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On some new sequence spaces defined by infinite matrix and modulus

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Abstract
The goal of this paper is to introduce and study some properties of some sequence spaces that are defined using the φ-function and the generalized three parametric real matrix A. Also, we define $\mathbf{A}$-statistical convergence.

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1 Introduction and background

Let s denote the set of all real and complex sequences $x = (x_k)$. By l_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\|x\| = \sup_n |x_n|$, respectively. A linear functional L on l_∞ is said to be a Banach limit [1] if it has the following properties:

(1) $L(x) \geq 0$ if $n \geq 0$ (*i.e.*, $x_n \geq 0$ for all n),
(2) $L(e) = 1$, where $e = (1, 1, \ldots)$,
(3) $L(Dx) = L(x)$, where the shift operator D is defined by $D(x_n) = \{x_{n+1}\}$.

Let B be the set of all Banach limits on l_∞. A sequence $x \in \ell_\infty$ is said to be almost convergent if all Banach limits of x coincide. Let $\hat{c}$ denote the space of almost convergent sequences. Lorentz [2] has shown that

$$\hat{c} = \Big\{ x \in l_\infty : \lim_m t_{m,n}(x) \text{ exists uniformly in } n \Big\},$$

where

$$t_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

By a lacunary $\theta = (k_r)$, $r = 0, 1, 2, \ldots$, where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and $h_r = k_r - k_{r-1}$.The ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r.

The space of lacunary strongly convergent sequences N_θ was defined by Freedman *et al.* [3] as follows:

$$N_\theta = \Big\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{k \in I_r} |x_k - le| = 0 \text{ for some } l \Big\}.$$

There is a strong connection between N_θ and the space w of strongly Cesàro summable sequences which is defined by

$$w = \left\{ x = (x_k) : \lim_n \frac{1}{n} \sum_{k=0}^{n} |x_k - le| = 0 \text{ for some } l \right\}.$$

In the special case where $\theta = (2^r)$, we have $N_\theta = w$.

More results on lacunary strong convergence can be seen from [4–11].

Ruckle [12] used the idea of a modulus function f to construct a class of FK spaces

$$L(f) = \left\{ x = (x_k) : \sum_{k=1}^{\infty} f\big(|x_k|\big) < \infty \right\}.$$

The space $L(f)$ is closely related to the space l_1 which is an $L(f)$ space with $f(x) = x$ for all real $x \geq 0$.

Maddox [13] introduced and examined some properties of the sequence spaces $w_0(f)$, $w(f)$ and $w_\infty(f)$ defined using a modulus f, which generalized the well-known spaces w_0, w and w_∞ of strongly summable sequences.

Recently Savaş [14] generalized the concept of strong almost convergence by using a modulus f and examined some properties of the corresponding new sequence spaces. Waszak [15] defined the lacunary strong (A, φ)-convergence with respect to a modulus function.

Following Ruckle, a modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that

(i) $f(x) = 0$ if and only if $x = 0$,

(ii) $f(x + y) \leq f(x) + f(x)$ for all $x, y \geq 0$,

(iii) f increasing,

(iv) f is continuous from the right at zero.

Since $|f(x) - f(y)| \leq f(|x - y|)$, it follows from condition (iv) that f is continuous on $[0, \infty)$.

By a φ-function we understood a continuous non-decreasing function $\varphi(u)$ defined for $u \geq 0$ and such that $\varphi(0) = 0$, $\varphi(u) > 0$ for $u > 0$ and $\varphi(u) \to \infty$ as $u \to \infty$.

A φ-function φ is called no weaker than a φ-function ψ if there are constants $c, b, k, l > 0$ such that $c\psi(lu) \leq b\varphi(ku)$ (for all large u) and we write $\psi \prec \varphi$.

φ-functions φ and ψ are called equivalent and we write $\varphi \sim \psi$ if there are positive constants b_1, b_2, c, k_1, k_2, l such that $b_1\varphi(k_1 u) \leq c\psi(lu) \leq b_2\varphi(k_2 u)$ (for all large u).

A φ-function φ is said to satisfy (Δ_2)-condition (for all large u) if there exists a constant $K > 1$ such that $\varphi(2u) \leq K\varphi(u)$.

In the present paper, we introduce and study some properties of the following sequence space that is defined using the φ-function and the generalized three parametric real matrix.

2 Main results

Let φ and f be a given φ-function and a modulus function, respectively. Moreover, let $\mathbf{A} = (a_{nk}(i))$ be the generalized three parametric real matrix, and let a lacunary sequence θ be given. Then we define

$$N_\theta^0(\mathbf{A}, \varphi, f) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i)\varphi\big(|x_k|\big) \right| \right) = 0 \text{ uniformly in } i \right\}.$$

If $x \in N_\theta^0(\mathbf{A},\varphi,f)$, the sequence x is said to be lacunary strong $(\mathbf{A},\varphi)$-convergent to zero with respect to a modulus f. When $\varphi(x) = x$, for all x, we obtain

$$N_\theta^0(\mathbf{A},f) = \left\{x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\big(|x_k|\big)\right|\right) = 0 \text{ uniformly in } i\right\}.$$

If we take $f(x) = x$, we write

$$N_\theta^0(\mathbf{A},\varphi) = \left\{x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n\in I_r} \left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi\big(|x_k|\big)\right| = 0 \text{ uniformly in } i\right\}.$$

If we take $\mathbf{A} = I$ and $\varphi(x) = x$ respectively, then we have [16]

$$N_\theta^0 = \left\{x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n\in I_r} f\big(|x_k|\big) = 0 \text{ uniformly in } i\right\}.$$

If we define the matrix $A = (a_{nk}(i))$ as follows: for all i,

$$a_{nk}(i) := \begin{cases} \frac{1}{n} & \text{if } n \geq k, \\ 0 & \text{otherwise,} \end{cases}$$

then we have

$$N_\theta^0(\mathbf{C},\varphi,f) = \left\{x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n\in I_r} f\left(\left|\frac{1}{n}\sum_{k=1}^{n} \varphi\big(|x_k|\big)\right|\right) = 0 \text{ uniformly in } i\right\},$$

$$a_{nk}(i) := \begin{cases} \frac{1}{n} & \text{if } i \leq k \leq i+n-1, \\ 0 & \text{otherwise,} \end{cases}$$

then we have

$$N_\theta^0(\hat{c},\varphi,f) = \left\{x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n\in I_r} f\left(\left|\frac{1}{n}\sum_{k=i}^{i+n} \varphi\big(|x_k|\big)\right|\right) = 0 \text{ uniformly in } i\right\}.$$

We are now ready to write the following theorem.

Theorem 2.1 *Let $\mathbf{A} = (a_{nk}(i))$ be the generalized three parametric real matrix, and let the φ-function $\varphi(u)$ satisfy the condition (Δ_2). Then the following conditions are true.*

(a) *If $x = (x_k) \in w(\mathbf{A},\varphi,f)$ and α is an arbitrary number, then $\alpha x \in w(\mathbf{A},\varphi,f)$.*

(b) *If $x, y \in w(\mathbf{A},\varphi,f)$, where $x = (x_k)$, $y = (y_k)$ and α, β are given numbers, then $\alpha x + \beta y \in w(\mathbf{A},\varphi,f)$.*

The proof is a routine verification by using standard techniques and hence is omitted.

Theorem 2.2 *Let f be any modulus function, and let the generalized three parametric real matrix A and the sequence θ be given. If*

$$w(\mathbf{A},\varphi,f) = \left\{x = (x_k) : \lim_m \frac{1}{m} \sum_{n=1}^{m} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi\big(|x_k|\big)\right|\right) = 0 \textit{ uniformly in } i\right\},$$

then the following relations are true.

(a) *If* $\liminf_r q_r > 1$, *then we have* $w(A,\varphi,f) \subseteq N_\theta^0(\mathbf{A},\varphi,f)$.
(b) *If* $\sup_r q_r < \infty$, *then we have* $N_\theta^0(\mathbf{A},\varphi,f) \subseteq w(A,\varphi,f)$.
(c) $1 < \liminf_r q_r \le \limsup_r q_r < \infty$, *then we have* $N_\theta^0(\mathbf{A},\varphi,f) = w(\mathbf{A},\varphi,f)$.

Proof (a) Let us suppose that $x \in w(A,\varphi,f)$. There exists $\delta > 0$ such that $q_r > 1 + \delta$ for all $r \ge 1$, and we have $h_r/k_r \ge \delta/(1+\delta)$ for sufficiently large r. Then, for all i,

$$\begin{aligned}
&\frac{1}{k_r}\sum_{n=1}^{k_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) \\
&\quad\ge \frac{1}{k_r}\sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) \\
&\quad= \frac{h_r}{k_r}\frac{1}{h_r}\sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) \\
&\quad\ge \frac{\delta}{1+\delta}\frac{1}{h_r}\sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}\varphi(|x_k|)\right|\right).
\end{aligned}$$

Hence, $x \in N_\theta^0(\mathbf{A},\varphi,f)$.

(b) If $\limsup_r q_r < \infty$, then there exists $M > 0$ such that $q_r < M$ for all $r \ge 1$. Let $x \in N_\theta^0(\mathbf{A},\varphi,f)$ and ε be an arbitrary positive number, then there exists an index j_0 such that for every $j \ge j_0$ and all i,

$$R_j = \frac{1}{h_j}\sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) < \varepsilon.$$

Thus, we can also find $K > 0$ such that $R_j \le K$ for all $j = 1, 2, \ldots$. Now, let m be any integer with $k_{r-1} \le m \le k_r$, then we obtain, for all i,

$$I = \frac{1}{m}\sum_{n=1}^{m} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) \le \frac{1}{k_{r-1}}\sum_{n=1}^{k_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) = I_1 + I_2,$$

where

$$\begin{aligned}
I_1 &= \frac{1}{k_{r-1}}\sum_{j=1}^{j_0}\sum_{n\in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right), \\
I_2 &= \frac{1}{k_{r-1}}\sum_{j=j_0+1}^{m}\sum_{n\in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right).
\end{aligned}$$

It is easy to see that

$$\begin{aligned}
I_1 &= \frac{1}{k_{r-1}}\sum_{j=1}^{j_0}\sum_{n\in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) \\
&= \frac{1}{k_{r-1}}\left(\sum_{n\in I_1} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right) + \cdots + \sum_{n\in I_{j_0}} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&\leq \frac{1}{k_{r-1}}(h_1R_1 + \cdots + h_{j_0}R_{j_0}) \\
&\leq \frac{1}{k_{r-1}} j_0 k_{j_0} \sup_{1\leq i\leq j_0} R_i \\
&\leq \frac{j_0 k_{j_0}}{k_{r-1}} K.
\end{aligned}
$$

Moreover, we have, for all i,

$$
\begin{aligned}
I_2 &= \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} \sum_{n\in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}\varphi(|x_k|)\right|\right) \\
&= \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} \frac{1}{h_j} \sum_{n\in I_j} f\left(\left|\sum_{k=1}^{\infty} a_{nk}\varphi(|x_k|)\right|\right) h_j \\
&\leq \varepsilon \frac{1}{k_{r-1}} \sum_{j=j_0+1}^{m} h_j \\
&\leq \varepsilon \frac{k_r}{k_{r-1}} \\
&= \varepsilon q_r < \varepsilon \cdot M.
\end{aligned}
$$

Thus $I \leq \frac{j_0 k_{j_0}}{k_{r-1}} K + \varepsilon \cdot M$. Finally, $x \in w(A, \psi, f)$.

The proof of (c) follows from (a) and (b). This completes the proof. □

We now prove the following theorem.

Theorem 2.3 *Let f be a modulus function. Then $N_\theta^0(A,\varphi) \subset N_\theta^0(A,\varphi,f)$.*

Proof Let $x \in N_\theta^0(A,\varphi)$. Let $\varepsilon > 0$ be given and choose $0 < \delta < 1$ such that $f(x) < \varepsilon$ for every $x \in [0,\delta]$. We can write

$$
\frac{1}{h_r} \sum_{n\in I_r} f\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right| = S_1 + S_2,
$$

where $S_1 = \frac{1}{h_r} \sum_{n\in I_r} f(|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)|)$, and this sum is taken over

$$
\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right| \leq \delta
$$

and

$$
S_2 = \frac{1}{h_r} \sum_{n\in I_r} f\left(\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right|\right),
$$

and this sum is taken over

$$
\left|\sum_{k=1}^{\infty} a_{nk}(i)\varphi(|x_k|)\right| > \delta.
$$

By the definition of the modulus f, we have $S_1 = \frac{1}{h_r}\sum_{n\in I_r} f(\delta) = f(\delta) < \varepsilon$ and further

$$S_2 = f(1)\frac{1}{\delta}\frac{1}{h_r}\sum_{n\in I_r}\sum_{k=1}^{\infty} a_{nk}(i)\varphi\bigl(|x_k|\bigr).$$

Therefore we have $x \in N_\theta^0(\mathbf{A},\varphi,f)$.

This completes the proof. □

3 A-Statistical convergence

The idea of convergence of a real sequence was extended to statistical convergence by Fast [17] (see also Schoenberg [18]) as follows: If $\mathbb{N}$ denotes the set of natural numbers and $K \subset \mathbb{N}$, then $K(m,n)$ denotes the cardinality of the set $K \cap [m,n]$. The upper and lower natural density of the subset K is defined by

$$\overline{d}(K) = \lim_{n\to\infty}\sup\frac{K(1,n)}{n} \quad \text{and} \quad \underline{d}(K) = \lim_{n\to\infty}\inf\frac{K(1,n)}{n}.$$

If $\overline{d}(K) = \underline{d}(K)$, then we say that the natural density of K exists and it is denoted simply by $d(K)$. Clearly, $d(K) = \lim_{n\to\infty}\frac{K(1,n)}{n}$.

A sequence (x_k) of real numbers is said to be statistically convergent to L if for arbitrary $\varepsilon > 0$, the set $K(\varepsilon) = \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ has natural density zero. Statistical convergence turned out to be one of the most active areas of research in summability theory after the work of Fridy [19] and Šalát [20].

In another direction, a new type of convergence, called lacunary statistical convergence, was introduced in [21] as follows.

A sequence $(x_k)_{n\in\mathbb{N}}$ of real numbers is said to be lacunary statistically convergent to L (or S_θ-convergent to L) if for any $\varepsilon > 0$,

$$\lim_{r\to\infty}\frac{1}{h_r}\bigl|\bigl\{k \in I_r : |x_k - L| \geq \varepsilon\bigr\}\bigr| = 0,$$

where $|A|$ denotes the cardinality of $A \subset \mathbb{N}$. In [21] the relation between lacunary statistical convergence and statistical convergence was established among other things. Moreover, Kolk [22] defined A-statistical convergence by using non-negative regular summability matrix.

In this section we define (A,φ)-statistical convergence by using the generalized three parametric real matrix and the φ-function $\varphi(u)$.

Let θ be a lacunary sequence, and let $\mathbf{A} = (a_{nk}(i))$ be the generalized three parametric real matrix; let the sequence $x = (x_k)$, the φ-function $\varphi(u)$ and a positive number $\varepsilon > 0$ be given. We write, for all i,

$$K_\theta^r\bigl((A,\varphi),\varepsilon\bigr) = \left\{n \in I_r : \sum_{k=1}^{\infty} a_{nk}(i)\varphi\bigl(|x_k|\bigr) \geq \varepsilon\right\}.$$

The sequence x is said to be $(\mathbf{A},\varphi)$-statistically convergent to a number zero if for every $\varepsilon > 0$,

$$\lim_r \frac{1}{k_r}\mu\bigl(K_\theta^r\bigl((A,\varphi),\varepsilon\bigr)\bigr) = 0 \quad \text{uniformly in } n,$$

where $\mu(K_\theta^r((A,\varphi),\varepsilon))$ denotes the number of elements belonging to $K_\theta^r((\mathbf{A},\varphi),\varepsilon)$. We denote by $S_\theta^0(\mathbf{A},\varphi)$ the set of sequences $x=(x_k)$ which are lacunary $(\mathbf{A},\varphi)$-statistical convergent to zero. We write

$$S_\theta^0(\mathbf{A},\varphi)=\left\{x=(x_k):\lim_r \frac{1}{h_r}\mu\bigl(K_\theta^r\bigl((A,\varphi),\varepsilon\bigr)\bigr)=0 \text{ uniformly in } i\right\}.$$

Theorem 3.1 *If $\psi \prec \varphi$, then $S_\theta^0(A,\psi)\subset S_\theta^0(A,\varphi)$.*

Proof By assumption we have $\psi(|x_k|)\le b\varphi(c|x_k|)$ and we have, for all i,

$$\sum_{k=1}^\infty a_{nk}(i)\psi\bigl(|x_k|\bigr)\le b\sum_{k=1}^\infty a_{nk}(i)\varphi\bigl(c|x_k|\bigr)\le L\sum_{k=1}^\infty a_{nk}(i)\varphi\bigl(|x_k|\bigr)$$

for $b,c>0$, where the constant L is connected with the properties of φ. Thus, the condition $\sum_{k=1}^\infty a_{nk}(i)\varphi(|x_k|)\ge 0$ implies the condition $\sum_{k=1}^\infty a_{nk}(i)\varphi(|x_k|)\ge\varepsilon$, and finally we get

$$\mu\bigl(K_\theta^r\bigl((A,\varphi),\varepsilon\bigr)\bigr)\subset\mu\bigl(K_\theta^r\bigl((A,\psi),\varepsilon\bigr)\bigr)$$

and

$$\lim_r\frac{1}{h_r}\mu\bigl(K_\theta^r\bigl((A,\varphi),\varepsilon\bigr)\bigr)\le\lim_r\frac{1}{h_r}\mu\bigl(K_\theta^r\bigl((A,\psi),\varepsilon\bigr)\bigr).$$

This completes the proof. □

We finally prove the following theorem.

Theorem 3.2 (a) *If the matrix A, the sequence θ and functions f and φ are given, then*

$$N_\theta^0\bigl((A,\varphi),f\bigr)\subset S_\theta^0(A,\varphi).$$

(b) *If the φ-function $\varphi(u)$ and the matrix A are given, and if the modulus function f is bounded, then*

$$S_\theta^0(A,\varphi)\subset N_\theta^0\bigl((A,\varphi),f\bigr).$$

(c) *If the φ-function $\varphi(u)$ and the matrix A are given, and if the modulus function f is bounded, then*

$$S_\theta^0(A,\varphi)=N_\theta^0\bigl((A,\varphi),f\bigr).$$

Proof (a) Let f be a modulus function, and let ε be a positive number. We write the following inequalities:

$$\begin{aligned}&\frac{1}{h_r}\sum_{n\in I_r}f\left(\left|\sum_{k=1}^\infty a_{nk}(i)\varphi\bigl(|x_k|\bigr)\right|\right)\\&\quad\ge\frac{1}{h_r}\sum_{n\in I_r^1}f\left(\left|\sum_{k=1}^\infty a_{nk}(i)\varphi\bigl(|x_k|\bigr)\right|\right)\end{aligned}$$

$$\geq \frac{1}{h_r} f(\varepsilon) \sum_{n \in I_r^1} 1$$

$$\geq \frac{1}{h_r} f(\varepsilon) \mu\bigl(K_\theta^r(A,\varphi), \varepsilon\bigr),$$

where

$$I_r^1 = \left\{ n \in I_r : \sum_{k=1}^{\infty} a_{nk}(i) \varphi\bigl(|x_k|\bigr) \geq \varepsilon \right\}.$$

Finally, if $x \in N_\theta^0((A,\varphi), f)$, then $x \in S_\theta^0(A,\varphi)$.

(b) Let us suppose that $x \in S_\theta^0(A,\varphi)$. If the modulus function f is a bounded function, then there exists an integer L such that $f(x) < L$ for $x \geq 0$. Let us take

$$I_r^2 = \left\{ n \in I_r : \sum_{k=1}^{\infty} a_{nk}(i) \varphi\bigl(|x_k|\bigr) < \varepsilon \right\}.$$

Thus we have

$$\begin{aligned}
&\frac{1}{h_r} \sum_{n \in I_r} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i) \varphi\bigl(|x_k|\bigr) \right| \right) \\
&\quad \leq \frac{1}{h_r} \sum_{n \in I_r^1} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i) \varphi\bigl(|x_k|\bigr) \right| \right) \\
&\qquad + \frac{1}{h_r} \sum_{n \in I_r^2} f\left(\left| \sum_{k=1}^{\infty} a_{nk}(i) \varphi\bigl(|x_k|\bigr) \right| \right) \\
&\quad \leq \frac{1}{h_r} M \mu\bigl(K_\theta^r\bigl((A,\varphi), \varepsilon\bigr)\bigr) + f(\varepsilon).
\end{aligned}$$

Taking the limit as $\varepsilon \to 0$, we obtain that $x \in N_\theta^0(A, \varphi, f)$.

The proof of (c) follows from (a) and (b).

This completes the proof. □

Competing interests
The author declares that they have no competing interests.

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