_{n+j}(t,x)}{\partial x_k}\right)^2\mathrm{d}x \\
&\quad\le-\sum_{k=1}^{l}\int_{\Omega}\frac{\xi_{jk}}{l_k^2}\big(E_{n+j}(t,x)\big)^2\,\mathrm{d}x.
\end{aligned}\tag{3.4}$$

By applying Lemma 3.2, (3.1)-(3.4), conditions (H_1)-(H_3) and the Hölder inequality, and noting the robustness of parameter intervals, we get

$$\begin{aligned}
\big(\|E_i(t,\cdot)\|_2^2\big)^{\Delta}&\le-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}\|E_i(t,\cdot)\|_2^2+2(m_i-\underline{b}_i)\|E_i(t,\cdot)\|_2^2 \\
&\quad+2\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\|E_{n+j}(t-\tau,\cdot)\|_2\|E_i(t,\cdot)\|_2 \\
&\quad+2\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)\|E_\nu(t-\tau,\cdot)\|_2\|E_i(t,\cdot)\|_2 \\
&\quad+2\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\int_0^{+\infty}k_{i\nu}(s)\|E_\nu(t-s,\cdot)\|_2\|E_i(t,\cdot)\|_2\Delta s \\
&\quad+\mu(t)\big\|\big(E_i(t,\cdot)\big)^{\Delta}\big\|_2^2 \\
&\le-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}\|E_i(t,\cdot)\|_2^2+2(m_i-\underline{b}_i)\|E_i(t,\cdot)\|_2^2 \\
&\quad+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\big[\|E_{n+j}(t-\tau,\cdot)\|_2^2+\|E_i(t,\cdot)\|_2^2\big] \\
&\quad+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)\big[\|E_\nu(t-\tau,\cdot)\|_2^2+\|E_i(t,\cdot)\|_2^2\big] \\
&\quad+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\left[\int_0^{+\infty}k_{i\nu}(s)\|E_\nu(t-s,\cdot)\|_2^2\Delta s+\|E_i(t,\cdot)\|_2^2\right] \\
&\quad+\mu(t)Q(t)\big\|\big(E_i(t,\cdot)\big)\big\|_2^2 \\
&=\Bigg[-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}| \\
&\quad+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)+\mu(t)Q(t)\Bigg]\|E_i(t,\cdot)\|_2^2 \\
&\quad+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\times\|E_{n+j}(t-\tau,\cdot)\|_2^2
\end{aligned}$$

$$\begin{aligned}&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)\times\big\|E_\nu(t-\tau,\cdot)\big\|_2^2\\&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\times\int_0^{+\infty}k_{i\nu}(s)\big\|E_\nu(t-s,\cdot)\big\|_2^2\Delta s,\end{aligned}\tag{3.5}$$

where $\|(E_i(t,\cdot))^\Delta\|_2^2=Q(t)\|E_i(t,\cdot)\|_2^2$, $Q(t)\geq 0$, $i=1,2,\ldots,n$.

Similar to the arguments of (3.5), we obtain

$$\begin{aligned}\big(\big\|E_{n+j}(t,\cdot)\big\|_2^2\big)^\Delta\leq{}&\Bigg[-\sum_{k=1}^{l}\frac{2\underline{\xi}_{jk}}{l_k^2}+2(m_{n+j}-\underline{\eta}_j)+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|\\&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|+|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)+\mu(t)R(t)\Bigg]\big\|E_{n+j}(t,\cdot)\big\|_2^2\\&+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|\times\big\|E_i(t-\tau,\cdot)\big\|_2^2\\&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|\big)\times\big\|E_{n+\varrho}(t-\tau,\cdot)\big\|_2^2\\&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)\times\int_0^{+\infty}\kappa_{j\varrho}(s)\big\|E_{n+\varrho}(t-s,\cdot)\big\|_2^2\Delta s,\end{aligned}\tag{3.6}$$

where $\|(E_{n+j}(t,\cdot))^\Delta\|_2^2=R(t)\|E_{n+j}(t,\cdot)\|_2^2$, $R(t)\geq 0$, $j=1,2,\ldots,m$.

If the first inequality of condition (H_3) holds, there exists one positive number $\varsigma>0$ (may be sufficiently small) such that

$$\begin{aligned}&-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\\&\quad+\sum_{\nu=1}^{n}\beta_i\big(|\overline{p}_{\nu i}|+|\overline{q}_{\nu i}|\big)e_{1\oplus1}(\tau,0)+\sum_{\nu=1}^{n}\beta_i\big(|\overline{r}_{\nu i}|+|\overline{w}_{\nu i}|\big)\int_0^{+\infty}k_{\nu i}(s)e_{1\oplus1}(s,0)\Delta s\\&\quad+\sum_{j=1}^{m}\gamma_i|\overline{\zeta}_{ji}|e_{1\oplus1}(\tau,0)+\varsigma<0,\quad i=1,2,\ldots,n.\end{aligned}\tag{3.7}$$

Now we consider the functions

$$\begin{aligned}h_i(z_i)={}&z_i\oplus z_i-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\\&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)+\sum_{\nu=1}^{n}\beta_i\big(|\overline{p}_{\nu i}|+|\overline{q}_{\nu i}|\big)e_{1\oplus1}(\tau,0)\\&+\sum_{\nu=1}^{n}\beta_i\big(|\overline{r}_{\nu i}|+|\overline{w}_{\nu i}|\big)\int_0^{+\infty}k_{\nu i}(s)e_{1\oplus1}(s,0)\Delta s+\sum_{j=1}^{m}\gamma_i|\overline{\zeta}_{ji}|e_{1\oplus1}(\tau,0)\\&+\frac{\max\{e_{z_i\oplus z_i}(\sigma(t),0),e_{(\theta(z_i)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(z_i)\mu(t)Q(t)}{e_{z_i\oplus z_i}(\sigma(t),0)},\end{aligned}\tag{3.8}$$

where $\theta(z_i) = \int_0^{z_i} (e^{z_i - s}/(z_i - s)^2)\,\mathrm{d}s$, $i = 1, 2, \ldots, n$. From (3.7) we achieve $h_i(0) < -\varsigma < 0$ and $h_i(z_i)$ is continuous for $z_i \in [0, +\infty)$. Moreover, $h_i(z_i) \to +\infty$ as $z_i \to +\infty$, thereby there exist constants $\overline{\epsilon}_i \in (0, +\infty)$ such that $h_i(\overline{\epsilon}_i^*) = 0$ and $h_i(\overline{\epsilon}_i) < 0$ for $\overline{\epsilon}_i \in (0, \overline{\epsilon}_i^*) \cap (0, 1)$. Choosing $\overline{\epsilon} = \min_{1 \le i \le n} \overline{\epsilon}_i$, obviously $1 > \overline{\epsilon} > 0$, we have, for $i = 1, 2, \ldots, n$,

$$\begin{aligned} h_i(\overline{\epsilon}) = {} & \overline{\epsilon} \oplus \overline{\epsilon} - \sum_{k=1}^{l} \frac{2\underline{a}_{ik}}{l_k^2} + 2(m_i - \underline{b}_i) + \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| \\ & + \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}| + |\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) + \sum_{\nu=1}^{n} \beta_i \bigl(|\overline{p}_{\nu i}| + |\overline{q}_{\nu i}|\bigr) e_{1\oplus 1}(\tau, 0) \\ & + \sum_{\nu=1}^{n} \beta_i \bigl(|\overline{r}_{\nu i}| + |\overline{w}_{\nu i}|\bigr) \int_0^{+\infty} k_{\nu i}(s) e_{1\oplus 1}(s, 0) \Delta s + \sum_{j=1}^{m} \gamma_i |\overline{\zeta}_{ji}| e_{1\oplus 1}(\tau, 0) \\ & + \frac{\max\{e_{\overline{\epsilon}\oplus\overline{\epsilon}}(\sigma(t), 0), e_{(\theta(\overline{\epsilon})-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t, 0)\} \theta(\overline{\epsilon}) \mu(t) Q(t)}{e_{\overline{\epsilon}\oplus\overline{\epsilon}}(\sigma(t), 0)} \le 0. \end{aligned} \tag{3.9}$$

Similar to the above arguments of (3.7)-(3.9), we can always choose $0 < \overline{\overline{\epsilon}} < 1$ such that for $j = 1, 2, \ldots, m$,

$$\begin{aligned} & \overline{\overline{\epsilon}} \oplus \overline{\overline{\epsilon}} - \sum_{k=1}^{l} \frac{2\underline{\xi}_{jk}}{l_k^2} + 2(m_{n+j} - \underline{\eta}_j) + \sum_{i=1}^{n} \gamma_i |\overline{\zeta}_{ji}| \\ & + \sum_{\varrho=1}^{m} \delta_\varrho \bigl(|\overline{\lambda}_{j\varrho}| + |\overline{\pi}_{j\varrho}| + |\overline{\rho}_{j\varrho}| + |\overline{\sigma}_{j\varrho}|\bigr) + \sum_{\varrho=1}^{m} \delta_j \bigl(|\overline{\lambda}_{\varrho j}| + |\overline{\pi}_{\varrho j}|\bigr) e_{1\oplus 1}(\tau, 0) \\ & + \sum_{\varrho=1}^{m} \delta_j \bigl(|\overline{\rho}_{\varrho j}| + |\overline{\sigma}_{\varrho j}|\bigr) \int_0^{+\infty} \kappa_{\varrho j}(s) e_{1\oplus 1}(s, 0) \Delta s + \sum_{i=1}^{n} \alpha_j |\overline{c}_{ij}| e_{1\oplus 1}(\tau, 0) \\ & + \frac{\max\{e_{\overline{\overline{\epsilon}}\oplus\overline{\overline{\epsilon}}}(\sigma(t), 0), e_{(\theta(\overline{\overline{\epsilon}})-1)\mu(t)R(t)\|E_i(t,\cdot)\|_2^2}(t, 0)\} \theta(\overline{\overline{\epsilon}}) \mu(t) R(t)}{e_{\overline{\overline{\epsilon}}\oplus\overline{\overline{\epsilon}}}(\sigma(t), 0)} \le 0. \end{aligned} \tag{3.10}$$

Thus, taking $\epsilon = \min\{\overline{\epsilon}, \overline{\overline{\epsilon}}\}$, we derive, for $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$,

$$\begin{aligned} & \epsilon \oplus \epsilon - \sum_{k=1}^{l} \frac{2\underline{a}_{ik}}{l_k^2} + 2(m_i - \underline{b}_i) + \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| \\ & + \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}| + |\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) + \sum_{\nu=1}^{n} \beta_i \bigl(|\overline{p}_{\nu i}| + |\overline{q}_{\nu i}|\bigr) e_{1\oplus 1}(\tau, 0) \\ & + \sum_{\nu=1}^{n} \beta_i \bigl(|\overline{r}_{\nu i}| + |\overline{w}_{\nu i}|\bigr) \int_0^{+\infty} k_{\nu i}(s) e_{1\oplus 1}(s, 0) \Delta s + \sum_{j=1}^{m} \gamma_i |\overline{\zeta}_{ji}| e_{1\oplus 1}(\tau, 0) \\ & + \frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t), 0), e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t, 0)\} \theta(\epsilon) \mu(t) Q(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t), 0)} \le 0 \end{aligned} \tag{3.11}$$

and

$$\begin{aligned} & \epsilon \oplus \epsilon - \sum_{k=1}^{l} \frac{2\underline{\xi}_{jk}}{l_k^2} + 2(m_{n+j} - \underline{\eta}_j) + \sum_{i=1}^{n} \gamma_i |\overline{\zeta}_{ji}| \\ & + \sum_{\varrho=1}^{m} \delta_\varrho \bigl(|\overline{\lambda}_{j\varrho}| + |\overline{\pi}_{j\varrho}| + |\overline{\rho}_{j\varrho}| + |\overline{\sigma}_{j\varrho}|\bigr) + \sum_{\varrho=1}^{m} \delta_j \bigl(|\overline{\lambda}_{\varrho j}| + |\overline{\pi}_{\varrho j}|\bigr) e_{1\oplus 1}(\tau, 0) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{\varrho=1}^{m}\delta_j\big(|\overline{\rho}_{\varrho j}|+|\overline{\sigma}_{\varrho j}|\big)\int_0^{+\infty}\kappa_{\varrho j}(s)e_{1\oplus 1}(s,0)\Delta s+\sum_{i=1}^{n}\alpha_j|\overline{c}_{ij}|e_{1\oplus 1}(\tau,0)\\
&+\frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t),0),e_{(\theta(\epsilon)-1)\mu(t)R(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(\epsilon)\mu(t)R(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\le 0. \qquad (3.12)
\end{aligned}
$$

Take the Lyapunov functional $V(t)$ as follows:

$$
V(t)=V\big(t,E(t)\big)=V_1(t)+V_2(t), \qquad (3.13)
$$

where

$$
\begin{aligned}
V_1(t)=\sum_{i=1}^{n}\Bigg\{&e_{\epsilon\oplus\epsilon}(t,0)\big\|E_i(t,\cdot)\big\|_2^2+e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\\
&+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\int_{t-\tau}^{t}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_{n+j}(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)\int_{t-\tau}^{t}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_\nu(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\int_0^{+\infty}k_{i\nu}(s)\bigg[\int_{t-s}^{t}e_{\epsilon\oplus\epsilon}\big(\sigma(s+r),0\big)\big\|E_\nu(r,\cdot)\big\|_2^2\Delta r\bigg]\Delta s\Bigg\},\\
V_2(t)=\sum_{j=1}^{m}\Bigg\{&e_{\epsilon\oplus\epsilon}(t,0)\big\|E_{n+j}(t,\cdot)\big\|_2^2+e_{(\theta(\epsilon)-1)\mu(t)R(t)\|E_{n+j}(t,\cdot)\|_2^2}(t,0)\\
&+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|\int_{t-\tau}^{t}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_i(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|\big)\int_{t-\tau}^{t}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_{n+\varrho}(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)\int_0^{+\infty}\kappa_{j\varrho}(s)\bigg[\int_{t-s}^{t}e_{\epsilon\oplus\epsilon}\big(\sigma(s+r),0\big)\big\|E_{n+\varrho}(r,\cdot)\big\|_2^2\Delta r\bigg]\Delta s\Bigg\}.
\end{aligned}
$$

Calculating $D^+V_1^\Delta(t)$ along (2.1) associated with (3.5) and noting that $(\mathrm{d}/\mathrm{d}z)[e_z(t,s)]=[\int_s^t 1/(1+\mu(\tau)z)\Delta\tau]e_z(t,s)>0$ if and only if $z\in\mathcal{R}^+$ (that is, $e_z(t,s)$ is increasing with respect to z if and only if $z\in\mathcal{R}^+$), we have

$$
\begin{aligned}
D^+V_1^\Delta(t)=\sum_{i=1}^{n}\Bigg\{&(\epsilon\oplus\epsilon)e_{\epsilon\oplus\epsilon}(t,0)\big\|E_i(t,\cdot)\big\|_2^2+e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\big(\big\|E_i(t,\cdot)\big\|_2^2\big)^\Delta\\
&+\big(\theta(\epsilon)-1\big)\mu(t)Q(t)\big\|E_i(t,\cdot)\big\|_2^2e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\\
&+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|e_{\epsilon\oplus\epsilon}\big(\sigma(t+\tau),0\big)\big\|E_{n+j}(t,\cdot)\big\|_2^2\\
&-\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\big\|E_{n+j}(t-\tau,\cdot)\big\|_2^2
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}|\bigr) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t+\tau),0\bigr) \bigl\|E_\nu(t,\cdot)\bigr\|_2^2 \\
&- \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}|\bigr) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \bigl\|E_\nu(t-\tau,\cdot)\bigr\|_2^2 \\
&+ \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \int_0^{+\infty} k_{i\nu}(s) e_{\epsilon\oplus\epsilon}\bigl(\sigma(s+t),0\bigr) \bigl\|E_\nu(t,\cdot)\bigr\|_2^2 \Delta s \\
&- \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \int_0^{+\infty} k_{i\nu}(s) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \bigl\|E_\nu(t-s,\cdot)\bigr\|_2^2 \Delta s \Biggr\} \\
\le{}& \sum_{i=1}^{n} \Biggl\{ (\epsilon \oplus \epsilon) e_{\epsilon\oplus\epsilon}(t,0) \bigl\|E_i(t,\cdot)\bigr\|_2^2 + e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \Biggl[\Biggl(-\sum_{k=1}^{l} \frac{2\underline{a}_{ik}}{l_k^2} + 2(m_i - \underline{b}_i) \\
&+ \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| + \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}| + |\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) + \mu(t)Q(t) \Biggr) \bigl\|E_i(t,\cdot)\bigr\|_2^2 \\
&+ \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| \times \bigl\|E_{n+j}(t-\tau,\cdot)\bigr\|_2^2 + \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}|\bigr) \times \bigl\|E_\nu(t-\tau,\cdot)\bigr\|_2^2 \\
&+ \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \times \int_0^{+\infty} k_{i\nu}(s) \bigl\|E_\nu(t-s,\cdot)\bigr\|_2^2 \Delta s, \Biggr] \\
&+ \bigl(\theta(\epsilon)-1\bigr)\mu(t)Q(t) \bigl\|E_i(t,\cdot)\bigr\|_2^2 e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0) \\
&+ \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| e_{\epsilon\oplus\epsilon}\bigl(\sigma(t+\tau),0\bigr) \bigl\|E_{n+j}(t,\cdot)\bigr\|_2^2 \\
&- \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \bigl\|E_{n+j}(t-\tau,\cdot)\bigr\|_2^2 \\
&+ \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}|\bigr) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t+\tau),0\bigr) \bigl\|E_\nu(t,\cdot)\bigr\|_2^2 \\
&- \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}|\bigr) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \bigl\|E_\nu(t-\tau,\cdot)\bigr\|_2^2 \\
&+ \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \int_0^{+\infty} k_{i\nu}(s) e_{\epsilon\oplus\epsilon}\bigl(\sigma(s+t),0\bigr) \bigl\|E_\nu(t,\cdot)\bigr\|_2^2 \Delta s \\
&- \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \int_0^{+\infty} k_{i\nu}(s) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \bigl\|E_\nu(t-s,\cdot)\bigr\|_2^2 \Delta s \Biggr\} \\
\le{}& \sum_{i=1}^{n} \Biggl\{ (\epsilon \oplus \epsilon) e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \bigl\|E_i(t,\cdot)\bigr\|_2^2 + e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr) \Biggl[-\sum_{k=1}^{l} \frac{2\underline{a}_{ik}}{l_k^2} + 2(m_i - \underline{b}_i) \\
&+ \sum_{j=1}^{m} \alpha_j |\overline{c}_{ij}| + \sum_{\nu=1}^{n} \beta_\nu \bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}| + |\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \Biggr] \bigl\|E_i(t,\cdot)\bigr\|_2^2 \\
&+ \mu(t)Q(t) \bigl\|E_i(t,\cdot)\bigr\|_2^2 \times \max\bigl\{ e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr), e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0) \bigr\} \\
&+ \bigl(\theta(\epsilon)-1\bigr)\mu(t)Q(t) \bigl\|E_i(t,\cdot)\bigr\|_2^2
\end{aligned}
$$

$$
\begin{aligned}
&\times \max\left\{e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big), e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\right\} \\
&+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|e_{\epsilon\oplus\epsilon}(\tau,0)e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\big\|E_{n+j}(t,\cdot)\big\|_2^2 \\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)e_{\epsilon\oplus\epsilon}(\tau,0)e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\big\|E_\nu(t,\cdot)\big\|_2^2 \\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\int_0^{+\infty}k_{i\nu}(s)e_{\epsilon\oplus\epsilon}(s,0)e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\big\|E_\nu(t,\cdot)\big\|_2^2\Delta s\Bigg\} \\
\le{}& e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\sum_{i=1}^{n}\Bigg\{\big\|E_i(t,\cdot)\big\|_2^2\Bigg[\epsilon\oplus\epsilon-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i) \\
&+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big) \\
&+\frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t),0), e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(\epsilon)\mu(t)Q(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\Bigg] \\
&+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|e_{\epsilon\oplus\epsilon}(\tau,0)\big\|E_{n+j}(t,\cdot)\big\|_2^2 \\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)e_{\epsilon\oplus\epsilon}(\tau,0)\big\|E_\nu(t,\cdot)\big\|_2^2 \\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\int_0^{+\infty}k_{i\nu}(s)e_{\epsilon\oplus\epsilon}(s,0)\big\|E_\nu(t,\cdot)\big\|_2^2\Delta s\Bigg\} \\
\le{}& e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\sum_{i=1}^{n}\Bigg\{\big\|E_i(t,\cdot)\big\|_2^2\Bigg[\epsilon\oplus\epsilon-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}| \\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)+\sum_{\nu=1}^{n}\beta_i\big(|\overline{p}_{\nu i}|+|\overline{q}_{\nu i}|\big)e_{1\oplus1}(\tau,0) \\
&+\sum_{\nu=1}^{n}\beta_i\big(|\overline{r}_{\nu i}|+|\overline{w}_{\nu i}|\big)\int_0^{+\infty}k_{\nu i}(s)e_{1\oplus1}(s,0)\Delta s \\
&+\frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t),0), e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(\epsilon)\mu(t)Q(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\Bigg]\Bigg\} \\
&+e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\sum_{j=1}^{m}\sum_{i=1}^{n}\alpha_j|\overline{c}_{ij}|e_{1\oplus1}(\tau,0)\big\|E_{n+j}(t,\cdot)\big\|_2^2.
\end{aligned}
\tag{3.14}
$$

By applying (3.6), we can similarly calculate $D^+V_2^\Delta(t)$ along (2.1) as follows:

$$
\begin{aligned}
D^+V_2^\Delta(t)\le{}& e_{\epsilon\oplus\epsilon}\big(\sigma(t),0\big)\sum_{j=1}^{m}\Bigg\{\big\|E_{n+j}(t,\cdot)\big\|_2^2\Bigg[\epsilon\oplus\epsilon-\sum_{k=1}^{l}\frac{2\underline{\xi}_{jk}}{l_k^2}+2(m_{n+j}-\underline{\eta}_j)+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}| \\
&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|+|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)+\sum_{\varrho=1}^{m}\delta_j\big(|\overline{\lambda}_{\varrho j}|+|\overline{\pi}_{\varrho j}|\big)e_{1\oplus1}(\tau,0)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{\varrho=1}^{m}\delta_j\bigl(|\overline{\rho}_{\varrho j}|+|\overline{\sigma}_{\varrho j}|\bigr)\int_0^{+\infty}\kappa_{\varrho j}(s)e_{1\oplus 1}(s,0)\Delta s\\
&+\frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t),0),e_{(\theta(\epsilon)-1)\mu(t)R(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(\epsilon)\mu(t)R(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\Biggr]\Biggr\}\\
&+e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr)\sum_{i=1}^{n}\sum_{j=1}^{m}\gamma_i|\overline{\zeta}_{ji}|e_{1\oplus 1}(\tau,0)\bigl\|E_i(t,\cdot)\bigr\|_2^2. \qquad (3.15)
\end{aligned}
$$

From (3.11)-(3.15), we get

$$
\begin{aligned}
D^+V(t) &= D^+V\bigl(t,E(t)\bigr)\\
&= D^+V_1(t)+D^+V_2(t)\\
&\le e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr)\sum_{i=1}^{n}\Biggl\{\bigl\|E_i(t,\cdot)\bigr\|_2^2\Biggl[\epsilon\oplus\epsilon-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\\
&\quad+\sum_{\nu=1}^{n}\beta_\nu\bigl(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\bigr)+\sum_{\nu=1}^{n}\beta_i\bigl(|\overline{p}_{\nu i}|+|\overline{q}_{\nu i}|\bigr)e_{1\oplus 1}(\tau,0)\\
&\quad+\sum_{\nu=1}^{n}\beta_i\bigl(|\overline{r}_{\nu i}|+|\overline{w}_{\nu i}|\bigr)\int_0^{+\infty}k_{\nu i}(s)e_{1\oplus 1}(s,0)\Delta s+\sum_{j=1}^{m}\gamma_i|\overline{\zeta}_{ji}|e_{1\oplus 1}(\tau,0)\\
&\quad+\frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t),0),e_{(\theta(\epsilon)-1)\mu(t)Q(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(\epsilon)\mu(t)Q(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\Biggr]\Biggr\}\\
&\quad+e_{\epsilon\oplus\epsilon}\bigl(\sigma(t),0\bigr)\sum_{j=1}^{m}\Biggl\{\bigl\|E_{n+j}(t,\cdot)\bigr\|_2^2\Biggl[\epsilon\oplus\epsilon-\sum_{k=1}^{l}\frac{2\underline{\xi}_{jk}}{l_k^2}+2(m_{n+j}-\underline{\eta}_j)\\
&\quad+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|+\sum_{\varrho=1}^{m}\delta_\varrho\bigl(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|+|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\bigr)\\
&\quad+\sum_{\varrho=1}^{m}\delta_j\bigl(|\overline{\lambda}_{\varrho j}|+|\overline{\pi}_{\varrho j}|\bigr)e_{1\oplus 1}(\tau,0)\\
&\quad+\sum_{\varrho=1}^{m}\delta_j\bigl(|\overline{\rho}_{\varrho j}|+|\overline{\sigma}_{\varrho j}|\bigr)\int_0^{+\infty}\kappa_{\varrho j}(s)e_{1\oplus 1}(s,0)\Delta s+\sum_{i=1}^{n}\alpha_j|\overline{c}_{ij}|e_{1\oplus 1}(\tau,0)\\
&\quad+\frac{\max\{e_{\epsilon\oplus\epsilon}(\sigma(t),0),e_{(\theta(\epsilon)-1)\mu(t)R(t)\|E_i(t,\cdot)\|_2^2}(t,0)\}\theta(\epsilon)\mu(t)R(t)}{e_{\epsilon\oplus\epsilon}(\sigma(t),0)}\Biggr]\Biggr\}\le 0. \qquad (3.16)
\end{aligned}
$$

Note that (3.16) means that the Lyapunov functional $V(t,E(t))$ is monotone decreasing with respect to $t\in[0,+\infty)_{\mathbb{T}}$. Therefore, in the light of (3.13) we get, for $t\in[0,+\infty)_{\mathbb{T}}$,

$$
\begin{aligned}
&e_{\epsilon\oplus\epsilon}(t,0)\bigl\|E(t,\cdot)\bigr\|^2\\
&\quad=e_{\epsilon\oplus\epsilon}(t,0)\sum_{i=1}^{n}\bigl\|E_i(t,\cdot)\bigr\|_2^2+e_{\epsilon\oplus\epsilon}(t,0)\sum_{j=1}^{m}\bigl\|E_{n+j}(t,\cdot)\bigr\|_2^2\le V\bigl(t,e(t)\bigr)\le V\bigl(0,e(0)\bigr)\\
&\quad=\sum_{i=1}^{n}\Biggl\{\bigl\|E_i(0,\cdot)\bigr\|_2^2+1+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\bigl(\sigma(s+\tau),0\bigr)\bigl\|E_{n+j}(s,\cdot)\bigr\|_2^2\Delta s
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_\nu(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\int_{0}^{+\infty}k_{i\nu}(s)\biggl[\int_{-s}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+r),0\big)\big\|E_\nu(r,\cdot)\big\|_2^2\Delta r\biggr]\Delta s\Biggr\}\\
&+\sum_{j=1}^{m}\Biggl\{\big\|E_{n+j}(0,\cdot)\big\|_2^2+1+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_i(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|\big)\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\big\|E_{n+\varrho}(s,\cdot)\big\|_2^2\Delta s\\
&+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)\int_{0}^{+\infty}\kappa_{j\varrho}(s)\biggl[\int_{-s}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+r),0\big)\big\|E_{n+\varrho}(r,\cdot)\big\|_2^2\Delta r\biggr]\Delta s\Biggr\}\\
\le{}&\sum_{i=1}^{n}\|\tilde{\phi}_i-\phi_i\|_1^2+n+\sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|\|\tilde{\varphi}_{n+j}-\varphi_{n+j}\|_1^2\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\Delta s\\
&+\sum_{i=1}^{n}\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|\big)\|\tilde{\phi}_\nu-\phi_\nu\|_1^2\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\Delta s\\
&+\sum_{i=1}^{n}\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\|\tilde{\phi}_\nu-\phi_\nu\|_1^2\int_{0}^{+\infty}k_{i\nu}(s)\biggl[\int_{-s}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+r),0\big)\Delta r\biggr]\Delta s\\
&+\sum_{j=1}^{m}\|\tilde{\varphi}_j-\varphi_j\|_1^2+m+\sum_{j=1}^{m}\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|\|\tilde{\phi}_i-\phi_i\|_1^2\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\Delta s\\
&+\sum_{j=1}^{m}\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|\big)\|\tilde{\varphi}_\varrho-\varphi_\varrho\|_1^2\int_{-\tau}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+\tau),0\big)\Delta s\\
&+\sum_{j=1}^{m}\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)\|\tilde{\varphi}_\varrho-\varphi_\varrho\|_1^2\int_{0}^{+\infty}\kappa_{j\varrho}(s)\biggl[\int_{-s}^{0}e_{\epsilon\oplus\epsilon}\big(\sigma(s+r),0\big)\Delta r\biggr]\Delta s\\
\triangleq{}&M^2,
\end{aligned}
$$

which implies that

$$\big\|E(t,\cdot)\big\|\le Me_{\ominus\epsilon}(t,0). \tag{3.17}$$

Obviously, $M>1$. According to Definition 2.5, we conclude that the controlled slave system (2.1)-(2.3) is globally robustly exponentially synchronous with the master system (1.1)-(1.3) on the time scale $[0,+\infty)_{\mathbb{T}}$. The proof is complete. □

When the time scale $\mathbb{T}=\mathbb{R}$ and $\mathbb{T}=\mathbb{Z}$, we will obtain the following two important corollaries.

Corollary 3.1 *Assume that the following* (H_4)-(H_6) *hold. Then the master system* (1.4)-(1.6) *and its controlled slave system are globally robustly exponentially synchronous.*

(H_4) The neurons activation f_j, F_i, g_i and G_j are Lipschitz continuous, that is, there exist positive constants α_j, β_i, γ_i and δ_j such that $|f_j(\xi)-f_j(\eta)|\le\alpha_j|\xi-\eta|$, $|F_i(\xi)-F_i(\eta)|\le\beta_i|\xi-$

$\eta|$, $|g_i(\xi) - g_i(\eta)| \le \gamma_i|\xi - \eta|$, $|G_j(\xi) - G_j(\eta)| \le \delta_j|\xi - \eta|$ for any $\xi, \eta \in \mathbb{R}$, $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$.

(H_5) The delay kernels $k_{ij}, \kappa_{ji} : [0, +\infty) \to [0, +\infty)$ ($i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$) are real-valued non-negative continuous functions and satisfy the following conditions:

$$\int_0^\infty k_{ij}(s)\,\mathrm{d}s = 1, \qquad \int_0^\infty sk_{ij}(s)\,\mathrm{d}s < \infty, \qquad \int_0^\infty \kappa_{ji}(s)\,\mathrm{d}s = 1, \qquad \int_0^\infty s\kappa_{ji}(s)\,\mathrm{d}s < \infty$$

and there exist constants $\omega_1 > 0$, $\omega_2 > 0$ such that

$$\int_0^\infty k_{ij}(s)e^{s\omega_1}\,\mathrm{d}s < \infty, \qquad \int_0^\infty \kappa_{ji}(s)e^{s\omega_2}\,\mathrm{d}s < \infty.$$

(H_6) The following conditions are always satisfied:

$$\begin{aligned}
&-\sum_{k=1}^{l} \frac{2\underline{a}_{ik}}{l_k^2} + 2(m_i - \underline{b}_i) + \sum_{j=1}^{m} \alpha_j|\overline{c}_{ij}| + \sum_{\nu=1}^{n} \beta_\nu\bigl(|\overline{p}_{i\nu}| + |\overline{q}_{i\nu}| + |\overline{r}_{i\nu}| + |\overline{w}_{i\nu}|\bigr) \\
&\quad + \sum_{\nu=1}^{n} \beta_i\bigl(|\overline{p}_{\nu i}| + |\overline{q}_{\nu i}|\bigr)e^{2\tau} + \sum_{\nu=1}^{n} \beta_i\bigl(|\overline{r}_{\nu i}| + |\overline{w}_{\nu i}|\bigr)\int_0^{+\infty} k_{\nu i}(s)e^{2s}\,\mathrm{d}s \\
&\quad + \sum_{j=1}^{m} \gamma_i|\overline{\zeta}_{ji}|e^{2\tau} < 0, \quad i = 1, 2, \dots, n; \\
&-\sum_{k=1}^{l} \frac{2\underline{\xi}_{jk}}{l_k^2} + 2(m_{n+j} - \underline{\eta}_j) + \sum_{i=1}^{n} \gamma_i|\overline{\zeta}_{ji}| + \sum_{\varrho=1}^{m} \delta_\varrho\bigl(|\overline{\lambda}_{j\varrho}| + |\overline{\pi}_{j\varrho}| + |\overline{\rho}_{j\varrho}| + |\overline{\sigma}_{j\varrho}|\bigr) \\
&\quad + \sum_{\varrho=1}^{m} \delta_j\bigl(|\overline{\lambda}_{\varrho j}| + |\overline{\pi}_{\varrho j}|\bigr)e^{2\tau} + \sum_{\varrho=1}^{m} \delta_j\bigl(|\overline{\rho}_{\varrho j}| + |\overline{\sigma}_{\varrho j}|\bigr)\int_0^{+\infty} \kappa_{\varrho j}(s)e^{2s}\,\mathrm{d}s \\
&\quad + \sum_{i=1}^{n} \alpha_j|\overline{c}_{ij}|e^{2\tau} < 0, \quad j = 1, 2, \dots, m.
\end{aligned}$$

Corollary 3.2 *Assume that the following* (H_7)-(H_9) *hold. Then the master system* (1.7)-(1.9) *and its controlled slave system are globally robustly exponentially synchronous.*

(H_7) The neurons activation f_j, F_i, g_i and G_j are Lipschitz continuous, that is, there exist positive constants α_j, β_i, γ_i and δ_j such that $|f_j(\xi) - f_j(\eta)| \le \alpha_j|\xi - \eta|$, $|F_i(\xi) - F_i(\eta)| \le \beta_i|\xi - \eta|$, $|g_i(\xi) - g_i(\eta)| \le \gamma_i|\xi - \eta|$, $|G_j(\xi) - G_j(\eta)| \le \delta_j|\xi - \eta|$ for any $\xi, \eta \in \mathbb{R}$, $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$.

(H_8) The delay kernels $k_{ij}, \kappa_{ji} : \mathbb{Z}^+ \to [0, +\infty)$ ($i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$) are real-valued non-negative rd-continuous functions and satisfy the following conditions:

$$\sum_{s=0}^{\infty} k_{ij}(s) = 1, \qquad \sum_{s=0}^{\infty} sk_{ij}(s) < \infty, \qquad \sum_{s=0}^{\infty} \kappa_{ji}(s) = 1, \qquad \sum_{s=0}^{\infty} s\kappa_{ji}(s) < \infty,$$

and there exist constants $\omega_1 > 0$, $\omega_2 > 0$ such that

$$\sum_{s=0}^{\infty} k_{ij}(s)(1 + \omega_1)^s < \infty, \qquad \sum_{s=0}^{\infty} \kappa_{ji}(s)(1 + \omega_2)^s < \infty.$$

(H_9) The following conditions are always satisfied:

$$\begin{aligned}
&-\sum_{k=1}^{l}\frac{2\underline{a}_{ik}}{l_k^2}+2(m_i-\underline{b}_i)+\sum_{j=1}^{m}\alpha_j|\overline{c}_{ij}|+\sum_{\nu=1}^{n}\beta_\nu\big(|\overline{p}_{i\nu}|+|\overline{q}_{i\nu}|+|\overline{r}_{i\nu}|+|\overline{w}_{i\nu}|\big)\\
&\quad+\sum_{\nu=1}^{n}\beta_i\big(|\overline{p}_{\nu i}|+|\overline{q}_{\nu i}|\big)4^{\tau}+\sum_{\nu=1}^{n}\beta_i\big(|\overline{r}_{\nu i}|+|\overline{w}_{\nu i}|\big)\sum_{s=0}^{+\infty}k_{\nu i}(s)4^{s}\\
&\quad+\sum_{j=1}^{m}\gamma_i|\overline{\zeta}_{ji}|4^{\tau}<0,\quad i=1,2,\ldots,n;\\
&-\sum_{k=1}^{l}\frac{2\underline{\xi}_{jk}}{l_k^2}+2(m_{n+j}-\underline{\eta}_j)+\sum_{i=1}^{n}\gamma_i|\overline{\zeta}_{ji}|+\sum_{\varrho=1}^{m}\delta_\varrho\big(|\overline{\lambda}_{j\varrho}|+|\overline{\pi}_{j\varrho}|+|\overline{\rho}_{j\varrho}|+|\overline{\sigma}_{j\varrho}|\big)\\
&\quad+\sum_{\varrho=1}^{m}\delta_j\big(|\overline{\lambda}_{\varrho j}|+|\overline{\pi}_{\varrho j}|\big)4^{\tau}+\sum_{\varrho=1}^{m}\delta_j\big(|\overline{\rho}_{\varrho j}|+|\overline{\sigma}_{\varrho j}|\big)\sum_{s=0}^{+\infty}\kappa_{\varrho j}(s)4^{s}\\
&\quad+\sum_{i=1}^{n}\alpha_j|\overline{c}_{ij}|4^{\tau}<0,\quad j=1,2,\ldots,m.
\end{aligned}$$

4 Illustrative example

Consider the following reaction-diffusion BAM recurrent FNNs on time scales:

$$\begin{cases}
u_i^{\Delta}(t,x)=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}(a_{ik}\frac{\partial u_i}{\partial x_k})-b_iu_i(t,x)+\sum_{j=1}^{m}c_{ij}f_j(v_j(t-\tau,x))+I_i\\
\qquad+\bigwedge_{j=1}^{n}p_{ij}F_j(u_j(t-\tau,x))+\bigwedge_{j=1}^{n}r_{ij}\int_0^{+\infty}k_{ij}(s)F_j(u_j(t-s,x))\Delta s\\
\qquad+\bigvee_{j=1}^{n}q_{ij}F_j(u_j(t-\tau,x))+\bigvee_{j=1}^{n}w_{ij}\int_0^{+\infty}k_{ij}(s)F_j(u_j(t-s,x))\Delta s\\
\qquad+\sum_{j=1}^{n}d_{ij}\mu_j+\bigwedge_{j=1}^{n}S_{ij}\mu_j+\bigvee_{j=1}^{n}T_{ij}\mu_j,\\
v_j^{\Delta}(t,x)=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}(\xi_{jk}\frac{\partial v_j}{\partial x_k})-\eta_jv_j(t,x)+\sum_{i=1}^{n}\zeta_{ji}g_i(u_i(t-\tau,x))+J_j\\
\qquad+\bigwedge_{i=1}^{m}\lambda_{ji}G_i(v_i(t-\tau,x))+\bigwedge_{i=1}^{m}\rho_{ji}\int_0^{+\infty}\kappa_{ji}(s)G_i(v_i(t-s,x))\Delta s\\
\qquad+\bigvee_{i=1}^{m}\pi_{ji}G_i(v_i(t-\tau,x))+\bigvee_{i=1}^{m}\sigma_{ji}\int_0^{+\infty}\kappa_{ji}(s)G_i(v_i(t-s,x))\Delta s\\
\qquad+\sum_{i=1}^{m}h_{ji}\nu_i+\bigwedge_{i=1}^{m}M_{ji}\nu_i+\bigvee_{i=1}^{m}N_{ji}\mu_i,
\end{cases}\tag{4.1}$$

subject to the following initial conditions

$$\begin{cases}
u_i(s,x)=\phi_i(s,x),\quad (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega,\\
v_j(s,x)=\varphi_j(s,x),\quad (s,x)\in[-\tau,0]_{\mathbb{T}}\times\Omega,
\end{cases}\tag{4.2}$$

and Dirichlet boundary conditions

$$\begin{cases}
u_i(t,x)=0,\quad (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega,\\
v_j(t,x)=0,\quad (t,x)\in[0,\infty)_{\mathbb{T}}\times\partial\Omega,
\end{cases}\tag{4.3}$$

where $n=m=l=2$, $f_j(v)=F_i(v)=g_i(v)=G_j(v)=\frac{e^{v}-e^{-v}}{e^{v}+e^{-v}}$ $(i,j=1,2)$, $k_{ij}(t)=\kappa_{ji}(t)=\frac{26}{27}(\frac{1}{3})^{t}$ $(i,j=1,2)$, $\mathbb{T}=\{3n:n=0,\pm1,\pm2,\ldots\}$, $\Omega=\{x:|x_i|<1,i=1,2\}$, $\tau=1$. $I=(I_1,I_2)$ and $J=(J_1,J_2)$ are the constant input vectors. $\mu=(\mu_1,\mu_2)$ and $\nu=(\nu_1,\nu_2)$ are the constant bias vectors. Obviously, $f_j(v)$, $F_i(v)$, $g_i(v)$ and $G_i(v)$ satisfy the Lipschitz condition with $\alpha_j=\beta_i=$

$\gamma_i = \delta_j = 1$. Let $(\underline{b}_1, \underline{b}_2) = (9.5, 10.5)$, $(\underline{\eta}_1, \underline{\eta}_2) = (8.5, 9)$,

$$\begin{pmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}, \qquad \begin{pmatrix} \overline{c}_{11} & \overline{c}_{12} \\ \overline{c}_{21} & \overline{c}_{22} \end{pmatrix} = \begin{pmatrix} 0.4 & 0.5 \\ 0.6 & 0.1 \end{pmatrix},$$
$$\begin{pmatrix} \overline{p}_{11} & \overline{p}_{12} \\ \overline{p}_{21} & \overline{p}_{22} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.5 \end{pmatrix}, \qquad \begin{pmatrix} \overline{q}_{11} & \overline{q}_{12} \\ \overline{q}_{21} & \overline{q}_{22} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.1 \\ 0.7 & 0.8 \end{pmatrix},$$
$$\begin{pmatrix} \overline{r}_{11} & \overline{r}_{12} \\ \overline{r}_{21} & \overline{r}_{22} \end{pmatrix} = \begin{pmatrix} 0.4 & 0.3 \\ 0.6 & 0.9 \end{pmatrix}, \qquad \begin{pmatrix} \overline{w}_{11} & \overline{w}_{12} \\ \overline{w}_{21} & \overline{w}_{22} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.1 \\ 0.8 & 0.3 \end{pmatrix},$$
$$\begin{pmatrix} \underline{\xi}_{11} & \underline{\xi}_{12} \\ \underline{\xi}_{21} & \underline{\xi}_{22} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.7 \end{pmatrix}, \qquad \begin{pmatrix} \overline{\zeta}_{11} & \overline{\zeta}_{12} \\ \overline{\zeta}_{21} & \overline{\zeta}_{22} \end{pmatrix} = \begin{pmatrix} 0.1 & 0.5 \\ 0.1 & 0.1 \end{pmatrix},$$
$$\begin{pmatrix} \overline{\lambda}_{11} & \overline{\lambda}_{12} \\ \overline{\lambda}_{21} & \overline{\lambda}_{22} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{pmatrix}, \qquad \begin{pmatrix} \overline{\pi}_{11} & \overline{\pi}_{12} \\ \overline{\pi}_{21} & \overline{\pi}_{22} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.2 \\ 0.6 & 0.4 \end{pmatrix},$$
$$\begin{pmatrix} \overline{\rho}_{11} & \overline{\rho}_{12} \\ \overline{\rho}_{21} & \overline{\rho}_{22} \end{pmatrix} = \begin{pmatrix} 0.5 & 0.2 \\ 0.7 & 0.8 \end{pmatrix}, \qquad \begin{pmatrix} \overline{\sigma}_{11} & \overline{\sigma}_{12} \\ \overline{\sigma}_{21} & \overline{\sigma}_{22} \end{pmatrix} = \begin{pmatrix} 0.3 & 0.1 \\ 0.9 & 0.2 \end{pmatrix}.$$

Take the controlled input vector $z(t,x) = (m_1E_1(t,x), m_2E_2(t,x), m_3E_3(t,x), m_4E_4(t,x))^T$, here $(m_1, m_2, m_3, m_4) = (5, 3, 2, 3)$. By a simple calculation, we have

$$\sigma(t) = t + 3, \qquad \mu(t) = 3, \qquad e_{1\oplus 1}(t,0) = \bigl(e_1(t,0)\bigr)^2 = 4^t,$$
$$\int_0^{+\infty} k_{ij}(s)\Delta s = \int_0^{+\infty} \kappa_{ji}(s)\Delta s = \sum_{s=0}^{+\infty} \frac{26}{27}\left(\frac{1}{3}\right)^{3s} = 1,$$
$$\int_0^{+\infty} sk_{ij}(s)\Delta s = \int_0^{+\infty} s\kappa_{ji}(s)\Delta s = \sum_{s=0}^{+\infty} \frac{78}{27}s\left(\frac{1}{3}\right)^{3s} = \frac{3}{26} < +\infty,$$
$$\begin{aligned} \int_0^{+\infty} se_{\omega}(s,0)k_{ij}(s)\Delta s = \int_0^{+\infty} se_{\omega}(s,0)\kappa_{ji}(s)\Delta s &= \sum_{s=0}^{+\infty} \frac{78}{27}s(1+3\omega)^s\left(\frac{1}{3}\right)^{3s} \\ &= \frac{27(1+3\omega)}{(26-3\omega)^2} < +\infty \quad (0 < \omega < 13), \end{aligned}$$
$$\begin{aligned} &-\sum_{k=1}^{2} \frac{2\underline{a}_{1k}}{l_k^2} + 2(m_1 - \underline{b}_1) + \sum_{j=1}^{2} \alpha_j|\overline{c}_{1j}| + \sum_{\nu=1}^{2} \beta_\nu\bigl(|\overline{p}_{1\nu}| + |\overline{q}_{1\nu}| + |\overline{r}_{1\nu}| + |\overline{w}_{1\nu}|\bigr) \\ &\quad + \sum_{\nu=1}^{2} \beta_i\bigl(|\overline{p}_{\nu 1}| + |\overline{q}_{\nu 1}|\bigr)4^\tau + \sum_{\nu=1}^{2} \beta_i\bigl(|\overline{r}_{\nu 1}| + |\overline{w}_{\nu 1}|\bigr)\sum_{s=0}^{+\infty} k_{\nu 1}(3s)4^s \\ &\quad + \sum_{j=1}^{2} \gamma_1|\overline{\zeta}_{j1}|4^\tau \approx -0.24 < 0, \end{aligned}$$
$$\begin{aligned} &-\sum_{k=1}^{2} \frac{2\underline{a}_{2k}}{l_k^2} + 2(m_2 - \underline{b}_2) + \sum_{j=1}^{2} \alpha_j|\overline{c}_{2j}| + \sum_{\nu=1}^{2} \beta_\nu\bigl(|\overline{p}_{2\nu}| + |\overline{q}_{2\nu}| + |\overline{r}_{2\nu}| + |\overline{w}_{2\nu}|\bigr) \\ &\quad + \sum_{\nu=1}^{2} \beta_i\bigl(|\overline{p}_{\nu 2}| + |\overline{q}_{\nu 2}|\bigr)4^\tau + \sum_{\nu=1}^{2} \beta_2\bigl(|\overline{r}_{\nu 2}| + |\overline{w}_{\nu 2}|\bigr)\sum_{s=0}^{+\infty} k_{\nu 2}(3s)4^s \end{aligned}$$

$$
\begin{aligned}
&\quad + \sum_{j=1}^{2} \gamma_2 |\overline{\zeta}_{j2}| 4^{\tau} \approx -0.78 < 0, \\
&-\sum_{k=1}^{2} \frac{2\underline{\xi}_{1k}}{l_k^2} + 2(m_3 - \underline{\eta}_1) + \sum_{i=1}^{2} \gamma_i |\overline{\zeta}_{1i}| + \sum_{\varrho=1}^{2} \delta_{\varrho} \bigl(|\overline{\lambda}_{1\varrho}| + |\overline{\pi}_{1\varrho}| + |\overline{\rho}_{1\varrho}| + |\overline{\sigma}_{1\varrho}|\bigr) \\
&\quad + \sum_{\varrho=1}^{2} \delta_1 \bigl(|\overline{\lambda}_{\varrho 1}| + |\overline{\pi}_{\varrho 1}|\bigr) 4^{\tau} + \sum_{\varrho=1}^{2} \delta_1 \bigl(|\overline{\rho}_{\varrho 1}| + |\overline{\sigma}_{\varrho 1}|\bigr) \sum_{s=0}^{+\infty} \kappa_{\varrho 1}(3s) 4^{s} \\
&\quad + \sum_{i=1}^{2} \alpha_1 |\overline{c}_{i1}| 4^{\tau} \approx -0.19 < 0, \\
&-\sum_{k=1}^{2} \frac{2\underline{\xi}_{2k}}{l_k^2} + 2(m_4 - \underline{\eta}_2) + \sum_{i=1}^{2} \gamma_i |\overline{\zeta}_{2i}| + \sum_{\varrho=1}^{2} \delta_{\varrho} \bigl(|\overline{\lambda}_{2\varrho}| + |\overline{\pi}_{2\varrho}| + |\overline{\rho}_{2\varrho}| + |\overline{\sigma}_{2\varrho}|\bigr) \\
&\quad + \sum_{\varrho=1}^{2} \delta_2 \bigl(|\overline{\lambda}_{\varrho 2}| + |\overline{\pi}_{\varrho 2}|\bigr) 4^{\tau} + \sum_{\varrho=1}^{2} \delta_2 \bigl(|\overline{\rho}_{\varrho 2}| + |\overline{\sigma}_{\varrho 2}|\bigr) \sum_{s=0}^{+\infty} \kappa_{\varrho 2}(3s) 4^{s} \\
&\quad + \sum_{i=1}^{2} \alpha_2 |\overline{c}_{i2}| 4^{\tau} \approx -1.03 < 0.
\end{aligned}
$$

Thus, conditions (H_1)-(H_3) are satisfied. It follows from Theorem 3.1 that the master system (4.1)-(4.3) and its controlled slave system are globally robustly exponentially synchronized.

Competing interests
The author declares to have no competing interests.

Author's contributions
The author read and approved the final manuscript.

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