Ma et al. *Advances in Difference Equations* 2013, 2013:321
http://www.advancesindifferenceequations.com/content/2013/1/321

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH **Open Access**

Exact three-wave solutions for the $(3+1)$-dimensional Boiti-Leon-Manna-Pempinelli equation

Hongcai Ma[1,2*], Yongbin Bai[1] and Aiping Deng[1]

*Correspondence: hongcaima@hotmail.com
[1]Department of Applied Mathematics, Donghua University, Shanghai, 201620, China
[2]Faculty of Science, Ningbo University, Ningbo, 315211, China

Abstract

In this paper, we consider a (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation. We employ the Hirota bilinear method to obtain the bilinear form of the (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation. Based on the bilinear form, we derive exact three-wave solutions by using an extended three-soliton method. In addition, we also get the trajectory of some solution with the help of MAPLE.

Keywords: (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation; extended three-soliton method; periodic wave; MAPLE

1 Introduction

Integrable systems and nonlinear evolution equations [1–9] have attracted much attention of mathematicians and physicists. Especially, exact solutions of nonlinear evolution equations play a pivotal role in the study of mathematical physical phenomena. Not only can these exact solutions describe many important phenomena in physics and other fields, but they can also help physicists to understand the mechanisms of the complicated physical phenomena. A variety of powerful methods have been employed to study nonlinear phenomena, such as the inverse scattering transform [10], the tanh function method [11], the extended tanh-function method [12], the homogeneous balance method [13], the auxiliary function method [14], and the exp-function method [15], the Pfaffian technique [16], the dressing method [17], the Bäcklund transformation method [18], the Darboux transformation [19], the generalized symmetry method, the tri-function method [20] and the G'/G-expansion method [21], the modified CK direct method [22].

Very recently, Dai *et al.* proposed a new technique called the three-wave approach to seek periodic solitary wave solutions for integrable equations [23]. The method is to use Frobenius' idea [24] to reduce the PDE into integrable ODEs. Frobenius' idea was successfully used to establish the transformed rational function method [25] and to solve the KPP equation [26]. In fact, the Tanh function method and the G'/G expansion method are special cases of the reduction idea raised in [26], say, the general Frobenius idea. Furthermore, a three-wave solution in $(3+1)$-dimension was obtained by using the multiple exp-function method [27, 28]. With the rapid development of computer technology and the help of symbolic computation, this approach is of utmost simplicity. Hence, it can be applied to many kinds of nonlinear evolution equations and higher-dimensional soliton equations. Zitian Li obtained periodic cross-kink wave solutions, doubly periodic solitary wave solutions and breather type of two-solitary wave solutions for the $(3+1)$-dimensional Jimbo-Miwa equation by this method [29]. Wang applied the method to a higher dimensional KdV-type equation [30].

The BLMP equation was first derived in [31]:

$$u_{yt} + u_{xxxy} - 3u_{xx}u_y - 3u_x u_{xy} = 0, \tag{1}$$

where $u = u(x, y, t)$ and subscripts represent partial differentiation with respect to the given variable. Boiti *et al.* [31] also discussed the Painlevé property, Lax pairs and some exact solutions of $(2+1)$-dimensional BLMP. Through the Bäcklund transformation, Bai and Zhao got some new solutions of the BLMP equation. By means of the multilinear variable separation approach, a general variable separation solution of the BLMP equation was derived in [32]. Liu proposed a simple Bäcklund transformation of a potential BLMP system by using the standard truncated Painlevé expansion and symbolic computation, and a solution of the potential BLMP system with three arbitrary functions was given in [33]. The symmetry, similarity reductions and new solutions of the $(2+1)$-dimensional BLMP equation were obtained in [34]. These solutions include rational function solutions, double-twisty function solutions, Jacobi oval function solutions and triangular cycle solutions. In [35], based on the binary Bell polynomials, the bilinear form for the BLMP equation was obtained. The new exact solutions were derived with an arbitrary function in y, and soliton interaction properties were discussed by the graphical analysis. The author in [36] discussed the BLMP equation and generalized breaking soliton equations by using the exponential function and obtained some new exact solutions of the equations. By using the modified Clarkson-Kruskal (CK) direct method, Li *et al.* [37] constructed a Bäcklund transformation of the $(2+1)$-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation. Laurent Delisle and Masoud Mosaddeghi proposed the study of the BLMP equation from two points of view: the classical and the super symmetric. They constructed new solutions of this equation from Wronskian formalism and the Hirota method in [38].

In this paper, we consider the $(3+1)$-dimensional Boiti-Leon-Manna-Pempinelli equation

$$u_{yt} + u_{zt} + u_{xxxy} + u_{xxxz} - 3u_x(u_{xy} + u_{xz}) - 3u_{xx}(u_y + u_z) = 0, \tag{2}$$

which was introduced by Darvishi in [39]. We apply the extended three-soliton method to the $(3+1)$-dimensional Boiti-Leon-Manna-Pempinelli equation, obtaining more exact solutions including a complexiton solution, periodic cross-kink solutions about it.

2 Methodology

In this section, we briefly highlight the main features of the extended three-soliton method. Let us consider a PDE for $u(x, z, t)$ in the form

$$P(u, u_t, u_x, u_z, u_{tt}, u_{tx}, u_{tz}, u_{xx}, u_{xz}, u_{zz}, \ldots) = 0, \tag{3}$$

where P is a polynomial in its arguments. The solution method will also work for systems of nonlinear equations and high-dimensional ones.

Step 1. Firstly, we introduce the D-operator which was proposed by Hirota [40] and defined as

$$D_t^m D_x^n a(t,x)\cdot b(t,x) = \frac{\partial^m}{\partial s^m}\frac{\partial^n}{\partial y^n} a(t+s,x+y)b(t-s,x-y)|_{s=0,y=0}. \tag{4}$$

By transformation $u = a\ln f$, $u = \frac{G(f)}{f}$ and D-operator definition, Eq. (3) can be turned into

$$F(D,D_t,D_x,D_z,D_{tt},D_{tx},D_{tz},\ldots)f\cdot f = 0, \tag{5}$$

where F is a polynomial in its arguments.

Step 2. To seek the three-wave solution of Eq. (3), let us consider the solution of Eq. (5) in the following form:

$$f = \cos(\xi) + a_{-1}\exp(-\theta) + a_1\exp(\theta) + a_2\sinh(\eta), \tag{6}$$

where $\xi = p_1(x+\gamma_1 z+\beta_1 y+\alpha_1 t)$, $\eta = p_2(x+\gamma_2 z+\beta_2 y+\alpha_2 t)$, $\theta = p_3(x+\gamma_3 z+\beta_3 y+\alpha_3 t)$ and p_i, α_i, β_i, γ_i $(i = 1,2,3)$ are free constants to be determined later.

Step 3. Substituting Eq. (6) into Eq. (5), and collecting the coefficient of $\sinh(\eta)\cos(\xi)$, $\sinh(\eta)\exp(\theta)$, $\sinh(\eta)\exp(-\theta)$, $\cosh(\eta)\sin(\xi)$, $\cosh(\eta)\exp(\theta)$, $\cosh(\eta)\exp(-\theta)$, $\sin(\xi)\exp(\theta)$, $\sin(\xi)\exp(-\theta)$, $\cos(\xi)\exp(\theta)$, $\cos(\xi)\exp(-\theta)$ to zero, we can derive a set of algebraic equations for a_{-1}, a_1, a_2, p_i, α_i, β_i, γ_i $(i = 1,2,3)$.

Step 4. Solving the set of algebraic equations defined by Step 3 with the help of MAPLE, we can derive parameters a_{-1}, a_1, a_2, p_i, α_i, β_i, γ_i $(i = 1,2,3)$. Therefore, we can obtain abundant exact multi-wave solutions of Eq. (3).

3 Exact three-wave solutions for the (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation

In this section, we consider the following (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation Eq. (2):

$$u_{yt} + u_{zt} + u_{xxxy} + u_{xxxz} - 3u_x(u_{xy} + u_{xz}) - 3u_{xx}(u_y + u_z) = 0,$$

or, equivalently,

$$(u_y + u_z)_t + (u_y + u_z)_{xxx} - 3u_x(u_y + u_z)_x - 3u_{xx}(u_y + u_z) = 0. \tag{7}$$

Under the dependent variable transformation,

$$u = -2(\ln f)_x, \tag{8}$$

where $f(x,y,z,t)$ is an unknown real function, system (2) is turned into

$$\begin{aligned} &-f_y f_t - f_z f_t - f_{xxx} f_y - 3f_{xxy} f_x + 3f_{xx} f_{xy} - f_{xxx} f_z - 3f_{xxz} f_x + 3f_{xx} f_{xz} \\ &\quad + f_{yt} f + f_{zt} f + f_{xxxy} f + f_{xxxz} f = 0. \end{aligned} \tag{9}$$

Equivalently, Eq. (9) can be mapped into the Hirota bilinear equation

$$\left(D_yD_t + D_zD_t + D_yD_x^3 + D_zD_x^3\right)f\cdot f = 0. \tag{10}$$

According to the methodology in Section 2, we can derive a set of algebraic equations for a_{-1}, a_1, a_2, p_i, α_i, β_i, γ_i $(i = 1, 2, 3)$

$$\begin{aligned}
&\sinh(\eta)\cos(\xi):\\
&-3p_2^2p_1^2\gamma_1^2 + p_2^2\alpha_2^2\beta_2 - p_1^2\alpha_1\gamma_1 + p_2^2\alpha_2\gamma_2 - p_1^2\alpha_1\beta_1 - 3p_1^2p_2^2\beta_2 - 3p_2^2p_1^2\beta_1 - 3p_1^2p_2^2\gamma_2\\
&\quad + p_1^4\beta_1 + p_2^4\beta_2 + p_1^4\gamma_1 + p_2^4\gamma_2 = 0,\\
&\sinh(\eta)\exp(\theta), \sinh(\eta)\exp(-\theta):\\
&3p_3^2p_2^2\gamma_2 + 3p_2^2p_3^2\gamma_3 + p_2^4\gamma_2 + p_3^4\beta_3 + 3p_3^2p_2^2\beta_2 + p_2^4\beta_2 + 3p_2^2p_3^2\beta_3 + p_3^4\gamma_3 + p_3^2\beta_3\gamma_3\\
&\quad + p_2^2\alpha_2\gamma_2 + p_2^2\alpha_2\beta_2 + p_3^2\gamma_3\alpha_3 = 0,\\
&\cosh(\eta)\sin(\xi):\\
&-3p_1^2\beta_1 + 3p_2^2\beta_2 + p_2^2\gamma_1 - 3p_1^2\gamma_1 + 3p_2^2\gamma_2 - p_1^2\gamma_2 + \beta_2\alpha_1 + \gamma_1\alpha_2 + \gamma_2\alpha_1\\
&\quad + \beta_1\alpha_2 - p_1^2\beta_2 + p_2^2\beta_1 = 0,\\
&\cosh(\eta)\exp(\theta), \cosh(\eta)\exp(-\theta):\\
&\beta_3\alpha_2 + 3p_2^2\beta_2 + \beta_2\alpha_3 + 3p_3^2\beta_3 + \gamma_3\alpha_2 + p_2^2\gamma_3 + \gamma_2\alpha_3 + p_3^2\gamma_2 + p_3^2\beta_2\\
&\quad + 3p_2^2\gamma_2 + p_2^2\beta_3 + 3p_3^2\gamma_3 = 0,\\
&\sin(\xi)\exp(\theta), \sin(\xi)\exp(-\theta):\\
&-\beta_1\alpha_3 + p_1^2\beta_3 + p_1^2\gamma_3 - p_3^2\gamma_1 + 3p_1^2\beta_1 + 3p_1^2\gamma_1 - p_3^2\beta_1 - \beta_3\alpha_1 - 3p_3^2\gamma_3\\
&\quad - \gamma_1\alpha_3 - 3p_3^2\beta_3 - \gamma_3\alpha_1 = 0,\\
&\cos(\xi)\exp(\theta), \cos(\xi)\exp(-\theta):\\
&p_3^2\beta_3\alpha_3 - p_1^2\alpha_1\gamma_1 - 3p_1^2p_3^2\beta_3 + p_3^2\gamma_3\alpha_3 - 3p_3^2p_1^2\beta_1 + p_1^4\beta_1 - 3p_1^2p_3^2\gamma_3 + p_3^4\beta_3\\
&\quad - 3p_3^2p_1^2\gamma_1 + p_1^4\gamma_1 - p_1^2\alpha_1\beta_1 + p_3^4\gamma_3 = 0,
\end{aligned}$$

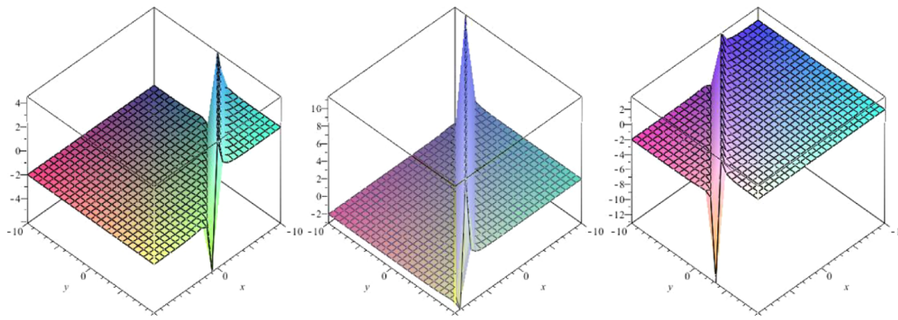
and constant term:

$$\begin{aligned}
&-a_2^2p_2^2\alpha_2\beta_2 - a_2^2p_2^2\alpha_2\gamma_2 - p_1^2\alpha_1\beta_1 - p_1^2\alpha_1\gamma_1 - 4a_2^2p_2^4\beta_2 - 4a_2^2p_2^4\gamma_2 + 16a_{-1}p_3^4a_1\beta_3\\
&\quad + 16a_{-1}p_3^4a_1\gamma_3 + 4a_{-1}p_3^2\beta_3a_1\alpha_3 + 4a_{-1}p_3^2\gamma_3a_1\alpha_3 + 4p_1^4\beta_1 + 4p_1^4\gamma_1 = 0.
\end{aligned}$$

Solving the above algebraic equations with the help of MAPLE gives the following solutions.

Case 1.

$$\begin{aligned}
&a_{-1} = a_{-1}, \qquad a_1 = a_1, \qquad a_2 = a_2, \qquad p_1 = 0, \qquad p_2 = 0, \qquad p_3 = p_3,\\
&\alpha_1 = \alpha_1, \qquad \alpha_2 = \alpha_2, \qquad \alpha_3 = -p_3^2,
\end{aligned}$$



(a) t=-8 (b) t=0 (c) t=8

Figure 1 The propagation of solution u_1 with $a_{-1} = -1$, $a_1 = 1$, $p_3 = 1$, $\gamma_3 = 1$, $z = 1$.

$$\beta_1 = -\frac{\beta_2\alpha_1 + \gamma_1\alpha_2 + \gamma_2\alpha_1}{\alpha_2}, \qquad \beta_2 = \beta_2, \qquad \beta_3 = -\gamma_3,$$
$$\gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$

In this case, we obtain the single soliton solution

$$u_1 = -2\frac{-a_{-1}p_3e^{-p_3[x+\gamma_3(z-y)-p_3^2t]} + a_1p_3e^{p_3[x+\gamma_3(z-y)-p_3^2t]}}{1 + a_{-1}e^{-p_3[x+\gamma_3(z-y)-p_3^2t]} + a_1e^{p_3[x+\gamma_3(z-y)-p_3^2t]}}, \tag{11}$$

where a_{-1}, a_1, p_3, γ_3 are free constants. The propagation of solution u_1 is described in Figure 1.

Case 2.

$$a_{-1} = a_{-1}, \qquad a_1 = a_1, \qquad a_2 = a_2, \qquad p_1 = p_1, \qquad p_2 = 0, \qquad p_3 = 0,$$
$$\alpha_1 = p_1^2, \qquad \alpha_2 = \alpha_2, \qquad \alpha_3 = \alpha_3,$$
$$\beta_1 = -\gamma_1, \qquad \beta_2 = -\frac{\beta_3\alpha_2 + \gamma_3\alpha_2 + \gamma_2\alpha_3}{\alpha_3}, \qquad \beta_3 = \beta_3,$$
$$\gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$

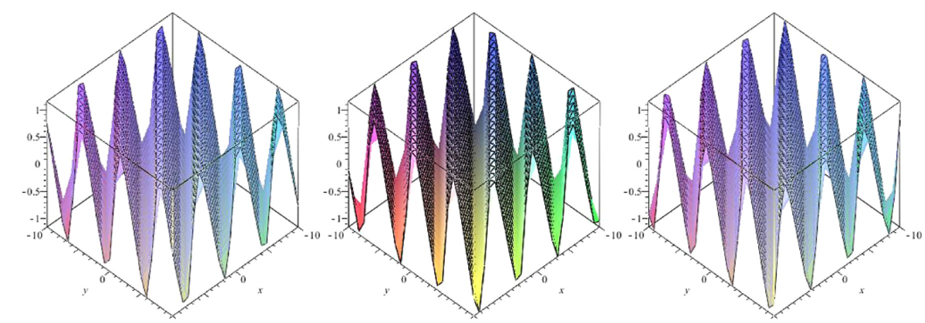
Then we obtain new periodic solutions as follows:

$$u_2 = \frac{2p_1\sin(p_1[x + \gamma_1(z-y) + p_1^2t])}{\cos(p_1[x + \gamma_1(z-y) + p_1^2t]) + a_{-1} + a_1}, \tag{12}$$

where a_{-1}, a_1, p_1, γ_1 are free constants. The propagation of solution u_2 is described in Figure 2.

Case 3.

$$a_{-1} = 0, \qquad a_1 = a_1, \qquad a_2 = a_2, \qquad p_1 = 0, \qquad p_2 = p_2, \qquad p_3 = p_3,$$
$$\alpha_1 = \alpha_1, \qquad \alpha_2 = -p_2^2, \qquad \alpha_3 = -p_3^2,$$
$$\beta_1 = \beta_1, \qquad \beta_2 = -\gamma_2, \qquad \beta_3 = -\gamma_3, \qquad \gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$



(a) t=-5 (b) t=0 (c) t=5

Figure 2 The propagation of solution u_2 with $a_{-1} = 1$, $a_1 = 1$, $p_1 = -1$, $\gamma_1 = 1$, $z = 5$.

We then obtain

$$u_3 = -2\frac{a_1p_3e^{p_3[x+\gamma_3(z-y)-p_3^2t]} + a_2p_2\sinh(p_2[x+\gamma_2(z-y)-p_2^2t])}{1 + a_1e^{p_3[x+\gamma_3(z-y)-p_3^2t]} + a_2\cosh(p_2[x+\gamma_2(z-y)-p_2^2t])}, \tag{13}$$

where $a_1, a_2, p_2, p_3, \gamma_3$ are arbitrary constants.

Case 4.

$$\begin{aligned}
&a_{-1} = \frac{1}{4a_1}, \quad a_1 = a_1, \quad a_2 = a_2, \quad p_1 = p_3i, \quad p_2 = 0, \quad p_3 = p_3,\\
&\alpha_1 = -p_3^2, \quad \alpha_2 = -3p_3^2, \quad \alpha_3 = -p_3^2,\\
&\beta_1 = -\gamma_1 - \beta_3 - \gamma_3, \quad \beta_2 = \beta_2, \quad \beta_3 = \beta_2, \quad \gamma_1 = \gamma_1, \quad \gamma_2 = \gamma_2, \quad \gamma_3 = \gamma_3.
\end{aligned}$$

We then obtain a complexiton solution

$$u_4 = -2\frac{-p_3i\sin(\xi) - \frac{p_3e^{-p_3(x+\gamma_3z+\beta_3y-p_3^2t)}}{4a_1} + a_1p_3e^{p_3(x+\gamma_3z+\beta_3y-p_3^2t)}}{\cos(\xi) + \frac{e^{-p_3(x+\gamma_3z+\beta_3y-p_3^2t)}}{4a_1} + a_1e^{p_3(x+\gamma_3z+\beta_3y-p_3^2t)}}, \tag{14}$$

where $\xi = ip_3(x + \gamma_1z + (-\gamma_1 - \beta_3 - \gamma_3)y - p_3^2t)$ and $a_1, p_3, \alpha_3, \beta_3, \gamma_1, \gamma_3$ are free constants.

Case 5.

$$\begin{aligned}
&a_{-1} = a_{-1}, \quad a_1 = a_1, \quad a_2 = a_2, \quad p_1 = p_1, \quad p_2 = 0,\\
&p_3 = p_3, \quad \alpha_1 = p_1^2, \quad \alpha_2 = \alpha_2,\\
&\alpha_3 = \alpha_3, \quad \beta_1 = -\gamma_1, \quad \beta_2 = -\gamma_2, \quad \beta_3 = -\gamma_3,\\
&\gamma_1 = \gamma_1, \quad \gamma_2 = \gamma_2, \quad \gamma_3 = \gamma_3.
\end{aligned}$$

We then obtain new periodic cross-kink solutions

$$u_5 = -2\frac{-p_1\sin(\xi_1) - a_{-1}p_3e^{-p_3(x+\gamma_3z+\beta_3y-p_3^2t)} + a_1p_3e^{p_3(x+\gamma_3z-\gamma_3y+\alpha_3t)}}{\cos(\xi_1) + a_{-1}e^{p_3(x+\gamma_3z-\gamma_3y+\alpha_3t)} + a_1e^{p_3(x+\gamma_3z-\gamma_3y+\alpha_3t)}}, \tag{15}$$

where $\xi_1 = p_1(x + \gamma_1z - \gamma_1y + p_1^2t)$ and $a_{-1}, a_1, p_1, p_3, \alpha_3, \gamma_3$ are free constants.

Case 6.

$$a_{-1} = -\frac{p_1^2\gamma_1 + p_1^2\beta_1 + a_2^2p_3^2\gamma_3 + a_2^2p_3^2\beta_3}{4(\beta_3+\gamma_3)p_3^2a_1}, \qquad a_1 = a_1, \qquad a_2 = a_2,$$
$$p_1 = p_1, \qquad p_2 = p_3, \qquad p_3 = p_3, \qquad \alpha_1 = -3p_3^2 + p_1^2, \qquad \alpha_2 = 3p_1^2 - p_3^2,$$
$$\alpha_3 = 3p_1^2 - p_3^2, \qquad \beta_1 = \beta_1, \qquad \beta_2 = -\gamma_2 - \gamma_3 - \beta_3, \qquad \beta_3 = \beta_3,$$
$$\gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$

We then obtain new periodic cross-kink solutions

$$u_6 = -2\frac{-p_1\sin(\xi_2) + \frac{p_1^2\gamma_1+p_1^2\beta_1+a_2^2p_3^2\gamma_3+a_2^2p_3^2\beta_3}{4(\beta_3+\gamma_3)p_3a_1}e^{-\theta_2} + a_1p_3e^{\theta_2} + a_2p_3\cosh(\eta_2)}{\cos(\xi_2) - \frac{p_1^2\gamma_1+p_1^2\beta_1+a_2^2p_3^2\gamma_3+a_2^2p_3^2\beta_3}{4(\beta_3+\gamma_3)p_3^2a_1}e^{-\theta_2} + a_1e^{\theta_2} + a_2\sinh(\eta_2)}, \tag{16}$$

where $\xi_2 = p_1(x + \gamma_1z + \beta_1y + (-3p_3^2 + p_1^2)t)$, $\theta_2 = p_3(x + \gamma_3z + \beta_3y + (3p_1^2 - p_3^2)t)$, $\eta_2 = p_3(x + \gamma_2z + (-\gamma_2 - \gamma_3 - \beta_3)y + (3p_1^2 - p_3^2)t)$ and $a_1, a_2, p_1, p_3, \beta_1, \beta_3, \gamma_1, \gamma_2, \gamma_3$ are free constants.

Case 7.

$$a_{-1} = 0, \qquad a_1 = a_1, \qquad a_2 = a_2, \qquad p_1 = ip_2, \qquad p_2 = p_2,$$
$$p_3 = p_3, \qquad \alpha_1 = -4p_2^2, \qquad \alpha_2 = -4p_2^2, \qquad \alpha_3 = -\frac{3p_3p_2^2 + p_3^3 - 3p_3^2p_2 + 3p_2^3}{p_3},$$
$$\beta_1 = -\frac{p_3\beta_3 + p_3\gamma_3 + p_2\gamma_1}{p_2}, \qquad \beta_2 = -\frac{p_3\beta_3 + p_2\gamma_2 + p_3\gamma_3}{p_2}, \qquad \beta_3 = \beta_3,$$
$$\gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$

We then obtain a new complexiton solution

$$u_7 = -2\frac{-ip_2\sin(\xi_3) + a_1p_3e^{p_3(x+\gamma_3z+\beta_3y-\frac{3p_3p_2^2+p_3^3-3p_3^2p_2+3p_2^3}{p_3}t)} + a_2p_2\cosh(\eta_3)}{\cos(\xi_3) + a_1e^{p_3(x+\gamma_3z+\beta_3y-\frac{3p_3p_2^2+p_3^3-3p_3^2p_2+3p_2^3}{p_3}t)} + a_2\sinh(\eta_3)}, \tag{17}$$

where $\xi_3 = p_1(x + \gamma_1z - \frac{p_3\beta_3+p_3\gamma_3+p_2\gamma_1}{p_2}y - 4p_2^2t)$, $\eta_3 = p_2(x + \gamma_2z - \frac{p_3\beta_3+p_2\gamma_2+p_3\gamma_3}{p_2}y - 4p_2^2t)$ and $a_1, a_2, p_2, p_3, \beta_3, \gamma_1, \gamma_2, \gamma_3$ are free constants.

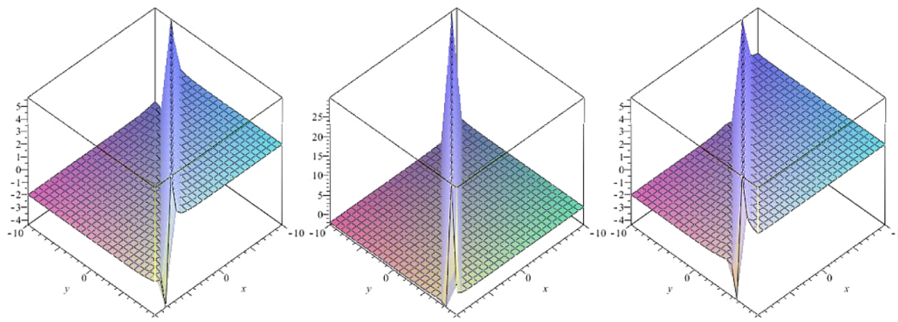
Case 8.

$$a_{-1} = a_{-1}, \qquad a_1 = a_1, \qquad a_2 = a_2, \qquad p_1 = p_1, \qquad p_2 = p_2,$$
$$p_3 = 0, \qquad \alpha_1 = p_1^2, \qquad \alpha_2 = -p_2^2, \qquad \alpha_3 = \alpha_3,$$
$$\beta_1 = -\gamma_1, \qquad \beta_2 = -\gamma_2, \qquad \beta_3 = \beta_3,$$
$$\gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$

We then obtain new periodic cross-kink solutions

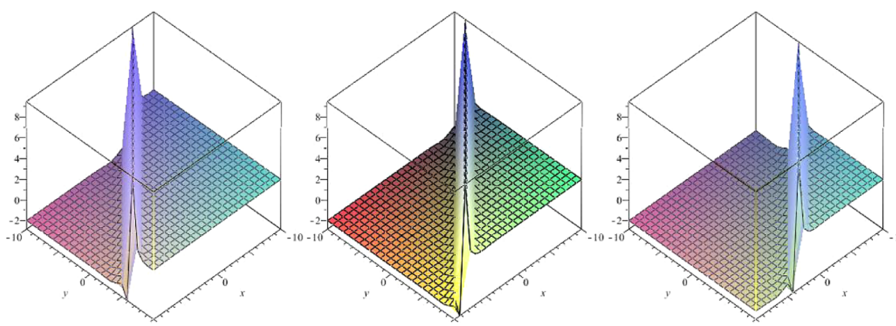
$$u_8 = -2\frac{-p_1\sin(p_1(x + \gamma_1z - \gamma_1y + p_1^2t)) + a_2p_2\cosh(p_2(x + \gamma_2z - \gamma_2y - p_2^2t))}{\cos(p_1(x + \gamma_1z - \gamma_1y + p_1^2t)) + a_{-1} + a_1 + a_2\sinh(p_2(x + \gamma_2z - \gamma_2y - p_2^2t))}, \tag{18}$$

where $a_{-1}, a_1, a_2, p_1, p_2, \alpha_3, \beta_3, \gamma_1, \gamma_2$ and γ_3 are free constants.



(a) t=0 (b) t=3 (c) t=5

Figure 3 The propagation of solution u_3 with $a_1 = 1$, $a_2 = 1$, $p_2 = 1$, $p_3 = 1$, $\gamma_2 = 1$, $\gamma_3 = 1$, $z = 1$.



(a) t=-5 (b) t=0 (c) t=5

Figure 4 The propagation of solution u_5 with $a_{-1} = 1$, $a_1 = 1$, $p_1 = 1$, $p_3 = 1$, α_1, $\gamma_1 = 1$, $\gamma_3 = 1$, $z = 1$.

Case 9.

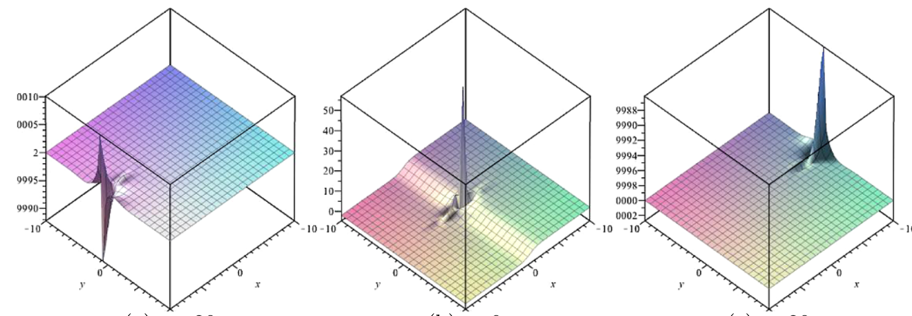
$$a_{-1} = \frac{(a_2^2\beta_1 + \beta_1 + \gamma_1 + a_2^2\gamma_1)p_2^2}{4(\beta_3 + \gamma_3)p_3^2 a_1} a_1 = a_1, \qquad a_2 = a_2, \qquad p_1 = ip_2,$$
$$p_2 = p_2, \qquad p_3 = p_3, \qquad \alpha_1 = -3p_3^2 - p_2^2, \qquad \alpha_2 = -3p_3^2 - p_2^2, \qquad \alpha_3 = -3p_2^2 - p_3^2,$$
$$\beta_1 = \beta_1, \qquad \beta_2 = -\beta_1 - \gamma_2 - \gamma_1, \qquad \beta_3 = \beta_3, \qquad \gamma_1 = \gamma_1, \qquad \gamma_2 = \gamma_2, \qquad \gamma_3 = \gamma_3.$$

We then obtain a complexiton solution

$$u_9 = -2\frac{-ip_2\sin(\xi_4) - \frac{a_2^2\beta_1+\beta_1+\gamma_1+a_2^2\gamma_1}{4(\beta_3+\gamma_3)p_3 a_1}e^{-\theta_4} + a_1 p_3 e^{\theta_4} + a_2 p_2\cosh(\eta_4)}{\cos(\xi_4) + \frac{a_2^2\beta_1+\beta_1+\gamma_1+a_2^2\gamma_1}{4(\beta_3+\gamma_3)p_3^2 a_1}e^{-\theta_4} + a_1 e^{\theta_4} + a_2\cosh(\eta_4)}, \tag{19}$$

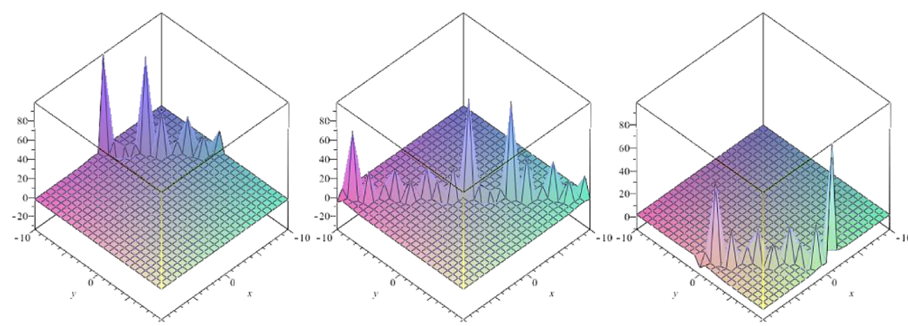
where $\xi_4 = ip_2(x + \gamma_1 z + \beta_1 y + (-3p_3^2 - p_2^2)t)$, $\theta_4 = p_3(x + \gamma_3 z + \beta_3 y + (-3p_2^2 - p_3^2)t)$, $\eta_4 = p_2(x + \gamma_2 z + (-\beta_1 - \gamma_2 - \gamma_1)y + (-3p_3^2 - p_2^2)t)$ and a_1, a_2, p_2, p_3, β_1, β_3, γ_1, γ_2, γ_3 are free constants. Figures 3, 4, 5, 6 described the solution of u_3, u_4, u_6 and u_8 respectively.

Remark 1 Noting if we set $\beta_i = -\gamma_i$ in Case 1 to Case 5 of the solutions above are special solutions of the equation, we can see that for an arbitrary function, $u(x, y - z, t)$ is also a solution. However, the other cases are different.



(a) t=-20 (b) t=0 (c) t=20

Figure 5 The propagation of solution u_6 with $a_1 = 1$, $a_2 = 1$, $p_1 = 1$, $p_3 = 1$, $\beta_1 = 1$, $\beta_3 = 1$, $\gamma_1 = 1$, $\gamma_2 = -1$, $\gamma_3 = 1$, $z = 1$.



(a) t=-10 (b) t=0 (c) t=10

Figure 6 The propagation of solution u_8 with $a_{-1} = 1$, $a_1 = 1$, $a_2 = 1$, $p_1 = 1$, $p_2 = 1$, $\gamma_1 = 1$, $\gamma_2 = -1$, $z = 1$.

Remark 2 Noting $\sinh(ix) = i\sin(x)$ and $\cos(ix) = \cosh(x)$, the solutions presented in this paper can be obtained by using the multiple exp-function. Furthermore, we can get an N-soliton solution just by modifying the ansatz and using the exp expanding method [27].

4 Conclusion

In this paper, we obtained three-wave solutions to the (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation with the extended three-soliton method. All the presented solutions show remarkable richness of the solution space of the (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation and also that the (3 + 1)-dimensional integrable system may have very rich dynamical behavior. The considered solutions are of complexiton type [41]. There is also a generalized theory of the Bell polynomials method which describes the generalized bilinear differential equations [42, 43]. To our knowledge, our solutions are novel. They cannot be obtained just through the simple generalization of the (2 + 1)-dimensional BLMP equation. In fact, the extended three-soliton method is entirely algorithmic and involves a large amount of tedious calculations. However, the method is direct, concise and effective. Therefore, we can apply the method to the variety of dynamics of a higher-dimensional nonlinear system and many other types of a nonlinear evolution equation in further work.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Acknowledgements
The authors are in debt to thank the anonymous referees for helpful suggestions. The work is supported by the National Natural Science Foundation of China (project No. 11371086), the Fund of Science and Technology Commission of Shanghai Municipality (project No. ZX201307000014) and the Fundamental Research Funds for the Central Universities.

Received: 11 July 2013 Accepted: 26 September 2013 Published: 18 Nov 2013

References
1. Korteweg, D, de Vries, G: On the change of form of long waves advancing in a rectangular canal, and on a new type of long stationary waves. Philos. Mag. **39**, 422-443 (1895)
2. Toda, K, Yu, S: The investigation into new equations in (2 + 1)-dimensions. J. Nonlinear Math. Phys. **8**, 272-277 (2001)
3. Yu, S, Toda, K: Lax pairs, Painlevè properties and exact solutions of the alogero Korteweg-de Vries equation and a new (2 + 1)-dimensional equation. J. Nonlinear Math. Phys. **7**, 1-13 (2000)
4. Li, C-X, Ma, WX, Liu, X-J, Zeng, Y-B: Wronskian solutions of the Boussinesq equation-solitons, negatons, positons and complexitons. Inverse Probl. **23**, 279-296 (2007)
5. Ma, WX, Maruno, K: Complexiton solutions of the Toda lattice equation. Physica A **343**, 219-237 (2004)
6. Ma, WX, You, Y: Solving the Korteweg-de Vries equation by its bilinear form: Wronskian solutions. Trans. Am. Math. Soc. **357**, 1753-1778 (2005)
7. Wazwaz, A-M: New higher-dimensional fifth-order nonlinear equations with multiple soliton solutions. Phys. Scr. **84**, 025007 (2011)
8. Wazwaz, A-M: A new (2 + 1)-dimensional Korteweg-de Vries equation and its extension to a new (3 + 1)-dimensional Kadomtsev-Petviashvili equation. Phys. Scr. **84**, 035010 (2011)
9. Asaad, MG, Ma, WX: Extended Gram-type determinant, wave and rational solutions to two (3 + 1)-dimensional nonlinear evolution equations. Appl. Math. Comput. **219**, 213-225 (2012)
10. Ablowitz, MJ, Clarkson, PA: Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, New York (1991)
11. Parkes, EJ, Duffy, BR: An automated tanh-function method for finding solitary wave solutions to non-linear evolution equations. Comput. Phys. Commun. **98**, 288-300 (1996)
12. Fan, E: Extended tanh-function method and its applications to nonlinear equations. Phys. Lett. A **277**, 212-218 (2000)
13. Wang, M, Zhou, Y, Li, Z: Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics. Phys. Lett. A **216**, 67-75 (1996)
14. Zhang, S, Xia, T: A generalized new auxiliary equation method and its applications to nonlinear partial differential equations. Phys. Lett. A **363**, 356-360 (2007)
15. He, J-H: An elementary introduction to recently developed asymptotic methods and nanomechanics in textile engineering. Int. J. Mod. Phys. B **22**, 3487-3578 (2008)
16. Hirota, R: Soliton solutions to the BKP equations. I. The Pfaffian technique. J. Phys. Soc. Jpn. **58**, 2285-2296 (1989)
17. Zakharov, V, Shabat, A: Integration of nonlinear equations of mathematical physics by the method of inverse scattering. II. Funct. Anal. Appl. **13**, 166-174 (1979)
18. Satsuma, J, Kaup, DJ: A Bäcklund transformation for a higher order Korteweg-de Vries equation. J. Phys. Soc. Jpn. **43**, 692-697 (1977)
19. Gu, C, Hu, H, Zhou, Z: Darboux Transformations in Integrable Systems: Theory and Their Applications to Geometry. Kluwer Academic, London (2005)
20. Yan, Z: The new tri-function method to multiple exact solutions of nonlinear wave equations. Phys. Scr. **78**, 035001 (2008)
21. Wang, M, Li, X, Zhang, J: The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A **372**, 417-423 (2008)
22. Lou, S, Ma, H-C: Non-Lie symmetry groups of (2 + 1)-dimensional nonlinear systems obtained from a simple direct method. J. Phys. A, Math. Gen. **38**, L129-L137 (2005)
23. Dai, Z, Lin, S, Fu, H, Zeng, X: Exact three-wave solutions for the KP equation. Appl. Comput. Math. **216**, 1599-1604 (2010)
24. Ma, WX, Wu, H, He, J-S: Partial differential equations possessing Frobenius integrable decompositions. Phys. Lett. A **364**, 29-32 (2007)
25. Ma, WX, Lee, J-H: A transformed rational function method and exact solutions to the 3 + 1 dimensional Jimbo-Miwa equation. Chaos Solitons Fractals **42**, 1356-1363 (2009)
26. Ma, WX, Fuchssteiner, B: Explicit and exact solutions to a Kolmogorov-Petrovskii-Piskunov equation. Int. J. Non-Linear Mech. **31**, 329-338 (1996)
27. Ma, WX, Huang, T, Zhang, Y: A multiple exp-function method for nonlinear differential equations and its application. Phys. Scr. **82**, 065003 (2010)
28. Ma, WX, Zhu, Z: Solving the (3 + 1)-dimensional generalized KP and BKP equations by the multiple exp-function algorithm. Appl. Comput. Math. **218**, 11871-11879 (2012)
29. Li, Z, Dai, Z, Liu, J: Exact three-wave solutions for the (3 + 1)-dimensional Jimbo-Miwa equation. Comput. Math. Appl. **61**, 2062-2066 (2011)
30. Wang, C, Dai, Z, Liang, L: Exact three-wave solution for higher dimensional KdV-type equation. Appl. Comput. Math. **216**, 501-505 (2010)
31. Boiti, M, Leon, JJ-P, Pempinelli, F: On the spectral transform of a Korteweg-de Vries equation in two spatial dimensions. Inverse Probl. **2**, 271-279 (1986)

32. Bai, C-J, Zhao, H: New solitary wave and Jacobi periodic wave excitations in (2 + 1)-dimensional Boiti-Leon-Manna-Pempinelli system. Int. J. Mod. Phys. B **22**, 2407-2420 (2008)
33. Liu, GT: Bäcklund transformation and new coherent structures of the potential BLMP system. J. Inn. Mong. Norm. Univ. **37**, 145-148 (2008)
34. Liu, N, Liu, X: Symmetries, new exact solutions and conservation laws of (2 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation. Chin. J. Quantum Electron. **25**, 546-552 (2008)
35. Luo, L: New exact solutions and Bäcklund transformation for Boiti-Leon-Manna-Pempinelli equation. Phys. Lett. A **375**, 1059-1063 (2011)
36. Zhang, LL: Exact solutions of breaking soliton equations and BLMP equation. J. Liaocheng Univ. Nat. Sci **21**, 35-38 (2008)
37. Li, Y, Li, D: New exact solutions for the (2 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation. Appl. Math. Sci. **6**, 579-587 (2012)
38. Delisle, L, Mosaddeghi, M: Classical and SUSY solutions of the Boiti-Leon-Manna-Pempinelli equation. J. Phys. A, Math. Theor. **46**, 115203 (2013)
39. Darvishi, M, Najafi, M, Kavitha, L, Venkatesh, M: Stair and step soliton solutions of the integrable (2 + 1) and (3 + 1)-Dimensional Boiti-Leon-Manna-Pempinelli Equations. Commun. Theor. Phys. **58**, 785-794 (2012)
40. Hirota, R: The Direct Method in Soliton Theory. Cambridge University Press, Cambridge (2004)
41. Ma, WX: Complexiton solutions to the Korteweg-de Vries equation. Phys. Lett. A **301**, 35-44 (2002)
42. Ma, WX: Bilinear equations, Bell polynomials and linear superposition principle. J. Phys. Conf. Ser. **411**, 012021 (2013)
43. Ma, WX: Generalized bilinear differential equations. Stud. Nonlinear Sci. **2**, 140-144 (2011)

10.1186/1687-1847-2013-321
Cite this article as: Ma et al.: Exact three-wave solutions for the (3 + 1)-dimensional Boiti-Leon-Manna-Pempinelli equation. *Advances in Difference Equations* 2013, 2013:321