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Power series method and approximate linear differential equations of second order

Soon-Mo Jung[1] and Hamdullah Şevli[2*]

*Correspondence:
hsevli@yahoo.com
[2]Department of Mathematics, Faculty of Sciences and Arts, Istanbul Commerce University, Uskudar, Istanbul, 34672, Turkey
Full list of author information is available at the end of the article

Abstract
In this paper, we will establish a theory for the power series method that can be applied to various types of linear differential equations of second order to prove the Hyers-Ulam stability.
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Keywords: power series method; approximate linear differential equation; simple harmonic oscillator equation; Hyers-Ulam stability; approximation

1 Introduction

Let X be a normed space over a scalar field $\mathbb{K}$, and let $I \subset \mathbb{R}$ be an open interval, where $\mathbb{K}$ denotes either $\mathbb{R}$ or $\mathbb{C}$. Assume that $a_0, a_1, \ldots, a_n : I \to \mathbb{K}$ and $g : I \to X$ are given continuous functions. If for every n times continuously differentiable function $y : I \to X$ satisfying the inequality

$$\left\| a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \cdots + a_1(x)y'(x) + a_0(x)y(x) + g(x) \right\| \le \varepsilon$$

for all $x \in I$ and for a given $\varepsilon > 0$, there exists an n times continuously differentiable solution $y_0 : I \to X$ of the differential equation

$$a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \cdots + a_1(x)y'(x) + a_0(x)y(x) + g(x) = 0$$

such that $\|y(x) - y_0(x)\| \le K(\varepsilon)$ for any $x \in I$, where $K(\varepsilon)$ is an expression of ε with $\lim_{\varepsilon \to 0} K(\varepsilon) = 0$, then we say that the above differential equation has the Hyers-Ulam stability. For more detailed definitions of the Hyers-Ulam stability, we refer the reader to [1–8].

Obłoza seems to be the first author who has investigated the Hyers-Ulam stability of linear differential equations (see [9, 10]). Thereafter, Alsina and Ger [11] proved the Hyers-Ulam stability of the differential equation $y'(x) = y(x)$. It was further proved by Takahasi *et al.* that the Hyers-Ulam stability holds for the Banach space valued differential equation $y'(x) = \lambda y(x)$ (see [12] and also [13–15]).

Moreover, Miura *et al.* [16] investigated the Hyers-Ulam stability of an nth-order linear differential equation. The first author also proved the Hyers-Ulam stability of various linear differential equations of first order (ref. [17–25]).

Recently, the first author applied the power series method to studying the Hyers-Ulam stability of several types of linear differential equations of second order (see [26–34]).

However, it was inconvenient that he had to alter and apply the power series method with respect to each differential equation in order to study the Hyers-Ulam stability. Thus, it is inevitable to develop a power series method that can be comprehensively applied to different types of differential equations.

In Sections 2 and 3 of this paper, we establish a theory for the power series method that can be applied to various types of linear differential equations of second order to prove the Hyers-Ulam stability.

Throughout this paper, we assume that the linear differential equation of second order of the form

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = 0, \tag{1}$$

for which $x = 0$ is an ordinary point, has the general solution $y_h : (-\rho_0, \rho_0) \to \mathbb{C}$, where ρ_0 is a constant with $0 < \rho_0 \leq \infty$ and the coefficients $p, q, r : (-\rho_0, \rho_0) \to \mathbb{C}$ are analytic at 0 and have power series expansions

$$p(x) = \sum_{m=0}^{\infty} p_m x^m, \qquad q(x) = \sum_{m=0}^{\infty} q_m x^m \quad \text{and} \quad r(x) = \sum_{m=0}^{\infty} r_m x^m$$

for all $x \in (-\rho_0, \rho_0)$. Since $x = 0$ is an ordinary point of (1), we remark that $p_0 \neq 0$.

2 Inhomogeneous differential equation

In the following theorem, we solve the linear inhomogeneous differential equation of second order of the form

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = \sum_{m=0}^{\infty} a_m x^m \tag{2}$$

under the assumption that $x = 0$ is an ordinary point of the associated homogeneous linear differential equation (1).

Theorem 2.1 *Assume that the radius of convergence of power series* $\sum_{m=0}^{\infty} a_m x^m$ *is* $\rho_1 > 0$ *and that there exists a sequence* $\{c_m\}$ *satisfying the recurrence relation*

$$\sum_{k=0}^{m} \big[(k+2)(k+1)c_{k+2}p_{m-k} + (k+1)c_{k+1}q_{m-k} + c_k r_{m-k}\big] = a_m \tag{3}$$

for any $m \in \mathbb{N}_0$. *Let* ρ_2 *be the radius of convergence of power series* $\sum_{m=0}^{\infty} c_m x^m$ *and let* $\rho_3 = \min\{\rho_0, \rho_1, \rho_2\}$, *where* $(-\rho_0, \rho_0)$ *is the domain of the general solution to* (1). *Then every solution* $y : (-\rho_3, \rho_3) \to \mathbb{C}$ *of the linear inhomogeneous differential equation* (2) *can be expressed by*

$$y(x) = y_h(x) + \sum_{m=0}^{\infty} c_m x^m$$

for all $x \in (-\rho_3, \rho_3)$, *where* $y_h(x)$ *is a solution of the linear homogeneous differential equation* (1).

Proof Since $x = 0$ is an ordinary point, we can substitute $\sum_{m=0}^{\infty} c_m x^m$ for $y(x)$ in (2) and use the formal multiplication of power series and consider (3) to get

$$\begin{aligned}
&p(x)y''(x) + q(x)y'(x) + r(x)y(x) \\
&\quad = \sum_{m=0}^{\infty}\sum_{k=0}^{m} p_{m-k}(k+2)(k+1)c_{k+2}x^m + \sum_{m=0}^{\infty}\sum_{k=0}^{m} q_{m-k}(k+1)c_{k+1}x^m \\
&\qquad + \sum_{m=0}^{\infty}\sum_{k=0}^{m} r_{m-k}c_k x^m \\
&\quad = \sum_{m=0}^{\infty}\sum_{k=0}^{m}\bigl[(k+2)(k+1)c_{k+2}p_{m-k} + (k+1)c_{k+1}q_{m-k} + c_k r_{m-k}\bigr]x^m \\
&\quad = \sum_{m=0}^{\infty} a_m x^m
\end{aligned}$$

for all $x \in (-\rho_3, \rho_3)$. That is, $\sum_{m=0}^{\infty} c_m x^m$ is a particular solution of the linear inhomogeneous differential equation (2), and hence every solution $y : (-\rho_3, \rho_3) \to \mathbb{C}$ of (2) can be expressed by

$$y(x) = y_h(x) + \sum_{m=0}^{\infty} c_m x^m,$$

where $y_h(x)$ is a solution of the linear homogeneous differential equation (1). □

For the most common case in applications, the coefficient functions $p(x)$, $q(x)$, and $r(x)$ of the linear differential equation (1) are simple polynomials. In such a case, we have the following corollary.

Corollary 2.2 *Let $p(x)$, $q(x)$, and $r(x)$ be polynomials of degree at most $d \geq 0$. In particular, let d_0 be the degree of $p(x)$. Assume that the radius of convergence of power series $\sum_{m=0}^{\infty} a_m x^m$ is $\rho_1 > 0$ and that there exists a sequence $\{c_m\}$ satisfying the recurrence formula*

$$\sum_{k=m_0}^{m}\bigl[(k+2)(k+1)c_{k+2}p_{m-k} + (k+1)c_{k+1}q_{m-k} + c_k r_{m-k}\bigr] = a_m \tag{4}$$

for any $m \in \mathbb{N}_0$, where $m_0 = \max\{0, m-d\}$. If the sequence $\{c_m\}$ satisfies the following conditions:

(i) $\lim_{m\to\infty} c_{m-1}/mc_m = 0$,
(ii) *there exists a complex number L such that* $\lim_{m\to\infty} c_m/c_{m-1} = L$ *and* $p_{d_0} + Lp_{d_0-1} + \cdots + L^{d_0-1}p_1 + L^{d_0}p_0 \neq 0$,

then every solution $y : (-\rho_3, \rho_3) \to \mathbb{C}$ of the linear inhomogeneous differential equation (2) *can be expressed by*

$$y(x) = y_h(x) + \sum_{m=0}^{\infty} c_m x^m$$

for all $x \in (-\rho_3, \rho_3)$, *where* $\rho_3 = \min\{\rho_0, \rho_1\}$ *and* $y_h(x)$ *is a solution of the linear homogeneous differential equation* (1).

Proof Let m be any sufficiently large integer. Since $p_{d+1} = p_{d+2} = \cdots = 0$, $q_{d+1} = q_{d+2} = \cdots = 0$ and $r_{d+1} = r_{d+2} = \cdots = 0$, if we substitute $m - d + k$ for k in (4), then we have

$$a_m = \sum_{k=0}^{d} \bigl[(m-d+k+2)(m-d+k+1)c_{m-d+k+2}p_{d-k} + (m-d+k+1)c_{m-d+k+1}q_{d-k} + c_{m-d+k}r_{d-k}\bigr].$$

By (i) and (ii), we have

$$\begin{aligned}
&\limsup_{m\to\infty} |a_m|^{1/m} \\
&= \limsup_{m\to\infty} \Biggl| \sum_{k=0}^{d} (m-d+k+2)(m-d+k+1)c_{m-d+k+2} \\
&\quad \times \biggl(p_{d-k} + \frac{q_{d-k}}{(m-d+k+2)} \frac{c_{m-d+k+1}}{c_{m-d+k+2}} \\
&\quad + \frac{r_{d-k}}{(m-d+k+2)(m-d+k+1)} \frac{c_{m-d+k}}{c_{m-d+k+1}} \frac{c_{m-d+k+1}}{c_{m-d+k+2}}\biggr)\Biggr|^{1/m} \\
&= \limsup_{m\to\infty} \Biggl| \sum_{k=0}^{d} (m-d+k+2)(m-d+k+1)c_{m-d+k+2}p_{d-k} \Biggr|^{1/m} \\
&= \limsup_{m\to\infty} \Biggl| \sum_{k=d-d_0}^{d} (m-d+k+2)(m-d+k+1)c_{m-d+k+2}p_{d-k} \Biggr|^{1/m} \\
&= \limsup_{m\to\infty} \bigl|(m-d_0+2)(m-d_0+1)c_{m-d_0+2}\bigl(p_{d_0} + Lp_{d_0-1} + \cdots + L^{d_0}p_0\bigr)\bigr|^{1/m} \\
&= \limsup_{m\to\infty} \bigl|\bigl(p_{d_0} + Lp_{d_0-1} + \cdots + L^{d_0}p_0\bigr)(m-d_0+2)(m-d_0+1)\bigr|^{1/m} \\
&\quad \times \bigl(|c_{m-d_0+2}|^{1/(m-d_0+2)}\bigr)^{(m-d_0+2)/m} \\
&= \limsup_{m\to\infty} |c_{m-d_0+2}|^{1/(m-d_0+2)},
\end{aligned}$$

which implies that the radius of convergence of the power series $\sum_{m=0}^{\infty} c_m x^m$ is ρ_1. The rest of this corollary immediately follows from Theorem 2.1. □

In many cases, it occurs that $p(x) \equiv 1$ in (1). For this case, we obtain the following corollary.

Corollary 2.3 *Let* ρ_3 *be a distance between the origin* 0 *and the closest one among singular points of* $q(z)$, $r(z)$, *or* $\sum_{m=0}^{\infty} a_m z^m$ *in a complex variable* z. *If there exists a sequence* $\{c_m\}$ *satisfying the recurrence relation*

$$(m+2)(m+1)c_{m+2} + \sum_{k=0}^{m} \bigl[(k+1)c_{k+1}q_{m-k} + c_k r_{m-k}\bigr] = a_m \tag{5}$$

for any $m \in \mathbb{N}_0$, *then every solution* $y : (-\rho_3, \rho_3) \to \mathbb{C}$ *of the linear inhomogeneous differential equation*

$$y''(x) + q(x)y'(x) + r(x)y(x) = \sum_{m=0}^{\infty} a_m x^m \tag{6}$$

can be expressed by

$$y(x) = y_h(x) + \sum_{m=0}^{\infty} c_m x^m$$

for all $x \in (-\rho_3, \rho_3)$, *where* $y_h(x)$ *is a solution of the linear homogeneous differential equation* (1) *with* $p(x) \equiv 1$.

Proof If we put $p_0 = 1$ and $p_i = 0$ for each $i \in \mathbb{N}$, then the recurrence relation (3) reduces to (5). As we did in the proof of Theorem 2.1, we can show that $\sum_{m=0}^{\infty} c_m x^m$ is a particular solution of the linear inhomogeneous differential equation (6).

According to [35, Theorem 7.4] or [36, Theorem 5.2.1], there is a particular solution $y_0(x)$ of (6) in a form of power series in x whose radius of convergence is at least ρ_3. Moreover, since $\sum_{m=0}^{\infty} c_m x^m$ is a solution of (6), it can be expressed as a sum of both $y_0(x)$ and a solution of the homogeneous equation (1) with $p(x) \equiv 1$. Hence, the radius of convergence of $\sum_{m=0}^{\infty} c_m x^m$ is at least ρ_3.

Now, every solution $y : (-\rho_3, \rho_3) \to \mathbb{C}$ of (6) can be expressed by

$$y(x) = y_h(x) + \sum_{m=0}^{\infty} c_m x^m,$$

where $y_h(x)$ is a solution of the linear differential equation (1) with $p(x) \equiv 1$. □

3 Approximate differential equation

In this section, let $\rho_1 > 0$ be a constant. We denote by $\mathcal{C}$ the set of all functions $y : (-\rho_1, \rho_1) \to \mathbb{C}$ with the following properties:

(a) $y(x)$ is expressible by a power series $\sum_{m=0}^{\infty} b_m x^m$ whose radius of convergence is at least ρ_1;

(b) There exists a constant $K \geq 0$ such that $\sum_{m=0}^{\infty} |a_m x^m| \leq K |\sum_{m=0}^{\infty} a_m x^m|$ for any $x \in (-\rho_1, \rho_1)$, where

$$a_m = \sum_{k=0}^{m} \big[(k+2)(k+1)b_{k+2}p_{m-k} + (k+1)b_{k+1}q_{m-k} + b_k r_{m-k}\big]$$

for all $m \in \mathbb{N}_0$ and $p_0 \neq 0$.

Lemma 3.1 *Given a sequence* $\{a_m\}$, *let* $\{c_m\}$ *be a sequence satisfying the recurrence formula* (3) *for all* $m \in \mathbb{N}_0$. *If* $p_0 \neq 0$ *and* $n \geq 2$, *then* c_n *is a linear combination of* $a_0, a_1, \ldots, a_{n-2}$, c_0, *and* c_1.

Proof We apply induction on n. Since $p_0 \neq 0$, if we set $m = 0$ in (3), then

$$c_2 = \frac{1}{2p_0}a_0 - \frac{r_0}{2p_0}c_0 - \frac{q_0}{2p_0}c_1,$$

i.e., c_2 is a linear combination of a_0, c_0, and c_1. Assume now that n is an integer not less than 2 and c_i is a linear combination of $a_0, \dots, a_{i-2}$, c_0, c_1 for all $i \in \{2, 3, \dots, n\}$, namely,

$$c_i = \alpha_i^0 a_0 + \alpha_i^1 a_1 + \cdots + \alpha_i^{i-2} a_{i-2} + \beta_i c_0 + \gamma_i c_1,$$

where $\alpha_i^0, \dots, \alpha_i^{i-2}$, β_i, γ_i are complex numbers. If we replace m in (3) with $n-1$, then

$$\begin{aligned} a_{n-1} &= 2c_2 p_{n-1} + c_1 q_{n-1} + c_0 r_{n-1} \\ &\quad + 6c_3 p_{n-2} + 2c_2 q_{n-2} + c_1 r_{n-2} \\ &\quad + \cdots \\ &\quad + n(n-1)c_n p_1 + (n-1)c_{n-1} q_1 + c_{n-2} r_1 \\ &\quad + (n+1)nc_{n+1} p_0 + nc_n q_0 + c_{n-1} r_0 \\ &= (n+1)np_0 c_{n+1} + \bigl[n(n-1)p_1 + nq_0\bigr]c_n + \cdots \\ &\quad + (2p_{n-1} + 2q_{n-2} + r_{n-3})c_2 + (q_{n-1} + r_{n-2})c_1 + r_{n-1} c_0, \end{aligned}$$

which implies

$$\begin{aligned} c_{n+1} &= \frac{1}{(n+1)np_0}a_{n-1} - \frac{n(n-1)p_1 + nq_0}{(n+1)np_0}c_n - \cdots \\ &\quad - \frac{2p_{n-1} + 2q_{n-2} + r_{n-3}}{(n+1)np_0}c_2 - \frac{q_{n-1} + r_{n-2}}{(n+1)np_0}c_1 - \frac{r_{n-1}}{(n+1)np_0}c_0 \\ &= \alpha_{n+1}^0 a_0 + \alpha_{n+1}^1 a_1 + \cdots + \alpha_{n+1}^{n-1} a_{n-1} + \beta_{n+1} c_0 + \gamma_{n+1} c_1, \end{aligned}$$

where $\alpha_{n+1}^0, \dots, \alpha_{n+1}^{n-1}$, β_{n+1}, γ_{n+1} are complex numbers. That is, c_{n+1} is a linear combination of $a_0, a_1, \dots, a_{n-1}$, c_0, c_1, which ends the proof. □

In the following theorem, we investigate a kind of Hyers-Ulam stability of the linear differential equation (1). In other words, we answer the question whether there exists an exact solution near every approximate solution of (1). Since $x = 0$ is an ordinary point of (1), we remark that $p_0 \neq 0$.

Theorem 3.2 *Let* $\{c_m\}$ *be a sequence of complex numbers satisfying the recurrence relation* (3) *for all* $m \in \mathbb{N}_0$, *where* (b) *is referred for the value of* a_m, *and let* ρ_2 *be the radius of convergence of the power series* $\sum_{m=0}^{\infty} c_m x^m$. *Define* $\rho_3 = \min\{\rho_0, \rho_1, \rho_2\}$, *where* $(-\rho_0, \rho_0)$ *is the domain of the general solution to* (1). *Assume that* $y : (-\rho_1, \rho_1) \to \mathbb{C}$ *is an arbitrary function belonging to* $\mathcal{C}$ *and satisfying the differential inequality*

$$\bigl|p(x)y''(x) + q(x)y'(x) + r(x)y(x)\bigr| \le \varepsilon \tag{7}$$

for all $x \in (-\rho_3, \rho_3)$ and for some $\varepsilon > 0$. Let $\alpha_n^0, \alpha_n^1, \dots, \alpha_n^{n-2}$, β_n, γ_n be the complex numbers satisfying

$$c_n = \alpha_n^0 a_0 + \alpha_n^1 a_1 + \cdots + \alpha_n^{n-2} a_{n-2} + \beta_n c_0 + \gamma_n c_1 \tag{8}$$

for any integer $n \geq 2$. If there exists a constant $C > 0$ such that

$$\left|\alpha_n^0 a_0 + \alpha_n^1 a_1 + \cdots + \alpha_n^{n-2} a_{n-2}\right| \leq C|a_n| \tag{9}$$

for all integers $n \geq 2$, then there exists a solution $y_h : (-\rho_3, \rho_3) \to \mathbb{C}$ of the linear homogeneous differential equation (1) *such that*

$$\left|y(x) - y_h(x)\right| \leq CK\varepsilon$$

for all $x \in (-\rho_3, \rho_3)$, where K is the constant determined in (b).

Proof By the same argument presented in the proof of Theorem 2.1 with $\sum_{m=0}^{\infty} b_m x^m$ instead of $\sum_{m=0}^{\infty} c_m x^m$, we have

$$p(x)y''(x) + q(x)y'(x) + r(x)y(x) = \sum_{m=0}^{\infty} a_m x^m \tag{10}$$

for all $x \in (-\rho_3, \rho_3)$. In view of (b), there exists a constant $K \geq 0$ such that

$$\sum_{m=0}^{\infty} \left|a_m x^m\right| \leq K \left|\sum_{m=0}^{\infty} a_m x^m\right| \tag{11}$$

for all $x \in (-\rho_1, \rho_1)$.

Moreover, by using (7), (10), and (11), we get

$$\sum_{m=0}^{\infty} \left|a_m x^m\right| \leq K \left|\sum_{m=0}^{\infty} a_m x^m\right| \leq K\varepsilon$$

for any $x \in (-\rho_3, \rho_3)$. (That is, the radius of convergence of power series $\sum_{m=0}^{\infty} a_m x^m$ is at least ρ_3.)

According to Theorem 2.1 and (10), $y(x)$ can be written as

$$y(x) = y_h(x) + \sum_{n=0}^{\infty} c_n x^n \tag{12}$$

for all $x \in (-\rho_3, \rho_3)$, where $y_h(x)$ is a solution of the homogeneous differential equation (1). In view of Lemma 3.1, the c_n can be expressed by a linear combination of the form (8) for each integer $n \geq 2$.

Since $\sum_{n=0}^{\infty} c_n x^n$ is a particular solution of (2), if we set $c_0 = c_1 = 0$, then it follows from (8), (9), and (12) that

$$\left|y(x) - y_h(x)\right| \leq \sum_{n=0}^{\infty} \left|c_n x^n\right| \leq CK\varepsilon$$

for all $x \in (-\rho_3, \rho_3)$. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors declare that this paper is their original paper. All authors read and approved the final manuscript.

Author details
[1]Mathematics Section, College of Science and Technology, Hongik University, Sejong, 339-701, Republic of Korea.
[2]Department of Mathematics, Faculty of Sciences and Arts, Istanbul Commerce University, Uskudar, Istanbul, 34672, Turkey.

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