Lee *Advances in Difference Equations* 2014, **2014**:170
http://www.advancesindifferenceequations.com/content/2014/1/170

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

The ergodic shadowing property from the robust and generic view point

Manseob Lee*

*Correspondence:
lmsds@mokwon.ac.kr
Department of Mathematics,
Mokwon University, Daejeon,
302-729, Korea

Abstract

In this paper, we discuss that if a diffeomorphisms has the C^1-stably ergodic shadowing property in a closed set, then it is a hyperbolic elementary set. Moreover, C^1-generically: if a diffeomorphism has the ergodic shadowing property in a locally maximal closed set, then it is a hyperbolic basic set.

MSC: 34D30; 37C20

Keywords: ergodic shadowing; shadowing; locally maximal; generic; Anosov

1 Introduction

Let M be a closed C^∞ manifold, and let $\mathrm{Diff}(M)$ be the space of diffeomorphisms of M endowed with the C^1-topology. Denote by d the distance on M induced from a Riemannian metric $\|\cdot\|$ on the tangent bundle TM. Let $f \in \mathrm{Diff}(M)$. For $\delta > 0$, a sequence of points $\{x_i\}_{i=a}^{b}$ $(-\infty \leq a < b \leq \infty)$ in M is called a *δ-pseudo orbit* of f if $d(f(x_i), x_{i+1}) < \delta$ for all $a \leq i \leq b-1$. For given $x, y \in M$, we write $x \rightsquigarrow y$ if for any $\delta > 0$, there is a δ-pseudo orbit $\{x_i\}_{i=a}^{b}$ $(a < b)$ of f such that $x_a = x$ and $x_b = y$. Let Λ be a closed f-invariant set. We say that f has the *shadowing property* in Λ if for every $\epsilon > 0$ there is $\delta > 0$ such that, for any δ-pseudo orbit $\{x_i\}_{i=a}^{b} \subset \Lambda$ of f $(-\infty \leq a < b \leq \infty)$, there is a point $y \in \Lambda$ such that $d(f^i(y), x_i) < \epsilon$ for all $a \leq i \leq b-1$. If $\Lambda = M$, then f has the shadowing property. The shadowing property usually plays an important role in the investigation of stability theory and ergodic theory. For instance, Sakai [1] proved that if f has the C^1-robustly shadowing property, then f is structurally stable. Now we introduce the notion of the ergodic shadowing property which was introduced and studied by [2]. Lee has shown in [3] that if f belongs to the C^1-interior of the set of all diffeomorphisms having the ergodic shadowing property, then it is structurally stable diffeomorphisms. In [4], Lee showed that if f is local star condition and has the ergodic shadowing property on the homoclinic class, then it is hyperbolic. For any $\delta > 0$, a sequence $\xi = \{x_i\}_{i\in\mathbb{Z}}$ is a *δ-ergodic pseudo orbit* of f if for $Np_n^+(\xi, f, \delta) = \{i : d(f(x_i), x_{i+1}) \geq \delta\} \cap \{0, 1, \ldots, n-1\}$, and $Np_n^-(\xi, f, \delta) = \{-i : d(f^{-1}(x_{-i}), x_{-i-1}) \geq \delta\} \cap \{-n+1, \ldots, -1, 0\}$

$$\lim_{n\to\infty} \frac{\#Np_n^+(\xi, f, \delta)}{n} = 0 \quad \text{and} \quad \lim_{n\to-\infty} \frac{\#Np_n^-(\xi, f, \delta)}{n} = 0.$$

Here $\#A$ is the number of elements of the set A. We say that f has the *ergodic shadowing property* in Λ (or $f|_\Lambda$ has *ergodic shadowing*) if for any $\epsilon > 0$, there is a $\delta > 0$ such that every δ-ergodic pseudo orbit $\xi = \{x_i\}_{i\in\mathbb{Z}} \subset \Lambda$ of f there is a point $z \in \Lambda$ such that, for

$Ns_n^+(\xi,f,z,\epsilon) = \{i : d(f^i(z),x_i) \geq \epsilon\} \cap \{0,1,\ldots,n-1\}$, and $Ns_n^-(\xi,f,z,\epsilon) = \{-i : d(f^{-i}(z),x_{-i}) \geq \epsilon\} \cap \{-n+1,\ldots,-1,0\}$,

$$\lim_{n\to\infty} \frac{\#Ns_n^+(\xi,f,z,\epsilon)}{n} = 0 \quad \text{and} \quad \lim_{n\to-\infty} \frac{\#Ns_n^-(\xi,f,z,\epsilon)}{n} = 0.$$

Note that f has the ergodic shadowing property on Λ and f has the ergodic shadowing property in Λ are different notions. That is, the shadowing point is in M or Λ. In the first notion, the shadowing point is in M. In the second notion, the shadowing point is in Λ. In this paper we consider the latter case.

We say that Λ is *locally maximal* if there is a compact neighborhood U of Λ such that

$$\bigcap_{n\in\mathbb{Z}} f^n(U) = \Lambda_f(U) = \Lambda.$$

Now, we introduce the notion of the C^1-stably ergodic shadowing property in a closed set.

Definition 1.1 Let Λ be a closed f-invariant set. We say that f has the *C^1-stably ergodic shadowing property* in Λ if

(i) there is a neighborhood U of Λ and a C^1-neighborhood $\mathcal{U}(f)$ of f such that $\Lambda_f(U) = \Lambda = \bigcap_{n\in\mathbb{Z}} f^n(U)$ (that is, Λ is locally maximal);
(ii) for any $g \in \mathcal{U}(f)$, g has the ergodic shadowing property on $\Lambda_g(U) = \bigcap_{n\in\mathbb{Z}} g^n(U)$, where $\Lambda_g(U)$ is the *continuation* of Λ.

We say that Λ is *hyperbolic* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E^s \oplus E^u$ and there exist constants $C > 0$ and $0 < \lambda < 1$ such that

$$\|D_x f^n|_{E^s_x}\| \leq C\lambda^n \quad \text{and} \quad \|D_x f^{-n}|_{E^u_x}\| \leq C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$. If $\Lambda = M$, then f is Anosov. We say that Λ is a *basic set* (resp. *elementary set*) if $f|_\Lambda$ is transitive (resp. mixing) and locally maximal. Note that if Λ is hyperbolic, then we can easily show that there is a periodic point such that the orbit of the periodic point is dense in the set. Then we get the following.

Theorem 1.2 [5, Theorem 3.3] *Let Λ be a closed f-invariant set. If f has the C^1-stably ergodic shadowing property in Λ, then it is a hyperbolic elementary set.*

Corollary 1.3 *If f belongs to the C^1-interior of the set of all diffeomorphisms having the ergodic shadowing property, then it is transitive Anosov.*

We say that a subset $\mathcal{G} \subset \mathrm{Diff}(M)$ is *residual* if $\mathcal{G}$ contains the intersection of a countable family of open and dense subsets of $\mathrm{Diff}(M)$; in this case $\mathcal{G}$ is dense in $\mathrm{Diff}(M)$. A property P is said to be *C^1-generic* if P holds for all diffeomorphisms which belong to some residual subset of $\mathrm{Diff}(M)$. We use the terminology 'for C^1-generic f' to express 'there is a residual subset $\mathcal{G} \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G} \ldots$.' In [6], Abdenur and Díaz proved that if tame diffeomorphisms has the shadowing property, then it is hyperbolic. Still open is the question if C^1-generically: f is shadowable, then is it hyperbolic?

Recently, Ahn *et al.* [7] have given a partial answer which is C^1-generically: if a locally maximal homoclinic class is shadowing, then it is hyperbolic. Lee has shown in [8] that C^1-generically: if f has the limit shadowing property on the homoclinic class, then it is hyperbolic. Inspired by this, we consider that C^1-generically: f has the ergodic shadowing property in a locally maximal closed set. Then we have the following.

Theorem 1.4 *For C^1-generic f, if f has the ergodic shadowing property in a locally maximal closed set Λ, then it is a hyperbolic elementary set. Moreover, C^1-generically: if f has the ergodic shadowing property, then it is transitive Anosov.*

2 Proof of Theorem 1.4

Let $P(f)$ be the set of periodic points of f. If $f|_\Lambda$ is transitive, then every $p \in \Lambda \cap P(f)$ is saddle, that is, there is no eigenvalues of $D_p f^{\pi(p)}$ with modulus equal to 1, at least one of them is greater than 1, at least one of them is smaller than 1, where $\pi(p)$ is the minimum period of p.

Lemma 2.1 [2, Corollary 3.5] *If f has the ergodic shadowing property in Λ, then $f|_\Lambda$ is mixing.*

By Lemma 2.1, f has the ergodic shadowing property in Λ, then $f|_\Lambda$ is mixing, and so $f|_\Lambda$ is transitive. Thus $p \in \Lambda \cap P(f)$ is neither a sink nor a source.

Lemma 2.2 [2, Lemma 3.2] *If f has the ergodic shadowing property in Λ, then f has a finite shadowing property in Λ.*

We say that f has the *finite shadowing property* on Λ if for any $\epsilon > 0$ there is $\delta > 0$ such that, for any finite δ-pseudo orbit $\{x_0, x_1, \dots, x_n\} \subset \Lambda$, there is $y \in M$ such that $d(f^i(y), x_i) < \epsilon$ for all $0 \le i < n$. In [9, Lemma 1.1.1], Pilyugin showed that f has a finite shadowing shadowing property on Λ, then f has the shadowing property on Λ.

Lemma 2.3 *Let f have the ergodic shadowing in Λ and Λ be locally maximal in U. Then the shadowing point taken from Λ.*

Proof Let f have the ergodic shadowing property in Λ, and let U be a locally maximal of Λ. For any $\epsilon > 0$, let $\delta > 0$ be the number of the ergodic shadowing property of f. Take a sequence $\gamma = \{x_i\}_{i=0}^{n}$ ($n \ge 1$) such that γ is a δ-pseudo orbit of f and $\gamma \subset \Lambda$. As in the proof of [2, Lemma 3.1], there is a δ-pseudo orbit $\eta = \{x_i\}_{i=n}^{0}$ such that $\eta \subset \Lambda$. Then we set $\xi = \{\dots, \gamma, \eta, \gamma, \eta, \dots\}$ is a δ-ergodic pseudo orbit of f. Clear that $\xi \subset \Lambda$. Since f has the ergodic shadowing property in Λ, ξ can be ergodic shadowed by some point $y \in \Lambda$. By Lemma 2.2, there is $\gamma \in \xi$ such that $d(f^i(y), x_i) < \epsilon$ for $0 \le i \le n-1$. By [9, Lemma 1.1.1], f has the shadowing property on Λ. Since Λ is locally maximal in U, the shadowing point $y \in \Lambda$. □

Let $p \in P(f)$ be a hyperbolic saddle with period $\pi(p) > 0$. Then there are the local stable manifold $W^s_\epsilon(p)$ and the local unstable manifold $W^u_\epsilon(p)$ of p for some $\epsilon = \epsilon(p) > 0$. It is easily seen that if $d(f^n(x), f^n(p)) \le \epsilon$ for all $n \ge 0$, then $x \in W^s_\epsilon(p)$, and if $d(f^n(x), f^n(p)) \le \epsilon$ for all $n \le 0$, then $x \in W^u_\epsilon(p)$. The stable manifold $W^s(p)$ and the unstable manifold $W^u(p)$

defined as following. It is well known that if p is a hyperbolic periodic point of f with period k, then the sets

$$W^s(p) = \{x \in M : f^{kn}(x) \to p \text{ as } n \to \infty\} \quad \text{and}$$
$$W^u(p) = \{x \in M : f^{-kn}(x) \to p \text{ as } n \to \infty\}$$

are C^1-injectively immersed submanifolds of M.

Lemma 2.4 *Let $p, q \in P(f)$ be hyperbolic saddles. If f has the ergodic shadowing property in a closed set Λ, then $W^s(p) \cap W^u(q) \neq \emptyset$, and $W^u(p) \cap W^s(q) \neq \emptyset$.*

Proof Let $p, q \in P(f)$ be hyperbolic saddles, and let U be a locally maximal neighborhood of Λ. Suppose that f has the ergodic shadowing property in a locally maximal Λ. Since p and q are hyperbolic, there are $\epsilon(p) > 0$ and $\epsilon(q) > 0$ as in the above. Take $\epsilon = \min\{\epsilon(p), \epsilon(q)\}/4$ and let $0 < \delta \leq \epsilon$ be the number of the ergodic shadowing property of f. For simplicity, we may assume that $f(p) = p$ and $f(q) = q$. Since f has the ergodic shadowing property in Λ, $f|_\Lambda$ is chain transitive. Then we can construct a finite δ-pseudo orbit form p to q as follows: $x_0 = p$, $x_n = q$ ($n \geq 1$), and $d(f(x_i), x_{i+1}) < \delta$ for all $0 < i < n-1$. Put (i) $x_{-i} = f^{-i}(p)$, for all $i \leq 0$, and (ii) $x_{n+i} = f^i(q)$ for all $i \geq 0$. Then we have the sequence $\xi = \{x_i\}_{i \in \mathbb{Z}} = \{\ldots, p, x_1, x_2, \ldots, x_n, x_{n+1}, \ldots\}$. It is clearly a δ-ergodic pseudo orbit of f. Since f has the ergodic shadowing property in Λ and locally maximal, by Lemma 2.2, f has the finite shadowing property on Λ and so, by [9, Lemma 1.1.1], f has the shadowing property in Λ. By the shadowing property in Λ, we can show that $\mathrm{Orb}(y) \subset W^u(p) \cap W^s(q)$, and so $W^u(p) \cap W^s(q) \neq \emptyset$. The other case is similar. □

A diffeomorphism f is *Kupka-Smale* if their periodic points of f are hyperbolic and if $p, q \in P(f)$, then $W^s(p)$ is transversal to $W^u(q)$. Then it is C^1-residual in $\mathrm{Diff}(M)$. Denote by $\mathcal{KS}(M)$ the set of all Kupka-Smale diffeomorphisms. The following was proved by [10].

Lemma 2.5 [10, Lemma 2.4] *Let Λ be locally maximal in U, and let $\mathcal{U}(f)$ be given. If for any $g \in \mathcal{U}(f)$, $p \in \Lambda_g(U) \cap P(g)$ is not hyperbolic, then there is $g_1 \in \mathcal{U}(f)$ such that g_1 has two hyperbolic periodic points $p, q \in \Lambda_{g_1}(U)$ with different indices.*

Denote by $\mathcal{F}(M)$ the set of $f \in \mathrm{Diff}(M)$ such that there is a C^1 neighborhood $\mathcal{U}(f)$ of f such that, for any $g \in \mathcal{U}(f)$, every $p \in P(g)$ is hyperbolic. In [11], Hayashi proved that $f \in \mathcal{F}(M)$ if and only if f satisfies both Axiom A and the no-cycle condition. We say that f is *the local star condition diffeomorphism* if there exist a C^1-neighborhood $\mathcal{U}(f)$ and a neighborhood U of Λ such that, for any $g \in \mathcal{U}(f)$, every $p \in \Lambda_g(U) \cap P(g)$ is hyperbolic (see [12]). Denote by $\mathcal{F}(\Lambda)$ the set of all local star diffeomorphisms. Note that there are a C^1-neighborhood $\mathcal{U}(f)$ and a neighborhood U of p such that, for all $g \in \mathcal{U}(f)$, there is a unique hyperbolic periodic point $p_g \in U$ of g with the same period as p and $\mathrm{index}(p_g) = \mathrm{index}(p)$. Here $\mathrm{index}(p) = \dim E^s_p$, and the point p_g is called the *continuation* of p.

Lemma 2.6 [13, Lemma 2.2] *There is a residual set $\mathcal{G}_1 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_1$, if for any C^1-neighborhood $\mathcal{U}(f)$ of f, there exists $g \in \mathcal{U}(f)$ such that two hyperbolic periodic points $p_g, q_g \in P(g)$ with $\mathrm{index}(p_g) \neq \mathrm{index}(q_g)$, then f has two hyperbolic periodic points $p, q \in P(f)$ with $\mathrm{index}(p) \neq \mathrm{index}(q)$.*

Lemma 2.7 *There is a residual set $\mathcal{G}_2 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_2$, if f has the ergodic shadowing property in a locally maximal Λ, then for any $p, q \in \Lambda \cap P(f)$*

$$\mathrm{index}(p) = \mathrm{index}(q).$$

Proof Let $f \in \mathcal{G}_2 = \mathcal{G}_1 \cap \mathcal{KS}(M)$, and let $p, q \in \Lambda \cap P(f)$ be hyperbolic saddles. Suppose that f has the ergodic shadowing property in a locally maximal Λ. Then by Lemma 2.4 $W^s(p) \cap W^u(q) \neq \emptyset$ and $W^u(p) \cap W^s(q) \neq \emptyset$. Since $f \in \mathcal{G}_2$, $W^s(p) \pitchfork W^u(q) \neq \emptyset$ and $W^u(p) \pitchfork W^s(q) \neq \emptyset$. This means that $p \sim q$ and so $\mathrm{index}(p) = \mathrm{index}(q)$. □

Let p be a periodic point of f. For $0 < \delta < 1$, we say that p has a δ-*weak eigenvalue* if $Df^{\pi(p)}(p)$ has an eigenvalue λ such that $(1-\delta)^{\pi(p)} < |\lambda| < (1+\delta)^{\pi(p)}$. We say that a periodic point has a *real spectrum* if all of its eigenvalues are real and *simple spectrum* if all its eigenvalues have multiplicity one. Denote by $P_h(f)$ the set of all hyperbolic periodic points of f.

Lemma 2.8 [14, Lemma 5.1] *There is a residual set $\mathcal{G}_3 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_3$:*

- *For any $\delta > 0$, if for any C^1-neighborhood $\mathcal{U}(f)$ of f there exist $g \in \mathcal{U}(f)$ and $p_g \in P_h(g)$ with a δ-weak eigenvalue, then there is $p \in P_h(f)$ with a 2δ-weak eigenvalue.*
- *For any $\delta > 0$, if $q \in P_h(f)$ with a δ-weak eigenvalue and a real spectrum, then there is $p \in P_h(f)$ with a δ-weak eigenvalue with a simple real spectrum.*

Lemma 2.9 *There is a residual set $\mathcal{G}_4 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_4$, if f has the ergodic shadowing property in a locally maximal Λ, then there exists $\eta > 0$ such that, for any $q \in \Lambda \cap P_h(f)$, q has no η-weak eigenvalues.*

Proof Let $f \in \mathcal{G}_4 = \mathcal{G}_2 \cap \mathcal{G}_3$ have the ergodic shadowing property in a locally maximal Λ. We will derive a contradiction. Suppose that, for any $\eta > 0$, there is $q \in \Lambda \cap P_h(f)$ such that q has an η-weak eigenvalue. By Franks' lemma, there is g C^1-close to f such that p is not hyperbolic. By Franks' lemma and Lemma 2.5, there is h C^1-nearby g and C^1-close to f such that h has tow hyperbolic periodic points q_h, γ_h with different indices. Since $f \in \mathcal{G}_1$, and it is locally maximal, by Lemma 2.6 f has two hyperbolic periodic points q, γ in Λ. Since f has the ergodic shadowing property in Λ, this is a contradiction by Lemma 2.7. □

Proposition 2.10 *There is a residual set $\mathcal{G}_4 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_4$, if f has the ergodic shadowing property in Λ, then $f \in \mathcal{F}(\Lambda)$.*

Proof Let $f \in \mathcal{G}_4$ have the ergodic shadowing property in a locally maximal Λ. Suppose by contradiction that $f \notin \mathcal{F}(\Lambda)$. Then there are g C^1-close to f and $q \in P(g)$ such that q has a η-weak eigenvalue. Then by Lemma 2.9, we get a contradiction. Thus $f \in \mathcal{F}(\Lambda)$. □

Proposition 2.11 [15, Proposition A] *Let $f \in \mathcal{G}_4$, and let Λ be locally maximal. If f has the ergodic shadowing property in Λ, then there are $m > 0$, $C \geq 1$ and $\lambda \in (0,1)$ such that, for any $p \in \Lambda \cap P(f)$ with $\pi(q) > m$, we have*

$$\prod_{i=0}^{\pi(p)-1} \left\| Df^m|_{E^s(f^{mi}(x))} \right\| \leq C\lambda^{\pi(p)},$$

$$\prod_{i=0}^{\pi(p)-1} \left\| Df^m|_{E^s(f^{mi}(x))} \right\| \le C\lambda^{\pi(p)} \quad \textit{and}$$

$$\left\| Df^m|_{E^s(x)} \right\| \cdot \left\| Df^{-m}|_{E^u(f^m(x))} \right\| \le \lambda,$$

where $\pi(p)$ is the period of p.

Remark 2.12 By Pugh's closing lemma, there is a residual set $\mathcal{G}_6 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_6$, if $f|_\Lambda$ is transitive, then there is a periodic orbit p_n such that $\mathrm{Orb}(p_n) \to \Lambda$ in Hausdorff metric.

Lemma 2.13 [16, Theorem 3.8] *There is residual set $\mathcal{G}_5 \subset \mathrm{Diff}(M)$ such that, for any $f \in \mathcal{G}_5$, for any ergodic measure μ of f, there is a sequence of the periodic point p_n such that $\mu_{p_n} \to \mu$ in weak* topology and $\mathrm{Orb}(p_n) \to \mathrm{Supp}(\mu)$ in Hausdorff metric.*

The following was proved by Mañé [17]. Denote by $\mathcal{M}(f|_\Lambda)$ the set of invariant probabilities on the Borel σ-algebra of Λ endowed with the weak* topology.

Lemma 2.14 *Let $\Lambda \subset M$ be a closed f-invariant set of f and $E \subset T_\Lambda M$ be a continuous invariant subbundle. If there is $m > 0$ such that*

$$\int \log \left\| Df^m|_E \right\| d\mu < 0$$

for every ergodic $\mu \in \mathcal{M}(f^m|_\Lambda)$, then E is contracting.

Proof of Theorem 1.4 Let $f \in \mathcal{G}_4 \cap \mathcal{G}_5 \cap \mathcal{G}_6$ have the ergodic shadowing property in a locally maximal Λ. Then by Proposition 2.11, we know that Λ admits a dominated splitting $T_\Lambda M = E \oplus F$. Since f has the ergodic shadowing property in Λ, by Lemma 2.1, Remark 2.12, and Lemma 2.13, there is a sequence of periodic points such that $\mathrm{Orb}(p_n) \to \mathrm{Supp}(\mu) = \Lambda$ in the Hausdorff metric. By Proposition 2.11, we have

$$\int \left\| Df^m|_E \right\| d\mu = \lim_{n\to\infty} \int \left\| Df^m|_E \right\| d\mu_{p_n} < 0.$$

By Lemma 2.14, E is contracting. Similarly, we can show that F is expanding. □

Corollary 2.15 *For C^1-generic f, if f has the ergodic shadowing property, then f is transitive Anosov.*

Competing interests
The author declares that they have no competing interests.

Acknowledgements
This work is supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (No. 2014R1A1A1A05002124).

Received: 6 January 2014 Accepted: 29 May 2014 Published: 23 June 2014

References

1. Sakai, K: Pseudo orbit tracing property and strong transversality of diffeomorphisms on closed manifolds. Osaka J. Math. **31**, 373-386 (1994)
2. Fakhari, A, Ghane, FH: On shadowing: ordinary and ergodic. J. Math. Anal. Appl. **364**, 151-155 (2010)
3. Lee, M: Diffeomorphisms with robustly ergodic shadowing. Dyn. Contin. Discrete Impuls. Syst., Ser. A Math. Anal. **20**, 747-753 (2013)
4. Lee, M: The ergodic shadowing property and homoclinic classes. J. Inequal. Appl. **2014**, 90 (2014)
5. Barzanouni, A, Honary, B: C^1-Stable ergodic shadowable invariant sets and hyperbolicity. Gen. Math. Notes **9**, 1-6 (2012)
6. Abdenur, F, Díaz, LJ: Pseudo-orbit shadowing in the C^1 topology. Discrete Contin. Dyn. Syst. **17**, 223-245 (2007)
7. Ahn, J, Lee, K, Lee, M: Homoclinic classes with shadowing. J. Inequal. Appl. **2012**, 97 (2012)
8. Lee, M: Usual limit shadowable homoclinic classes of generic diffeomorphisms. Adv. Differ. Equ. **2012**, 91 (2012)
9. Pilyugin, S: Shadowing in Dynamical Systems. Lect. Notes in Math., vol. 1706. Springer, Berlin (1999)
10. Sakai, K, Sumi, N, Yamamoto, K: Diffeomorphisms satisfying the specification property. Proc. Am. Math. Soc. **138**, 315-321 (2009)
11. Hayashi, S: Diffeomorphisms in $\mathcal{F}^1(M)$ satisfy Axiom A. Ergod. Theory Dyn. Syst. **12**, 233-253 (1992)
12. Dai, X: Dominated splitting of differentiable dynamics with C^1-topological weak-star property. J. Math. Soc. Jpn. **64**, 1249-1295 (2012)
13. Lee, M, Lee, S: Robustly transitive sets with generic diffeomorphisms. Commun. Korean Math. Soc. **28**, 581-587 (2013)
14. Arbieto, A: Periodic orbits and expansiveness. Math. Z. **269**, 801-807 (2011)
15. Yang, D, Gan, S: Expansive homoclinic classes. Nonlinearity **22**, 729-733 (2009)
16. Abdenur, F, Bonatti, C, Crovisier, C: Non-uniform hyperbolicity for C^1-generic diffeomorphisms. Isr. J. Math. **183**, 1-60 (2011)
17. Mañé, R: A proof of the C^1 stability conjecture. Publ. Math. Inst. Hautes Études Sci. **66**, 161-210 (1987)

doi:10.1186/1687-1847-2014-170
Cite this article as: Lee: The ergodic shadowing property from the robust and generic view point. *Advances in Difference Equations* 2014 **2014**:170.