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The solutions of one type q-difference functional system

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Abstract

In this paper, we study the functional system on q-difference equations, our results can give estimates on the proximity functions and the counting functions of the solutions of q-difference equations system. This implies that solutions have a relatively large number of poles. The main results in this paper concern q-difference equations to the system of q-difference equations.

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1 Introduction and main results

A function $f(z)$ is called meromorphic if it is analytic in the complex plane $\mathbb{C}$ except at isolate poles. In what follows, we assume that the reader is familiar with the basic notion of Nevanlinna's value distribution theory, see [1] and [2].

Let us consider the q-difference polynomial case. Let $d_j \in \mathbb{C}$ for $j = 1, \dots, n$, and let I_q be a finite set of multi-indexes $\gamma = (\gamma_0, \dots, \gamma_n)$. A q-difference polynomial of a meromorphic function $w(z)$ is defined as follows:

$$\begin{aligned} P(z,w) &= P\bigl(z, w(qz), w\bigl(q^2 z\bigr), \dots, w\bigl(q^n z\bigr)\bigr) \\ &= \sum_{\gamma \in I_q} a_\gamma(z) w(z)^{\gamma_0} w(qz)^{\gamma_1} \cdots w\bigl(q^n z\bigr)^{\gamma_n}, \end{aligned} \tag{1.1}$$

where $q \in \mathbb{C}\{0\}$, and the coefficients $a_\gamma(z)$ are small meromorphic functions with respect to $w(z)$ such that $T(r, a_\gamma) = o(T(r,w))$ on a logarithmic density 1, denoted by $S_q(r,w)$. The total degree of $P(z,w)$ in $w(z)$ and the q-shifts of $w(z)$ is denoted by $\deg_w^q(P)$, and the order of zero of $P(z, x_0, x_1, \dots, x_n)$, as a function of x_0 at $x_0 = 0$, is denoted as $\operatorname{ord}_0^q(P)$, which can be found, *e.g.*, in [3]. Moreover, the weight of difference polynomial (1.1) is defined by

$$K_q(P) = \max_{\gamma \in I_q} \left\{ \sum_{j=1}^{n} \gamma_j \right\},$$

where γ and I_q are the same as in (1.1) above. The q-difference polynomial $P(z,w)$ is said to be homogeneous with respect to $w(z)$ if the degree $d_\gamma = \gamma_0 + \cdots + \gamma_n$ of each term in the sum (1.1) is non-zero and the same for all $\gamma \in I_q$.

We recall the following result of Zhang *et al.* [4, Theorem 1].

Theorem A *Let $w(z)$ be a zero-order meromorphic solution of*

$$H(z,w)P(z,w) = Q(z,w),$$

where $P(z,w)$ is a homogeneous q-difference polynomial with polynomial coefficients, and $H(z,w)$ and $Q(z,w)$ are polynomials in $w(z)$ with polynomial coefficients having no common factors. If

$$\max\left\{\deg_w^q(H), \deg_w^q(Q) - \deg_w^q(P)\right\} > \min\left\{\deg_w^q(P), \operatorname{ord}_0^q(Q)\right\} - \operatorname{ord}_0^q(P),$$

then $N(r,w) \neq S_q(r,w)$, where $\operatorname{ord}_0^q(P)$ denotes the order of zero of $P(z,x_0,x_1,\dots,x_n)$, as a function of x_0 at $x_0 = 0$.

Now let us introduce some notation. Let $q_j \in \mathbb{C} \setminus \{0,\}$ for $j = 1,\dots,n$, and let I and J be a finite set of multi-indexes $I = (i_0,\dots,i_n)$ and $J = (j_0,\dots,j_n)$. Two q-difference polynomials of a meromorphic function $w(z)$ are defined as follows:

$$\begin{aligned}\Omega_1(z,w_1,w_2) &= \Omega_1\bigl(z,w_1(z),w_2(z),w_1(q_1z),w_2(q_1z),\dots,w_1(q_nz),w_2(q_nz)\bigr)\\ &= \sum_{i\in I} a_i(z)\prod_{k=1}^{2} w_k(z)^{k_{i_0}} w_k(q_1z)^{k_{i_1}}\cdots w_k(q_nz)^{k_{i_n}}\end{aligned}$$

and

$$\begin{aligned}\Omega_2(z,w_1,w_2) &= \Omega_2\bigl(z,w_1(z),w_2(z),w_1(q_1z),w_2(q_1z),\dots,w_1(q_nz),w_2(q_nz)\bigr)\\ &= \sum_{j\in J} b_j(z)\prod_{k=1}^{2} w_k(z)^{k_{i_0}} w_k(q_1z)^{k_{i_1}}\cdots w_k(q_nz)^{k_{i_n}},\end{aligned}$$

where the coefficients $a_i(z)$ and $b_j(z)$ are small with respect to $w_1(z)$ and $w_2(z)$ in the sense that $T(r,a_i) = o(T(r,w_k))$ and $T(r,b_j) = o(T(r,w_k))$, $k = 1,2$, on a set of logarithmic density 1, as r tends to infinity outside of an exceptional set E of finite logarithmic measure

$$\lim_{r\to\infty}\int_{E\cap[1,r)}\frac{dt}{t} < \infty.$$

The weights of $\Omega_1(z,w_1,w_2)$ and $\Omega_2(z,w_1,w_2)$ in $w_1(z)$, $w_2(z)$ are denoted by

$$\lambda_{11} = \max_i\left\{\sum_{l=0}^{n} i_{1l}\right\},\qquad \lambda_{12} = \max_i\left\{\sum_{l=0}^{n} i_{2l}\right\}$$

and

$$\lambda_{21} = \max_j\left\{\sum_{l=0}^{n} i_{1l}\right\},\qquad \lambda_{22} = \max_j\left\{\sum_{l=0}^{n} i_{2l}\right\}.$$

The purpose of this paper is to study the problem of the properties of Nevanlinna counting functions and proximity functions of meromorphic solutions of a type of systems of

q-difference equations of the following form:

$$\begin{cases} \Omega_1(z, w_1, w_2) = R_1(z, w_1), \\ \Omega_2(z, w_1, w_2) = R_2(z, w_2), \end{cases} \tag{1.2}$$

where

$$R_1(z, w_1) = \frac{P_1(z, w_1)}{Q_1(z, w_1)} = \frac{\sum_{i=0}^{p_1} a_i(z) w_1^i}{\sum_{j=0}^{q_1} b_j(z) w_1^j}$$

and

$$R_2(z, w_2) = \frac{P_2(z, w_2)}{Q_2(z, w_2)} = \frac{\sum_{i=0}^{p_2} c_i(z) w_2^i}{\sum_{j=0}^{q_2} d_j(z) w_2^j},$$

the coefficients $\{a_i(z)\}$, $\{b_i(z)\}$, $\{c_i(z)\}$, $\{d_i(z)\}$ are meromorphic functions and small functions. The order of zero of $\Omega_j(z, x_0, \ldots, x_n)$, as a function of x_0 at $x_0 = 0$, is denoted by $\operatorname{ord}_0(\Omega_j)$. The q-difference polynomial $\Omega_k(z, w_1, w_2)$, $k = 1, 2$, is said to be homogeneous with respect to $w_k(z)$ if the degree $d_k = i_{k0} + \cdots + i_{kn}$ of each term in the sum is non-zero and the same for all $i \in I$. Finally, the order of growth of a meromorphic solution (w_1, w_2) is defined by

$$\rho(w_1, w_2) = \max\{\rho(w_1), \rho_2(w_2)\},$$

where

$$\rho(w_k) = \limsup_{r \to \infty} \frac{\log T(r, w_k)}{\log r}, \quad k = 1, 2.$$

In this paper, the main results are as follows.

Theorem 1 *Let* (w_1, w_2) *be a zero-order meromorphic solution of system* (1.2), *where* $\Omega_k(z, w_1, w_2)$ $(k = 1, 2)$ *are homogeneous q-difference polynomials in* w_1 *and* w_2, *respectively, with meromorphic coefficients, and* $P_k(z, w_k)$ *and* $Q(z, w_k)$, $k = 1, 2$, *are polynomials in* $w_k(z)$ *with meromorphic coefficients having no common factors. If*

$$\max\{q_1, p_1 - \lambda_{11}\} > \min\{\lambda_{11}, \operatorname{ord}_{w_1}(P_1)\} - \operatorname{ord}_{w_1}(\Omega_1) + \lambda_{12} \tag{1.3}$$

and

$$\max\{q_2, p_2 - \lambda_{22}\} > \min\{\lambda_{22}, \operatorname{ord}_{w_2}(P_2)\} - \operatorname{ord}_{w_2}(\Omega_2) + \lambda_{21}, \tag{1.4}$$

then $N(r, w_1) = S_q(r, w_1)$ *and* $N(r, w_2) = S_q(r, w_2)$ *cannot hold both at the same time, possibly outside of an exceptional set of finite logarithmic measure.*

Theorem 2 *Let* (w_1, w_2) *be a zero-order meromorphic solution of system* (1.2), *where* $\Omega_k(z, w_1, w_2)$ $(k = 1, 2)$ *are homogeneous q-difference polynomials in* w_1 *and* w_2, *respectively,* *with meromorphic coefficients, and $P_k(z, w_k)$ and $Q(z, w_k)$, $k = 1, 2$, are polynomials in $w_k(z)$ with meromorphic coefficients having no common factors,*

$$A = 2\lambda_{11} - \bigl(\max\{p_1, q_1 + \lambda_{11}\} - \min\bigl\{\lambda_{11}, \operatorname{ord}_{w_1}(\Omega_1)\bigr\}\bigr)$$

and

$$B = 2\lambda_{22} - \bigl(\max\{p_2, q_2 + \lambda_{22}\} - \min\bigl\{\lambda_{22}, \operatorname{ord}_{w_2}(\Omega_2)\bigr\}\bigr).$$

If $A < 0$, $B < 0$ and $AB > 9\lambda_{21}\lambda_{12}$, then $m(r, w_k) = S_q(r, w_k)$ $(k = 1, 2)$, where r runs to infinity outside of an exceptional set of finite logarithmic measure.

2 Some lemmas

Lemma 1 ([5], Theorem 1.2) *Let $f(z)$ be a non-constant zero-order meromorphic function, and $q \in \mathbb{C} \setminus \{0\}$. Then*

$$m\left(r, \frac{f(qz)}{f(z)}\right) = S_q(r, f).$$

Lemma 2 ([6], Lemma 4) *If $T : \mathbb{R}^+ \to \mathbb{R}^+$ is a piecewise continuous increasing function such that*

$$\lim_{r\to\infty} \frac{\log T(r)}{\log r} = 0,$$

then the set

$$E := \bigl\{r : T(C_1 r) \ge C_2 T(r)\bigr\}$$

has logarithmic density 0 for all $C_1 > 1$ and $C_2 > 1$.

3 Proof of Theorem 1

Since $\Omega_k(z, w_1, w_2)$ are homogeneous in w_1 and w_2, respectively, it follows by Lemma 1 that

$$m\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) \le \lambda_{12} m(r, w_2) + S_q(r, w_1) \tag{3.1}$$

and

$$m\left(r, \frac{\Omega_2(z, w_1, w_2)}{w_2^{\lambda_{22}}}\right) \le \lambda_{21} m(r, w_1) + S_q(r, w_2) \tag{3.2}$$

for all r outside of an exceptional set of finite logarithmic measure. Moreover, from (1.2), we have

$$\begin{aligned} T\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) &= T\left(r, \frac{P_1(z, w_1)}{Q_1(z, w_1) w_1^{\lambda_{11}}}\right) \\ &= \bigl(\max\{p_1, q_1 + \lambda_{11}\} - \min\bigl\{\lambda_{11}, \operatorname{ord}_{w_1}(P_1)\bigr\}\bigr) T(r, w_1) \\ &\quad + S_q(r, w_1) \end{aligned} \tag{3.3}$$

and

$$T\left(r, \frac{\Omega_2(z, w_1, w_2)}{w_2^{\lambda_{22}}}\right) = T\left(r, \frac{P_2(z, w_2)}{Q_2(z, w_2)w_2^{\lambda_{22}}}\right)$$
$$= \left(\max\{p_2, q_2 + \lambda_{22}\} - \min\left\{\lambda_{22}, \operatorname{ord}_{w_2}(P_2)\right\}\right) T(r, w_2)$$
$$+ S_q(r, w_2), \tag{3.4}$$

where r approaches infinity outside of an exceptional set of finite logarithmic measure. By combining (3.1) and (3.3), (3.2) and (3.4), respectively, it follows that

$$N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) \geq \left(1 + \lambda_{12} + \lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)\right) T(r, w_1)$$
$$- \lambda_{12} m(r, w_2) + S_q(r, w_1) \tag{3.5}$$

and

$$N\left(r, \frac{\Omega_2(z, w_1, w_2)}{w_2^{\lambda_{22}}}\right) \geq \left(1 + \lambda_{21} + \lambda_{22} - \operatorname{ord}_{w_2}(\Omega_2)\right) T(r, w_1)$$
$$- \lambda_{21} m(r, w_1) + S_q(r, w_2). \tag{3.6}$$

From Lemma 2, we have

$$N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\operatorname{ord}_{w_1}(\Omega_1(z, w_1, w_2))}}\right)$$
$$\leq \left(\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)\right) N(qr, w_1) + \lambda_{12} N(qr, w_2) + S_q(r, w_1)$$
$$= \left(\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)\right) N(r, w_1) + \lambda_{12} N(r, w_2) + S_q(r, w_1) + S_q(r, w_2)$$

and

$$N\left(r, \frac{\Omega_2(z, w_1, w_2)}{w_1^{\operatorname{ord}_{w_2}(\Omega_2(z, w_1, w_2))}}\right)$$
$$\leq \left(\lambda_{22} - \operatorname{ord}_{w_2}(\Omega_2)\right) N(qr, w_2) + \lambda_{21} N(qr, w_1) + S_q(r, w_2)$$
$$= \left(\lambda_{22} - \operatorname{ord}_{w_2}(\Omega_2)\right) N(r, w_2) + \lambda_{11} N(r, w_1) + S_q(r, w_1) + S_q(r, w_2).$$

Therefore,

$$N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) \leq N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\operatorname{ord}_{w_1}(\Omega_1(z, w_1, w_2))}}\right) + N\left(r, \frac{1}{w_1^{\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)}}\right)$$
$$\leq \left(\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)\right) N(r, w_1) + \lambda_{12} N(r, w_2)$$
$$+ T\left(r, \frac{1}{w_1^{\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)}}\right) + S_q(r, w_1) + S_q(r, w_2)$$
$$\leq \left(\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)\right) N(r, w_1) + \lambda_{12} N(r, w_2)$$
$$+ \left(\lambda_{11} - \operatorname{ord}_{w_1}(\Omega_1)\right) T(r, w_1) + S_q(r, w_2) + S_q(r, w_2) \tag{3.7}$$

and

$$\begin{aligned} N\left(r, \frac{\Omega_2(z,w_1,w_2)}{w_2^{\lambda_{22}}}\right) &\le N\left(r, \frac{\Omega_2(z,w_1,w_2)}{w_2^{\mathrm{ord}_{w_2}(\Omega_2(z,w_1,w_2))}}\right) + N\left(r, \frac{1}{w_2^{\lambda_{22}-\mathrm{ord}_{w_2}(\Omega_2)}}\right) \\ &\le \left(\lambda_{22} - \mathrm{ord}_{w_2}(\Omega_2)\right)N(r,w_2) + \lambda_{21}N(r,w_1) \\ &\quad + T\left(r, \frac{1}{w_2^{\lambda_{22}-\mathrm{ord}_{w_2}(\Omega_2)}}\right) + S_q(r,w_1) + S_q(r,w_2) \\ &\le \left(\lambda_{22} - \mathrm{ord}_{w_2}(\Omega_2)\right)N(r,w_2) + \lambda_{21}N(r,w_1) \\ &\quad + \left(\lambda_{22} - \mathrm{ord}_{w_2}(\Omega_2)\right)T(r,w_2) + S_q(r,w_2) + S_q(r,w_2). \end{aligned} \tag{3.8}$$

Combining (3.5) and (3.7), (3.6) and (3.8), respectively, we have

$$\begin{aligned} &\left(1 + \lambda_{12} + \lambda_{11} - \mathrm{ord}_{w_1}(\Omega_1)\right)T(r,w_1) \\ &\quad < \left(\lambda_{11} - \mathrm{ord}_{w_1}(\Omega_1)\right)N(r,w_1) + \lambda_{12}T(r,w_2) \\ &\qquad + \left(\lambda_{11} - \mathrm{ord}_{w_1}(\Omega_1)\right)T(r,w_1) + S_q(r,w_1) + S_q(r,w_2) \end{aligned} \tag{3.9}$$

and

$$\begin{aligned} &\left(1 + \lambda_{21} + \lambda_{22} - \mathrm{ord}_{w_2}(\Omega_2)\right)T(r,w_2) \\ &\quad < \left(\lambda_{22} - \mathrm{ord}_{w_2}(\Omega_2)\right)N(r,w_2) + \lambda_{21}T(r,w_1) \\ &\qquad + \left(\lambda_{22} - \mathrm{ord}_{w_2}(\Omega_2)\right)T(r,w_2) + S_q(r,w_1) + S_q(r,w_2). \end{aligned} \tag{3.10}$$

Suppose that $N(r,w_1) = S_q(r,w_1)$ and $N(r,w_2) = S_q(r,w_2)$, according to (3.9) and (3.10), we have

$$(1 + \lambda_{12})T(r,w_1) < \lambda_{12}T(r,w_2) + S_q(r,w_1) + S_q(r,w_2)$$

and

$$(1 + \lambda_{21})T(r,w_2) < \lambda_{21}T(r,w_1) + S_q(r,w_1) + S_q(r,w_2).$$

That is,

$$\left(1 + \lambda_{12} + o(1)\right)T(r,w_1) < \left(\lambda_{12} + o(1)\right)T(r,w_2) \tag{3.11}$$

and

$$\left(1 + \lambda_{21} + o(1)\right)T(r,w_2) < \left(\lambda_{12} + o(1)\right)T(r,w_1). \tag{3.12}$$

By (3.11) and (3.12), we conclude that

$$1 + \lambda_{12} + 1 + \lambda_{21} + o(1) < \lambda_{12} + \lambda_{21},$$

which is impossible, we prove the assertion.

4 Proof of Theorem 2

It follows by Lemma 1 that

$$m\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) \le \lambda_{12} m(r, w_2) + S_q(r, w_1) \tag{4.1}$$

and

$$m\left(r, \frac{\Omega_2(z, w_1, w_2)}{w_2^{\lambda_{22}}}\right) \le \lambda_{21} m(r, w_1) + S_q(r, w_2) \tag{4.2}$$

for all r outside of an exceptional set of finite logarithmic measure.

Suppose now that $(w_1(z), w_2(z))$ is a finite-order meromorphic solution of (1.2). Denoting $C = \max_{j=1,\dots,n}\{|c_j|\}$ in $\Omega_1(z, w_1, w_2)$ and $\Omega_2(z, w_1, w_2)$, by Lemma 2, we obtain

$$\begin{aligned} N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) &\le \lambda_{11}\left(N\big(|q|r, w_1\big) + N\left(r, \frac{1}{w_1}\right)\right) \\ &\quad + \lambda_{12}\left(N\big(|q|r, w_2\big) + N\left(r, \frac{1}{w_2}\right)\right) \\ &\quad + \lambda_{12} N(r, w_2) + S_q(r, w_1) + S_q(r, w_2) \\ &= \lambda_{11}\left(N(r, w_1) + N\left(r, \frac{1}{w_1}\right)\right) + \lambda_{12}\left(N(r, w_2) + N\left(r, \frac{1}{w_2}\right)\right) \\ &\quad + \lambda_{12} N(r, w_2) + S_q(r, w_1) + S_q(r, w_2) \end{aligned} \tag{4.3}$$

for all r outside of a set E of finite logarithmic measure. By (4.1) and (4.3), we have

$$\begin{aligned} N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) &\le \lambda_{11}\left(N(r, w_1) + N\left(r, \frac{1}{w_1}\right)\right) \\ &\quad + \lambda_{12}\left(N(r, w_2) + N\left(r, \frac{1}{w_2}\right)\right) + S_q(r, w_1) + S_q(r, w_2) \\ &\le \lambda_{12}\big(2T(r, w_1) - m(r, w_1)\big) + \lambda_{12}\big(3T(r, w_2) - 2m(r, w_2)\big) \\ &\quad + S_q(r, w_1) + S_q(r, w_2) \end{aligned} \tag{4.4}$$

for all $r \notin E$. On the other hand, by (4.1) and (4.3),

$$\begin{aligned} &N\left(r, \frac{\Omega_1(z, w_1, w_2)}{w_1^{\lambda_{11}}}\right) + \lambda_{12} m(r, w_2) \\ &\quad \ge T\left(r, \frac{P_1(r, w_1)}{w_1^{\lambda_{11}} Q_1 r, w_1}\right) \\ &\quad = \big(\max\{p_1, q_1 + \lambda_{11}\} - \min\big\{\lambda_{11}, \operatorname{ord}_{w_1}(\Omega_1)\big\}\big) T(r, w_1) + S_q(r, w_1), \end{aligned} \tag{4.5}$$

where r lies outside of a set F of finite logarithmic measure. Combining inequalities (4.4) and (4.5) with the assumption in Theorem 2, we have

$$\begin{aligned} &\big(\max\{p_1, q_1 + \lambda_{11}\} - \min\big\{\lambda_{11}, \operatorname{ord}_{w_1}(\Omega_1)\big\}\big) T(r, w_1) \\ &\quad - \lambda_{12} m(r, w_2) + S_q(r, w_1) + S_q(r, w_2) \end{aligned}$$

$$\begin{aligned}&\le \lambda_{11}\bigl(2T(r,w_1)-m(r,w_1)\bigr)+\lambda_{12}\bigl(3T(r,w_2)-2m(r,w_2)\bigr)\\&\quad+S_q(r,w_1)+S_q(r,w_2).\end{aligned} \tag{4.6}$$

Similarly, we obtain

$$\begin{aligned}&\bigl(\max\{p_2,q_2+\lambda_{22}\}-\min\bigl\{\lambda_{22},\operatorname{ord}_{w_2}(\Omega_2)\bigr\}\bigr)T(r,w_2)\\&\quad-\lambda_{21}m(r,w_1)+S_q(r,w_1)+S_q(r,w_2)\\&\le \lambda_{22}\bigl(2T(r,w_2)-m(r,w_2)\bigr)+\lambda_{21}\bigl(3T(r,w_1)-2m(r,w_1)\bigr)\\&\quad+S_q(r,w_1)+S_q(r,w_2).\end{aligned} \tag{4.7}$$

By (4.6) and (4.7), we obtain

$$\begin{aligned}&\lambda_{11}m(r,w_1)\\&\quad\le \bigl(2\lambda_{11}-\bigl(\max\{p_1,q_1+\lambda_{11}\}-\min\bigl\{\lambda_{11},\operatorname{ord}_{w_1}(\Omega_1)\bigr\}\bigr)+o(1)\bigr)T(r,w_1)\\&\qquad+\bigl(3\lambda_{12}+o(1)\bigr)T(r,w_2)\end{aligned} \tag{4.8}$$

and

$$\begin{aligned}&\bigl(\bigl(\max\{p_2,q_2+\lambda_{22}\}-\min\bigl\{\lambda_{22},\operatorname{ord}_{w_2}(\Omega_2)\bigr\}\bigr)-2\lambda_{22}+o(1)\bigr)T(r,w_2)\\&\quad\le \bigl(3\lambda_{21}+o(1)\bigr)T(r,w_1)-2\lambda_{21}m(r,w_2).\end{aligned} \tag{4.9}$$

Combining (4.8) and (4.9), we have

$$\begin{aligned}&\lambda_{11}m(r,w_1)\\&\quad\le \bigl(2\lambda_{11}-\bigl(\max\{p_1,q_1+\lambda_{11}\}-\min\bigl\{\lambda_{11},\operatorname{ord}_{w_1}(\Omega_1)\bigr\}\bigr)+o(1)\bigr)T(r,w_1)\\&\qquad+\frac{3\lambda_{12}(3\lambda_{21}+o(1))T(r,w_1)-6\lambda_{12}\lambda_{21}m(r,w_1)}{(\max\{p_2,q_2+\lambda_{22}\}-\min\{\lambda_{22},\operatorname{ord}_{w_2}(\Omega_2)\})-2\lambda_{22}},\end{aligned}$$

that is,

$$\left(\lambda_{11}-\frac{6\lambda_{12}\lambda_{21}}{B}\right)m(r,w_1)\le\left(A-\frac{9\lambda_{12}\lambda_{21}+o(1)}{B}\right)T(r,w_1), \tag{4.10}$$

where $A=2\lambda_{11}-(\max\{p_1,q_1+\lambda_{11}\}-\min\{\lambda_{11},\operatorname{ord}_{w_1}(\Omega_1)\})$ and $B=2\lambda_{22}-(\max\{p_2,q_2+\lambda_{22}\}-\min\{\lambda_{22},\operatorname{ord}_{w_2}(\Omega_2)\})$. Combining the assumption and (4.10), we have

$$m(r,w_1)=S_q(r,w_1)$$

for all r outside of $E\cup F$, a set of finite logarithmic measure.

Similarly, we obtain

$$m(r,w_2)=S_q(r,w_2)$$

for all r outside of $E\cup F$, we have proved the assertion.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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