

RESEARCH

Open Access

Relations between Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ and Exton's function X_8

Junesang Choi^{1*} and Arjun K Rathie²

*Correspondence:

junesang@mail.dongguk.ac.kr

¹Department of Mathematics,
Dongguk University, Gyeongju,
Korea

Full list of author information is
available at the end of the article

Abstract

Very recently Choi *et al.* derived some interesting relations between Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ and the Srivastava function $F^{(3)}[x, y, z]$ by simply splitting Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ into eight parts. Here, in this paper, we aim at establishing eleven new and interesting transformations between Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ and Exton's function X_8 in the form of a single result. Our results presented here are derived with the help of two general summation formulae for the terminating ${}_2F_1(2)$ series which were very recently obtained by Kim *et al.* and also include the relationship between $F_A^{(3)}(x, y, z)$ and X_8 due to Exton.

MSC: Primary 33C20; secondary 44A45

Keywords: gamma function; hypergeometric functions of several variables; multiple Gaussian hypergeometric series; Exton's triple hypergeometric series; Gauss's hypergeometric functions; Lauricella's triple hypergeometric functions

1 Introduction and preliminaries

In the usual notation, let \mathbb{C} denote the set of *complex* numbers. For

$$\alpha_j \in \mathbb{C} \quad (j = 1, \dots, p) \quad \text{and} \quad \beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^- \quad (\mathbb{Z}_0^- := \mathbb{Z}^- \cup \{0\} = \{0, -1, -2, \dots\}),$$

the *generalized hypergeometric series* ${}_pF_q$ with p numerator parameters $\alpha_1, \dots, \alpha_p$ and q denominator parameters β_1, \dots, β_q is defined by (see, for example, [1, Chapter 4]; see also [2, p.73]):

$${}_pF_q \left[\begin{matrix} \alpha_1, \dots, \alpha_p \\ \beta_1, \dots, \beta_q \end{matrix} ; z \right] = \sum_{n=0}^{\infty} \frac{\prod_{j=1}^p (\alpha_j)_n}{\prod_{j=1}^q (\beta_j)_n} \frac{z^n}{n!} = {}_pF_q(\alpha_1, \dots, \alpha_p; \beta_1, \dots, \beta_q; z)$$

$$(p, q \in \mathbb{N}_0 := \mathbb{N} \cup \{0\} = \{0, 1, 2, \dots\}; p \leq q + 1; p \leq q \text{ and } |z| < \infty;$$

$$p = q + 1 \text{ and } |z| < 1; p = q + 1, |z| = 1 \text{ and } \Re(\omega) > 0), \quad (1.1)$$

where

$$\omega := \sum_{j=1}^q \beta_j - \sum_{j=1}^p \alpha_j \quad (\alpha_j \in \mathbb{C} \ (j = 1, \dots, p); \beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^- \ (j = 1, \dots, q)) \quad (1.2)$$

and $(\lambda)_\nu$ is the Pochhammer symbol or the *shifted factorial* since

$$(1)_n = n! \quad (n \in \mathbb{N}_0),$$

which is defined (for $\lambda, \nu \in \mathbb{C}$), in terms of the familiar gamma function Γ , by

$$(\lambda)_\nu := \frac{\Gamma(\lambda + \nu)}{\Gamma(\lambda)} = \begin{cases} 1 & (\nu = 0; \lambda \in \mathbb{C} \setminus \{0\}), \\ \lambda(\lambda + 1) \cdots (\lambda + n - 1) & (\nu = n \in \mathbb{N}; \lambda \in \mathbb{C}), \end{cases} \quad (1.3)$$

it being *understood conventionally* that $(0)_0 := 1$.

By using the notations as described in [3, p.33, Equation 1.4(1)] (with $n = 3$) (see also [4, p.114, Equation (1)], [5, Equation (2.1)] and [6, p.60, Equation 1.7(1)]), Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ is defined by

$$\begin{aligned} F_A^{(3)}(x, y, z) &= F_A^{(3)}(a, b_1, b_2, b_3; c_1, c_2, c_3; x, y, z) \\ &:= \sum_{m, n, p=0}^{\infty} \frac{(a)_{m+n+p} (b_1)_m (b_2)_n (b_3)_p}{(c_1)_m (c_2)_n (c_3)_p} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^p}{p!} \\ &\quad (|x| + |y| + |z| < 1; c_j \in \mathbb{C} \setminus \mathbb{Z}_0^- \ (j = 1, 2, 3)). \end{aligned} \quad (1.4)$$

Motivated essentially by the works by Lardner [7] and Carlson [8], by simply splitting Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ into eight parts, very recently Choi *et al.* [5] presented several relationships between $F_A^{(3)}(x, y, z)$ and the Srivastava function $F^{(3)}[x, y, z]$ (see also [9]). The widely-investigated Srivastava's triple hypergeometric function $F^{(3)}[x, y, z]$, which was introduced over four decades ago by Srivastava [10, p.428] (see also [5], [3, p.44, Equation 1.5(14)] and [6, p.69, Equation 1.7(39)]), provides an interesting unification (and generalization) of Lauricella's 14 triple hypergeometric functions F_1, \dots, F_{14} (see [11], [12, pp.113-114]) and Srivastava's three functions H_A, H_B and H_C (see [13, pp.99-100]; see also [14-16], [3, p.43] and [6, pp.60-68]).

Exton's triple hypergeometric function X_8 (see [17]; see also [3, p.84, Entry 41a and p.101, List 41a]) is defined by

$$\begin{aligned} X_8(a, b_1, b_2; c_1, c_2, c_3; x, y, z) &:= \sum_{m, n, p=0}^{\infty} \frac{(a)_{2m+n+p} (b_1)_n (b_2)_p}{(c_1)_m (c_2)_n (c_3)_p} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^p}{p!} \\ &\quad (2\sqrt{|x|} + |y| + |z| < 1; c_j \in \mathbb{C} \setminus \mathbb{Z}_0^- \ (j = 1, 2, 3)). \end{aligned} \quad (1.5)$$

In fact, in 1982 Exton [17] published a very interesting and useful research paper in which he encountered a number of triple hypergeometric functions of second order whose series representations involve such products as $(a)_{2m+2n+p}$ and $(a)_{2m+n+p}$ and introduced a set of 20 distinct triple hypergeometric functions X_1 to X_{20} and also gave their integral representations of Laplacian type which include the confluent hypergeometric functions ${}_0F_1, {}_1F_1$, a Humbert function ψ_1 and a Humbert function ϕ_2 in their kernels. It is not out of place to mention here that Exton's functions X_1 to X_{20} have been studied a lot until today; see, for example, the works [12, 18-23] and [24]. Moreover, Exton [17] presented a large number of very interesting transformation formulas and reducible cases with the

help of two known results which are called in the literature Kummer's first and second transformations or theorems.

Here, in this paper, we aim at establishing eleven new and interesting transformations between Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ and Exton's function X_8 in the form of a single result. Our results presented here are derived with the help of two general summation formulae for the terminating ${}_2F_1(2)$ series which were very recently obtained by Kim *et al.* [25] and also include the relationship between $F_A^{(3)}(x, y, z)$ and X_8 due to Exton [17].

2 Results required

For our purpose, we require the following results recently obtained by Kim *et al.* [25]:

$${}_2F_1 \left[\begin{matrix} -2n, \alpha; \\ 2\alpha + j; \end{matrix} 2 \right] = \mathcal{A}_j \frac{\Gamma(\alpha)\Gamma(1-\alpha)(\frac{1}{2})_n(\alpha + [\frac{j+1}{2}]_n)}{\Gamma(\alpha + \frac{1}{2}j + \frac{1}{2}|j|)\Gamma(1-\alpha - [\frac{j+1}{2}])_n(\alpha + \frac{1}{2}j + \frac{1}{2})_n} \quad (2.1)$$

and

$${}_2F_1 \left[\begin{matrix} -2n-1, \alpha; \\ 2\alpha + j; \end{matrix} 2 \right] = \frac{\mathcal{B}_j}{2\alpha + j} \frac{\Gamma(-\alpha)\Gamma(\alpha+1)(\frac{3}{2})_n(1+\alpha + [\frac{j}{2}]_n)}{\Gamma(\alpha + \frac{1}{2}j + \frac{1}{2}|j|)\Gamma(-\alpha - [\frac{j}{2}])_n(\alpha + \frac{1}{2}j + \frac{1}{2})_n(\alpha + \frac{1}{2}j + 1)_n}, \quad (2.2)$$

where $n \in \mathbb{N}_0$, $j = 0, \pm 1, \dots, \pm 5$, $[x]$ is the greatest integer less than or equal to x and its modulus is denoted by $|x|$, and the coefficients \mathcal{A}_j and \mathcal{B}_j are given in Table 1.

3 Main transformation formulae

The results to be established here are as follows:

$$\begin{aligned} & (1+2x)^{-a} F_A^{(3)} \left(a, c_1, b_1, b_2; 2c_1 + j, c_2, c_3; \frac{4x}{1+2x}, \frac{y}{1+2x}, \frac{z}{1+2x} \right) \\ &= \frac{\Gamma(c_1)\Gamma(1-c_1)}{\Gamma(c_1 + \frac{j}{2} + \frac{1}{2}|j|)\Gamma(1-c_1 - [\frac{j+1}{2}])} \\ & \times \sum_{n,p,r=0}^{\infty} \mathcal{C}_j \frac{(a)_{2r+n+p}(b_1)_n(b_2)_p(c_1 + [\frac{j+1}{2}])_r}{(c_2)_n(c_3)_p(c_1 + \frac{j}{2})_r(c_1 + \frac{j}{2} + \frac{1}{2})_r} \frac{x^{2r}}{r!} \frac{y^n}{n!} \frac{z^p}{p!} \\ & - \frac{2ax}{2c_1 + j} \frac{\Gamma(-c_1)\Gamma(1+c_1)}{\Gamma(c_1 + \frac{j}{2} + \frac{1}{2}|j|)\Gamma(-c_1 - [\frac{j}{2}])} \\ & \times \sum_{n,p,r=0}^{\infty} \mathcal{D}_j \frac{(a+1)_{2r+n+p}(b_1)_n(b_2)_p(1+c_1 + [\frac{j}{2}])_r}{(c_2)_n(c_3)_p(c_1 + \frac{j}{2} + \frac{1}{2})_r(c_1 + \frac{j}{2} + 1)_r} \frac{x^{2r}}{r!} \frac{y^n}{n!} \frac{z^p}{p!} \\ & (j = 0, \pm 1, \dots, \pm 5), \end{aligned} \quad (3.1)$$

where the coefficients \mathcal{C}_j and \mathcal{D}_j can be obtained by simply changing n and α into r and c_1 , respectively, in Table 1 of \mathcal{A}_j and \mathcal{B}_j .

Table 1 Contiguous relation coefficients

j	\mathcal{A}_j	\mathcal{B}_j
5	$-4(1-\alpha-2n)^2 + 2(1-\alpha)(1-\alpha-2n) + (1-\alpha)^2 + 22(1-\alpha-2n) - 13(1-\alpha) - 20$	$4(\alpha+2n)^2 - 2(1-\alpha)(\alpha+2n) + (1-\alpha)^2 + 34(\alpha+2n) + (1-\alpha) + 62$
4	$2(\alpha+1+2n)(\alpha+3+2n) - \alpha(\alpha+3)$	$4(\alpha+2n+3)$
3	$-\alpha-4n-2$	$-3\alpha-4n-6$
2	$-\alpha-1-2n$	-2
1	-1	1
0	1	0
-1	1	1
-2	$1-\alpha-2n$	2
-3	$1-\alpha-4n$	$3-3\alpha-4n$
-4	$2(1-2\alpha-n)(3-\alpha-2n) - (1-\alpha)(4-\alpha)$	$4(1-\alpha-2n)$
-5	$4(1-\alpha-2n)^2 - 2(1-\alpha)(1-\alpha-2n) - (1-\alpha)^2 + 8(1-\alpha-2n) + 7\alpha - 7$	$4(\alpha+2n)^2 + 2(1-\alpha)(\alpha+2n) - (1-\alpha)^2 - 16(\alpha+2n) + \alpha - 1$

Proof For convenience and simplicity, by denoting the left-hand side of (3.1) by S and using the series definition of $F_A^{(3)}(x, y, z)$ as given in (1.4), after a little simplification, we have

$$S = \sum_{m,n,p=0}^{\infty} \frac{(a)_{m+n+p}(c_1)_m(b_1)_n(b_2)_p}{(2c_1+j)_m(c_2)_n(c_3)_p} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^p}{p!} 2^{2m} (1+2x)^{-(a+m+n+p)}.$$

Using the binomial theorem (see, for example, [2, p.58]) for the last factor, we get

$$S = \sum_{m,n,p,r=0}^{\infty} \frac{(a)_{m+n+p}(c_1)_m(b_1)_n(b_2)_p}{(2c_1+j)_m(c_2)_n(c_3)_p} \frac{x^{m+r}}{m!r!} \frac{y^n}{n!} \frac{z^p}{p!} 2^{2m+r} (-1)^r (a+m+n+p)_r.$$

Using the identity $(a)_{m+n+p}(a+m+n+p)_r = (a)_{m+n+p+r}$, after a little simplification, we obtain

$$S = \sum_{m,n,p,r=0}^{\infty} \frac{(a)_{m+n+p+r}(c_1)_m(b_1)_n(b_2)_p}{(2c_1+j)_m(c_2)_n(c_3)_p} \frac{x^{m+r}}{m!r!} \frac{y^n}{n!} \frac{z^p}{p!} 2^{2m+r} (-1)^r.$$

Now using the following well-known formal manipulation of double series (see [2, p.56]; for other manipulations, see also [26, Eq. (1.4)]):

$$\sum_{r=0}^{\infty} \sum_{m=0}^{\infty} A(m, r) = \sum_{r=0}^{\infty} \sum_{m=0}^r A(m, r-m),$$

after a little simplification, we have

$$S = \sum_{n,p,r=0}^{\infty} \sum_{m=0}^r \frac{(a)_{n+p+r}(c_1)_m(b_1)_n(b_2)_p}{(2c_1+j)_m(c_2)_n(c_3)_p} \frac{x^r}{r!} \frac{y^n}{n!} \frac{z^p}{p!} \frac{2^{m+r} (-1)^{r-m}}{(r-m)!}.$$

Using the following formula:

$$(r-m)! = \frac{(-1)^m r!}{(-r)_m} \quad (0 \leq m \leq r; r, m \in \mathbb{N}_0),$$

after a little simplification, we get

$$S = \sum_{n,p,r=0}^{\infty} \frac{(a)_{n+p+r}(b_1)_n(b_2)_p (-2)^r}{(c_2)_n(c_3)_p} \frac{x^r}{r!} \frac{y^n}{n!} \frac{z^p}{p!} \sum_{m=0}^r \frac{(c_1)_m (-r)_m}{(2c_1+j)_m m!} 2^m.$$

Using the definition of ${}_pF_q$ in (1.1) for the inner series, we obtain

$$S = \sum_{n,p,r=0}^{\infty} \frac{(a)_{n+p+r}(b_1)_n(b_2)_p(-2)^r}{(c_2)_n(c_3)_p} \frac{x^r}{r!} \frac{y^n}{n!} \frac{z^p}{p!} {}_2F_1 \left[\begin{matrix} -r, c_1; \\ 2c_1 + j; \end{matrix} \middle| 2 \right].$$

Separating r into even and odd integers, we have

$$S = \sum_{n,p,r=0}^{\infty} \frac{(a)_{n+p+2r}(b_1)_n(b_2)_p 2^{2r}}{(c_2)_n(c_3)_p} \frac{x^{2r}}{(2r)!} \frac{y^n}{n!} \frac{z^p}{p!} {}_2F_1 \left[\begin{matrix} -2r, c_1; \\ 2c_1 + j; \end{matrix} \middle| 2 \right] \\ + \sum_{n,p,r=0}^{\infty} \frac{(a)_{n+p+2r+1}(b_1)_n(b_2)_p(-2)^{2r+1}}{(c_2)_n(c_3)_p} \frac{x^{2r+1}}{(2r+1)!} \frac{y^n}{n!} \frac{z^p}{p!} {}_2F_1 \left[\begin{matrix} -2r-1, c_1; \\ 2c_1 + j; \end{matrix} \middle| 2 \right].$$

Making use of the following identity:

$$(\alpha)_{2r} = 2^{2r} (\alpha)_r \left(\frac{\alpha}{2} \right)_r,$$

after a little simplification, we get

$$S = \sum_{n,p,r=0}^{\infty} \frac{(a)_{n+p+2r}(b_1)_n(b_2)_p}{(c_2)_n(c_3)_p \left(\frac{1}{2}\right)_r} \frac{x^{2r}}{r!} \frac{y^n}{n!} \frac{z^p}{p!} {}_2F_1 \left[\begin{matrix} -2r, c_1; \\ 2c_1 + j; \end{matrix} \middle| 2 \right] \\ - 2xa \sum_{n,p,r=0}^{\infty} \frac{(a+1)_{n+p+2r}(b_1)_n(b_2)_p}{(c_2)_n(c_3)_p \left(\frac{3}{2}\right)_r} \frac{x^{2r}}{r!} \frac{y^n}{n!} \frac{z^p}{p!} {}_2F_1 \left[\begin{matrix} -2r-1, c_1; \\ 2c_1 + j; \end{matrix} \middle| 2 \right].$$

Finally, using the known results (2.1) and (2.2), after a little simplification, we easily arrive at the right-hand side of (3.1). This completes the proof of (3.1). \square

4 Special cases

In our main formula (3.1), if we take $j = 0, \pm 1$ and ± 2 , after a little simplification, and interpret the respective resulting right-hand sides with the definition of Exton's triple hypergeometric series X_8 given in (1.5), we get the following very interesting relations between $F_A^{(3)}(x, y, z)$ and X_8 :

The case $j = 0$.

$$(1+2x)^{-a} F_A^{(3)} \left(a, c_1, b_1, b_2; 2c_1, c_2, c_3; \frac{4x}{1+2x}, \frac{y}{1+2x}, \frac{z}{1+2x} \right) \\ = X_8 \left(a, b_1, b_2; c_1 + \frac{1}{2}, c_2, c_3; x^2, y, z \right). \quad (4.1)$$

The case $j = 1$.

$$(1+2x)^{-a} F_A^{(3)} \left(a, c_1, b_1, b_2; 2c_1 + 1, c_2, c_3; \frac{4x}{1+2x}, \frac{y}{1+2x}, \frac{z}{1+2x} \right) \\ = X_8 \left(a, b_1, b_2; c_1 + \frac{1}{2}, c_2, c_3; x^2, y, z \right) \\ - \frac{2ax}{2c_1 + 1} X_8 \left(a, b_1, b_2; c_1 + \frac{3}{2}, c_2, c_3; x^2, y, z \right). \quad (4.2)$$

The case $j = -1$.

$$\begin{aligned} & (1+2x)^{-a} F_A^{(3)} \left(a, c_1, b_1, b_2; 2c_1 - 1, c_2, c_3; \frac{4x}{1+2x}, \frac{y}{1+2x}, \frac{z}{1+2x} \right) \\ &= X_8 \left(a, b_1, b_2; c_1 - \frac{1}{2}, c_2, c_3; x^2, y, z \right) \\ &+ \frac{2ax}{2c_1 - 1} X_8 \left(a, b_1, b_2; c_1 + \frac{1}{2}, c_2, c_3; x^2, y, z \right). \end{aligned} \quad (4.3)$$

The case $j = 2$.

$$\begin{aligned} & (1+2x)^{-a} F_A^{(3)} \left(a, c_1, b_1, b_2; 2c_1 + 2, c_2, c_3; \frac{4x}{1+2x}, \frac{y}{1+2x}, \frac{z}{1+2x} \right) \\ &= X_8 \left(a, b_1, b_2; c_1 + \frac{3}{2}, c_2, c_3; x^2, y, z \right) \\ &+ \frac{ax}{c_1 + 1} X_8 \left(a + 1, b_1, b_2; c_1 + \frac{3}{2}, c_2, c_3; x^2, y, z \right) \\ &+ \frac{4a(a+1)x^2}{(c_1 + 1)(2c_1 + 3)} X_8 \left(a + 2, b_1, b_2; c_1 + \frac{5}{2}, c_2, c_3; x^2, y, z \right). \end{aligned} \quad (4.4)$$

The case $j = -2$.

$$\begin{aligned} & (1+2x)^{-a} F_A^{(3)} \left(a, c_1, b_1, b_2; 2c_1 - 2, c_2, c_3; \frac{4x}{1+2x}, \frac{y}{1+2x}, \frac{z}{1+2x} \right) \\ &= X_8 \left(a, b_1, b_2; c_1 - \frac{1}{2}, c_2, c_3; x^2, y, z \right) \\ &+ \frac{ax}{c_1 - 1} X_8 \left(a + 1, b_1, b_2; c_1 - \frac{1}{2}, c_2, c_3; x^2, y, z \right) \\ &+ \frac{4a(a+1)x^2}{(c_1 - 1)(2c_1 - 1)} X_8 \left(a + 2, b_1, b_2; c_1 + \frac{1}{2}, c_2, c_3; x^2, y, z \right). \end{aligned} \quad (4.5)$$

Remark Clearly, Equation (4.1) is Exton's result (see [17]) and Equations (4.2) to (4.5) are closely related to it. The other special cases of (3.1) can also be expressed in terms of X_8 in a similar manner.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors have equal contributions to each part of this paper. All authors have read and approved the final manuscript.

Author details

¹Department of Mathematics, Dongguk University, Gyeongju, Korea. ²Department of Mathematics, School of Mathematical & Physical Sciences, Central University of Kerala, Riverside Transit Campus, Padannakad P.O. Nileshevar, Kasaragod, 671 328, India.

Acknowledgements

Dedicated to Professor Hari M Srivastava.

This paper was supported by the Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology (2012-0002957).

Received: 14 December 2012 Accepted: 21 January 2013 Published: 14 February 2013

References

1. Erdélyi, A, Magnus, W, Oberhettinger, F, Tricomi, FG: Higher Transcendental Functions, vol. I. McGraw-Hill, New York (1953)
2. Rainville, ED: Special Functions. Macmillan Co., New York (1960). Reprinted by Chelsea, New York (1971)
3. Srivastava, HM, Karlsson, PW: Multiple Gaussian Hypergeometric Series. Horwood, Chichester (1985)
4. Appell, P, Kampé de Fériet, J: Fonctions Hypergeometriques et Hyperspheriques; Polynomes d'Hermite. Gauthier-Villars, Paris (1926)
5. Choi, J, Hasanov, A, Srivastava, HM: Relations between Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ and the Srivastava function $F^{(3)}[x, y, z]$. Integral Transforms Spec. Funct. **23**(1), 69-82 (2012)
6. Srivastava, HM, Manocha, HL: A Treatise on Generating Functions. Horwood, Chichester (1984)
7. Lardner, TJ: Relations between ${}_0F_3$ and Bessel functions. SIAM Rev. **11**, 69-72 (1969)
8. Carlson, BC: Some extensions of Lardner's relations between ${}_0F_3$ and Bessel functions. SIAM J. Math. Anal. **1**(2), 232-242 (1970)
9. Choi, J, Kim, YS, Hasanov, A: Relations between the hypergeometric function of Appell F_3 and Kampé de Fériet functions. Miskolc Math. Notes **12**(2), 131-148 (2011)
10. Srivastava, HM: Generalized Neumann expansions involving hypergeometric functions. Proc. Camb. Philos. Soc. **63**, 425-429 (1967)
11. Lauricella, G: Sulle funzioni ipergeometriche a più variabili. Rend. Circ. Mat. Palermo **7**, 111-158 (1893)
12. Kim, YS, Choi, J, Rathie, AK: Remark on two results by Padmanabham for Exton's triple hypergeometric series X_8 . Honam Math. J. **27**(4), 603-608 (2005)
13. Srivastava, HM: Hypergeometric functions of three variables. Ganita **15**(2), 97-108 (1964)
14. Choi, J, Hasanov, A, Srivastava, HM, Turaev, M: Integral representations for Srivastava's triple hypergeometric functions. Taiwan. J. Math. **15**, 2751-2762 (2011)
15. Srivastava, HM: Some integrals representing triple hypergeometric functions. Rend. Circ. Mat. Palermo **16**, 99-115 (1967)
16. Turaev, M: Decomposition formulas for Srivastava's hypergeometric function H_A on Saran functions. J. Comput. Appl. Math. **233**, 842-846 (2009)
17. Exton, H: Hypergeometric functions of three variables. J. Indian Acad. Math. **4**, 113-119 (1982)
18. Choi, J, Hasanov, A, Turaev, M: Decomposition formulas and integral representations for some Exton hypergeometric functions. J. Chungcheong Math. Soc. **24**(4), 745-758 (2011)
19. Choi, J, Hasanov, A, Turaev, M: Linearly independent solutions for the hypergeometric Exton functions X_1 and X_2 . Honam Math. J. **32**(2), 223-229 (2010)
20. Choi, J, Hasanov, A, Turaev, M: Certain integral representations of Euler type for the Exton function X_5 . Honam Math. J. **32**(3), 389-397 (2010)
21. Choi, J, Hasanov, A, Turaev, M: Certain integral representations of Euler type for the Exton function X_2 . J. Korean Soc. Math. Edu., Ser. B, Pure Appl. Math. **17**(4), 347-354 (2010)
22. Kim, YS, Rathie, AK: On an extension formulas for the triple hypergeometric X_8 due to Exton. Bull. Korean Math. Soc. **44**(4), 743-751 (2007)
23. Kim, YS, Rathie, AK, Choi, J: Another method for Padmanabham's transformation formula for Exton's triple hypergeometric series X_8 . Commun. Korean Math. Soc. **24**(4), 517-521 (2009)
24. Lee, SW, Kim, YS: An extension of the triple hypergeometric series by Exton. Honam Math. J. **31**(1), 61-71 (2009)
25. Kim, YS, Rakha, MA, Rathie, AK: Generalization of Kummer's second summation theorem with applications. Comput. Math. Math. Phys. **50**(3), 387-402 (2010)
26. Choi, J: Notes on formal manipulations of double series. Commun. Korean Math. Soc. **18**(4), 781-789 (2003)

doi:10.1186/1687-1847-2013-34

Cite this article as: Choi and Rathie: Relations between Lauricella's triple hypergeometric function $F_A^{(3)}(x, y, z)$ and Exton's function X_8 . *Advances in Difference Equations* 2013 **2013**:34.

Submit your manuscript to a SpringerOpen[®] journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at ► springeropen.com