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Weakly mixing sets and transitive sets for non-autonomous discrete systems

Lei Liu[1,2*] and Yuejuan Sun[1]

*Correspondence: mathliulei@163.com
[1]School of Mathematics and Information Science, Shangqiu Normal University, Shangqiu, Henan 476000, P.R. China
[2]Department of Mathematics, University of Science and Technology of China, Hefei, Anhui 230026, P.R. China

Abstract
In this paper we mainly study the weakly mixing sets and transitive sets of non-autonomous discrete systems. Some basic concepts are introduced for non-autonomous discrete systems, including a weakly mixing set and a transitive set. We discuss the basic properties of weakly mixing sets and transitive sets of non-autonomous discrete systems. Also, we investigate the relationship between two conjugated non-autonomous discrete systems on weakly mixing sets and transitive sets.

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Keywords: non-autonomous discrete system; weakly mixing set; transitive set

1 Introduction

Throughout this paper $\mathbb{N}$ denotes the set of all positive integers, and let $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. Let X be a topological space, let $f_n : X \to X$ for each $n \in \mathbb{N}$ be a continuous map, and let $f_{1,\infty}$ denote the sequence $(f_1, f_2, \ldots, f_n, \ldots)$. The pair $(X, f_{1,\infty})$ is referred to as a non-autonomous discrete system [1]. Define

$$f_1^n(x) := f_n \circ f_{n-1} \circ \cdots \circ f_2 \circ f_1, \quad n \in \mathbb{N},$$

and $f_1^0 := \mathrm{id}_X$, the identity on X. In particular, when $f_{1,\infty}$ is a constant sequence $(f, f, \ldots, f, \ldots)$, the pair $(X, f_{1,\infty})$ is just a classical discrete dynamical system (autonomous discrete dynamical system) (X, f). The orbit initiated from $x \in X$ under $f_{1,\infty}$ is defined by the set

$$\gamma(x, f_{1,\infty}) = \{x, f_1(x), f_1^2(x), \ldots, f_1^n(x), \ldots\}.$$

Its long-term behaviors are determined by its limit sets.

Topological transitivity, weak mixing and sensitive dependence on initial conditions (see [1–4]) are global characteristics of topological dynamical systems. Let (X, f) be a topological dynamical system. (X, f) is *topologically transitive* if for any nonempty open subsets U and V of X there exists $n \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$. (X, f) is (*topologically*) *mixing* if for any nonempty open subsets U and V of X, there exists $N \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$ for all $n \in \mathbb{N}$ with $n \geq N$. (X, f) is (*topologically*) *weakly mixing* if for any nonempty open subsets U_1, U_2, V_1 and V_2 of X, there exists $n \in \mathbb{N}$ such that $f^n(U_1) \cap V_1 \neq \emptyset$ and $f^n(U_2) \cap V_2 \neq \emptyset$. It follows from these definitions that mixing implies weak mixing which in turn implies transitivity.

Blanchard introduced overall properties and partial properties in [5]. For example, sensitive dependence on initial conditions, Devaney chaos (see [6]), weak mixing, mixing and more belong to overall properties; Li-Yorke chaos (see [7]) and positive entropy (see [2, 8]) belong to partial properties. Weak mixing is an overall property, it is stable under semi-conjugate maps and implies Li-Yorke chaos. By [9], we know that a weakly mixing system always contains a dense uncountable scrambled set. In [10], Blanchard and Huang introduced the concepts of weakly mixing set and partial weak mixing, derived from a result given by Xiong and Yang [11] and showed that 'partial weak mixing implies Li-Yorke chaos' and 'Li-Yorke chaos cannot imply partial weak mixing'. Let A be a closed subset of X but not a singleton. Then A is a *weakly mixing set* of X if and only if for any $k \in \mathbb{N}$, any choice of nonempty open subsets $V_1, V_2, \ldots, V_k$ of A and nonempty open subsets $U_1, U_2, \ldots, U_k$ of X with $A \cap U_i \neq \emptyset$, $i = 1, 2, \ldots, k$, there exists $m \in \mathbb{N}$ such that $f^m(V_i) \cap U_i \neq \emptyset$ for $1 \leq i \leq k$. (X, f) is called *partial weak mixing* if X contains a weakly mixing subset. Next, Oprocha and Zhang [12] extended the notion of weakly mixing set and gave the concept of 'transitive set' and discussed its basic properties. Let A be a nonempty subset of X. A is called a *transitive set* of (X, f) if for any choice of a nonempty open subset V^A of A and a nonempty open subset U of X with $A \cap U \neq \emptyset$, there exists $n \in \mathbb{N}$ such that $f^n(V^A) \cap U \neq \emptyset$.

In past ten years, a large number of papers have been devoted to dynamical properties in non-autonomous discrete systems. Kolyada and Snoha [1] gave the definition of topological entropy in non-autonomous discrete systems; Kolyada *et al.* [13] discussed minimality of non-autonomous discrete systems; Kempf [14] and Canovas [15] studied ω-limit sets in non-autonomous discrete systems. Krabs [16] discussed stability in non-autonomous discrete systems; Huang *et al.* [17, 18] studied topological pressure and pre-image entropy of non-autonomous discrete systems. Shi and Chen [19] and Oprocha and Wilczynski [20] and Canovas [21] discussed chaos in non-autonomous discrete systems, respectively. Kuang and Cheng [22] studied fractal entropy of non-autonomous systems. In this paper, we extend the notions of weakly mixing set and transitive set and give the definitions of transitive set and weakly mixing set for a non-autonomous discrete system. We discuss the basic properties of weakly mixing sets and transitive sets for non-autonomous discrete systems. Moreover, we investigate the weakly mixing sets and transitive sets for the conjugated non-autonomous discrete systems and obtain that if a system has a transitive set (a weakly mixing set), then the conjugated system has a transitive set (a weakly mixing set).

2 Preliminaries

In the present paper, $\overline{A}$ and $\operatorname{int}(A)$ denote the closure and interior of the set A, respectively. f_1^n denotes $f_n \circ f_{n-1} \circ \cdots \circ f_2 \circ f_1$, *i.e.*, $f_1^n = f_n \circ f_{n-1} \circ \cdots \circ f_2 \circ f_1$ for any $n \in \mathbb{N}$. We define

$$(f_k)^n = \underbrace{f_k \circ f_k \circ \cdots \circ f_k}_{n}$$

for any $k, n \in \mathbb{N}$.

A non-autonomous discrete dynamical system $(X, f_{1,\infty})$ is said to be point transitive if there exists a point $x \in X$, the orbit of x is dense in X, *i.e.*, $\overline{\gamma(x, f_{1,\infty})} = X$, and x is called a transitive point of $(X, f_{1,\infty})$. $(X, f_{1,\infty})$ is said to be topologically transitive if for any two nonempty open sets U and V of X, there exists $k \in \mathbb{N}$ such that $f_1^k(U) \cap V \neq \emptyset$. $(X, f_{1,\infty})$ is said to be weakly mixing if for any nonempty open sets U_i and V_i of X for $i = 1,2$, there exists $k \in \mathbb{N}$ such that $f_1^k(U_i) \cap V_i \neq \emptyset$ for $i = 1,2$.

Definition 2.1 [13] Let $(X, f_{1,\infty})$ be a non-autonomous discrete system. The set $A \subseteq X$ is said to be invariant if $f_1^n(A) \subseteq A$ for any $n \in \mathbb{N}$.

Definition 2.2 Let $(X, f_{1,\infty})$ be a non-autonomous discrete system and A be a nonempty closed subset of X. A is called a transitive set of $(X, f_{1,\infty})$ if for any choice of a nonempty open set V^A of A and a nonempty open set U of X with $A \cap U \neq \emptyset$, there exists $n \in \mathbb{N}$ such that $f_1^n(V^A) \cap U \neq \emptyset$.

Remark If $(X, f_{1,\infty})$ is topologically transitive, then X is a transitive set of $(X, f_{1,\infty})$.

Definition 2.3 Let $(X, f_{1,\infty})$ be a non-autonomous discrete system and A be a nonempty closed subset of X but not a singleton. A is called a weakly mixing set of $(X, f_{1,\infty})$ if for any $k \in \mathbb{N}$, any choice of nonempty open subsets $V_1^A, V_1^A, \ldots, V_k^A$ of A and nonempty open subsets $U_1, U_2, \ldots, U_k$ of X with $A \cap U_i \neq \emptyset$, $i = 1,2,\ldots,k$, there exists $n \in \mathbb{N}$ such that $f_1^n(V_i^A) \cap U_i \neq \emptyset$ for each $1 \leq i \leq k$.

According to the definitions of transitive set and weakly mixing set of a non-autonomous discrete system, we have the following results.

Result 1. If A is a weakly mixing set of $(X, f_{1,\infty})$, then A is a transitive set of $(X, f_{1,\infty})$.

Result 2. If $a \in X$ is a transitive point of $(X, f_{1,\infty})$, then $\{a\}$ is a transitive set of $(X, f_{1,\infty})$.

Example 2.1 Let

$$f_n(x) = \begin{cases} \frac{2n}{n-2}x & \text{if } 0 \leq x \leq \frac{n-2}{2n}, \\ 1 & \text{if } \frac{n-2}{2n} \leq x \leq \frac{n+2}{2n}, \\ \frac{2n}{n-2}(1-x) & \text{if } \frac{n+2}{2n} \leq x \leq 1, \end{cases} \qquad n = 3,4,\ldots$$

and $f_1 = f_2 = \mathrm{id}$, the identity on $[0,1]$.

Observe that the given sequence converges uniformly to the tent map

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq \frac{1}{2}, \\ 2(1-x) & \text{if } \frac{1}{2} \leq x \leq 1, \end{cases}$$

which is known to be topologically transitive on $I = [0,1]$ from [6, 8]. We can easily prove that $[0, \frac{1}{2}]$ is a transitive set of $(X, f_{1,\infty})$.

Figure 1 and Figure 2 denote the tent map f and the 2nd iterate f^2 of the tent map f, respectively.

Definition 2.4 [23] Let (X, τ) be a topological space and A be a nonempty set of X. A is a regular closed set of X if $A = \overline{\mathrm{int}(A)}$.

We easily prove that A is a regular closed set if and only if $\mathrm{int}(V^A) \neq \emptyset$ for any nonempty set V^A of A.

Figure 1 The tent map *f*.

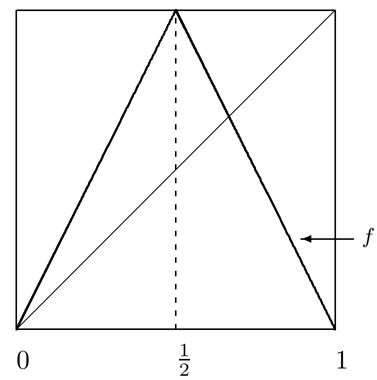
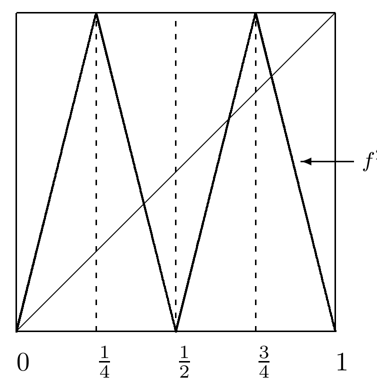


Figure 2 The 2nd iterate f^2 of the tent map *f*.



Definition 2.5 [24] Let (X,τ) be a topological space. A and B are two nonempty subsets of X. B is dense in A if $A \subseteq \overline{A \cap B}$.

In fact, we easily prove that B is dense in A if and only if $V^A \cap B \neq \emptyset$ for any nonempty open set V^A of A.

3 Main results

In this section, we discuss the properties of transitive sets and weakly mixing sets for non-autonomous discrete systems.

Proposition 3.1 *Let $(X,f_{1,\infty})$ be a non-autonomous discrete system and A be a nonempty closed set of X. Then the following conditions are equivalent.*

(1) *A is a transitive set of $(X,f_{1,\infty})$.*
(2) *Let V^A be a nonempty open subset of A and U be a nonempty open subset of X with $A \cap U \neq \emptyset$. Then there exists $n \in \mathbb{N}$ such that $V^A \cap (f_1^n)^{-1}(U) \neq \emptyset$.*
(3) *Let U be a nonempty open set of X with $U \cap A \neq \emptyset$. Then $\bigcup_{n\in\mathbb{N}}(f_1^n)^{-1}(U)$ is dense in A.*

Proof (1) $\Rightarrow$ (2) Let A be a transitive set of $(X,f_{1,\infty})$. Then, for any choice of a nonempty open set V^A of A and a nonempty open set U of X with $A \cap U \neq \emptyset$, there exists $n \in \mathbb{N}$ such that $f_1^n(V^A) \cap U \neq \emptyset$. Since $f_1^n(V^A \cap (f_1^n)^{-1}(U)) = f_1^n(V^A) \cap U$, it follows that $V^A \cap (f_1^n)^{-1}(U) \neq \emptyset$.

(2) $\Rightarrow$ (3) Let V^A be any nonempty open set of A and U be a nonempty open set of X with $A \cap U \neq \emptyset$. By the assumption of (2), there exists $n \in \mathbb{N}$ such that $V^A \cap (f_1^n)^{-1}U \neq \emptyset$.

Furthermore, we have

$$V^A \cap \bigcup_{n\in\mathbb{N}} (f_1^n)^{-1} U = \bigcup_{n\in\mathbb{N}} \bigl(V^A \cap (f_1^n)^{-1}(U)\bigr) \neq \emptyset.$$

Therefore, $\bigcup_{n\in\mathbb{N}}(f_1^n)^{-1}(U)$ is dense in A.

(3) $\Rightarrow$ (1) Let V^A be any nonempty open set of A and U be a nonempty open set of X with $A \cap U \neq \emptyset$. Since $\bigcup_{n\in\mathbb{N}}(f_1^n)^{-1}(U)$ is dense in A, it follows that $V^A \cap \bigcup_{n\in\mathbb{N}}(f_1^n)^{-1}(U) \neq \emptyset$. Furthermore, there exists $n \in \mathbb{N}$ such that $V^A \cap (f_1^n)^{-1}(U) \neq \emptyset$. As $f_1^n(V^A \cap (f_1^n)^{-1}(U)) = f_1^n(V^A) \cap U$, we have $f_1^n(V^A) \cap U \neq \emptyset$. Therefore, A is a transitive set of $(X, f_{1,\infty})$. $\square$

Corollary 3.1 *Let (X,f) be a classical dynamical system and A be a nonempty closed set of X. Then the following conditions are equivalent.*

(1) *A is a transitive set of (X,f).*

(2) *Let V^A be a nonempty open subset of A and U be a nonempty open subset of X with $A \cap U \neq \emptyset$. Then there exists $n \in \mathbb{N}$ such that $V^A \cap f^{-n}(U) \neq \emptyset$.*

(3) *Let U be a nonempty open set of X with $A \cap U \neq \emptyset$. Then $\bigcup_{n\in\mathbb{N}} f^{-n}(U)$ is dense in A.*

Proposition 3.2 *Let $(X, f_{1,\infty})$ be a non-autonomous discrete system, where (X,d) is a metric space and A is a nonempty closed subset of X. Then the following conditions are equivalent.*

(1) *A is a transitive set of $(X, f_{1,\infty})$.*

(2) *Let $a, x \in A$ and $\varepsilon, \delta > 0$. Then there exists $n \in \mathbb{N}$ such that $(A \cap B(a,\varepsilon)) \cap (f_1^n)^{-1}(B(x,\varepsilon)) \neq \emptyset$.*

(3) *Let $a, x \in A$ and $\varepsilon > 0$. Then there exists $n \in \mathbb{N}$ such that $(A \cap B(a,\varepsilon)) \cap (f_1^n)^{-1}(B(x,\varepsilon)) \neq \emptyset$.*

Proof (1) $\Rightarrow$ (2) By the definition of transitive set, (2) is obtained easily.

(2) $\Rightarrow$ (3) Obviously.

(3) $\Rightarrow$ (1) Let V^A be any nonempty open set of A and U be a nonempty open set of X with $A \cap U \neq \emptyset$, then there exist $a, x \in A$ and $\varepsilon > 0$ such that $A \cap B(a,\varepsilon) \subseteq V^A$ and $B(x,\varepsilon) \subseteq U$. By the assumption of (3), there exists $n \in \mathbb{N}$ such that $(A \cap B(a,\varepsilon)) \cap (f_1^n)^{-1}(B(x,\varepsilon)) \neq \emptyset$, further, $V^A \cap (f_1^n)^{-1}(U) \neq \emptyset$. Therefore, A is a transitive set of $(X, f_{1,\infty})$. $\square$

Proposition 3.3 *Let $(X, f_{1,\infty})$ be a non-autonomous discrete system and A is a transitive set of $(X, f_{1,\infty})$. Then*:

(1) *U is dense in A if U is a nonempty open set of X satisfying $A \cap U \neq \emptyset$ and $(f_1^n)^{-1}(U) \subseteq U$ for every $n \in \mathbb{N}$.*

(2) *$E = A$ or E is nowhere dense in A if E is a closed invariant subset of X and $E \subseteq A$.*

(3) *$\bigcup_{n\in\mathbb{N}} f_1^n(A)$ is dense in A if A is a regular closed set of X.*

Proof (1) Since $(f_1^n)^{-1}(U) \subseteq U$ for every $n \in \mathbb{N}$, we have $\bigcup_{n\in\mathbb{N}}(f_1^n)^{-1}(U) \subseteq U$. By Proposition 3.1, we have that U is dense in A.

(2) Let $E \neq A$. Since E is a closed set of X and $E \subseteq A$, it follows that $U = X \setminus E$ is an open set of X and $U \cap A \neq \emptyset$. Moreover, E is an invariant subset of X, we have $f_1^n(E) \subseteq E$ for every $n \in \mathbb{N}$. Furthermore,

$$\bigl(f_1^n\bigr)^{-1}(U) = \bigl(f_1^n\bigr)^{-1}(X \setminus E) = \bigl(f_1^n\bigr)^{-1}(X) \setminus \bigl(f_1^n\bigr)^{-1}(E) \subseteq X \setminus E = U \quad \text{for every } n \in \mathbb{N}.$$

By the result of (1), U is dense in A. Therefore, E is nowhere dense in A.

(3) Let V^A be a nonempty open set of A. Since A is a regular closed set of X, it follows that $\mathrm{int}(V^A) \neq \emptyset$ and $\mathrm{int}(A) \neq \emptyset$. Moreover, A is a transitive set of $(X, f_{1,\infty})$, there exists $n \in \mathbb{N}$ such that $f_1^n(\mathrm{int}(A)) \cap \mathrm{int}(V^A) \neq \emptyset$. Furthermore, we have $f_1^n(A) \cap V^A \neq \emptyset$, which implies that $\bigcup_{n\in\mathbb{N}} f_1^n(A)$ is dense in A. □

Theorem 3.1 *Let $(X, f_{1,\infty})$ be a non-autonomous discrete system and A be a nonempty closed invariant set of X. Then A is a transitive set of $(X, f_{1,\infty})$ if and only if $(A, f_{1,\infty})$ is topologically transitive.*

Proof Necessity. Let V^A and U^A be two nonempty open subsets of A. For a nonempty open subset U^A of A, there exists an open set U of X such that $U^A = U \cap A$. Since A is a transitive set of $(X, f_{1,\infty})$, there exists $n \in \mathbb{N}$ such that $f_1^n(V^A) \cap U \neq \emptyset$. Moreover, A is invariant, *i.e.*, $f_1^n(A) \subseteq A$ for every $n \in \mathbb{N}$, which implies that $f_1^n(V^A) \subseteq A$. Therefore, $f_1^n(V^A) \cap A \cap U \neq \emptyset$, *i.e.*, $f_1^n(V^A) \cap U^A \neq \emptyset$. This shows that $(A, f_{1,\infty})$ is topologically transitive.

Sufficiency. Let V^A be a nonempty open set of A and U be a nonempty open set of X with $A \cap U \neq \emptyset$. Since U is an open set of X and $A \cap U \neq \emptyset$, it follows that $U \cap A$ is a nonempty open set of A. As $(A, f_{1,\infty})$ is topologically transitive, there exists $n \in \mathbb{N}$ such that $f_1^n(V^A) \cap (U \cap A) \neq \emptyset$, which implies that $f_1^n(V^A) \cap U \neq \emptyset$. This shows that A is a transitive set of $(X, f_{1,\infty})$. □

Theorem 3.2 *Let $(X, f_{1,\infty})$ be topologically transitive and A be a regular closed set of X. Then A is a transitive set of $(X, f_{1,\infty})$.*

Proof Let V^A be a nonempty set of A and U be a nonempty set of X with $A \cap U \neq \emptyset$. Since A is a regular closed set and $(X, f_{1,\infty})$ is topologically transitive, there exists $n \in \mathbb{N}$ such that $f_1^n(\mathrm{int}(V^A)) \cap U \neq \emptyset$, which implies that $f_1^n(V^A) \cap U \neq \emptyset$. Therefore, A is a transitive set of $(X, f_{1,\infty})$. □

Corollary 3.2 *Let $(X, f_{1,\infty})$ be a non-autonomous discrete system. Then $(X, f_{1,\infty})$ is topologically transitive if and only if every nonempty regular closed set of X is a transitive set of $(X, f_{1,\infty})$.*

Definition 3.1 Let $(X, f_{1,\infty})$ and $(Y, g_{1,\infty})$ be two non-autonomous discrete systems, and let $h : X \to Y$ be a continuous map and

$$g_n\big(h(x)\big) = h\big(f_n(x)\big) \quad \text{for any } n \in \mathbb{N}, x \in X.$$

(1) If $h : X \to Y$ is a surjective map, then $f_{1,\infty}$ and $g_{1,\infty}$ are said to be topologically semi-conjugate.
(2) If $h : X \to Y$ is a homeomorphism, then $f_{1,\infty}$ and $g_{1,\infty}$ are said to be topologically conjugate.

Example 3.1 Let $f_n : R \to R$ with $f_n(x) = nx$ for all $n \in \mathbb{N}$ and $x \in R$, where R is a real line, and $g_n : S^1 \to S^1$ with $g_n(e^{i\theta}) = e^{in\theta}$ for all $n \in \mathbb{N}$, where S^1 is the unite circle. Define $h : R \to S^1$ by $h(x) = e^{2\pi ix}$. It can be easily verified that h is a continuous surjective map and $h \circ f_n = g_n \circ h$. Therefore, $(R, f_{1,\infty})$ and $(S^1, g_{1,\infty})$ are topologically semi-conjugate.

Theorem 3.3 *Let $(X,f_{1,\infty})$ and $(Y,g_{1,\infty})$ be two non-autonomous discrete systems, and let $h: X \to Y$ be a semi-conjugate map. A is a nonempty closed subset of X and $h(A)$ is a closed subset of Y. Then*:

(1) *If A is a transitive set of $(X,f_{1,\infty})$, then $h(A)$ is a transitive set of $(Y,g_{1,\infty})$.*
(2) *If A is a weakly mixing set of $(X,f_{1,\infty})$ and $h(A)$ is not a singleton, then $h(A)$ is a weakly mixing set of $(Y,g_{1,\infty})$.*

Proof (1) Let $V^{h(A)}$ be a nonempty open set of $h(A)$ and U be a nonempty open set of Y with $h(A)\cap U \neq \emptyset$. Since $h(A)$ is a subspace of Y, there exists an open set V of Y such that $V^{h(A)} = V \cap h(A)$. Furthermore,

$$A \cap h^{-1}\bigl(V^{h(A)}\bigr) = A \cap h^{-1}\bigl(V \cap h(A)\bigr) = A \cap h^{-1}(V).$$

Hence, $A \cap h^{-1}(V^{h(A)})$ is an open subset of A. Since

$$h\bigl(A \cap h^{-1}\bigl(V^{h(A)}\bigr)\bigr) = h(A) \cap V^{h(A)} = V^{h(A)} \neq \emptyset,$$

then we have $A \cap h^{-1}(V^{h(A)}) \neq \emptyset$. Moreover, $U \cap h(A) \neq \emptyset$, which implies that $h^{-1}(U) \cap A \neq \emptyset$. Since A is a transitive set of $(X,f_{1,\infty})$, there exists $n \in \mathbb{N}$ such that $h^{-1}(V^{h(A)}) \cap A \cap (f_1^n)^{-1}(h^{-1}(U)) \neq \emptyset$. As h is a semi-conjugate map, *i.e.*, $g_k(h(x)) = h(f_k(x))$ for every $k \in \mathbb{N}$, $x \in X$, we have $h^{-1}(g_k)^{-1}(x) = (f_k)^{-1}h^{-1}(x)$ for every $k \in \mathbb{N}$, $x \in X$. Therefore, $h^{-1}(V^{h(A)} \cap (g_1^n)^{-1}(U)) \neq \emptyset$, which implies that $V^{h(A)} \cap (g_1^n)^{-1}U \neq \emptyset$. This shows that $h(A)$ is a transitive set of $(Y,g_{1,\infty})$.

(2) Suppose that A is a weakly mixing set of $(X,f_{1,\infty})$ and $h(A)$ is a closed subset of Y but not a singleton. Fix $k \in \mathbb{N}$. If $V_1^{h(A)}, V_2^{h(A)}, \ldots, V_k^{h(A)}$ are nonempty open subsets of $h(A)$ and $U_1, U_2, \ldots, U_k$ are nonempty open subsets of Y with $h(A) \cap U_i \neq \emptyset$, $i = 1,2,\ldots,k$. Since $h(A)$ is a subspace of Y, there exists an open subset V_i of Y such that $V_i^{h(A)} = V_i \cap h(A)$ for each $i = 1,2,\ldots,k$. But

$$A \cap h^{-1}\bigl(V_i^{h(A)}\bigr) = A \cap h^{-1}\bigl(V_i \cap h(A)\bigr) = A \cap h^{-1}(V_i),$$

then $A \cap h^{-1}(V_i^{h(A)})$ $(i = 1,2,\ldots,k)$ are open subsets of A. Since

$$h\bigl(A \cap h^{-1}\bigl(V_i^{h(A)}\bigr)\bigr) = h(A) \cap V_i^{h(A)} = V_i^{h(A)} \neq \emptyset,$$

it follows that $A \cap h^{-1}(V_i^{h(A)}) \neq \emptyset$ for each $i = 1,2,\ldots,k$. Furthermore, $h^{-1}(U_i)$ is a nonempty open subset of X with $h^{-1}(U_i) \cap A \neq \emptyset$ for each $i = 1,2,\ldots,k$. Since A is a weakly mixing set of $(X,f_{1,\infty})$, there exists $n \in \mathbb{N}$ such that $(h^{-1}(V_i^{h(A)}) \cap A) \cap (f_1^n)^{-1}(h^{-1}(U_i)) \neq \emptyset$. As h is a semi-conjugate map, *i.e.*, $g_m(h(x)) = h(f_m(x))$ for every $m \in \mathbb{N}$, $x \in X$, we have $h^{-1}(g_m)^{-1}(x) = (f_m)^{-1}h^{-1}(x)$ for every $m \in \mathbb{N}$, $x \in X$. Furthermore, $h^{-1}(V_i^{h(A)} \cap (g_1^n)^{-1}U_i) \neq \emptyset$ for each $i = 1,2,\ldots,k$, which implies that $V_i^{h(A)} \cap (g_1^n)^{-1}U_i \neq \emptyset$ for each $i = 1,2,\ldots,k$. This shows that $h(A)$ is a weakly mixing set of $(Y,g_{1,\infty})$. □

Corollary 3.3 *Let $(X,f_{1,\infty})$ and $(Y,g_{1,\infty})$ be two non-autonomous discrete systems, and let $h: X \to Y$ be a conjugate map. Then*:

(1) *$(X,f_{1,\infty})$ has a transitive set if and only if so is $(Y,g_{1,\infty})$.*
(2) *$(X,f_{1,\infty})$ has a weakly mixing set if and only if so is $(Y,g_{1,\infty})$.*

Definition 3.2 Let $(X, f_{1,\infty})$ be a non-autonomous discrete system. $f_{1,\infty}$ is a k-periodic discrete system if there exists $k \in \mathbb{Z}^+$ such that $f_{n+k}(x) = f_n(x)$ for any $x \in X$ and $n \in \mathbb{Z}^+$.

Let $(X, f_{1,\infty})$ be a k-periodic discrete system for any $k \in \mathbb{Z}^+$. Define $g =: f_k \circ f_{k-1} \circ \cdots \circ f_1$, we say that (X, g) is an induced autonomous discrete system by a k-periodic discrete system $(X, f_{1,\infty})$.

From Definition 3.2, we easily obtain the following proposition.

Proposition 3.4 *Let $(X, f_{1,\infty})$ be a k-periodic non-autonomous discrete system where (X, d) is a metric space, $g = f_k \circ f_{k-1} \circ \cdots \circ f_1$, (X, g) is its induced autonomous discrete system. Then*:

(1) *If (X, g) has a transitive set, then so is $(X, f_{1,\infty})$.*
(2) *If (X, g) has a weakly mixing set, then so is $(X, f_{1,\infty})$.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
LL (the first author) carried out the study of weakly mixing sets and transitive sets for non-autonomous discrete systems and drafted the manuscript. YS (the second author) helped to draft the manuscript. All authors read and approved the final manuscript.

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