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Minimum distance estimation for fractional Ornstein-Uhlenbeck type process

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Abstract

We consider a one-dimensional linear stochastic differential equation defined as $dX_t = \theta X_t\,dt + \varepsilon\,dB_t^H$, $X_0 = x_0$, with θ the unknown drift parameter, where $\{B_t^H, 0 \le t \le T\}$ is a fractional Brownian motion with $\varepsilon > 0$. The consistency and the asymptotic distribution of the minimum Skorohod distance estimator θ_ε^* of θ based on the observation $\{X_t, 0 \le t \le T\}$ is studied as $T \to +\infty$.

Keywords: long-range dependence; minimum distance estimation; consistency; asymptotic distribution

Introduction

Stochastic models having long-range dependence phenomena have been paid much attention to in view of their applications in signal processing, computer networks, and mathematical finance (see [1, 2]). The long-range dependence phenomenon is said to occur in a stationary time series $\{X_n, n \ge 0\}$ if the autocovariance functions $\rho(n) := \operatorname{cov}(X_k, X_{k+1})$ satisfy

$$\lim_{n\to\infty} \frac{\rho(n)}{cn^{-\alpha}} = 1$$

for some constant c and $\alpha \in (0,1)$. In this case, the dependence between X_k and X_{k+n} decays slowly as $n \to \infty$ and $\sum_{n=1}^{\infty} \rho(n) = \infty$.

Fractional Brownian motions are a special class of long memory processes when the Hurst parameter $H > \frac{1}{2}$. When one implements the fractional Ornstein-Uhlenbeck model, it is important to estimate the parameters in the model.

In case of diffusion type processes driven by fractional Brownian motions, the most important methods are either maximum likelihood estimation (MLE) or least square estimation (LSE). Substantial progress has been made in this direction. The problem of parameter estimation in a simple linear model driven by a fractional Brownian motion was studied in [3] in the continuous case. For the case of discrete data, the problem of parameter estimation was studied in [4, 5]. Hu and Nualart [6] studied a least squares estimator for the Ornstein-Uhlenbeck process driven by fractional Brownian motion and derived the asymptotic normality of by using Malliavin calculus. The MLE of the drift parameter has also been extensively studied. Kleptsyna and Le Breton [7] considered one-dimensional homogeneous linear stochastic differential equation driven by a fractional Brownian motion in place of the usual Brownian motion. The asymptotic behavior of the maximum likelihood estimator of the drift parameter was analyzed. Tudor and Viens [8] applied the techniques of stochastic integration with respect to fractional Brownian motion and the theory of regularity and supremum estimation for stochastic processes to study the MLE for the drift parameter of stochastic processes satisfying stochastic equations driven by a fractional Brownian motion with any level of Hölder-regularity (any Hurst parameter). Moreover, in recent years, there has been increased interest in studying the asymptotic properties of the MLE for the drift parameter in some fractional diffusion systems (see [9–11]). However, MLE has some shortcomings; its expressions of likelihood function are not explicitly computable. Moreover, MLE are not robust, which means that the properties of MLE will be changed by a slight perturbation.

In order to overcome this difficulty, the minimum distance estimation approach is proposed. For a more comprehensive discussion of the properties of the minimum distance estimation, we refer to Millar [12]. In this direction, the parameter estimation for Ornstein-Uhlenbeck process driven by Brownian motions is well developed. Kutoyants and Pilibossian [13] and Kutoyants [14] proved that $\varepsilon^{-1}(\theta_\varepsilon^* - \theta_0)$ converge in probability to the random variable ζ_T with L_1, L_2 or supremum norm and he also proved that ζ_T is asymptotically normal when $\theta_0 > 0$ as $T \to +\infty$. Hénaff [15] established the same results in the general case of a norm in some Banach space of functions on $[0, T]$. Diop and Yode [16] studied the minimum Skorohod distance estimation for a stochastic differential equation driven by a centered Lévy process. However, there have been very few studies on the minimum distance estimate for the fractional Ornstein-Uhlenbeck process. Prakasa Rao [17] studied the minimum L_1-norm estimator θ_ε^* of the drift parameter of a fractional Ornstein-Uhlenbeck type process and proved that $\varepsilon^{-1}(\theta_\varepsilon^* - \theta)$ converges in probability under $\mathbb{P}_{\theta_0}$ to a random variable ζ. Our main motivation is to obtain the minimum Skorohod distance estimator of Ornstein-Uhlenbeck process driven by fractional Brownian motions and study the asymptotic properties of this estimator.

Preliminaries

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a stochastic basis satisfying the usual conditions, *i.e.*, a filtered probability space with a filtration. $\{\mathcal{F}_t\}_{t\geq 0}$ is right continuous and $\mathcal{F}_0$ contains every $\mathbb{P}$-null set. Suppose that the processes discussed in the following are $\{\mathcal{F}_t\}_{t\geq 0}$-adapted. Further the natural filtration of a process is understood as the $\mathbb{P}$-completion of the filtration generated by this process.

Consider the parameter estimation problem for a special fractional process, *i.e.*, fractional Ornstein-Uhlenbeck type process $X = \{X_t, 0 \leq t \leq T\}$, which satisfies the following stochastic integral equation:

$$X_t = x_0 + \theta \int_0^t X_s\, ds + \varepsilon B_t^H, \quad 0 \leq t \leq T, \tag{1}$$

where the drift parameter $\theta \in \Theta = (\theta_1, \theta_2) \subseteq \mathbb{R}$ is unknown, $\varepsilon > 0$, and $B^H = \{B_t^H(t), 0 \leq t \leq T\}$ is a scalar fractional Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$. For a fractional Brownian motion B^H with Hurst parameter $H \in (\frac{1}{2}, 1)$, we mean that it is a continuous and centered Gaussian process with the covariance function

$$E\big(B_s^H B_t^H\big) = \frac{1}{2}\big[s^{2H} + t^{2H} - |s-t|^{2H}\big], \quad t \geq 0, s \geq 0. \tag{2}$$

By [18] (see Definitions 1.5.1 and 1.5.2, p.11), we introduce the following.

Definition 1 We say that an $\mathbb{R}^d$-valued random process $X = (X_t)_{t\geq 0}$ is self-similar or satisfies the property of self-similarity if for every $a > 0$ there exists $b > 0$ such that

$$\mathrm{Law}(X_{at}, t \geq 0) = \mathrm{Law}(bX_t, t \geq 0), \tag{3}$$

where $\mathrm{Law}(\cdot)$ denotes the law of random variable $\cdot$.

Remark 1 Note that (3) means that the two processes X_{at} and X_{bt} have the same finite-dimensional distribution functions, *i.e.*, for every choice $t_0, \dots, t_n$ in $\mathbb{R}$,

$$P(X_{at_0} \leq x_0, \dots, X_{at_n} \leq x_n) = P(X_{bt_0} \leq x_0, \dots, X_{bt_n} \leq x_n)$$

for every $x_0, \dots, x_n$ in $\mathbb{R}$.

Definition 2 If $b = a^H$ in the above definition, then we say that $X = (X_t)_{t\geq 0}$ is a self-similar process with Hurst index H or that it satisfies the property of (statistical) self-similar with Hurst index H. The quantity $D = 1/H$ is called the statistical fractal dimension of X.

Remark 2 Note that the law of a Gaussian random variance is determined by its expectation value and variation. By (2), it is easy to see that B^H is a self-similar process with Hurst index H. Let

$$B_T^{H*} := \sup_{0\leq t\leq T} B_t^H. \tag{4}$$

Then we conclude from the fact that B^H is a self-similar process with Hurst index H that

$$\mathrm{Law}\big(B_{at}^{H*}\big) = \mathrm{Law}\big(a^H B_t^{H*}\big), \quad a > 0, t \geq 0. \tag{5}$$

Let $x_t(\theta)$ be the solution of the above differential equation with $\varepsilon = 0$. It is obvious that

$$x_t(\theta) = x_0 e^{\theta t}, \quad 0 \leq t \leq T. \tag{6}$$

Let

$$K_H(t,s) = H(2H-1)\frac{d}{ds}\int_s^t r^{H-\frac{1}{2}}(r-s)^{H-\frac{3}{2}}\,dr, \quad 0 \leq s \leq t. \tag{7}$$

Define the minimum Skorohod distance estimator

$$\theta_\varepsilon^* := \arg\min_{\theta\in\Theta} \rho\big(X, x(\theta)\big), \tag{8}$$

the Skorohod distance $\rho(\cdot,\cdot)$

$$\rho(x,y) := \inf_{\lambda\in\Delta([0,T])}\Big(H(\lambda) + \sup_{t\in[0,T]}\big|x\big(\lambda(t)\big) - y(t)\big|\Big) \tag{9}$$

on the Skorohod space $\mathcal{D}([0,T],\mathbb{R})$.

Here

$$\Delta([0,T]) := \{\lambda | t \in [0,T], \lambda(t) \in [0,T]\}, \tag{10}$$

$\Delta([0,T])$ is continuous, strictly increasing such that $\lambda(0) = 0$ and $\lambda(T) = T$. Let

$$H(\lambda) := \sup_{s,t\in[0,T],s\neq t} \left| \log\left(\frac{\lambda(s)-\lambda(t)}{s-t}\right)\right| < \infty. \tag{11}$$

Note that the space $\mathcal{D}([0,T],\mathbb{R})$ consists of functions which are right continuous with left limits on $[0,T]$. The uniform metric coincides with Skorohod distance when relativized to $\mathcal{C}([0,T],\mathbb{R})$ the space of continuous functions on $[0,T]$.

Denote θ_0 the true parameter of θ and $\mathbb{P}_{\theta_0}^{(\varepsilon)}$ be the probability measure induced by the process $\{X_t\}$.

The following two lemmas due to Novikov and Valkeila [19] and Kutoyants and Pilibossian [13] play an important role in the limit analysis below.

Lemma 1 *Let $T > 0$, $B_T^{H*} = \sup_{0\leq t\leq T} B_t^H$ and $\{B_t^H(t), 0 \leq t \leq T\}$ be a fractional Brownian motion with Hurst parameter H, then for every $p > 0$,*

$$\mathbf{E}(B_T^{H*})^p = K(p,H)T^{pH}, \tag{12}$$

where $K(p,H) = \mathbf{E}(B_1^{H*})^p$.

Lemma 2 *Let $Z_\varepsilon(u)$, $\varepsilon > 0$, $u \in \mathbb{R}$ be a sequence of continuous functions and $Z_0(u)$ a convex function which admits a unique minimum $\xi \in \mathbb{R}$. Let L_ε, $\varepsilon > 0$ be a sequence of positive numbers such that $L_\varepsilon \to +\infty$ as $\varepsilon \to 0$. We suppose that*

$$\lim_{\varepsilon\to 0} \sup_{|u|<L_\varepsilon} \left|Z_\varepsilon(u) - Z_0(u)\right| = 0,$$

then

$$\lim_{\varepsilon\to 0} \arg\min_{|u|<L_\varepsilon} Z_\varepsilon(u) = \xi,$$

where if there are several minima of Z_ε, we choose an arbitrary one.

For any $\delta > 0$, define

$$g(\delta) := \inf_{|\theta-\theta_0|>\delta} \left\|x(\theta) - x(\theta_0)\right\|_\infty. \tag{13}$$

Note that $g(\delta) > 0$ for any $\delta > 0$.

Introduce the random variable

$$\zeta := \arg\min_{u\in\mathbb{R}} \rho\big(Y(\theta_0), ux'(\theta_0)\big), \tag{14}$$

where $x'(\theta_0) = x_0 te^{\theta_0 t}$ is the derivative of $x_t(\theta_0)$ with respect to θ_0.

It can be obtained from (1) that

$$X_t - x_t(\theta_0) = \varepsilon e^{\theta_0 t} \int_0^t e^{-\theta_0 s} \, dB_s^H. \tag{15}$$

Let

$$Y_t = e^{\theta_0 t} \int_0^t e^{-\theta_0 s} \, dB_s^H. \tag{16}$$

Note that $\{Y_t, 0 \leq t \leq T\}$ is a Gaussian process and can be interpreted as the 'derivative' of the process $\{X_t, 0 \leq t \leq T\}$ with respect to ε.

Now we investigate the parameter estimation problem of parameter θ based on the observation of a fractional Ornstein-Uhlenbeck type process $X = \{X_t, 0 \leq t \leq T\}$ satisfying the following stochastic differential equation:

$$dX_t = \theta X_t \, dt + \varepsilon \, dB_t^H, \quad 0 \leq t \leq T, X_0 = x_0, \tag{17}$$

where the drift parameter $\theta \in \Theta = (\theta_1, \theta_2) \subseteq \mathbb{R}$ is unknown and T is a fixed time. We will study its consistency as $\varepsilon \to 0$.

Consistency

Theorem 1 *For every* $p > 0$, $\delta > 0$, *we have*

$$\mathbb{P}_{\theta_0}^{(\varepsilon)}\left(\left|\theta_\varepsilon^* - \theta_0\right| > \delta\right) \leq 2^p T^{pH} K(p,H) e^{|\theta_0 T|p} \left(g(\delta)\right)^{-p} \varepsilon^p = O\left(\left(g(\delta)\right)^{-p} \varepsilon^p\right). \tag{18}$$

Proof Let the set

$$\mathcal{A}_0 = \left\{\omega : \inf_{|\theta - \theta_0| < \delta} \rho\left(X, x(\theta)\right) > \inf_{|\theta - \theta_0| > \delta} \rho\left(X, x(\theta)\right)\right\}. \tag{19}$$

Fix $\delta > 0$,

$$\mathbb{P}_{\theta_0}^{(\varepsilon)}\left(\left|\theta_\varepsilon^* - \theta_0\right| > \delta\right) = \mathbb{P}_{\theta_0}^{(\varepsilon)}(\mathcal{A}_0). \tag{20}$$

In fact, for $\omega \in \mathcal{A}_0$,

$$\begin{aligned} \inf_{|\theta - \theta_0| < \delta} \rho\left(X(\omega), x(\theta)\right) &> \inf_{|\theta - \theta_0| > \delta} \rho\left(X(\omega), x(\theta)\right) \\ &\geq \inf_{\theta \in \omega} \rho\left(X(\omega), x(\theta)\right) = \rho\left(X(\omega), x\left(\theta_\varepsilon^*\right)\right), \end{aligned}$$

thus $|\theta_\varepsilon^*(\omega) - \theta_0| > \delta$.

Conversely, if $|\theta_\varepsilon^*(\omega) - \theta_0| > \delta$, then

$$\begin{aligned} \rho\left(X(\omega), x\left(\theta_\varepsilon^*\right)\right) &\leq \inf_{|\theta - \theta_0| > \delta} \rho\left(X(\omega), x(\theta)\right) \\ &< \inf_{|\theta - \theta_0| < \delta} \rho\left(X(\omega), x(\theta)\right). \end{aligned}$$

Since

$$\begin{aligned}
&\rho\big(X, x(\theta_0)\big) \le \big\|X - x(\theta_0)\big\|_\infty, \\
&\rho\big(x(\theta), x(\theta_0)\big) = \big\|x(\theta) - x(\theta_0)\big\|_\infty, \\
&\inf_{|\theta-\theta_0|<\delta} \rho\big(x(\theta), x(\theta_0)\big) = 0,
\end{aligned}$$

then, for all $\delta > 0$,

$$\begin{aligned}
&\mathbb{P}_{\theta_0}^{(\varepsilon)}\big(\big|\theta_\varepsilon^* - \theta_0\big| > \delta\big) \\
&\quad \le \mathbb{P}_{\theta_0}^{(\varepsilon)}\Big(\inf_{|\theta-\theta_0|<\delta} \rho\big(X, x(\theta)\big) > \inf_{|\theta-\theta_0|>\delta} \big|\rho\big(X, x(\theta_0)\big) - \rho\big(x(\theta_0), x(\theta)\big)\big|\Big) \\
&\quad \le \mathbb{P}_{\theta_0}^{(\varepsilon)}\Big(\inf_{|\theta-\theta_0|<\delta} \rho\big(X, x(\theta)\big) > \inf_{|\theta-\theta_0|>\delta} \rho\big(x(\theta_0), x(\theta)\big) - \rho\big(X, x(\theta_0)\big)\Big) \\
&\quad \le \mathbb{P}_{\theta_0}^{(\varepsilon)}\Big(\inf_{|\theta-\theta_0|<\delta} \rho\big(x(\theta), x(\theta_0)\big) + 2\rho\big(X, x(\theta_0)\big) > \inf_{|\theta-\theta_0|>\delta} \rho\big(x(\theta_0), x(\theta)\big)\Big) \\
&\quad \le \mathbb{P}_{\theta_0}^{(\varepsilon)}\left(\big\|x(\theta) - x(\theta_0)\big\|_\infty > \frac{g(\delta)}{2}\right).
\end{aligned}$$

Since the process $\{X_t\}$ satisfies the stochastic differential equation (1), it follows that

$$X_t - x_t(\theta_0) = x_0 + \theta_0 \int_0^t X_s\,ds + \varepsilon B_t^H - x_t(\theta_0) = \theta_0 \int_0^t \big(X_s - x_s(\theta_0)\big)\,ds + \varepsilon B_t^H. \tag{21}$$

Then

$$\big|X_t - x_t(\theta_0)\big| = \left|\theta_0 \int_0^t \big(X_s - x_s(\theta_0)\big)\,ds + \varepsilon B_t^H\right| \le |\theta_0| \int_0^t \big|X_s - x_s(\theta_0)\big|\,ds + \varepsilon\big|B_t^H\big|. \tag{22}$$

Applying the Gronwall-Bellman lemma, we obtain

$$\sup_{0\le t\le T} \big|X - x_t(\theta_0)\big| = \big\|X - x(\theta_0)\big\|_\infty \le \varepsilon e^{|\theta_0 T|} \sup_{0\le t\le T} \big|B_t^H\big|. \tag{23}$$

Hence,

$$\begin{aligned}
\mathbb{P}_{\theta_0}^{(\varepsilon)}\left(\big\|x(\theta) - x(\theta_0)\big\|_\infty > \frac{g(\delta)}{2}\right) &\le P\left(\sup_{0\le t\le T} \big|B_t^H\big| \ge \frac{g(\delta)}{2\varepsilon e^{|\theta_0 T|}}\right) \\
&= P\left(B_t^{H*} \ge \frac{g(\delta)}{2\varepsilon e^{|\theta_0 T|}}\right).
\end{aligned} \tag{24}$$

Applying Lemma 1 and Chebyshev's inequality, for all $p > 1$, we get

$$\begin{aligned}
\mathbb{P}_{\theta_0}^{(\varepsilon)}\big(\big|\theta_\varepsilon^* - \theta_0\big| > \delta\big) &\le \mathbb{E}\big(B_t^{H*}\big)^p \left(\frac{2\varepsilon e^{|\theta_0 T|}}{g(\delta)}\right)^p \\
&= 2^p T^{pH} K(p,H) e^{|\theta_0 T|p} \big(g(\delta)\big)^{-p} \varepsilon^p \\
&= O\big(\big(g(\delta)\big)^{-p} \varepsilon^p\big).
\end{aligned} \tag{25}$$

This completes the proof. □

Remark 3 As a consequence of the above theorem, we obtain the result that θ_ε^* converges in probability to θ_0 under $P_{\theta_0}^{(\varepsilon)}$-measure as $\varepsilon \to 0$. Furthermore, the rate of convergence is of order $O(\varepsilon^p)$ for every $p > 0$.

Asymptotic distribution

Theorem 2 *As $\varepsilon \to 0$, the random variable $\varepsilon^{-1}(\theta_\varepsilon^* - \theta_0)$ converges in probability to a random variable whose probability distribution is the same as that of ζ under P_{θ_0}.*

Proof Denote $x_t'(\theta) = x_0 t e^{\theta t}$ and let

$$Z_\varepsilon(u) = \rho\bigl(Y, \varepsilon^{-1}\bigl(x(\theta_0 + \varepsilon u) - x(\theta_0)\bigr)\bigr), \tag{26}$$

$$Z_0(u) = \rho\bigl(Y, u x'(\theta_0)\bigr). \tag{27}$$

Furthermore, let

$$\begin{aligned} &u_\varepsilon^* = \varepsilon^{-1}\bigl(\theta_\varepsilon^* - \theta_0\bigr), \qquad A_\varepsilon = \bigl\{\omega : \bigl|\theta_\varepsilon^* - \theta_0\bigr| < \delta_\varepsilon\bigr\}, \quad \delta_\varepsilon = \varepsilon^\tau, \tau \in \left(\frac{1}{2}, 1\right), \\ &L_\varepsilon = \varepsilon^{\tau - 1}. \end{aligned} \tag{28}$$

For the random variable u_ε^*, we get

$$Z_\varepsilon\bigl(u_\varepsilon^*\bigr) = \min_{|u| < L_\varepsilon} Z_\varepsilon(u), \quad \omega \in A_\varepsilon. \tag{29}$$

Also we define the random variable

$$\zeta_\varepsilon := \arg\min_{|u| < L_\varepsilon} Z_0(u). \tag{30}$$

Note that, with probability 1, we get

$$\begin{aligned} &\sup_{|u|<L_\varepsilon} \bigl|Z_\varepsilon(u) - Z_0(u)\bigr| \\ &\quad = \sup_{|u|<L_\varepsilon} \Bigl| \inf_{\lambda \in \Delta([0,T])} \bigl\| Y_\lambda - \varepsilon^{-1}\bigl(x(\theta_0 + \varepsilon u) - x(\theta_0)\bigr) \bigr\|_\infty - \inf_{\lambda \in \Delta([0,T])} \bigl\| Y_\lambda - u x'(\theta_0) \bigr\|_\infty \Bigr| \\ &\quad = \sup_{|u|<L_\varepsilon} \left| \inf_{\lambda \in \Delta([0,T])} \left\| Y_\lambda - u x'(\theta_0) - \frac{1}{2}\varepsilon u^2 x''(\tilde{\theta}) \right\|_\infty - \inf_{\lambda \in \Delta([0,T])} \bigl\| Y_\lambda - u x'(\theta_0) \bigr\|_\infty \right| \\ &\quad \le \sup_{|u|<L_\varepsilon} \left[\frac{1}{2}\varepsilon u^2 \sup_{0 \le t \le T} \bigl|x''(\tilde{\theta})\bigr| \right] \\ &\quad \le \frac{\varepsilon L_\varepsilon^2}{2} |x_0| T^2 e^{(|\theta_0| + \varepsilon L_\varepsilon)T} \\ &\quad = \frac{\varepsilon^{2\tau - 1}}{2} |x_0| T^2 e^{(|\theta_0| + \varepsilon^\tau)T}, \end{aligned}$$

where $\tilde{\theta} = \theta_0 + t\varepsilon u \in (\theta_0, \theta_0 + \varepsilon u)$, $t \in (0,1)$. From Lemma 2, we get $\{Z_0(u), -\infty < u < +\infty\}$ has a unique minimum u^* with probability 1.

Furthermore, we can choose the interval $[-L, L]$ such that

$$\mathbb{P}_{\theta_0}^{(\varepsilon)}\bigl\{u_\varepsilon^* \in (-L, L)\bigr\} \ge 1 - \beta g(L)^{-p} \tag{31}$$

and

$$\mathbb{P}\left\{u^* \in (-L, L)\right\} \geq 1 - \beta g(L)^{-p}, \tag{32}$$

where $\beta > 0$. The processes $\{Z_\varepsilon(u), u \in [-L, L]\}$, and $\{Z_0(u), u \in [-L, L]\}$, satisfy the Lipschitz conditions and $Z_\varepsilon(u)$ converges uniformly to $Z_0(u)$ on $u \in [-L, L]$, so the minimizer of $Z_\varepsilon(u)$ converges to the minimizer of $Z_0(u)$. This completes the proof. □

Remark 4 It is not clear what the distribution of ζ is. It would be interesting to say something about the distribution of ζ through simulation studies even if an explicit computation of the distribution seems to be difficult.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the manuscript. All authors read and approved the final manuscript.

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