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Robust H_∞ synchronization of chaotic systems with input saturation and time-varying delay

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Abstract

This paper investigates drive-response robust synchronization of chaotic systems with disturbance, time-varying delay and input saturation via state feedback control. Sufficient conditions for achieving the synchronization of two chaotic systems are derived on the basis of the Lyapunov theory and the linear matrix inequality (LMI) technique, which is not only to guarantee the asymptotic synchronization but also to attenuate the effects of the perturbation on the overall error system to a prescribed level. Finally, an illustrative numerical simulation is also given to demonstrate the effectiveness of the proposed scheme.

Keywords: chaotic system; robust synchronization; saturation; time-varying delay

1 Introduction

In 1963, Lorenz found the first chaotic attractor in a three-dimensional autonomous system when he studied the atmosphere convection [1]. Since then, more chaotic systems have been constructed, such as Chua's circuit, logistic map, Chen system, and generalized Lorenz system (see [2–6]), and their complex behaviors have also been widely studied. Nowadays, there has been considerable interest in the control of chaos in nonlinear dynamical systems, and many different techniques, such as OGY method [7], PC technique [8], backstepping approach [9], adaptive control [10], fuzzy control [11], digital control [12], state feedback control [13], time-delay feedback control [14], sampled driving signals [15], and observer-based approach [16], have been proposed to control chaos. Since the pioneering work by Pecora and Carroll [17] who originally proposed the drive-response concept for achieving the synchronization of coupled chaotic systems, chaotic synchronization has received considerable attention due to its potential applications in physics, biology, and engineering and has become an important topic in control theory [18, 19].

However, all of these works and many others in the literature have focused on the study of chaotic synchronization between two chaotic systems without model uncertainties and external disturbance. In real physical systems, some noise or disturbance always exists, which may cause instability and poor performance. Therefore, how to reduce the effect of the noise or disturbance in synchronization process for chaotic systems has become an important issue, see [20–22]. On the other hand, there has been increasing interest in time-delay chaotic systems since the chaos phenomenon in time-delay systems was first found by Mackey and Glass [23]. For chaotic systems with time-delay and disturbance, several works have proposed the problem for various chaotic systems in the literature [24–26]. In [24], an adaptive control law was derived and applied to achieve the state lag-synchronization of two nonidentical time-delayed chaotic systems with unknown parameters. In [25], an output coupling and feedback scheme were proposed to achieve the robust synchronization of noise-perturbed chaotic systems with multiple time-delays. An impulse control was proposed by Qian and Cao [26] to synchronize two nonidentical chaotic systems with time-varying delay. Most of them are based on the fact that the time-delay is a constant, while, in real world applications, the time-delay is also varying over time. Hence the study of chaotic synchronization with time-varying delay is an important topic.

Besides, in a practical chaos system, there exist not only disturbance and varying-time delay but also the input saturation. Many literature works are based on the assumption that the actuator will not be saturated during the control process, but actuator will saturate due to its physical limitations in practice. Due to its high sensitivity to system parameters, the presence of saturation of control input may cause serious influence on system stability and performance. Hence, the derivation of controller with input saturation is an important problem. In [27], an adaptive sliding mode control scheme for Lorenz chaos subject saturating input was presented. Rehan studied the synchronization and anti-synchronization of chaotic oscillators under input saturation via simple state feedback control in [28], and the design of dynamic controller and static anti-windup compensator for Lipschitz nonlinear systems under input saturation was described in [29] and [30]. However, most of them studied the normal chaotic system without time-delay and the inner uncertainty and the external disturbance. Motivated by the above discussion, in this paper we investigate the synchronization of chaotic systems with disturbance and varying time-delay under input saturation. Based on the Lyapunov stability theory, a robust controller is designed and its robustness and stability are analytically proved. Finally, we present a numerical simulation to demonstrate the feasibility and usefulness.

This paper is organized as follows. Section 2 provides the system description. In Section 3, LMI-based conditions for chaotic synchronization are developed. In Section 4, a numerical example is given to illustrate the main result. Finally, conclusion is made in Section 5.

Standard notation is used in this paper. For a matrix M, the ith row is denoted by $M_{(i)}$. For a vector $u \in R^m$, $\mathrm{sat}(u) = \mathrm{sign}(u_{(i)}) \min(\bar{u}_{(i)}, |u_{(i)}|)$ represents the classical nonlinear saturation function, where $\bar{u}_{(i)} > 0$ denotes the ith bound on the saturation.

2 System description and preliminaries

Consider a class of uncertain chaotic systems with time-varying delay which is described by

$$\frac{dx}{dt} = (A + \Delta A)x(t) + (B + \Delta B)x\bigl(t - \tau(t)\bigr) + f(x), \tag{1}$$

where $x \in R^n$ is the state vector. The vector $f(\cdot) \in R^n$ is a continuous nonlinear vector function satisfying the Lipschitz condition $\|f(x_1) - f(x_2)\| \le \rho\|x_1 - x_2\|$ (1a), where ρ is a positive constant. $\tau(t)$ denotes the varying time-delay. A and B are known real constant matrices with suitable dimensions. ΔA and ΔB are perturbation matrices representing parametric uncertainties and are assumed to be of the following form:

$$\Delta A = H_1 F(t) E_1, \qquad \Delta B = H_2 F(t) E_2, \tag{2}$$

where H_1, H_2, E_1, and E_2 are known real constant matrices with appropriate dimensions, $F(t) \in R^{n\times n}$ is an unknown real and possibly time-varying matrix satisfying

$$F^T(t)F(t) \le I. \tag{3}$$

The uncertainties ΔA and ΔB are said to be admissible if both (2) and (3) hold. Eq. (1) is considered as the drive system and the controlled response system is given by the following differential Eq. (4):

$$\frac{dy}{dt} = (A + \Delta A)y(t) + (B + \Delta B)y\bigl(t - \tau(t)\bigr) + f(y) + C\,\mathrm{sat}(u) + w(t), \tag{4}$$

where $y(t) \in R^n$, $u \in R^m$ and $w(t) \in R^n$ are the state, the input, and the external disturbance vectors for the response system, respectively, and $\mathrm{sat}(u) \in R^m$ represents the saturated input. $C \in R^{n\times m}$ represents a constant matrix.

Define the synchronization error as $e(t) = (x(t) - y(t)) \in R^n$. Subtracting the drive system (1) from the response system (4) yields the dynamical system

$$\frac{de}{dt} = (A + \Delta A)e(t) + (B + \Delta B)e\bigl(t - \tau(t)\bigr) + f(x) - f(y) - C\,\mathrm{sat}(u) - w(t). \tag{5}$$

This paper aims at designing the controller to not only asymptotically synchronize between the drive and the response systems but also to guarantee a prescribed performance of the external perturbation attenuation γ.

Before presenting the main result, we introduce the following definition.

Definition [25] For the synchronization error system (5), it is said to have the H_∞ synchronization with the external perturbation attenuation γ if the following conditions are satisfied:

(i) With $w(t) = 0$, the dynamics error system (5) is asymptotically stable.
(ii) Given a desired positive scalar γ and under the zero-initial condition, the following performance index is satisfied:

$$J = \int_0^\infty \bigl(e^T(t)e(t) - \gamma^2 w^T(t)w(t)\bigr)\,dt \le 0. \tag{6}$$

For a positive definite diagonal matrix $W \in R^{m\times m}$, the saturation nonlinearity satisfies the classical global sector condition [31] given by

$$\phi^T(u)W\bigl(u - \phi(u)\bigr) \ge 0, \tag{7}$$

where $\phi(u) = u - \mathrm{sat}(u)$ represents the dead zone nonlinearity. This sector condition can be used to design a global controller for synchronization of nonlinear systems under input saturation. However, if global results cannot be achieved, a more general sector condition can be utilized to design a local synchronization controller. Define the following associated set:

$$S(\bar{u}) = \{\nu \in R^m; -\bar{u} \leq u - \nu \leq \bar{u}\}, \tag{8}$$

where $\bar{u} \in R^m$ represents the bound on saturation. If (8) holds, the local sector condition

$$\phi^T(u)W(\nu - \phi(u)) \geq 0 \tag{9}$$

is satisfied.

In dealing with this study, the following assumptions and lemmas are necessary for the sake of convenience.

Assumption 1 The time-delay $\tau(t)$ is a bounded and continuously differentiable function such that $0 \leq \tau(t) \leq \mu_1$ and $0 < \dot{\tau}(t) \leq \mu < 1$.

Lemma 1 [32] *Given any vector x, y of appropriate dimensions and a positive number ε, the following inequality holds*:

$$2x^T y \leq \frac{1}{\varepsilon}x^T x + \varepsilon y^T y. \tag{10}$$

Lemma 2 [33] *Let S_{11} be a regular $n \times n$ matrix, S_{12} can be an $n \times q$ matrix, and S_{22} is a regular matrix. Let a Hermitian matrix S be represented as $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix}$.*

Then the matrix S is positive definite if and only if the matrices S_{11} and $S_{22} - S_{12}^T S_{11}^{-1} S_{12}$ are positive definite.

3 Chaotic synchronization

To synchronize the drive-response systems (1) and (4), the following state feedback control law is considered:

$$u = Fe, \tag{11}$$

where $F \in R^{m\times n}$. By using $\phi(u) = u - \mathrm{sat}(u)$, (5) and (12), the overall closed-loop system becomes

$$\frac{de}{dt} = (A - CF + \Delta A)e(t) + (B + \Delta B)e(t - \tau(t)) + f(x) - f(y) + C\phi(u) - w(t). \tag{12}$$

Theorem 1 *Consider the drive-response systems* (1) *and* (2) *satisfying Assumption* 1 *and condition* (1a). *Given a scalar $\gamma > 0$ and a matrix $Q = Q^T > 0$, if there exist a matrix $X_1 = X_1^T > 0 \in R^{n\times n}$, a matrix $R = R^T > 0 \in R^{n\times n}$, a diagonal matrix $U \in R^{m\times m}$, a matrix $X_2 \in R^{m\times n}$, a matrix $X_3 \in R^{m\times n}$, and scalars $\varepsilon_i > 0$ $(i = 1, 2, 3)$ satisfying the following linear matrix inequalities* (*LMIS*):

$$-Q + R < 0, \tag{13}$$

$$\begin{bmatrix} X_1 & (X_{2(i)} - X_{3(i)})^T \\ * & \frac{\bar{u}_{(i)}^2}{\delta} \end{bmatrix} \geq 0, \quad i = 1, 2, \dots, n, \tag{14}$$

$$\begin{bmatrix} \Xi & BX_1 & X_3^T + BU & -X_1 & H_1 & H_2 & X_1Q & X_1 & X_1E_1^T \\ * & -R + \varepsilon_2 E_2^T E_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2U & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{\varepsilon_1}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\varepsilon_2}I & 0 & 0 & 0 \\ * & * & * & * & * & * & -(1-\mu)Q & 0 & 0 \\ * & * & * & * & * & * & * & -I + \varepsilon_3\rho^2 I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0 \tag{15}$$

with $\Xi = X_1A^T + AX_1 - CX_2 - X_2{}^TC^T + \frac{1}{\varepsilon_3}I$, *then the overall closed-loop system with Eq.* (5) *is* H_∞ *synchronized with the disturbance attenuation level* γ.

Proof Choose the following Lyapunov functional candidate:

$$V(t) = V_1(t) + V_2(t),$$

where $V_1(t) = e^T(t)Pe(t)$, $V_2(t) = \frac{1}{1-\mu}\int_{t-\tau(t)}^t e^T(s)Re(s)\,ds$, $P = P^T$, $R = R^T$.

First, evaluating the time derivative of $V_1(t)$ along the trajectory given in Eq. (13) gives

$$\begin{aligned} V_1(t) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\ &= e^T(t)\big(A^TP + PA - F^TC^TP - PCF\big)e(t) \\ &\quad + e^T(t)\big(\Delta A^TP + P\Delta A\big)e(t) \\ &\quad + e^T\big(t-\tau(t)\big)B^TPe(t) + e^T(t)PBe\big(t-\tau(t)\big) \\ &\quad + e^T\big(t-\tau(t)\big)\Delta B^TPe(t) + e^T(t)P\Delta Be\big(t-\tau(t)\big) \\ &\quad + \big(f(x)-f(y)\big)^TPe(t) + e^T(t)P\big(f(x)-f(y)\big) \\ &\quad + \phi^T(u)C^TPe(t) + e^T(t)PC\phi(u) - w^T(t)Pe(t) - e^T(t)Pw(t). \end{aligned} \tag{16}$$

By using Lemma 1 and the Lipschitz condition, we have

$$e^T(t)\big(\Delta A^TP + P\Delta A\big)e(t) \leq \frac{1}{\varepsilon_1}e^T(t)PH_1H_1{}^TPe(t) + \varepsilon_1 e^T(t)E_1^TE_1e(t), \tag{17}$$

$$\begin{aligned} &e^T\big(t-\tau(t)\big)\Delta B^TPe(t) + e^T(t)P\Delta Be\big(t-\tau(t)\big) \\ &\quad \leq \frac{1}{\varepsilon_2}e^T(t)PH_2H_2{}^TPe(t) + \varepsilon_2 e^T\big(t-\tau(t)\big)E_2^TE_2e\big(t-\tau(t)\big), \end{aligned} \tag{18}$$

$$\big(f(x)-f(y)\big)^TPe(t) + e^T(t)P\big(f(x)-f(y)\big) \leq e^T(t)\left(\frac{1}{\varepsilon_3}PP + \varepsilon_3\rho I\right)e(t). \tag{19}$$

Substituting Eqs. (18), (19), and (20) into Eq. (17) results in

$$\begin{aligned} \dot{V}_1(t) &\leq e^T(t)\big(A^TP + PA - F^TC^TP - PCF\big)e(t) \\ &\quad + e^T(t)\left(\varepsilon_1E_1^TE_1 + \frac{1}{\varepsilon_1}PH_1H_1{}^TP + \frac{1}{\varepsilon_2}PH_2H_2{}^TP + \frac{1}{\varepsilon_3}PP + \varepsilon_3\rho I\right)e(t) \end{aligned}$$

$$
\begin{aligned}
&+\varepsilon_2 e^T\big(t-\tau(t)\big)E_2^T E_2 e\big(t-\tau(t)\big)\\
&+e^T\big(t-\tau(t)\big)B^T Pe(t)+e^T(t)PBe\big(t-\tau(t)\big)\\
&+\phi^T(u)C^T Pe(t)+e^T(t)PC\phi(u)\\
&-w^T(t)Pe(t)-e^T(t)Pw(t).
\end{aligned}\tag{20}
$$

By using $u = Fe$, we take $v = Ge$, then the local sector conditions (8) and (9) can be rewritten as

$$S(\bar{u}) = \{v \in R^n; -\bar{u} \le (F-G)e \le \bar{u}\},\tag{21}$$

$$\phi^T(u)W\big(Ge-\phi(u)\big) \ge 0.\tag{22}$$

Consider the set $\varepsilon(P,\delta) = \{e(t) \in R^n; e^T(t)Pe(t) \le \delta\}$, then LMI (15) is obtained by including the region $e^T(t)Pe(t) \le \delta$ into $S(\bar{u})$. Hence the region $S(\bar{u})$ in (22) remains valid, which further implies that the sector condition (23) is satisfied. By using (23), we have

$$
\begin{aligned}
\dot{V}_1(t) \le{}& e^T(t)\big(A^T P+PA-F^T C^T P-PCF\big)e(t)\\
&+e^T(t)\left(\varepsilon_1 E_1^T E_1+\frac{1}{\varepsilon_1}PH_1H_1{}^T P+\frac{1}{\varepsilon_2}PH_2H_2{}^T P+\frac{1}{\varepsilon_3}PP+\varepsilon_3\rho I\right)e(t)\\
&+\varepsilon_2 e^T\big(t-\tau(t)\big)E_2^T E_2 e\big(t-\tau(t)\big)+e^T\big(t-\tau(t)\big)B^T Pe(t)+e^T(t)PBe\big(t-\tau(t)\big)\\
&+e^T(t)\big(PC+G^T W\big)\phi(u)+\phi^T(u)\big(WG+C^T P\big)e(t)-2\phi^T(u)W\phi(u)\\
&-w^T(t)Pe(t)-e^T(t)Pw(t).
\end{aligned}\tag{23}
$$

By using Assumption 1, we have

$$\dot{V}_2(t) \le \frac{1}{1-\mu}e^T(t)Re(t)-e^T\big(t-\tau(t)\big)Re\big(t-\tau(t)\big).\tag{24}$$

In order to obtain LMI (16), we give a matrix $Q = Q^T$ such that the following inequality holds:

$$-Q+R<0.\tag{25}$$

Then we have

$$
\begin{aligned}
\dot{V}(t) ={}& \dot{V}_1(t)+\dot{V}_2(t)\\
\le{}& e^T(t)\big(A^T P+PA-F^T C^T P-PCF\big)e(t)\\
&+e^T(t)\left(\frac{1}{1-\mu}Q+\varepsilon_1 E_1^T E_1+\frac{1}{\varepsilon_1}PH_1H_1{}^T P+\frac{1}{\varepsilon_2}PH_2H_2{}^T P+\frac{1}{\varepsilon_3}PP+\varepsilon_3\rho I\right)e(t)\\
&+\frac{1}{1-\mu}e^T(t)(-Q+R)e(t)+e^T\big(t-\tau(t)\big)\big(-R+\varepsilon_2 E_2^T E_2\big)e\big(t-\tau(t)\big)\\
&+e^T\big(t-\tau(t)\big)B^T Pe(t)+e^T(t)PBe\big(t-\tau(t)\big)\\
&+e^T(t)\big(PC+G^T W\big)\phi(u)+\phi^T(u)\big(WG+C^T P\big)e(t)-2\phi^T(u)W\phi(u)
\end{aligned}
$$

$$\begin{aligned}
&- w^T(t)Pe(t) - e^T(t)Pw(t) \\
&\le e^T(t)\big(A^TP + PA - F^TC^TP - PCF\big)e(t) \\
&+ e^T(t)\left(\frac{1}{1-\mu}Q + \varepsilon_1 E_1^T E_1 + \frac{1}{\varepsilon_1}PH_1{H_1}^TP + \frac{1}{\varepsilon_2}PH_2{H_2}^TP + \frac{1}{\varepsilon_3}PP + \varepsilon_3\rho I\right)e(t) \\
&+ e^T\big(t-\tau(t)\big)\big(-R + \varepsilon_2 E_2^T E_2\big)e\big(t-\tau(t)\big) \\
&+ e^T\big(t-\tau(t)\big)B^TPe(t) + e^T(t)PBe\big(t-\tau(t)\big) \\
&+ e^T(t)\big(PC + G^TW\big)\phi(u) + \phi^T(u)\big(WG + C^TP\big)e(t) - 2\phi^T(u)W\phi(u) \\
&- w^T(t)Pe(t) - e^T(t)Pw(t).
\end{aligned} \tag{26}$$

Define a functional $J(e(t), w(t))$ as follows:

$$J\big(e(t), w(t)\big) = \dot{V}(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t). \tag{27}$$

Substituting (27) into (28) yields

$$J\big(e(t), w(t)\big) \le \begin{pmatrix} e(t) \\ e(t-\tau(t)) \\ \phi(u) \\ w(t) \end{pmatrix}^T \Omega \begin{pmatrix} e(t) \\ e(t-\tau(t)) \\ \phi(u) \\ w(t) \end{pmatrix},$$

where

$$\Omega = \begin{bmatrix} \Omega_1 & PB & G^TW + PC & P \\ * & -R + \varepsilon_2 E_2^T E_2 & 0 & 0 \\ * & * & -2W & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix},$$

$$\begin{aligned}
\Omega_1 &= A^TP + PA - F^TC^TP - PCF \\
&\quad + \frac{1}{1-\mu}Q + \varepsilon_1 E_1^T E_1 + \frac{1}{\varepsilon_1}PH_1{H_1}^TP + \frac{1}{\varepsilon_2}PH_2{H_2}^TP + \frac{1}{\varepsilon_3}PP + \varepsilon_3\rho I + I.
\end{aligned}$$

From the above, if the following inequality holds:

$$\Omega = \begin{bmatrix} \Omega_1 & PB & G^TW + PC & P \\ * & -R + \varepsilon_2 E_2^T E_2 & 0 & 0 \\ * & * & -2W & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{28}$$

where

$$\begin{aligned}
\Omega_1 &= A^TP + PA - F^TC^TP - PCF \\
&\quad + \frac{1}{1-\mu}Q + \varepsilon_1 E_1^T E_1 + \frac{1}{\varepsilon_1}PH_1{H_1}^TP + \frac{1}{\varepsilon_2}PH_2{H_2}^TP + \frac{1}{\varepsilon_3}PP + \varepsilon_3\rho I + I.
\end{aligned}$$

Applying the Schur complement and congruence transform by using diag $(P^{-1}, I, W^{-1}, I, I, I, I, I, I, I)$, and further, substituting $P^{-1} = X_1$, $W^{-1} = U$, $X_2 = FX_1$, $X_3 = GX_1$, LMI (16) is

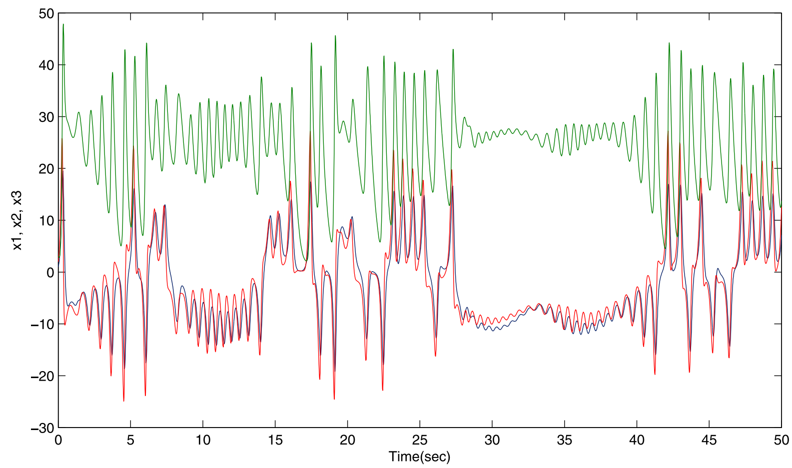

Figure 1 Chaotic behavior of the Lorenz system with time-delays.

obtained. Then integrating the function in (26) yields

$$V(\infty) - V(0) + \int_0^\infty \left(e^T(t)e(t) - \gamma^2 w^T(t)w(t)\right) dt \le 0.$$

With the zero-initial condition, we have

$$\int_0^\infty \left(e^T(t)e(t) - \gamma^2 w^T(t)w(t)\right) dt \le 0,$$

which completes the proof of Theorem 1. □

4 Examples and simulation results

To demonstrate the validity of the proposed synchronization approach with input saturation and time-delays, we consider the Lorenz chaotic system with:

$$A = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2/3 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0 & 0.1 & 0 \\ 0.1 & 0 & 0.2 \end{bmatrix}, \qquad H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 2 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \qquad f(x) = \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix}, \qquad \bar{u} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}.$$

The Lipschitz constant is chosen as $\rho = 1$, and the parameter δ is given as $\delta = 1$. The disturbance is selected as $w(t) = (0.1\sin 10t \quad 0.2\sin 20t \quad 0.1\sin 30t)^T$. For convenience, we choose $\tau(t) = 1$.

The chaotic behavior of Lorenz system with time-delays is shown in Figures 1 and 2.

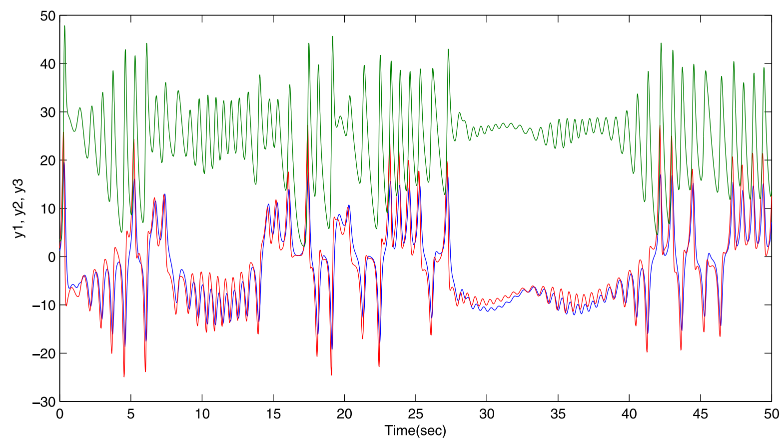

Figure 2 Chaotic behavior of the Lorenz system with time-delays and disturbance.

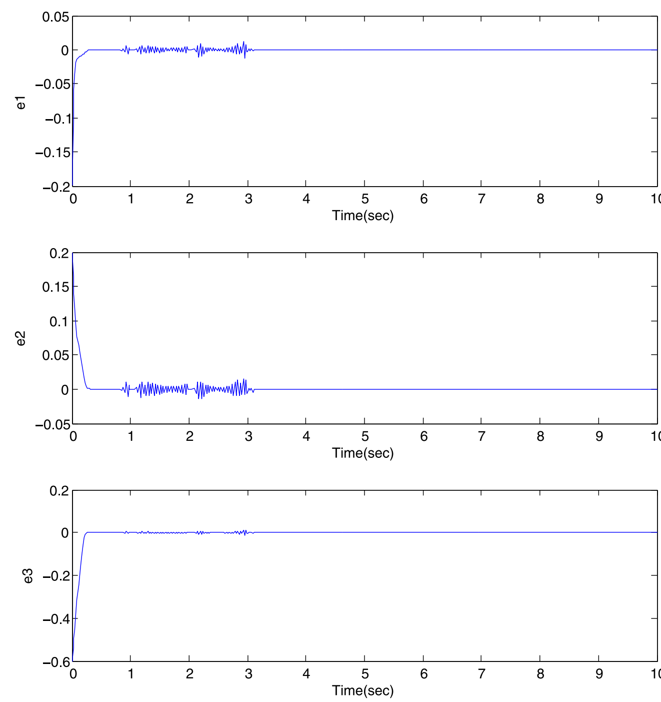

Figure 3 Synchronization error of the Lorenz system without the disturbance $w(t)$.

By applying the conditions in Theorem 1 with $\varepsilon_1 = \frac{1}{4}$, $\varepsilon_2 = \frac{1}{3}$, $\varepsilon_3 = \frac{1}{5}$, and the disturbance attenuation $\gamma = 0.5$, we can obtain the following matrices:

$$F = \begin{bmatrix} 29.4692 & 16.4469 & 1.5622 \\ 25.9050 & 39.8604 & 7.1042 \\ 1.6037 & 4.9806 & 28.5541 \end{bmatrix}, \qquad G = \begin{bmatrix} 28.6094 & 16.1171 & 1.4984 \\ 25.5716 & 39.5272 & 7.0591 \\ 1.5686 & 4.9206 & 27.6124 \end{bmatrix},$$

$$P = \begin{bmatrix} 4.7185 & 2.7094 & 0.1772 \\ 2.7094 & 4.1320 & 0.7937 \\ 0.1772 & 0.7937 & 4.1079 \end{bmatrix}, \qquad R = \begin{bmatrix} 1.1866 & -0.0227 & 0.0027 \\ -0.0227 & 1.6894 & -0.0052 \\ 0.0027 & -0.0052 & 1.1703 \end{bmatrix}.$$

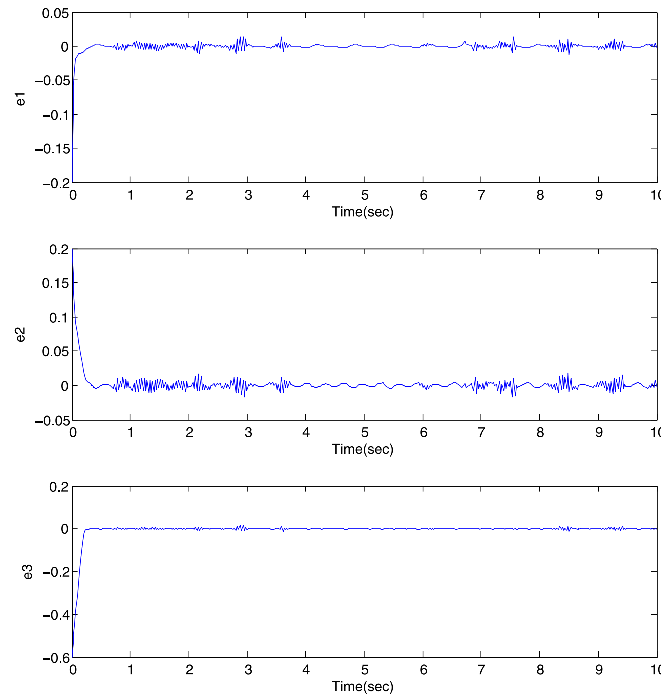

Figure 4 Synchronization error of the Lorenz system with the disturbance $w(t)$.

Applying the controller $u = Fe$ without the disturbance signal, the synchronization error between the drive system and the response system with the initial conditions $x_0 = (1 \quad 2 \quad 3)^T$ and $y_0 = (1.2 \quad 1.8 \quad 3.6)^T$, respectively, is shown in Figure 3, which implies that the synchronization error converges to zero. Figure 4 shows that the effect of the disturbance $w(t)$ on the dynamic error system has been reduced within a prescribed level to $\gamma = 0.5$ by the control gain F.

5 Conclusions

The problem of robust H_∞ synchronization for an uncertain chaotic system with time-varying delay and input saturation has been presented. Based on the Lyapunov theory and the LMI technique, the sufficient condition has been derived not only to guarantee the asymptotic synchronization but also to ensure a prescribed perturbation attenuation performance. Finally, a simulation example is presented to verify the validity of the proposed method.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors have made the same contribution. All authors read and approved the final manuscript.

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