Ju et al. *Advances in Difference Equations* 2014, 2014:280
http://www.advancesindifferenceequations.com/content/2014/1/280

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH

Open Access

An impulsive prey-predator system with stage-structure and Holling II functional response

Zhixiang Ju[1], Yuanfu Shao[1*], Weili Kong[2], Xiangmin Ma[1] and Xianjia Fang[1]

*Correspondence:
shaoyuanfu@163.com
[1]School of Science, Guilin University of Technology, Guilin, Guangxi 541004, P.R. China
Full list of author information is available at the end of the article

Abstract
Taking into account that individual organisms usually go through immature and mature stages, in this paper, we investigate the dynamics of an impulsive prey-predator system with a Holling II functional response and stage-structure. Applying the comparison theorem and some analysis techniques, the sufficient conditions of the global attractivity of a mature predator periodic solution and the permanence are investigated. Examples and numerical simulations are shown to verify the validity of our results.

Keywords: stage-structure; impulsive; global attractivity; permanence

1 Introduction

The Food and Agriculture Organization of the United Nations reported that, with the development of modern science and technology, many methods have been used for pest control, such as chemical pesticides and biological control (*i.e.*, suppress the pests by natural enemies). Although great progress has been made in the Integrated Pest Management (IPM), people still cannot completely exterminate them all. For the IPM strategy on an ecosystem, the predators are released periodically every time T and periodic catching or spraying pesticides is also applied. Hence the predator and prey abruptly experience a change of state. In fact, many evolution processes are characterized by the fact that at certain moments their stage changes abruptly. Consequently, it is natural to assume that these processes act in the form of impulses. Impulsive methods have been applied in almost every field of the applied sciences. On the other hand, the purpose of IPM is to gain the biggest benefit with the minimum expense; see references [1–7]. For example, some authors [7] proposed an IPM predator-prey model concerning periodic biological and chemical management. It implied that the chemical pesticide is the most effective method which can eliminate a great quantity of pests in a short time. In recent work, biologists realized that appropriate human harvesting and stocking has vital significance on the permanent of biological resource. Jiang *et al.* [8] considered an impulsive prey-predator system with Holling type II functional response and state feedback control as follows:

$$\begin{cases} \left.\begin{aligned} \dot{x}(t) &= rx(t)(1-x(t)) - \tfrac{ax(t)y(t)}{1+x(t)}, \\ \dot{y}(t) &= \tfrac{ax(t)y(t)}{1+x(t)} - by(t), \end{aligned}\right\} & x \neq h, \\ \left.\begin{aligned} \Delta x(t) &= -px(t), \\ \Delta y(t) &= qy(t) + \tau, \end{aligned}\right\} & x = h, \end{cases}$$

where $x(t)$, $y(t)$ represent the densities of the prey and the predator, respectively. For the parameters $r, a, b > 0$, r is the intrinsic growth rate of the prey, $\frac{axy}{1+x}$ is the Holling II function response, b denotes the death rate of the predator, $p \in (0,1)$, $h > 0$, $q > 0$, $\tau \geq 0$. One obtained the complex dynamics of the system.

However, in the real world, the development of an individual organism usually goes through two stages on the time: immaturity and maturity. Some stage-structured models for the prey-predator system consisting of immature and mature individuals were analyzed in [9–12]. For example, a stage-structured prey-predator model with impulsive stocking on prey and continuous harvesting on predator was considered in [10]. Song and Chen [11] studied optimal harvesting and stability for a two species competitive system with stage structure. Shao and Dai [12] considered a predator-prey model with time delay and impulsive harvesting on prey and stocking on the immature predator. Actually, as the literature [13, 14] pointed out, stage-structured differential equations exhibit much more complicated behaviors than ordinary ones since time delays could cause a stable equilibrium to become unstable and cause the population to fluctuate. Therefore, it is important to consider the dynamics of a prey-predator system with stage-structure; see [15] and references cited therein.

On the other hand, with food safety gaining importance, green food is being paid more and more attention to. In order to plant green food, one can use a periodic harvesting or stocking prey or predator, instead of using high toxic or high residues pesticide.

Based on the above discussion, in this paper, we consider a stage-structured prey-predator model with Holling II functional response and impulsive catching or poisoning the immature prey and stocking of the mature predator as follows:

$$\begin{cases} \left.\begin{aligned} &\dot{x}_1(t) = rx_2(t) - re^{-d_1\tau_1}x_2(t-\tau_1) - d_1x_1(t),\\ &\dot{x}_2(t) = re^{-d_1\tau_1}x_2(t-\tau_1) - \frac{kx_2(t)}{c+x_2(t)}y_2(t) - d_2x_2(t) - d_3x_2^2(t),\\ &\dot{y}_1(t) = \frac{\lambda kx_2(t)}{c+x_2(t)}y_2(t) - \lambda ke^{-d_4\tau_2}\frac{x_2(t-\tau_2)}{c+x_2(t-\tau_2)}y_2(t-\tau_2) - d_4y_1(t),\\ &\dot{y}_2(t) = \lambda ke^{-d_4\tau_2}\frac{x_2(t-\tau_2)}{c+x_2(t-\tau_2)}y_2(t-\tau_2) - d_5y_2(t), \end{aligned}\right\} \quad t \neq nT,\\ \left.\begin{aligned} &x_1(t^+) = (1-p)x_1(t),\\ &x_2(t^+) = x_2(t),\\ &y_1(t^+) = y_1(t),\\ &y_2(t^+) = y_2(t) + \mu, \end{aligned}\right\} \quad t = nT, \end{cases} \tag{1.1}$$

with initial conditions

$$\begin{aligned} &\bigl(\varphi_1(\xi), \varphi_2(\xi), \varphi_3(\xi), \varphi_4(\xi)\bigr) \in C_+ = C\bigl([-\tau, 0], R_+^4\bigr), \qquad \varphi_i(0) > 0, \quad i = 1,2,3,4,\\ &R_+^4 = \bigl\{x \in R^4 : x \geq 0\bigr\}, \qquad \tau = \max(\tau_1, \tau_2), \end{aligned}$$

where $x_1(t)$ ($x_2(t)$), $y_1(t)$ ($y_2(t)$) denote the densities of immature (mature) prey and immature (mature) predator, respectively. The parameters r, k, λ, d_1, d_2, d_3, d_4, d_5 are all positive constants, r denotes the birth rate of the immature prey, k is the maximum number of the mature prey that can be eaten by a mature predator per unit of time, λ represents the rate of conversing prey into predator (*i.e.*, the converse rate from mature prey to immature predator), d_1 ($d_1 > d_2$), d_2 are the mortality rates of the immature and mature prey, respectively, and d_4 ($d_4 > d_5$), d_5 are the mortality rates of the immature and mature predator, respectively, d_3 is the intra-specific competition rate of the mature prey, τ_1, τ_2 represent a constant time to reach maturity of prey and predator, respectively, μ (≥ 0) denotes the stocking amount of the mature predator, p ($0 \leq p < 1$) is the catching rate of the immature prey at $t = nT$, $n \in Z_+$, and $Z_+ = \{1, 2, \ldots\}$, T is the period of the impulsive effect.

In this paper, we aim to investigate the global attractivity of a mature predator periodic solution and the permanence of system (1.1). In agreement with the biological point of view, we only consider (1.1) in the biological sense, region $D = \{(x(t), y_1(t), y_2(t)) : x(t) \geq 0, y_1(t) \geq 0, y_2(t) \geq 0\}$.

The organization of the paper is as follows. In Section 2, some preliminaries and lemmas are given. In Section 3, sufficient conditions for the global attractivity of a mature predator survival periodic solution are obtained. In Section 4, the permanence of system (1.1) is investigated. Some examples and numerical simulations are given to illustrate the main results in Section 5. Finally, in Section 6, a brief conclusion is presented.

2 Preliminaries and lemmas

In this section, some definitions and lemmas are introduced.

Let $R_+ = [0, \infty)$, $R^4_+ = \{x \in R^4, x \geq 0\}$. Denote by $f = (f_1, f_2, f_3, f_4)^T$ the map defined by the right-hand side of system (1.1). Let $V : R_+ \times R^4_+ \to R_+$, if:

(i) V is continuous in $(nT, (n+1)T] \times R^4_+$, for each $x \in R^4_+$, $n \in Z_+$,

$$\lim_{(t,y)\to((n-1)T,x)} V(t,y) = V\big((n-1)T, x\big) \quad \text{and}$$

$$\lim_{(t,y)\to(nT^+,x)} V(t,y) = V\big(nT^+, x\big) \quad \text{exist;}$$

(ii) V is locally Lipschitzian in x, then V is said to belong to class V_0.

Definition 2.1 Let $V \in V_0$, $(t,x) \in (nT, (n+1)T] \times R^4_+$, $n \in Z_+$, the upper right derivative of $V(t,x)$ with respect to impulsive differential system (1.1) is defined as

$$D^+ V(t,x) = \lim_{h\to 0^+} \sup \frac{1}{h}\big[V\big(t+h, x+hf(t,x)\big) - V(t,x)\big].$$

Next, we give some important lemmas which will be useful for our main results.

Lemma 2.1 [5] *Consider the impulsive differential system*

$$\begin{cases} \dot{m}(t) \leq p(t)m(t) + q(t), & t \neq t_k, \\ m(t^+) \leq d_k m(t) + b_k, & t = t_k, \end{cases}$$

where $p, q \in C(R_+, R)$, $k \in Z_+$, $d_k \geq 0$, and b_k are constants.

Assume that:

(i) *the sequence $\{t_k\}$ satisfies $0 \leq t_0 < t_1 < t_2 < \cdots$, with $\lim_{t_k\to+\infty} t_k = \infty$;*

(ii) *$m \in pc^1(R_+, R)$ and $m(t)$ is left-continuous at t_k, $k \in Z_+$.*

Then we have

$$m(t) \leq m(t_0) \prod_{t_0<t_k<t} d_k \exp\left(\int_{t_0}^t p(x)\,ds\right) + \sum_{t_0<t_k<t} \left(\prod_{t_k<t_j<t} d_j \exp\left(\int_{t_k}^t p(s)\,ds\right)\right) b_k$$
$$+ \int_{t_0}^t \prod_{s<t_k<t} d_k \exp\left(\int_s^t p(\theta)\,d\theta\right) q(s)\,ds, \quad t \geq t_0.$$

Lemma 2.2 [6, 7] *Consider the following equation*:

$$\dot{u}(t) = au(t-\tau) - bu(t) - cu^2(t),$$

where a, b, c, *and* τ *are positive constants*, $u(t) > 0$ *for* $t \in [-\tau, 0]$. *We have*
(i) *if* $a < b$, *then* $\lim_{t\to+\infty} u(t) = 0$;
(ii) *if* $a > b$, *then* $\lim_{t\to+\infty} u(t) = \frac{a-b}{c}$.

Lemma 2.3 [7] *Consider the following system*:

$$\begin{cases} \dot{x}(t) = c - dx(t), & t \neq nT, \\ x(t^+) = x(t) + p, & t = nT. \end{cases} \tag{2.1}$$

System (2.1) *has a positive periodic solution* $x^*(t)$ *with period* T. *For any solution* $x(t)$ *of system* (2.1), *we have*

$$\left|x(t) - x^*(t)\right| \to 0 \quad \text{as } t \to \infty,$$

where

$$x^*(t) = \frac{c}{d} + \frac{pe^{-d(t-nT)}}{1-e^{-dT}}, \qquad x^*(0^+) = \frac{c}{d} + \frac{p}{1-e^{-dT}}, \quad nT < t \le (n+1)T.$$

Lemma 2.4 *Consider the following system*:

$$\begin{cases} \dot{u}(t) = c - du(t), & t \neq nT, \\ u(t^+) = (1-p)u(t), & t = nT. \end{cases} \tag{2.2}$$

Then system (2.2) *has a positive periodic solution* $u^*(t)$ *with period* T. *For any solution* $u(t)$ *of system* (2.2), *we have*

$$\left|u(t) - u^*(t)\right| \to 0 \quad \text{as } t \to \infty,$$

where

$$u^*(t) = \frac{c}{d}\left(1 - \frac{pe^{-d(t-nT)}}{1-(1-p)e^{-dT}}\right), \quad nT < t \le (n+1)T \quad \text{and}$$
$$u^*(0^+) = \frac{c}{d}\left(1 - \frac{p}{1-(1-p)e^{-dT}}\right).$$

Proof Integrating the first equation of (2.2) on $nT < t \le (n+1)T$, we have

$$u(t) = \frac{c}{d} + \left(u(nT^+) - \frac{c}{d}\right)e^{-d(t-nT)}, \quad nT < t < (n+1)T.$$

After the successive pulses, the stroboscopic map of system (2.2) is obtained as follows:

$$u\big((n+1)T^+\big) = (1-p)u\big((n+1)T\big) = (1-p)\left(\frac{c}{d} + \left(u(nT^+) - \frac{c}{d}\right)e^{-dT}\right). \tag{2.3}$$

There is a unique positive fixed point for (2.3), which is as follows:

$$\tilde{u}(t) = \frac{c}{d}\left(1 - \frac{p}{1-(1-p)e^{-dT}}\right).$$

This means that there is a positive periodic solution

$$u^*(t) = \frac{c}{d}\left(1 - \frac{pe^{-d(t-nT)}}{1-(1-p)e^{-dT}}\right),$$

with initial value $u^*(0^+) = \frac{c}{d}(1 - \frac{p}{1-(1-p)e^{-dT}})$, $nT < t \leq (n+1)T$.

Suppose $u(t)$ is an arbitrary solution of (2.2), then applying the iterative technique, we have

$$\begin{aligned} u(t) &= \frac{c}{d} + \left(\frac{c}{d}(1-p)\left(1-e^{-dT}\right) + \frac{c}{d}(1-p)^2\left(1-e^{-dT}\right)e^{-dT} + \cdots \right. \\ &\quad \left. + \frac{c}{d}(1-p)^n\left(1-e^{-dT}\right)e^{-(n-1)dT} + \frac{c}{d}u\left(0^+\right)(1-p)^n\left(1-e^{-dT}\right)e^{-ndT}\right)e^{-(t-nT)} \\ &= u^*(t) + (1-p)^n e^{-ndT}\left(u\left(0^+\right) - u^*\left(0^+\right)\right)e^{-(t-nT)}, \quad nT < t \leq (n+1)T. \end{aligned}$$

Hence, $\lim_{t\to\infty} |u(t) - u^*(t)| = 0$. The proof is completed. □

Lemma 2.5 *There is a positive constant M such that $x_i(t) \leq \frac{M}{\lambda}$, $y_i(t) \leq M$, $i = 1, 2$, for every solution $(x_1(t), x_2(t), y_1(t), y_2(t))$ of system* (1.1) *with t sufficiently large, and λ is a positive constant defined in system* (1.1).

Proof Define $V(t) = V_1(t) + V_2(t)$, $V_1(t) = \lambda(x_1(t) + x_2(t))$, $V_2(t) = y_1(t) + y_2(t)$.

If $t \neq nT$, by $d_1 > d_2$, $d_4 > d_5$, we let $d = \min(d_2, d_5)$, then

$$\begin{aligned} D^+V(t) + dV(t) &\leq D^+V(t) + d_2V_1(t) + d_5V_2(t) \\ &= -\lambda(d_1 - d_2)x_1(t) - (d_4 - d_5)y_1(t) + \lambda r x_2(t) - \lambda d_3 x_2^2(t) \\ &\leq \lambda r x_2(t) - \lambda d_3 x_2^2(t) \leq M_0 = \frac{\lambda r^2}{4d_3}. \end{aligned}$$

If $t = nT$, then

$$V\left(nT^+\right) = \lambda x_1(nT) + \lambda\mu + \lambda x_2(nT) + y_1(nT) + (1-p)y_2(nT) \leq V(nT) + \lambda\mu.$$

Hence, for $t \in (nT, (n+1)T]$, by using Lemma 2.1, we have

$$\begin{aligned} V(t) &\leq V(0)e^{-dt} + \int_0^t M_0 e^{-d(t-s)}\,ds + \sum_{0<nT<t} \lambda\mu e^{-(t-nT)} \\ &< V(0)e^{-dt} + \frac{M_0}{d}\left(1-e^{-dt}\right) + \lambda\mu\frac{e^{-d(t-T)}}{1-e^{dT}} + \lambda\mu\frac{e^{dT}}{e^{dT}-1} \\ &\to \frac{M_0}{d} + \frac{\lambda\mu e^{dT}}{e^{dT}-1}, \quad t\to\infty. \end{aligned}$$

It means that $V(t)$ is uniformly ultimately bounded. Therefore, according to the definition of $V(t)$, there is a constant

$$M = \frac{M_0}{d} + \frac{\lambda\mu e^{dT}}{e^{dT}-1} > 0, \tag{2.4}$$

such that $x_i(t) \le \frac{M}{\lambda}$, $y_i(t) \le M$, $i = 1, 2$, with t large enough. This completes the proof. □

3 Global attractivity of mature predator periodic solution

In this section, we shall demonstrate the existence and global attractivity of the mature predator survival periodic solution of system (1.1).

Firstly, by Lemmas 2.2, 2.3, and 2.4, we can easily obtain the existence of a predator-extinction periodic solution for system (1.1).

Theorem 3.1 *System* (1.1) *has a mature predator survival periodic solution* $(0,0,0,y_2^*(t))$. *For* $t \in (nT,(n+1)T]$, *and each solution* $(0,0,0,y_2(t))$ *of system* (1.1), *we have* $y_2(t) \to y_2^*(t)$ *as* $t \to \infty$, *where* $y_2^*(t) = \mu\frac{e^{-d_5(t-nT)}}{1-e^{-d_5T}}$ *for* $nT < t \le (n+1)T$, *and* $y_2^*(0^+) = \frac{\mu}{1-e^{-d_5T}}$.

Next, we give the conditions on the global attractivity of the predator-extinction periodic solution $(x^*(t),0,0)$ of the system (1.1).

Theorem 3.2 *The mature predator survival periodic solution* $(0,0,0,y_2^*(t))$ *of system* (1.1) *is globally attractive, if*

$$(\mathrm{A}_1) \quad \left(re^{-d_1\tau_1} - d_2\right)\left(c + \frac{M}{\lambda}\right) < k\mu\frac{e^{-d_5T}}{1-e^{-d_5T}}.$$

Proof Let $(x_1(t),x_2(t),y_1(t),y_2(t))$ be any solution of system (1.1). From the fourth and the eighth of system (1.1), we have

$$\begin{cases} \dot{y}_2(t) \ge -d_5y_2(t), & t \ne nT, \\ y_2(t^+) = y_2(t) + \mu, & t = nT. \end{cases} \tag{3.1}$$

Considering the auxiliary system of (3.1) as follows:

$$\begin{cases} \dot{z}_1(t) = -d_5z_1(t), & t \ne nT, \\ z_1(t^+) = z_1(t) + \mu, & t = nT. \end{cases} \tag{3.2}$$

Applying Lemma 2.3, we have

$$z_1^*(t) = \mu\frac{e^{-d_5(t-nT)}}{1-e^{-d_5T}} \quad \text{for } nT < t \le (n+1)T.$$

Then system (3.2) has a unique and globally attractive positive periodic solution. Applying the comparison theorem of the impulsive differential equation [16], there is a $n_0 \in Z_+$ and a sufficiently small positive constant ε such that

$$y_2(t) \ge z_1(t) \ge z_1^*(t) - \varepsilon = \mu\frac{e^{-d_5(t-nT)}}{1-e^{-d_5T}} - \varepsilon \ge \mu\frac{e^{-d_5T}}{1-e^{-d_5T}} - \varepsilon \triangleq \rho \tag{3.3}$$

for $t \geq n_0 T$. By Lemma 2.5 and (3.3), we have

$$\dot{x}_2(t) \leq re^{-d_1\tau_1}x_2(t-\tau_1) - \left(\frac{k\rho}{c+\frac{M}{\lambda}} + d_2\right)x_2(t) - d_3x_2^2(t),$$

when $t \geq n_0 T + \tau_1$. We consider the auxiliary impulsive differential equation

$$\dot{z}_2(t) = re^{-d_1\tau_1}z_2(t-\tau_1) - \left(\frac{k\rho}{c+\frac{M}{\lambda}} + d_2\right)z_2(t) - d_3z_2^2(t).$$

According to hypothesis (A_1), for the sufficiently small constant $\varepsilon > 0$, we can obtain

$$re^{-d_1\tau_1} < \frac{k\rho}{c+\frac{M}{\lambda}} + d_2.$$

Applying Lemma 2.2, we have $\lim_{t\to\infty} z_2(t) = 0$. Since $x_2(s) = z_2(s) = \varphi_2(s) > 0$ for all $s \in [-\tau_1, 0]$, applying the comparison theorem, we have $x_2(t) \to 0$ as $t \to \infty$. Without loss of generality, suppose that there is a constant $\varepsilon_1 > 0$ such that

$$x_2(t) < \varepsilon_1, \quad t \geq 0. \tag{3.4}$$

From the first and the fifth equations of (1.1) and (3.4), we have

$$\begin{cases} \dot{x}_1(t) \leq r\varepsilon_1 - d_1x_1(t), & t \neq nT, \\ x_1(t^+) = (1-p)x_1(t), & t = nT. \end{cases} \tag{3.5}$$

Consider the following auxiliary impulsive differential system of (3.5):

$$\begin{cases} \dot{z}_3(t) = r\varepsilon_1 - d_1z_3(t), & t \neq nT, \\ z_3(t^+) = (1-p)z_3(t), & t = nT. \end{cases} \tag{3.6}$$

Applying Lemma 2.4, we have

$$z_3(t) = \frac{r\varepsilon_1}{d_1}\left(1 - \frac{pe^{-d_1(t-nT)}}{1-(1-p)e^{-d_1T}}\right) \quad \text{for } nT < t \leq (n+1)T.$$

Taking into account the comparison theorem, for any small $\varepsilon_2 > 0$, there exists $t_1 > 0$ such that $x_1(t) \leq z_3(t) + \varepsilon_2$, $t > t_1$. Let $\varepsilon_1 \to 0$, then $z_3(t) \to 0$ and

$$x_1(t) \leq \varepsilon_2. \tag{3.7}$$

From the fourth and the eighth equations of system (1.1), we have

$$\begin{cases} \dot{y}_2(t) \leq \lambda ke^{-d_4\tau}\frac{\varepsilon_1M}{c+\varepsilon_1} - d_5y_2(t), & t \neq nT, \\ y_2(t^+) = y_2(t) + \mu, & t = nT. \end{cases} \tag{3.8}$$

Consider the auxiliary system of (3.8),

$$\begin{cases} \dot{z}_4(t) = \lambda ke^{-d_4\tau}\frac{\varepsilon_1M}{c+\varepsilon_1} - d_5z_4(t), & t \neq nT, \\ z_4(t^+) = z_4(t) + \mu, & t = nT. \end{cases} \tag{3.9}$$

By using Lemma 2.3, the unique positive periodic solution of system (3.9) is

$$z_4^*(t) = \lambda k e^{-d_4\tau} \frac{\varepsilon_1 M}{d_5(c+\varepsilon_1)} + \frac{\mu e^{-d_5(t-nT)}}{1-e^{-d_5 T}} \quad \text{for } nT < t \le (n+1)T.$$

By the comparison theorem, for sufficiently small constants $\varepsilon > 0$, there exists $t_2 > 0$ such that $y_2(t) \le z_4^*(t) + \varepsilon \triangleq \rho_1$, for all $t > t_2$. Let $\varepsilon_1 \to 0$, then $z_4^*(t) \to y_2^*(t)$ and we have $y_2(t) \le y_2^*(t) + \varepsilon$. On the other hand, we can conclude from (3.1), (3.2), and (3.3) that $y_2(t) \ge y_2^*(t) - \varepsilon$ for t large enough, which implies $y_2(t) \to y_2^*(t)$ as $t \to \infty$.

From the third and the seventh equations of system (1.1) and (3.3), (3.4), we have

$$\dot{y}_1(t) \le \lambda k \frac{\varepsilon_1 \rho_1}{c+\varepsilon_1} - d_4 y_1(t), \quad t \ge 0. \tag{3.10}$$

Consider the auxiliary system of (3.10),

$$\dot{z}_5(t) = \lambda k \frac{\varepsilon_1 \rho_1}{c+\varepsilon_1} - d_4 z_5(t), \quad t \ge 0. \tag{3.11}$$

By simple calculation, we have

$$z_5(t) = \frac{\lambda k \varepsilon_1 \rho_1}{d_4(c+\varepsilon_1)} + \left(z_5(0^+) - \frac{\lambda k \varepsilon_1 \rho_1}{d_4(c+\varepsilon_1)} \right) e^{-d_4 t}.$$

It follows from the comparison theorem that, for sufficiently small constants $\varepsilon_3 > 0$, there exists $t_3 > 0$, such that $y_1(t) \le z_5(t) + \varepsilon_3$ for all $t > t_3$. Let $\varepsilon_1 \to 0$, then $z_5(t) \to 0$, and we have

$$y_1(t) \le \varepsilon_3. \tag{3.12}$$

Since ε, ε_1, ε_2, ε_3 are arbitrary small, we obtain $x_1(t) \to 0$, $x_2(t) \to 0$, $y_1(t) \to 0$, as t is large enough. The proof is completed. □

4 Permanence of system (1.1)

In the real world, from the principle of ecosystem balance and saving resources, we only need to control the prey under the economic threshold level, and not to eradicate the prey totally. Thus we focus on the permanence of system (1.1).

First, we give the definition of permanence.

Definition 4.1 System (1.1) is said to be permanent if there exist positive constants m and M such that each positive solution $(x_1(t), x_2(t), y_1(t), y_2(t))$ of system (1.1) satisfies $m \le x_i(t)$, $y_i(t) \le M$, $i = 1, 2$, for t large enough.

Theorem 4.1 *Assume that*:

(A$_2$) $\quad re^{-d_1\tau_1} - \dfrac{kq}{c} - d_2 - d_3\dfrac{M}{\lambda} > 0,$

(A$_3$) $\quad d_5 - \dfrac{\lambda k m_2^*}{c + m_2^*} > 0,$

$$(\mathrm{A}_4)\quad m_2 - \frac{M}{\lambda}e^{-d_1\tau_1} > 0,$$

$$(\mathrm{A}_5)\quad \frac{m_2}{c+m_2}m_4 - \frac{M^2}{\lambda c + M}e^{-d_4\tau_2} > 0,$$

where M, m_2, m_4, q *are defined in* (2.4), (4.7), (4.10), (4.13), *respectively, then system* (1.1) *is permanent.*

Proof Firstly, we will prove that there exists a constant $m_2 > 0$ such that $x_2(t) > m_2$ for t sufficiently large. The second equation of (1.1) is equivalent to the following equality:

$$\begin{aligned}\dot{x}^2(t) &= \left(re^{-d_1\tau_1} - \frac{ky_2(t)}{c+x_2(t)} - d_2 - d_3x_2(t)\right)x_2(t) \\ &\quad - re^{-d_1\tau_1}\frac{d}{dt}\int_{t-\tau_1}^{t} x_2(s)\,ds.\end{aligned} \tag{4.1}$$

According to (4.1), we define

$$V(t) = x_2(t) + re^{-d_1\tau_1}\int_{t-\tau_1}^{t} x_2(s)\,ds.$$

Calculating the derivative of $V(t)$, we obtain

$$\dot{V}(t) = \left(re^{-d_1\tau_1} - \frac{ky_2(t)}{c+x_2(t)} - d_2 - d_3x_2(t)\right)x_2(t). \tag{4.2}$$

Applying Lemma 2.5, (4.2) can be re-written as follows:

$$\dot{V}(t) > \left(re^{-d_1\tau_1} - \frac{k}{c}y_2(t) - d_2 - d_3\frac{M}{\lambda}\right)x_2(t). \tag{4.3}$$

By hypothesis (A_2), there is an arbitrary small positive ε_4 such that

$$re^{-d_1\tau_1} > \frac{k}{c}(q+\varepsilon_4) + d_2 + d_3\frac{M}{\lambda}, \tag{4.4}$$

where $q = \frac{\mu}{1-e^{-(d_5-\frac{\lambda k m_2^*}{c+m_2^*})}}$.

Let m_2^* be determined as follows:

$$\frac{c}{k}\left(re^{-d_1\tau_1} - d_2 - d_3\frac{M}{\lambda}\right) = \frac{\mu}{1-e^{-(d_5-\frac{\lambda k m_2^*}{c+m_2^*})}}.$$

Then, for any $t_4 > 0$, it is impossible that $x_2(t) < m_2^*$ for all $t > t_4$. Suppose that the claim is invalid, then there is $t_4 > 0$ such that $x_2(t) < m_2^*$ for all $t_4 > 0$. It follows from the fourth and the eight equations of system (1.1) that

$$\begin{cases} \dot{y}_2(t) < -(d_5 - \frac{\lambda k m_2^*}{c+m_2^*})y_2(t), & t \neq nT, \\ y_2(t^+) = y_2(t) + \mu, & t = nT \end{cases} \tag{4.5}$$

for all $t > t_4 + \tau_2$. Consider the following auxiliary impulsive system of (4.5):

$$\begin{cases} \dot{z}_6(t) = -z_6(t)(d_5 - \frac{\lambda k m_2^*}{c+m_2^*}), & t \neq nT, \\ z_6(t^+) = z_6(t) + \mu, & t = nT. \end{cases} \tag{4.6}$$

By using Lemma 2.3, the unique positive periodic solution of (4.6) is

$$z_6(t) = \frac{\mu e^{-(d_5 - \frac{\lambda k m_2^*}{c+m_2^*})(t-nT)}}{1 - e^{-(d_5 - \frac{\lambda k m_2^*}{c+m_2^*})}}, \quad nT < t \leq (n+1)T.$$

This is globally asymptotically stable by hypothesis (A_3). Taking into account the comparison theorem of an impulsive differential equation, there exists t_5 ($> t_4 + \tau_2$) such that

$$y_2(t) \leq z_6(t) + \varepsilon_4.$$

For $t > t_5$, we have

$$z_6(t) \leq \frac{\mu}{1 - e^{-(d_5 - \frac{\lambda k m_2^*}{c+m_2^*})}} \triangleq q. \tag{4.7}$$

Then

$$y_2(t) \leq q + \varepsilon_4 \triangleq \sigma, \quad t \geq t_5. \tag{4.8}$$

According to (4.4), we have

$$re^{-d_1\tau_1} > \frac{k\sigma}{c} + d_2 + d_3\frac{M}{\lambda}.$$

By (4.3) and (4.8), we get

$$\dot{V}(t) > \left(re^{-d_1\tau_1} - \frac{k\sigma}{c} - d_2 - d_3\frac{M}{\lambda}\right)x_2(t), \quad t \geq t_5. \tag{4.9}$$

Let $x_2^m = \min_{t\in[t_1,t_1+\tau]} x_2(t)$.

We will show that $x_2(t) \geq x_2^m$ for all $t \geq t_5$. Otherwise, there exists a $T_0 > 0$ such that $x_2(t) \geq x_2^m$ for $t_5 \leq t \leq t_5 + \tau + T_0$, $x_2(t_5 + \tau + T_0) \geq x_2^m$ and $\dot{x}_2(t_5 + \tau + T_0) < 0$. From the second equation of system (1.1) and (4.8), we have

$$\dot{x}_2(t_5 + \tau + T_0) > \left(re^{-d_1\tau_1} - \frac{k\sigma}{c} - d_2 - d_3\frac{M}{\lambda}\right)x_2^m > 0.$$

This is a contradiction. Thus, we have $x_2(t) \geq x_2^m$, $t \geq t_5$.

By (4.4) and (4.9), we have

$$\dot{V}(t) > \left(re^{-d_1\tau_1} - \frac{k\sigma}{c} - d_2 - d_3\frac{M}{\lambda}\right)x_2^m, \quad t \geq t_5.$$

This means that $V(t) \to \infty$ as $t \to \infty$. It is a contradiction with $V(t) \leq \frac{M}{\lambda}(1 + r\tau_1 e^{-d_1\tau_1})$.

Therefore, for any $t_4 > 0$, the inequality $x_2(t) < m_2^*$ cannot hold for all $t > t_4$. So there exist the following two possibilities.

(i) If $x_2(t) \geq m_2^*$ holds for all t large enough, then our goal is obtained.

(ii) If $x_2(t)$ is oscillatory about m_2^*. Setting

$$m_2 = \min\left\{\frac{m_2^*}{2}, m_2^* e^{-(kM+d_2+d_3m_2^*)\tau_1}\right\}, \tag{4.10}$$

we prove that $x_2(t) \geq m_2$ for all t large enough. Suppose that there exist two positive constants γ, η such that $x_2(\gamma) = x_2(\gamma + \eta)$ and $x_2(t) < m_2^*$ for all $\gamma < t < \gamma + \eta$, where γ is large enough, and the inequality (4.8) holds true for $\gamma < t < \gamma + \eta$. Since $x_2(t)$ is continuous, bounded, and is not affected by impulses, we conclude that $x_2(t)$ is uniformly continuous. Hence, there exists a constant T_1 ($0 < T_1 < \tau_1$ and T_1 is independent of the choice of γ) such that $x_2(\gamma) > \frac{m_2^*}{2}$ for $\gamma \leq t \leq \gamma + T_1$. If $\eta \leq T_1$, our aim is obtained. If $T_1 < \eta \leq \tau_1$, from the second equation of (1.1), we obtain, for $\gamma < t < \gamma + \eta$, $\dot{x}_2(t) \geq -\frac{k}{c}x_2(t)y_2(t) - d_2x_2(t) - d_3x_2^2(t)$. According to the assumption $x_2(\gamma) = m_2^*$ and $x_2(t) < m_2^*$ for $\gamma < t < \gamma + \eta$, we have $\dot{x}_2(t) \geq -(\frac{k}{c}M + d_2 + d_3m_2^*)x_2(t)$ for $\gamma < t \leq \gamma + \eta \leq \gamma + \tau_1$. Then we derive that $x_2(t) \geq m_2^* e^{-(\frac{k}{c}M+d_2+d_3m_2^*)\tau_1}$. It is clear that $x_2(t) \geq m_2$ for $\gamma < t < \gamma + \eta$. If $\eta \geq \tau_1$, then we have $x_2(t) \geq m_2$ for $\gamma < t < \gamma + \tau_1$. The same arguments can be continued. We obtain $x_2(t) \geq m_2$ for $\gamma + \tau_1 < t < \gamma + \eta$. Since the interval $[\gamma, \gamma + \eta]$ is arbitrarily chosen, we get $x_2(t) \geq m_2$ for t large enough. In view of our arguments above, the choice of m_2 is independent of the positive solution of (1.1), which satisfies $x_2(t) \geq m_2$ for t large enough.

Next, by the first and the fifth equations of system (1.1), we have

$$\begin{cases} \dot{x}_1(t) \geq r(m_2 - \frac{M}{\lambda}e^{-d_1\tau_1}) - d_1x_1(t), & t \neq nT, \\ x_1(t^+) = (1-p)x_1(t), & t = nT. \end{cases} \tag{4.11}$$

Consider the auxiliary system of (4.11) as follows:

$$\begin{cases} \dot{z}_7(t) = r(m_2 - \frac{M}{\lambda}e^{-d_1\tau_1}) - d_1z_7(t), & t \neq nT, \\ z_7(t^+) = (1-p)z_7(t), & t = nT. \end{cases} \tag{4.12}$$

By hypothesis (A_4), and applying Lemma 2.4, we have

$$z_7(t) = \frac{r(m_2 - \frac{M}{\lambda}e^{-d_1\tau_1})}{d_1}\left(1 - \frac{pe^{-d_1(t-nT)}}{(1-p)e^{-d_1T}}\right).$$

By the comparison theorem, there exists a positive constant ε_5 sufficiently small such that $\dot{x}_1(t) \geq z_7(t) - \varepsilon_5$ as t is large enough. Taking into account the comparison theorem of an impulsive differential equation, we obtain

$$x_1(t) \geq \frac{r(m_2 - \frac{M}{\lambda}e^{-d_1\tau_1})}{d_1}\left(1 - \frac{p}{(1-p)e^{-d_1T}}\right) - \varepsilon_5 \triangleq m_1.$$

From (3.3), let $\rho \triangleq m_4$, then $y_2(t) \geq m_4$.

Finally, by the third equation of system (1.1), we have

$$\dot{y}_1(t) \geq \lambda k\left(\frac{m_2}{c+m_2}m_4 - \frac{M^2}{\lambda c+M}e^{-d_4\tau_2}\right) - d_4 y_1(t). \tag{4.13}$$

Consider the auxiliary system of (4.13),

$$\dot{z}_8(t) = \lambda k\left(\frac{m_2}{c+m_2}m_4 - \frac{M^2}{\lambda c+M}e^{-d_4\tau_2}\right) - d_4 z_8(t). \tag{4.14}$$

It is easy to calculate that

$$\begin{aligned} z_8(t) = {} & \frac{\lambda k(\frac{m_2}{c+m_2}m_4 - \frac{M^2}{\lambda c+M}e^{-d_4\tau_2})}{d_4} \\ & - \left(\lambda k\left(\frac{m_2}{c+m_2}m_4 - \frac{M^2}{\lambda c+M}e^{-d_4\tau_2}\right) - z_8\left(0^+\right)\right)e^{-d_4 t}. \end{aligned}$$

Applying the comparison theorem, by hypothesis (A_5), there exists a positive constant ε_6 small enough when t is large enough, such that

$$y_1(t) \geq z_8(t) - \varepsilon_6 \geq \frac{\lambda k(\frac{m_2}{c+m_2}m_4 - \frac{M^2}{\lambda c+M}e^{-d_4\tau_2})}{d_4} - \varepsilon_6 \triangleq m_3.$$

Then taking $m = \min\{m_1, m_2, m_3, m_4\}$, we have $x_i(t), y_i(t) \geq m$, $i = 1, 2$. Considering Lemma 2.5 and the above discussion, we can find that system (1.1) is permanent. This completes the proof. □

5 Numerical simulations

In this section, we give some examples and numerical simulations to show the effectiveness of the main results. In system (1.1), we let $r = 1$, $d_1 = 0.5$, $k = 1$, $c = 1$, $d_2 = 0.3$, $d_3 = 0.2$, $\lambda = 0.5$, $d_4 = 0.4$, $d_5 = 0.2$, $p = 0.3$, $\mu = 0.5$, $\tau_1 = 1$, $\tau_2 = 1$, $T = 1$. It is quite clear that the parameters satisfy the conditions of Theorem 3.2, so we can obtain the global attractivity of the mature predator survival periodic solution, which is shown by Figure 1.

We let $r = 1$, $d_1 = 0.5$, $k = 2$, $c = 1$, $d_2 = 0.3$, $d_3 = 0.2$, $\lambda = 1$, $d_4 = 0.2$, $d_5 = 0.1$, $p = 0.3$, $\mu = 0.1$, $\tau_1 = 1$, $\tau_2 = 1$, $T = 10$. By computation, the conditions of Theorem 4.1 are also satisfied, hence, by Theorem 4.1, system (1.1) is permanent; see Figure 2.

6 Conclusion

In this paper, by using the comparison theorem of an impulsive differential equation and some analysis techniques, we obtain the sufficient conditions of the mature predator survival periodic solution and permanence of system (1.1). Theorem 3.2 implies that increasing T and μ is propitious to the global attractivity of the mature predator survival periodic solution $(0, 0, 0, y_2^*(t))$. By Theorem 4.1, we may see that reducing T and μ plays an important role in the permanence of system (1.1). Combining the biological resource management, we believe that there exists a threshold value of economic benefits. Thus, it is unadvisable to make too much effort to destroy all the pest, and there must exist an optimal harvesting policy for system (1.1), that is, what we should do is to gain more, rather than wipe out all pest,so it is interesting for us to continue to study the optimal harvesting policy of system (1.1) in the near future.

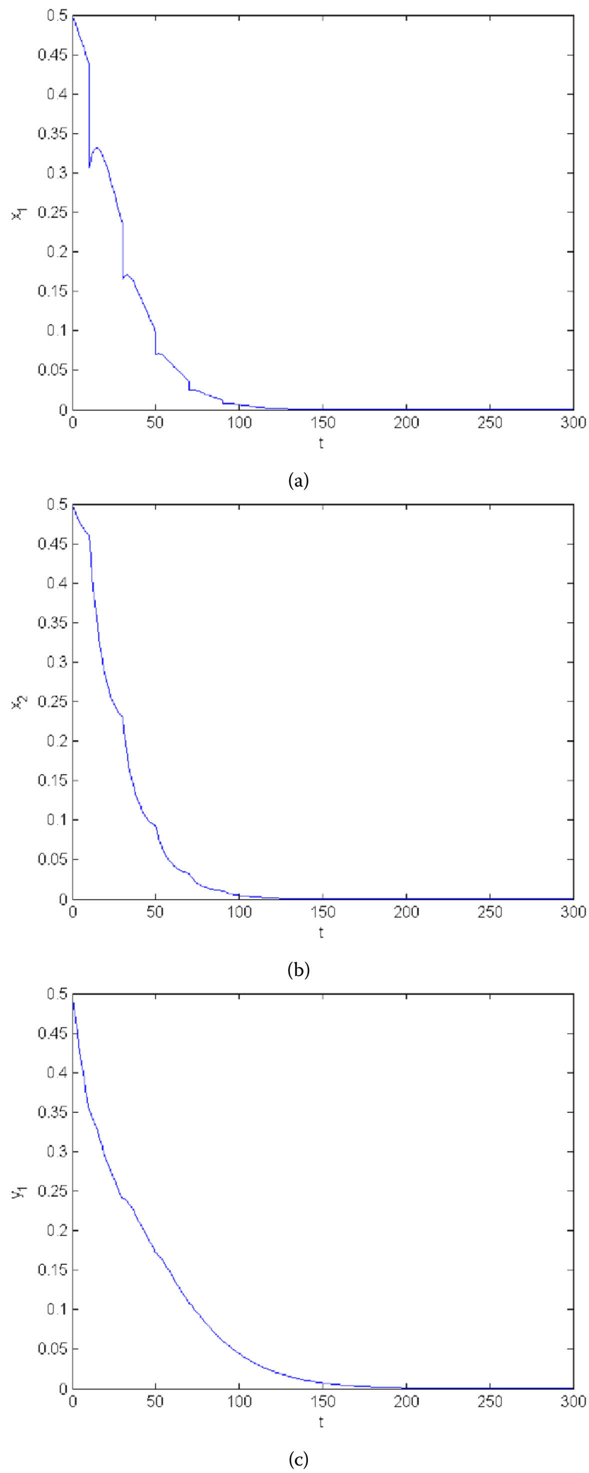
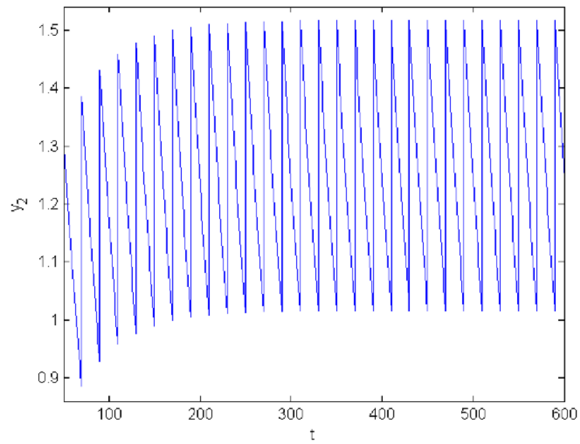
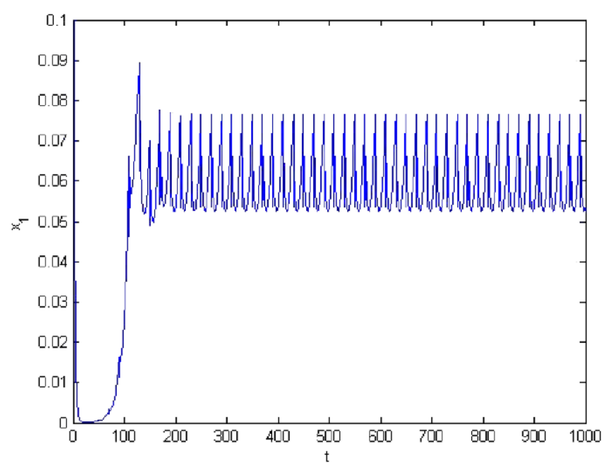


Figure 1 Dynamical behaviors of system (1.1) with $r = 1$, $d_1 = 0.5$, $k = 1$, $c = 1$, $d_2 = 0.3$, $d_3 = 0.2$, $\lambda = 0.5$, $d_4 = 0.4$, $d_5 = 0.2$, $p = 0.3$, $\mu = 0.5$, $\tau_1 = 1$, $\tau_2 = 1$, $T = 1$. (a) Time series of the immature prey population. **(b)** Time series of the mature prey population. **(c)** Time series of the immature predator population. **(d)** Time series of the mature predator population.

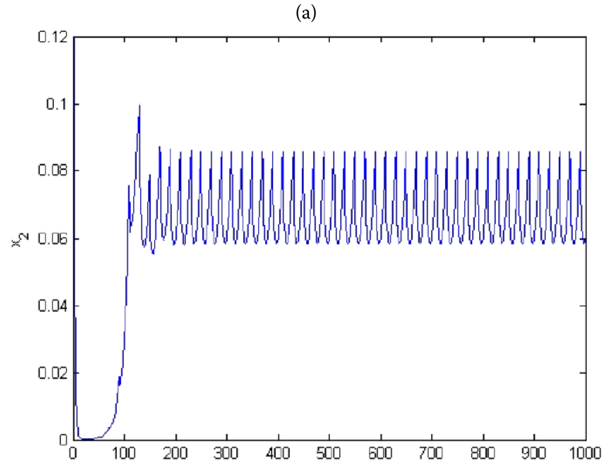


(d)

Figure 1 Continued



(a)



(b)

Figure 2 Dynamical behaviors of system (1.1) for $r = 1$, $d_1 = 0.5$, $k = 2$, $c = 1$, $d_2 = 0.3$, $d_3 = 0.2$, $\lambda = 1$, $d_4 = 0.2$, $d_5 = 0.1$, $p = 0.3$, $\mu = 0.1$, $\tau_1 = 1$, $\tau_2 = 1$, $T = 10$. (a) Time series of the immature prey population. **(b)** Time series of the mature prey population. **(c)** Time series of the immature predator population. **(d)** Time series of the mature predator population.

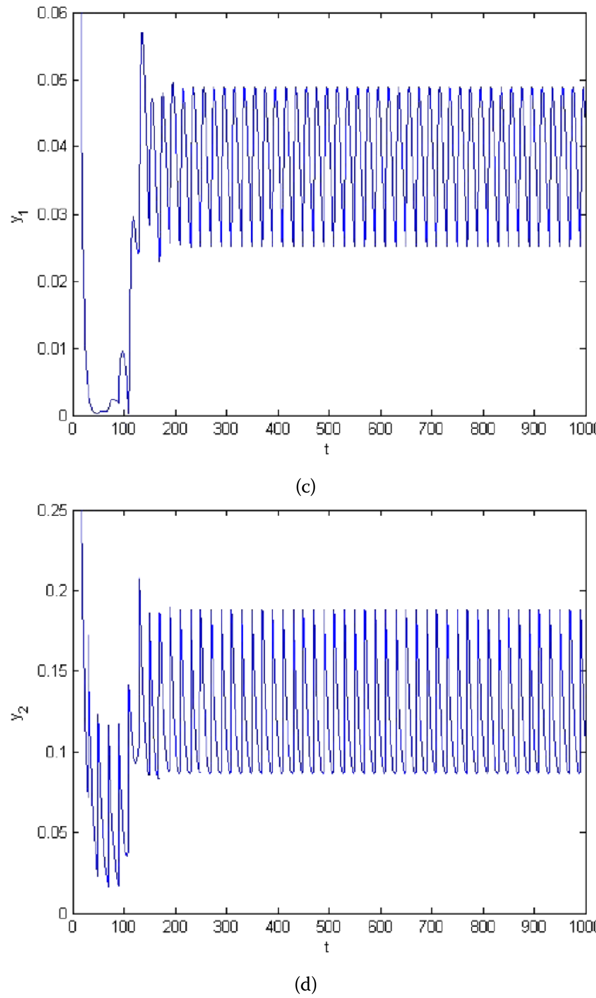


Figure 2 Continued

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Author details
[1]School of Science, Guilin University of Technology, Guilin, Guangxi 541004, P.R. China. [2]College of Teacher Education, Qujing Normal University, Qujing, Yunnan 655011, P.R. China.

Acknowledgements
This paper is supported by the National Natural Science Foundation of China (11161015, 11361012), and the Natural Science Foundation of Guangxi (2013GXNSFAA019003) and partially supported by the National High Technology Research and Development Program 863 under Grant No. 2013AA12A402.

Received: 7 May 2014 Accepted: 21 October 2014 Published: 31 Oct 2014

References
1. Liu, B, Zhang, YJ, Chen, LS: The dynamical behaviors of a Lotka-Volterra predator-prey model concerning integrated pest management. Nonlinear Anal., Real World Appl. **6**, 227-243 (2005)
2. Baek, HK: Qualitative analysis of Beddington-DeAngelis type impulsive predator-prey models. Nonlinear Anal., Real World Appl. **11**, 1312-1322 (2010)
3. Zhang, SW, Chen, LS: A study of predator-prey models with the Beddington-DeAngelis functional response and impulsive effect. Chaos Solitons Fractals **27**, 237-248 (2006)
4. Song, XY, Li, YF: Dynamic complexities of a Holling II two-prey one-predator system with impulsive effect. Chaos Solitons Fractals **33**, 463-478 (2007)

5. Georgescu, P, Morosanu, G: Impulsive perturbations of a three-trophic prey-dependent food chain system. Math. Comput. Model. **48**, 975-997 (2008)
6. Wang, WM, Wang, HL, Li, ZQ: The dynamic complexity of a three-species Beddington-type food chain with impulsive control strategy. Chaos Solitons Fractals **32**, 1772-1785 (2007)
7. Pei, YZ, Li, CG, Fan, SH: A mathematical model of a three species prey-predator system with impulsive control and Holling functional response. Appl. Math. Comput. **219**, 10945-10955 (2013)
8. Jiang, GR, Lu, QS, Qian, LN: Complex dynamics of a Holling type II prey-predator system with state feedback control. Chaos Solitons Fractals **31**, 448-461 (2007)
9. Shao, YF, Li, Y: Dynamical analysis of a stage structured predator-prey system with impulsive diffusion and generic functional response. Appl. Math. Comput. **220**, 472-481 (2013)
10. Jiang, XW, Song, Q, Hao, MY: Dynamics behaviors of a delayed stage-structured predator-prey model with impulsive effect. Appl. Math. Comput. **215**, 4221-4229 (2010)
11. Song, XY, Chen, LS: Optimal harvesting and stability for a two-species competitive system with stage structure. Math. Biosci. **170**, 173-186 (2001)
12. Shao, YF, Dai, BX: The dynamics of an impulsive delay predator-prey model with stage structure and Beddington-type functional response. Nonlinear Anal., Real World Appl. **11**, 3567-3576 (2010)
13. Kuang, Y: Delay Differential Equations: With Applications in Population Dynamics. Academic Press, New York (1993)
14. Li, YK, Kuang, Y: Periodic solutions of periodic delay Lotka-Volterra equations and systems. J. Math. Appl. **255**, 260-280 (2001)
15. Zhu, HT, Zhu, WD, Zhang, ZD: Persistence of competitive ecological mathematic model. J. Central South Univ. For. Technol. **3**(4), 214-218 (2011)
16. Lakshmikantham, V, Bainov, DD, Simeonov, P: Theory of Impulsive Differential Equations. World Scientific, Singapore (1989)

10.1186/1687-1847-2014-280
Cite this article as: Ju et al.: An impulsive prey-predator system with stage-structure and Holling II functional response. *Advances in Difference Equations* 2014, 2014:280