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Robust H_∞ control of switched stochastic systems with time delays under asynchronous switching

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Abstract

This paper investigates the problem of robust H_∞ control for a class of switched stochastic systems with time delays under asynchronous switching, where the asynchronous switching means that the switching of the controllers has a lag to the switching of system modes. The parameter uncertainties are allowed to be norm bounded. Firstly, by using the average dwell time approach, the stability criterion and H_∞ performance analysis for the underlying systems are developed. Then, based on the obtained results, sufficient conditions for the existence of admissible asynchronous switching controllers which guarantee that the resulting closed-loop systems are mean-square exponentially stable with H_∞ performance are derived. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

Keywords: switched stochastic systems; time delays; H_∞ control; average dwell time; asynchronous switching

1 Introduction

Switched systems are a kind of hybrid systems composed of a family of subsystems and a logical rule that orchestrates switching between these subsystems. Due to the physical properties or various environmental factors, many real-world systems can be modeled as switched systems, such as networked control systems [1, 2], robot control systems [3], and so on. Switched systems have drawn increasing attention during the past decades due to their wide applications. Common Lyapunov function method [4], multiple Lyapunov function method [5] and average dwell time approach [6, 7] have been proposed to study the stability of such systems.

It is worth pointing out that time delay phenomenon may cause systems to be unstable or have poor performance. Many scholars have devoted their energy to the study of switched systems with time delays, and some useful results have been proposed in [8–11]. The exponential stability and L_2-gain analysis for switched delay systems were investigated by employing the average time method in [8, 9]. The asymptotical stability and H_∞ control of switched delay systems were researched in [10]. The problem of delay-independent minimum dwell time was discussed in [11], and sufficient conditions were presented to guarantee the exponential stability of switched delay systems.

On the other hand, stochastic disturbance may not be ignored in some practical systems. Some useful results on stochastic delay systems have been established in [12, 13].

Moreover, the stability analysis of switched stochastic delay-free systems was investigated in [14]. Sufficient conditions of mean-square exponential stability for switched stochastic delay systems were presented in [15, 16]. In [17], the H_∞ control problems for continuous-time switched stochastic systems were considered. The $l_2 - l_\infty$ filtering problem for a class of nonlinear switched stochastic systems was addressed in [18]. It should be pointed out that the aforementioned results are based on a common assumption that the switching of the controller is synchronized with the switching of the system. However, as stated in [19–21], there inevitably exists asynchronous switching in actual operation (usually the switching of the controller lags behind that of the system). Thus, it is necessary to design asynchronously switched controllers for switched stochastic systems. Recently, some work on asynchronously switched control of switched stochastic delay-free systems has been done in [22–24]. Robust reliable control of switched stochastic systems under asynchronous switching was studied in [22], and robust H_∞ reliable control of switched stochastic nonlinear systems was researched in [23]. In [24], the problem of robust H_∞ filtering of switched stochastic delay-free systems under asynchronous switching was investigated, and the stabilization problem for a class of switched stochastic systems with time delays under asynchronous switching was addressed in [25]. However, to the best of our knowledge, the issue of asynchronously switched H_∞ control for switched stochastic systems with time delays has not yet been fully investigated to date, which motivates the present investigation.

In this paper, we are interested in investigating the robust H_∞ control problem for switched stochastic systems with time delays under asynchronous switching. The main contributions of this paper can be summarized as follows: (i) By constructing an appropriate Lyapunov-Krasovskii functional, the extended mean-square exponential stability result with H_∞ performance for the general switched stochastic delay systems is derived for the first time; (ii) The asynchronously switched H_∞ control problem for the underlying systems is studied, and sufficient conditions for the existence of a state feedback controller are formulated in a set of LMIs (linear matrix inequalities). Compared with the existing results presented in [22–25], the proposed conditions bring some convenience for solving the designed controllers.

The remainder of the paper is organized as follows. In Section 2, problem statement and some useful lemmas are given. In Section 3, a criterion of mean-square exponential stability with H_∞ performance for the general switched stochastic delay systems is developed by using the average dwell time approach. Then, sufficient conditions for the existence of admissible asynchronous switching H_∞ controllers are derived. In Section 4, a numerical example is given to illustrate the effectiveness of the proposed approach. Finally, concluding remarks are provided in Section 5.

Notations

In this paper, the superscript 'T' denotes the transpose, and the symmetric term in a matrix is denoted by *. The notation $X > Y$ ($X \geq Y$) means that matrix $X - Y$ is positive definite (positive semi-definite, respectively). R^n denotes the n-dimensional Euclidean space. $\|x(t)\|$ denotes the Euclidean norm. $L_2[t_0, \infty)$ is the space of square integrable vector-valued functions on $[t_0, \infty)$, and t_0 is the initial time. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. I is an identity matrix with appropriate dimension. $\operatorname{diag}\{a_i\}$ denotes a diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$.

2 Problem formulation and preliminaries

Consider the following switched stochastic systems with time-delays:

$$dx(t) = \left[\hat{A}_{\sigma(t)}x(t) + \hat{A}_{d\sigma(t)}x(t-\tau) + B_{\sigma(t)}u(t) + G_{\sigma(t)}v(t)\right]dt + \hat{D}_{\sigma(t)}x(t)\,dw(t), \tag{1a}$$

$$z(t) = M_{\sigma(t)}x(t), \tag{1b}$$

$$x(s) = \varphi(s), \quad s \in [t_0 - \tau, t_0], \tag{1c}$$

where $x(t) \in R^n$ is the state vector, $\varphi(s) \in R^n$ is the vector-valued initial function, $v(t) \in R^p$ is the disturbance input belonging to $L_2[t_0, \infty)$, $u(t) \in R^q$ is the control input, $w(t)$ is a one-dimensional zero-mean Wiener process on a probability space (Ω, F, P) satisfying

$$E\{dw(t)\} = 0, \qquad E\{dw^2(t)\} = dt, \tag{2}$$

where Ω is the sample space, F is σ-algebras of subsets of the sample space and P is the probability measure on F, $E\{\cdot\}$ is the expectation operator.

The switching signal $\sigma(t) : [0, \infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is a piecewise constant function of time, $\sigma(t) = i \in \underline{N}$ means that the ith subsystem is active; N is the number of subsystems. B_i, G_i and M_i, $i \in \underline{N}$, are real-valued matrices with appropriate dimensions. $\hat{A}_i$, $\hat{A}_{di}$ and $\hat{D}_i$ are uncertain real matrices with appropriate dimensions and can be written as

$$[\hat{A}_i \quad \hat{A}_{di} \quad \hat{D}_i] = [A_i \quad A_{di} \quad D_i] + H_i F_i(t)[E_{1i} \quad E_{2i} \quad E_{3i}], \tag{3}$$

where A_i, A_{di} and D_i are known real-value matrices with appropriate dimensions, $F_i(t)$ is unknown time-varying matrix satisfying

$$F_i^T(t)F_i(t) \le I. \tag{4}$$

For the sake of simplicity, $F_i(t)$ is written as F_i in this paper.

The system switching sequence can be described as $\sigma : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k)), \ldots\}$, where t_0 is the initial time and t_k denotes the kth switching instant.

Without loss of generality, denote $\sigma'(t)$ as the switching signal of the controller, and it can be described as

$$\sigma' : \left\{\left(t_0, \sigma'(t_0)\right), \left(t_1 + \Delta_1, \sigma'(t_1 + \Delta_1)\right), \ldots, \left(t_k + \Delta_k, \sigma'(t_k + \Delta_k)\right), \ldots\right\},$$
$$\sigma'(t_k + \Delta_k) \in \underline{N}, \quad k = 1, 2, \ldots,$$

where $\sigma'(t_0) = \sigma(t_0)$, $\sigma'(t_k + \Delta_k) = \sigma(t_k)$, $0 < \Delta_k < \inf_{k \ge 1}(t_{k+1} - t_k)$.

Considering the existence of asynchronous switching, the system control input is given by $u(t) = K_{\sigma'(t)}x(t)$.

Remark 1 The delayed period $\Delta_k > 0$ ($\Delta_k < 0$) denotes the time that the switching instant of the controller lags behind (exceeds) that of the system, and it is called the mismatched period. Throughout this paper, we only consider the case where $\Delta_k > 0$.

Remark 2 $\Delta_k < \inf_{k \ge 1}(t_{k+1} - t_k)$ guarantees that there always exists a period during which the controller and the system operate synchronously, and the period is said to be the matched period.

Definition 1 System (1a)-(1c) with $\nu(t) = 0$ is said to be exponentially stable in the mean-square sense under the switching signal $\sigma(t)$ if there exist scalars $\kappa > 0$ and $\alpha > 0$ such that the solution $x(t)$ of the system satisfies

$$E\{\|x(t)\|^2\} \le \kappa e^{-\alpha(t-t_0)} \sup_{t_0-\tau\le s\le t_0} E\{\|\varphi(s)\|^2\}, \quad \forall t \ge t_0. \tag{5}$$

Definition 2 [6] For any $T_2 > T_1 \ge t_0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval $[T_1, T_2)$. If

$$N_\sigma(T_1, T_2) \le N_0 + \frac{T_2 - T_1}{T_a} \tag{6}$$

holds for given $N_0 \ge 0$ and $T_a > 0$, then the constant T_a is called the average dwell time. As commonly used in the literature, we choose $N_0 = 0$.

Definition 3 [26] For any $\lambda > 0$ and $\gamma > 0$, system (1a)-(1c) is said to be exponentially stable in the mean-square sense and has a prescribed weighted H_∞ performance level γ if the following conditions are satisfied:

(a) When $\nu(t) = 0$, system (1a)-(1c) is exponentially stable in the mean-square sense;
(b) Under zero initial condition, the output $z(t)$ satisfies

$$E\left\{\int_{t_0}^{\infty} e^{-\lambda(s-t_0)} z^T(s)z(s)\,ds\right\} \le \gamma^2 \int_{t_0}^{\infty} \nu^T(s)\nu(s)\,ds, \quad \forall 0 \ne \nu(t) \in L_2[t_0, \infty). \tag{7}$$

Lemma 1 [27] *Let U, V, W and X be constant matrices of appropriate dimensions, and let X satisfy $X = X^T$, then for all $V^TV \le I$, $X + UVW + W^TV^TU^T < 0$ if and only if there exists a scalar $\varepsilon > 0$ such that*

$$X + \varepsilon UU^T + \varepsilon^{-1}W^TW < 0. \tag{8}$$

3 Main results

3.1 Stability and H_∞ performance analysis

Consider the following switched stochastic delay systems:

$$dx(t) = \left[f_{\sigma(t)}\big(t, x(t), x(t-\tau)\big) + g_{\sigma(t)}\big(t, \nu(t)\big)\right]dt + h_{\sigma(t)}\big(t, x(t)\big)\,dw(t), \tag{9a}$$

$$z(t) = \eta_{\sigma(t)}\big(t, x(t)\big), \tag{9b}$$

$$x(s) = \varphi(s), \quad s \in [t_0 - \tau, t_0]. \tag{9c}$$

Consider the Lyapunov function $V(t, x(t)) = V_{\sigma(t)}(t, x(t))$ (which is written as $V_{\sigma(t)}(t)$ in what follows). When the ith subsystem is activated, according to the *Itô* formula, along the trajectory of the ith subsystem, we have

$$E\{dV_i(t)\} = E\{\mathcal{L}V_i(t)\,dt\}, \tag{10}$$

$$\begin{aligned}\mathcal{L}V_i(t) = {} & V_{it}(t) + V_{ix}(t)\left[f_i\big(t, x(t), x(t-\tau)\big) + g_i\big(t, \nu(t)\big)\right] \\ & + \frac{1}{2}\operatorname{trace}\left[h_i^T\big(t, x(t)\big)V_{ixx}(t)h_i\big(t, x(t)\big)\right],\end{aligned} \tag{11}$$

where $V_{it}(t) = \frac{\partial V_i(t)}{\partial t}$, $V_{ix}(t) = (\frac{\partial V_i(t)}{\partial x_1}, \ldots, \frac{\partial V_i(t)}{\partial x_n})$, $V_{ixx}(t) = (\frac{\partial^2 V_i(t)}{\partial x_k\,\partial x_l})_{n\times n}$.

Let $T_\downarrow(t_a, t_b)$ and $T_\uparrow(t_a, t_b)$ denote unions of the dispersed intervals during which the Lyapunov function is decreasing and increasing within the time interval $[t_a, t_b)$, and let $T^-(t_a, t_b)$ and $T^+(t_a, t_b)$ represent the length of $T_\downarrow(t_a, t_b)$ and $T_\uparrow(t_a, t_b)$, respectively. Denote $\Gamma(t) = z^T(t)z(t) - \gamma^2 v^T(t)v(t)$.

Lemma 2 *Consider system* (9a)-(9c), *for given scalars* $\alpha > 0$ *and* $\beta > 0$, *if there exist* C^1 *functions* $V(t) = V_{\sigma(t)}(t)$, *and positive scalars* κ_1 *and* κ_2 *such that*

$$\kappa_1 E\{\|x(t)\|^2\} \le E\{V_{\sigma(t)}(t)\} \le \kappa_2 \sup_{t-\tau \le s \le t} E\{\|x(s)\|^2\}, \tag{12}$$

$$\begin{cases} \mathcal{L}V_{\sigma(t)}(t) + \alpha V_{\sigma(t)}(t) + \Gamma(t) \le 0, & t \in T_\downarrow(t_0, t), \\ \mathcal{L}V_{\sigma(t)}(t) - \beta V_{\sigma(t)}(t) + \Gamma(t) \le 0, & t \in T_\uparrow(t_0, t), \end{cases} \tag{13}$$

hold, then under the average dwell time scheme

$$\frac{T^-(t_0, t)}{T^+(t_0, t)} \ge \frac{\beta + \lambda}{\alpha - \lambda}, \quad 0 < \lambda < \alpha, \tag{14}$$

$$T_a > T_a^* = \frac{\ln \mu}{\lambda}, \tag{15}$$

the system is exponentially stable in the mean-square sense and has a prescribed weighted H_∞ *performance level* γ, *and* $\mu \ge 1$ *satisfies*

$$E\{V_{\sigma(t_k)}(t_k)\} \le \mu E\{V_{\sigma(t_k^-)}(t_k^-)\}, \tag{16}$$

where t_k $(k = 1, 2, \ldots)$ *is the kth switching instant.*

Proof From (10) and (13), it holds that

$$E\left\{\frac{d}{dt}\left(e^{\alpha t} V_{\sigma(t)}(t)\right)\right\} \le -E\{e^{\alpha t}\Gamma(t)\}, \quad t \in T_\downarrow(t_0, t), \tag{17}$$

$$E\left\{\frac{d}{dt}\left(e^{-\beta t} V_{\sigma(t)}(t)\right)\right\} \le -E\{e^{-\beta t}\Gamma(t)\}, \quad t \in T_\uparrow(t_0, t). \tag{18}$$

For any $t \in [t_l, t_{l+1})$, integrating both sides of (17) and (18), we have

$$E\{V_{\sigma(t)}(t)\} \le e^{-\alpha T^-(t_l,t)+\beta T^+(t_l,t)} E\{V_{\sigma(t)}(t_l)\} - E\left\{\int_{t_l}^t e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\right\}. \tag{19}$$

Then, according to (16), (19) and $N_\sigma(t_0, t)$ in Definition 2, one obtains that

$$\begin{aligned} &E\{V_{\sigma(t)}(t)\} \\ &\quad \le \mu e^{-\alpha T^-(t_l,t)+\beta T^+(t_l,t)} E\{V_{\sigma(t_l^-)}(t_l^-)\} - E\left\{\int_{t_l}^t e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\right\} \\ &\quad \le \mu e^{-\alpha T^-(t_{l-1},t)+\beta T^+(t_{l-1},t)} E\{V_{\sigma(t_{l-1})}(t_{l-1})\} - \mu E\left\{\int_{t_{l-1}}^{t_l} e^{-\alpha T^-(s,t_l)+\beta T^+(s,t_l)}\Gamma(s)\,ds\right\} \\ &\qquad - E\left\{\int_{t_l}^t e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\right\} \end{aligned}$$

$$
\begin{aligned}
&\le \mu^2 e^{-\alpha T^-(t_{l-1},t)+\beta T^+(t_{l-1},t)} E\bigl\{V_{\sigma(t_{l-1}^-)}\bigl(t_{l-1}^-\bigr)\bigr\} - \mu E\biggl\{\int_{t_{l-1}}^{t_l} e^{-\alpha T^-(s,t_l)+\beta T^+(s,t_l)}\Gamma(s)\,ds\biggr\} \\
&\quad - E\biggl\{\int_{t_l}^{t} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\} \\
&\le \cdots \\
&\le \mu^{N_\sigma(t_0,t)} e^{-\alpha T^-(t_0,t)+\beta T^+(t_0,t)} E\bigl\{V_{\sigma(t_0)}(t_0)\bigr\} - \mu^{N_\sigma(t_0,t)} E\biggl\{\int_{t_0}^{t_1} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\} \\
&\quad - \cdots - \mu E\biggl\{\int_{t_{l-1}}^{t_l} e^{-\alpha T^-(s,t_l)+\beta T^+(s,t_l)}\Gamma(s)\,ds\biggr\} - E\biggl\{\int_{t_l}^{t} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\} \\
&\le \mu^{N_\sigma(t_0,t)} e^{-\alpha T^-(t_0,t)+\beta T^+(t_0,t)} E\bigl\{V_{\sigma(t_0)}(t_0)\bigr\} \\
&\quad - E\biggl\{\int_{t_0}^{t} \mu^{N_\sigma(s,t)} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\} \\
&\le e^{-(\lambda-\frac{\ln\mu}{T_a})(t-t_0)} E\bigl\{V_{\sigma(t_0)}(t_0)\bigr\} - E\biggl\{\int_{t_0}^{t} \mu^{N_\sigma(s,t)} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\}.
\end{aligned}
\tag{20}
$$

In order to prove that system (9a)-(9c) is exponentially stable in the mean-square sense and has a prescribed weighted H_∞ performance level γ, two conditions in Definition 3 should be satisfied.

(a) When $v(t) = 0$, noticing that $\Gamma(t) = z^T(t)z(t)$, it can be obtained from (20) that

$$
\begin{aligned}
E\bigl\{V_{\sigma(t)}(t)\bigr\} &\le e^{-(\lambda-\frac{\ln\mu}{T_a})(t-t_0)} E\bigl\{V_{\sigma(t_0)}(t_0)\bigr\} \\
&\quad - E\biggl\{\int_{t_0}^{t} \mu^{N_\sigma(s,t)} e^{-\alpha T^-(s,t)+\beta T^+(s,t)} z^T(s)z(s)\,ds\biggr\} \\
&\le e^{-(\lambda-\frac{\ln\mu}{T_a})(t-t_0)} E\bigl\{V_{\sigma(t_0)}(t_0)\bigr\}.
\end{aligned}
$$

Then, according to Definition 1, we can obtain from (12) that system (9a)-(9c) is exponentially stable in the mean-square sense.

(b) Under zero initial condition, it follows from (20) that

$$
E\biggl\{\int_{t_0}^{t} \mu^{N_\sigma(s,t)} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\} \le 0. \tag{21}
$$

Multiplying both sides of (21) by $\mu^{-N_\sigma(t_0,t)}$ leads to

$$
E\biggl\{\int_{t_0}^{t} \mu^{-N_\sigma(t_0,s)} e^{-\alpha T^-(s,t)+\beta T^+(s,t)}\Gamma(s)\,ds\biggr\} \le 0. \tag{22}
$$

From (14), we get that there exists a scalar function $\lambda^* = \phi(s)$ satisfying

$$
\frac{T^-(s,t)}{T^+(s,t)} = \frac{\beta + \lambda^*}{\alpha - \lambda^*}, \tag{23}
$$

where $0 < \lambda^* < \alpha$.

Notice that $\Gamma(t) = z^T(t)z(t) - \gamma^2 v^T(t)v(t)$, then combining (22) and (23) yields

$$E\left\{\int_{t_0}^{t} \mu^{-N_\sigma(t_0,s)} e^{-\lambda^*(t-s)} z^T(s)z(s)\,ds\right\} \le \gamma^2 \int_{t_0}^{t} \mu^{-N_\sigma(t_0,s)} e^{-\lambda^*(t-s)} v^T(s)v(s)\,ds. \tag{24}$$

By Definition 2, we have

$$E\left\{\int_{t_0}^{t} e^{-\lambda^*(t-s)-\lambda(s-t_0)} z^T(s)z(s)\,ds\right\} \le \gamma^2 \int_{t_0}^{t} e^{-\lambda^*(t-s)} v^T(s)v(s)\,ds. \tag{25}$$

Integrating both sides of (25) from $t = t_0$ to ∞, inequality (7) is obtained.

The proof is completed. □

Remark 3 Note that the stability analysis of switched systems with stable and unstable subsystems has been studied in [7], and the proposed method is extended to system (9a)-(9c) in the paper. In Lemma 2, all the active subsystems during the time interval $T_\uparrow(t_0, t)$ are required to be unstable (but bounded), and all the active subsystems during the time interval $T_\downarrow(t_0, t)$ are required to be stable. By limiting the lower bound of average dwell time, the stability of system (9a)-(9c) is guaranteed.

3.2 Robust H_∞ control

In this subsection, we focus on the robust H_∞ control for switched stochastic systems with time delays under asynchronous switching. Considering system (1a)-(1c), under the asynchronous switching controller $u(t) = K_{\sigma'(t)}x(t)$, the resulting closed-loop system is given by

$$dx(t) = \left[(\hat{A}_{\sigma(t)} + B_{\sigma(t)}K_{\sigma'(t)})x(t) + \hat{A}_{d\sigma(t)}x(t-\tau) + G_{\sigma(t)}v(t)\right]dt + \hat{D}_{\sigma(t)}x(t)\,dw(t), \tag{26a}$$

$$z(t) = M_{\sigma(t)}x(t), \tag{26b}$$

$$x(s) = \varphi(s), \quad s \in [t_0 - \tau, t_0]. \tag{26c}$$

The following theorem gives sufficient conditions for the existence of asynchronous robust H_∞ controller such that the closed-loop system (26a)-(26c) is exponentially stable in the mean-square sense and has a prescribed weighted H_∞ performance level γ.

Theorem 1 *Consider system* (1a)-(1c), *for given positive scalars α and β, if there exist two positive scalars ε_i and ε_{ij}, and matrices W_i, $X_i > 0$, $Y_i > 0$ and $Z_i > 0$ with appropriate dimensions such that $\forall i,j \in \underline{N}$, $i \neq j$,*

$$Z_i - \alpha Y_i - \alpha\tau Z_i > 0, \tag{27a}$$

$$\begin{bmatrix} \Sigma_{11}^i & A_{di}X_i & G_i & X_iM_i^T & X_iD_i^T & X_iE_{1i}^T & X_iE_{3i}^T \\ * & -Y_i & 0 & 0 & 0 & X_iE_{2i}^T & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{55}^i & 0 & 0 \\ * & * & * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \tag{27b}$$

$$\begin{bmatrix} \Sigma_{11}^{ij} & A_{dj}X_i & G_j & X_iM_j^T & X_iD_j^T & X_iE_{1j}^T & X_iE_{3j}^T \\ * & -Y_i & 0 & 0 & 0 & X_iE_{2j}^T & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{55}^{ij} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{ij}I & 0 \\ * & * & * & * & * & * & -\varepsilon_{ij}I \end{bmatrix} < 0 \tag{27c}$$

hold, then under the switching controller $u(t) = K_{\sigma'(t)}x(t)$, $K_i = W_iX_i^{-1}$, *and the average dwell time scheme*

$$\frac{T^-(t_0,t)}{T^+(t_0,t)} \geq \frac{\beta+\lambda}{\alpha-\lambda}, \quad 0 < \lambda < \alpha, \tag{28}$$

$$T_a > T_a^* = \frac{\ln\mu}{\lambda}, \tag{29}$$

the resulting closed-loop system (26a)-(26c) *is exponentially stable in the mean-square sense and has a prescribed weighted* H_∞ *performance level* γ, *where* $\mu \geq 1$ *satisfies*

$$\begin{aligned} &X_j^{-1} \leq \mu X_i^{-1}, \qquad X_j^{-1}Y_jX_j^{-1} \leq \mu X_i^{-1}Y_iX_i^{-1}, \\ &X_j^{-1}Z_jX_j^{-1} \leq \mu X_i^{-1}Z_iX_i^{-1}, \quad \forall i,j \in \underline{N}, i \neq j, \end{aligned} \tag{30}$$

and

$$\begin{aligned} &\Sigma_{11}^i = X_iA_i^T + W_i^TB_i^T + A_iX_i + B_iW_i + \alpha X_i + Y_i + \tau Z_i + \varepsilon_iH_iH_i^T, \\ &\Sigma_{55}^i = \varepsilon_iH_iH_i^T - X_i, \\ &\Sigma_{11}^{ij} = X_iA_j^T + W_i^TB_j^T + A_jX_i + B_jW_i - \beta X_i + Y_i + \tau Z_i + \varepsilon_{ij}H_jH_j^T, \\ &\Sigma_{55}^{ij} = \varepsilon_{ij}H_jH_j^T - X_i. \end{aligned}$$

Proof Assume that the ith subsystem is activated at the switching instant t_{k-1}, and the jth subsystem is activated at the switching instant t_k. Because there exists asynchronous switching, the ith controller is active during the interval $[t_{k-1}+\Delta_{k-1}, t_k+\Delta_k)$, and the jth controller is active during the interval $[t_k+\Delta_k, t_{k+1}+\Delta_{k+1})$.

(1) When $t \in [t_{k-1}+\Delta_{k-1}, t_k)$, system (26a) can be described as

$$dx(t) = \left[(\hat{A}_i + B_iK_i)x(t) + \hat{A}_{di}x(t-\tau) + G_iv(t)\right]dt + \hat{D}_ix(t)\,dw(t). \tag{31}$$

Consider the following Lyapunov functional candidate:

$$V_i(t) = \sum_{g=1}^{3} V_{g,i}(t), \tag{32}$$

where

$$\begin{aligned} &V_{1,i}(t) = x^T(t)P_ix(t), \qquad V_{2,i}(t) = \int_{t-\tau}^{t} x^T(s)Q_ix(s)\,ds, \\ &V_{3,i}(t) = \int_0^{\tau}\int_{t-\theta}^{t} x^T(s)R_ix(s)\,ds\,d\theta, \end{aligned}$$

P_i, Q_i and R_i are symmetric positive definite matrices with appropriate dimensions to be determined.

By the *Itô* formula, we have

$$dV_i(t) = \sum_{g=1}^{3} \mathcal{L}V_{g,i}(t)\,dt + 2x^T(t)P_i\hat{D}_i x(t)\,dw(t), \tag{33}$$

where

$$\begin{aligned}
\mathcal{L}V_{1,i}(t) &= 2x^T(t)P_i\big[(\hat{A}_i + B_iK_i)x(t) + \hat{A}_{di}x(t-\tau) + G_i\nu(t)\big] \\
&\quad + x^T(t)\hat{D}_i^T P_i\hat{D}_i x(t), \\
\mathcal{L}V_{2,i}(t) &= x^T(t)Q_ix(t) - x^T(t-\tau)Q_ix(t-\tau), \\
\mathcal{L}V_{3,i}(t) &= \tau x^T(t)R_ix(t) - \int_{t-\tau}^{t} x^T(s)R_ix(s)\,ds.
\end{aligned}$$

From (32), we obtain

$$\begin{aligned}
V_i(t) &= \sum_{g=1}^{3} V_{g,i}(t) \\
&= x^T(t)P_ix(t) + \int_{t-\tau}^{t} x^T(s)Q_ix(s)\,ds + \int_0^{\tau}\int_{t-\theta}^{t} x^T(s)R_ix(s)\,ds\,d\theta \\
&\leq x^T(t)P_ix(t) + \int_{t-\tau}^{t} x^T(s)Q_ix(s)\,ds + \tau\int_{t-\tau}^{t} x^T(s)R_ix(s)\,ds.
\end{aligned}$$

Under the condition

$$R_i - \alpha Q_i - \alpha\tau R_i > 0, \tag{34}$$

we get that

$$\begin{aligned}
&\mathcal{L}V_i(t) + \alpha V_i(t) + \Gamma(t) \\
&\quad\leq 2x^T(t)P_i\big[(\hat{A}_i + B_iK_i)x(t) + \hat{A}_{di}x(t-\tau) + G_i\nu(t)\big] \\
&\qquad + x^T(t)\hat{D}_i^T P_i\hat{D}_i x(t) + x^T(t)(Q_i + \tau R_i)x(t) \\
&\qquad + \alpha x^T(t)P_ix(t) - x^T(t-\tau)Q_ix(t-\tau) + x^T(t)M_i^T M_i x(t) - \gamma^2\nu^T(t)\nu(t) \\
&\qquad - \int_{t-\tau}^{t} x^T(s)(R_i - \alpha Q_i - \alpha\tau R_i)x(s)\,ds \\
&\quad\leq 2x^T(t)P_i\big[(\hat{A}_i + B_iK_i)x(t) + \hat{A}_{di}x(t-\tau) + G_i\nu(t)\big] \\
&\qquad + x^T(t)\hat{D}_i^T P_i\hat{D}_i x(t) + x^T(t)(Q_i + \tau R_i)x(t) \\
&\qquad + \alpha x^T(t)P_ix(t) - x^T(t-\tau)Q_ix(t-\tau) + x^T(t)M_i^T M_i x(t) - \gamma^2\nu^T(t)\nu(t) \\
&\quad= \xi^T(t)\Theta_i\xi(t),
\end{aligned}$$

where $\xi^T(t) = [x^T(t)\ x^T(t-\tau)\ v^T(t)]$,

$$\Theta_i = \begin{bmatrix} (\hat{A}_i + B_iK_i)^T P_i + P_i(\hat{A}_i + B_iK_i) + \alpha P_i + Q_i + \tau R_i + \hat{D}_i^T P_i \hat{D}_i + M_i^T M_i & P_i\hat{A}_{di} & P_iG_i \\ * & -Q_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix}.$$

(2) When $t \in [t_k, t_k + \Delta_k)$, system (26a) can be written as

$$dx(t) = \left[(\hat{A}_j + B_jK_i)x(t) + \hat{A}_{dj}x(t-\tau) + G_jv(t)\right]dt + \hat{D}_jx(t)\,dw(t). \tag{35}$$

Following the step line in (1), we have

$$\mathcal{L}V_i(t) - \beta V_i(t) + \Gamma(t) \le \xi^T(t)\Theta_{ij}\xi(t), \tag{36}$$

where

$$\Theta_{ij} = \begin{bmatrix} (\hat{A}_j + B_jK_i)^T P_i + P_i(\hat{A}_j + B_jK_i) - \beta P_i + Q_i + \tau R_i + \hat{D}_j^T P_i \hat{D}_j + M_j^T M_j & P_i\hat{A}_{dj} & P_iG_j \\ * & -Q_i & 0 \\ * & * & -\gamma^2 I \end{bmatrix}.$$

From (32), one obtains $\kappa_1 E\{\|x(t)\|^2\} \le E\{V_i(t)\} \le \kappa_2 \sup_{t-\tau\le s\le t} E\{\|x(s)\|^2\}$, where

$$\kappa_1 = \min_{\forall i\in\underline{N}} \lambda_{\min}(P_i),$$

$$\kappa_2 = \max_{\forall i\in\underline{N}} \lambda_{\max}(P_i) + \tau \max_{\forall i\in\underline{N}} \lambda_{\max}(Q_i) + \frac{\tau^2}{2} \max_{\forall i\in\underline{N}} \lambda_{\max}(R_i).$$

If $\Theta_i < 0$ and $\Theta_{ij} < 0$, we have

$$\begin{cases} \mathcal{L}V_{\sigma(t)}(t) + \alpha V_{\sigma(t)}(t) + \Gamma(t) \le 0, & t \in [t_0, t_1) \bigcup_{k=1,2,\ldots} [t_{k-1} + \Delta_{k-1}, t_k), \\ \mathcal{L}V_{\sigma(t)}(t) - \beta V_{\sigma(t)}(t) + \Gamma(t) \le 0, & t \in \bigcup_{k=1,2,\ldots} [t_k, t_k + \Delta_k). \end{cases} \tag{37}$$

By the Schur complement, one obtains that $\Theta_i < 0$ is equivalent to

$$\begin{bmatrix} (\hat{A}_i + B_iK_i)^T P_i + P_i(\hat{A}_i + B_iK_i) + \alpha P_i + Q_i + \tau R_i & P_i\hat{A}_{di} & P_iG_i & M_i^T & \hat{D}_i^T P_i \\ * & -Q_i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P_i \end{bmatrix} < 0. \tag{38}$$

Denoting $X_i = P_i^{-1}$ and using $\operatorname{diag}\{X_i, X_i, I, I, X_i\}$ to pre- and post-multiply the left term of (38), one obtains that

$$\hat{T}^i = \begin{bmatrix} \hat{T}_{11}^i & \hat{A}_{di}X_i & G_i & X_iM_i^T & X_i\hat{D}_i^T \\ * & -Y_i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -X_i \end{bmatrix} < 0, \tag{39}$$

where

$$\hat{T}_{11}^i = X_i\hat{A}_i^T + W_i^T B_i^T + \hat{A}_i X_i + B_i W_i + \alpha X_i + Y_i + \tau Z_i,$$
$$Y_i = X_i Q_i X_i, \qquad Z_i = X_i R_i X_i, \qquad W_i = K_i X_i.$$

From (3), we have

$$\hat{T}^i = T^i + \Delta T^i, \tag{40}$$

where

$$T^i = \begin{bmatrix} T_{11}^i & A_{di}X_i & G_i & X_i M_i^T & X_i D_i^T \\ * & -Y_i & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -X_i \end{bmatrix},$$

$$\Delta T^i = \begin{bmatrix} X_i E_{1i}^T F_i^T H_i^T + H_i F_i E_{1i} X_i & H_i F_i E_{2i} X_i & 0 & 0 & X_i E_{3i}^T F_i^T H_i^T \\ X_i E_{2i}^T F_i^T H_i^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ H_i F_i E_{3i} X_i & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$T_{11}^i = X_i A_i^T + W_i^T B_i^T + A_i X_i + B_i W_i + \alpha X_i + Y_i + \tau Z_i.$$

By Lemma 1, it can be obtained that (39) is equivalent to

$$T^i + \varepsilon_i \begin{bmatrix} H_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & H_i \end{bmatrix} \begin{bmatrix} H_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & H_i \end{bmatrix}^T + \varepsilon_i^{-1} \begin{bmatrix} X_i E_{1i}^T & X_i E_{3i}^T \\ X_i E_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_i E_{1i}^T & X_i E_{3i}^T \\ X_i E_{2i}^T & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T < 0.$$

According to the Schur complement, it follows that (39) is equivalent to (27b). Then from (38) and (39), we obtain that (27b) is equivalent to $\Theta_i < 0$. Similarly, it is easy to get that (27c) is equivalent to $\Theta_{ij} < 0$.

Noticing that $X_i = P_i^{-1}$, $Y_i = X_i Q_i X_i$ and $Z_i = X_i R_i X_i$, one obtains that (30) is equivalent to the following inequalities:

$$P_j \le \mu P_i, \qquad Q_j \le \mu Q_i, \qquad R_j \le \mu R_i, \quad \forall i,j \in \underline{N}, i \ne j. \tag{41}$$

By Lemma 2, system (26a)-(26c) is exponentially stable in the mean-square sense and has a prescribed weighted H_∞ performance level γ. The proof is completed. □

Remark 4 A stabilizing controller design method of switched stochastic delay systems under asynchronous switching has been proposed in [25], and sufficient conditions for the existence of designed controller have been derived in a set of matrix inequalities, but there is some difficulty in finding the feasible solution. However, the focus of our work is on the asynchronous switching H_∞ controller design, and this is also the major contribution of the paper. In addition, the method of H_∞ controller design presented in the paper is different from the ones proposed in [22–25], and it is much easier to solve the designed controller.

4 Numerical example

In this section, a numerical example is presented to illustrate the effectiveness of the proposed approach. Consider system (1a)-(1c) with the following parameters:

$$A_1 = \begin{bmatrix} -3 & 1 \\ 2 & -4 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -0.3 & 0 \\ 0.1 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1 & -0.2 \\ 0.3 & -0.2 \end{bmatrix}, \qquad G_1 = \begin{bmatrix} 1.3 \\ 1.5 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.3 \end{bmatrix}, \qquad E_{11} = [0.2 \quad 4], \qquad E_{21} = [0 \quad -1.8],$$

$$E_{31} = [0.1 \quad -0.1], \qquad H_1 = \begin{bmatrix} 1.1 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 3 \\ -4 & -2 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.5 & 0.1 \\ 0.2 & -0.8 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & -0.2 \end{bmatrix}, \qquad G_2 = \begin{bmatrix} 1.5 \\ 1.3 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \qquad E_{12} = [0 \quad 2.1], \qquad E_{22} = [-0.1 \quad 0],$$

$$E_{32} = [-0.1 \quad 0.1], \qquad H_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix},$$

$$F_1(t) = \sin t, \qquad F_2(t) = \cos t, \qquad v(t) = 12e^{-0.8t}, \qquad \Delta_1 = \Delta_2 = 0.2, \qquad \tau = 1.$$

Take $\alpha = 0.8$, $\beta = 1$ and $\gamma = 1$, then solving LMIs (27a)-(27c) in Theorem 1, we have

$$X_1 = \begin{bmatrix} 13.1772 & 0.7714 \\ 0.7714 & 2.6426 \end{bmatrix}, \qquad Y_1 = \begin{bmatrix} 41.0781 & 8.9258 \\ 8.9258 & 56.6342 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 4.8293 & 1.2775 \\ 1.2775 & 7.0308 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} -13.0662 & -41.4525 \\ -8.0096 & 37.6451 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 11.5472 & 0.6269 \\ 0.6269 & 2.5231 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 41.8473 & 5.8609 \\ 5.8609 & 56.5162 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 4.9917 & 0.8236 \\ 0.8236 & 6.8097 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} -6.4665 & -38.4972 \\ -13.2522 & 31.9777 \end{bmatrix},$$

$$\varepsilon_1 = 1.4777, \qquad \varepsilon_2 = 13.9145, \qquad \varepsilon_{12} = 15.0698, \qquad \varepsilon_{21} = 1.4354.$$

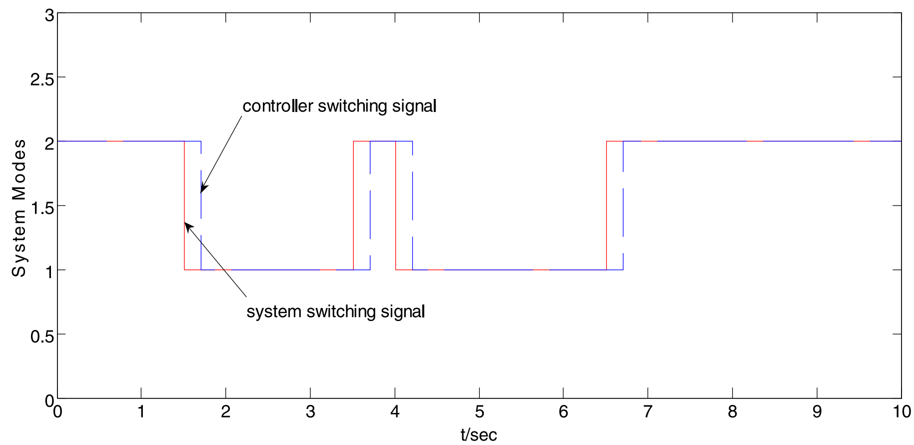

Figure 1 Switching signal.

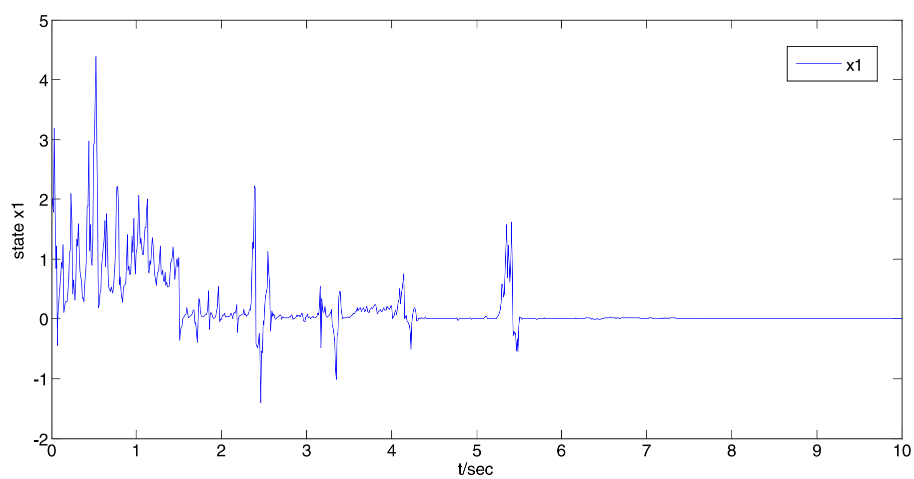

Figure 2 State x_1 of the closed-loop system.

Then the designed controller gain matrices can be obtained:

$$K_1 = \begin{bmatrix} -0.0758 & -15.6426 \\ -1.4657 & 14.6532 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.2720 & -15.3254 \\ -1.8608 & 13.1362 \end{bmatrix}.$$

From (30), we get $\mu = 1.9245$, and then from (28) and (29), it can be obtained that $T_a^* = 1.2683$. Thus, according to Theorem 1, under the average dwell time $T_a > 1.2683$, the designed controller can guarantee that the resulting closed-loop system is exponentially stable in the mean-square sense and has a prescribed weighted H_∞ performance level $\gamma = 1$.

Let $x(t) = [0,0]^T$, $t \in [-1,0)$, and $x(0) = [2,-2]^T$, and choose $T_a = 2$, simulation results are shown in Figures 1-3. Figure 1 depicts the switching signals of the system and the controller, respectively. Figures 2 and 3 show state trajectories of the closed-loop system, respectively.

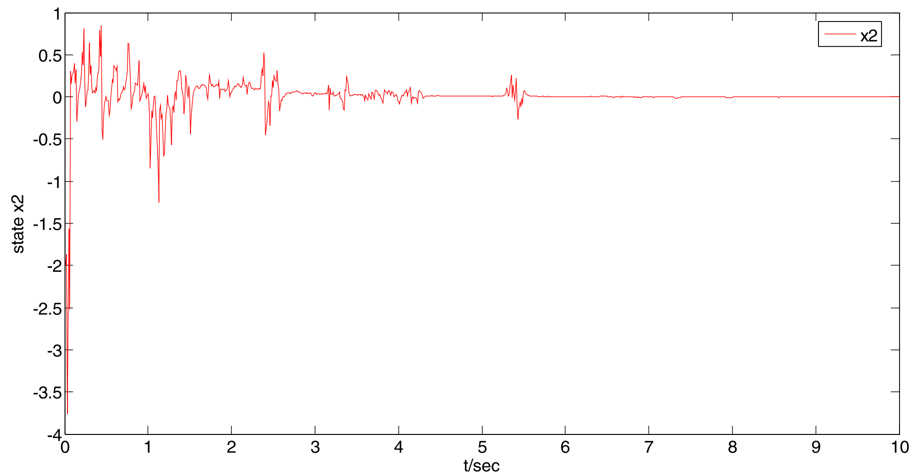

Figure 3 State x_2 of the closed-loop system.

5 Conclusions

In this paper, the robust H_∞ control problem for switched stochastic systems with time delays under asynchronous switching has been investigated. Based on the average dwell time approach, a criterion of mean-square exponential stability with H_∞ performance of switched stochastic delay systems is presented, and sufficient conditions for the existence of a robust H_∞ controller are derived. Finally, a numerical example is given to illustrate the effectiveness of the proposed approach.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
GC carried out the main results of this article and drafted the manuscript. ZX directed the study and helped inspection. Both the authors read and approved the final manuscript.

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