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Differential subordinations using the Ruscheweyh derivative and the generalized Sălăgean operator

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Abstract

In the present paper, we study the operator, using the Ruscheweyh derivative $R^m f(z)$ and the generalized Sălăgean operator $D_\lambda^m f(z)$, denote by $RD_{\lambda,\alpha}^m : \mathcal{A}_n \to \mathcal{A}_n$, $RD_{\lambda,\alpha}^m f(z) = (1-\alpha)R^m f(z) + \alpha D_\lambda^m f(z)$, $z \in U$, where $\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \cdots, z \in U\}$ is the class of normalized analytic functions. We obtain several differential subordinations regarding the operator $RD_{\lambda,\alpha}^m$.

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Keywords: differential subordination; convex function; best dominant; differential operator; generalized Sălăgean operator; Ruscheweyh derivative

1 Introduction

Denote by U the unit disc of the complex plane, $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{H}(U)$ the space of holomorphic functions in U.

Let $\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \cdots, z \in U\}$ and $\mathcal{H}[a,n] = \{f \in \mathcal{H}(U) : f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \cdots, z \in U\}$ for $a \in \mathbb{C}$ and $n \in \mathbb{N}$.

Denote by $K = \{f \in \mathcal{A}_n : \operatorname{Re} \frac{zf''(z)}{f'(z)} + 1 > 0, z \in U\}$ the class of normalized convex functions in U.

If f and g are analytic functions in U, we say that f is subordinate to g, written $f \prec g$, if there is a function w analytic in U, with $w(0) = 0$, $|w(z)| < 1$, for all $z \in U$ such that $f(z) = g(w(z))$ for all $z \in U$. If g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(U) \subseteq g(U)$.

Let $\psi : \mathbb{C}^3 \times U \to \mathbb{C}$, and let h be an univalent function in U. If p is analytic in U and satisfies the (second-order) differential subordination

$$\psi\left(p(z), zp'(z), z^2p''(z); z\right) \prec h(z), \quad z \in U, \tag{1.1}$$

then p is called a solution of the differential subordination. The univalent function q is called a dominant of the solutions of the differential subordination, or more simply a dominant, if $p \prec q$ for all p satisfying (1.1).

A dominant $\widetilde{q}$ that satisfies $\widetilde{q} \prec q$ for all dominants q of (1.1) is said to be the best dominant of (1.1). The best dominant is unique up to a rotation of U.

Definition 1.1 (Al-Oboudi [1]) For $f \in \mathcal{A}_n$, $\lambda \geq 0$ and $n, m \in \mathbb{N}$, the operator D_λ^m is defined by $D_\lambda^m : \mathcal{A}_n \to \mathcal{A}_n$,

$$D_\lambda^0 f(z) = f(z),$$
$$D_\lambda^1 f(z) = (1-\lambda) f(z) + \lambda z f'(z) = D_\lambda f(z),$$
$$\ldots,$$
$$D_\lambda^{m+1} f(z) = (1-\lambda) D_\lambda^m f(z) + \lambda z \big(D_\lambda^m f(z)\big)' = D_\lambda \big(D_\lambda^m f(z)\big), \quad z \in U.$$

Remark 1.1 If $f \in \mathcal{A}_n$ and $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $D_\lambda^m f(z) = z + \sum_{j=n+1}^{\infty} [1 + (j-1)\lambda]^m a_j z^j$, $z \in U$.

Remark 1.2 For $\lambda = 1$, in the definition above, we obtain the Sălăgean differential operator [2].

Definition 1.2 (Ruscheweyh [3]) For $f \in \mathcal{A}_n$, $n, m \in \mathbb{N}$, the operator R^m is defined by $R^m : \mathcal{A}_n \to \mathcal{A}_n$,

$$R^0 f(z) = f(z),$$
$$R^1 f(z) = z f'(z),$$
$$\ldots,$$
$$(m+1) R^{m+1} f(z) = z\big(R^m f(z)\big)' + m R^m f(z), \quad z \in U.$$

Remark 1.3 If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $R^m f(z) = z + \sum_{j=n+1}^{\infty} C_{m+j-1}^m a_j z^j$, $z \in U$.

Definition 1.3 [4] Let $\alpha, \lambda \geq 0$, $n, m \in \mathbb{N}$. Denote by $RD_{\lambda,\alpha}^m$ the operator given by $RD_{\lambda,\alpha}^m : \mathcal{A}_n \to \mathcal{A}_n$,

$$RD_{\lambda,\alpha}^m f(z) = (1-\alpha) R^m f(z) + \alpha D_\lambda^m f(z), \quad z \in U.$$

Remark 1.4 If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $RD_{\lambda,\alpha}^m f(z) = z + \sum_{j=n+1}^{\infty} \{\alpha[1 + (j-1)\lambda]^m + (1-\alpha) C_{m+j-1}^m\} a_j z^j$, $z \in U$.

This operator was studied also in [4–6] and [7].

Remark 1.5 For $\alpha = 0$, $RD_{\lambda,0}^m f(z) = R^m f(z)$, where $z \in U$ and for $\alpha = 1$, $RD_{\lambda,1}^m f(z) = D_\lambda^m f(z)$, where $z \in U$.

For $\lambda = 1$, we obtain $RD_{1,\alpha}^m f(z) = L_\alpha^m f(z)$, which was studied in [8–11].

For $m = 0$, $RD_{\lambda,\alpha}^0 f(z) = (1-\alpha) R^0 f(z) + \alpha D_\lambda^0 f(z) = f(z) = R^0 f(z) = D_\lambda^0 f(z)$, where $z \in U$.

Lemma 1.1 (Hallenbeck and Ruscheweyh [12, Th. 3.1.6, p.71]) *Let h be a convex function with $h(0) = a$, and let $\gamma \in \mathbb{C} \backslash \{0\}$ be a complex number with* $\operatorname{Re}\gamma \geq 0$. *If $p \in \mathcal{H}[a, n]$ and*

$$p(z) + \frac{1}{\gamma} z p'(z) \prec h(z), \quad z \in U,$$

then

$$p(z) \prec g(z) \prec h(z), \quad z \in U,$$

where $g(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t)t^{\gamma/n-1}\,dt$, $z \in U$.

Lemma 1.2 (Miller and Mocanu [12]) *Let g be a convex function in U, and let* $h(z) = g(z) + n\alpha z g'(z)$, *for* $z \in U$, *where* $\alpha > 0$ *and n is a positive integer.*

If $p(z) = g(0) + p_n z^n + p_{n+1} z^{n+1} + \cdots$, $z \in U$, *is holomorphic in U and*

$$p(z) + \alpha z p'(z) \prec h(z), \quad z \in U,$$

then

$$p(z) \prec g(z), \quad z \in U,$$

and this result is sharp.

2 Main results

Theorem 2.1 *Let g be a convex function,* $g(0) = 1$, *and let h be the function* $h(z) = g(z) + \frac{nz}{\delta} g'(z)$, *for* $z \in U$.

If $\alpha, \lambda, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\left(\frac{RD^m_{\lambda,\alpha} f(z)}{z}\right)^{\delta-1} \left(RD^m_{\lambda,\alpha} f(z)\right)' \prec h(z), \quad z \in U, \tag{2.1}$$

then

$$\left(\frac{RD^m_{\lambda,\alpha} f(z)}{z}\right)^{\delta} \prec g(z), \quad z \in U,$$

and this result is sharp.

Proof By using the properties of operator $RD^m_{\lambda,\alpha}$, we have

$$RD^m_{\lambda,\alpha} f(z) = z + \sum_{j=n+1}^{\infty} \left\{\alpha\left[1 + (j-1)\lambda\right]^m + (1-\alpha) C^m_{m+j-1}\right\} a_j z^j, \quad z \in U.$$

Consider $p(z) = (\frac{RD^m_{\lambda,\alpha} f(z)}{z})^{\delta} = (\frac{z + \sum_{j=n+1}^{\infty}\{\alpha[1+(j-1)\lambda]^m + (1-\alpha)C^m_{m+j-1}\} a_j z^j}{z})^{\delta} = 1 + p_{n\delta} z^{n\delta} + p_{n\delta+1} z^{n\delta+1} + \cdots$, $z \in U$.

We deduce that $p \in \mathcal{H}[1, n\delta]$.

Differentiating we obtain $(\frac{RD^m_{\lambda,\alpha} f(z)}{z})^{\delta-1} (RD^m_{\lambda,\alpha} f(z))' = p(z) + \frac{1}{\delta} z p'(z)$, $z \in U$.

Then (2.1) becomes

$$p(z) + \frac{1}{\delta} z p'(z) \prec h(z) = g(z) + \frac{nz}{\delta} g'(z) \quad \text{for } z \in U.$$

By using Lemma 1.2, we have

$$p(z) \prec g(z), \quad z \in U, \quad \text{i.e.,} \quad \left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta} \prec g(z), \quad z \in U. \qquad \square$$

Theorem 2.2 *Let h be a holomorphic function, which satisfies the inequality* $\operatorname{Re}(1+\frac{zh''(z)}{h'(z)}) > -\frac{1}{2}$, $z \in U$, *and* $h(0) = 1$.

If $\alpha, \lambda, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination

$$\left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta-1} \left(RD_{\lambda,\alpha}^m f(z)\right)' \prec h(z), \quad z \in U, \tag{2.2}$$

then

$$\left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta} \prec q(z), \quad z \in U,$$

where $q(z) = \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z h(t) t^{\frac{\delta}{n}-1}\,dt$. *The function q is convex, and it is the best dominant.*

Proof Let

$$\begin{aligned} p(z) &= \left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta} = \left(\frac{z + \sum_{j=n+1}^{\infty} \{\alpha[1+(j-1)\lambda]^m + (1-\alpha) C_{m+j-1}^m\} a_j z^j}{z}\right)^{\delta} \\ &= \left(1 + \sum_{j=n+1}^{\infty} \{\alpha\left[1+(j-1)\lambda\right]^m + (1-\alpha) C_{m+j-1}^m\} a_j z^{j-1}\right)^{\delta} = 1 + \sum_{j=n\delta}^{\infty} p_j z^j \end{aligned}$$

for $z \in U$, $p \in \mathcal{H}[1, n\delta]$.

Differentiating, we obtain $(\frac{RD_{\lambda,\alpha}^m f(z)}{z})^{\delta-1}(RD_{\lambda,\alpha}^m f(z))' = p(z) + \frac{1}{\delta} z p'(z)$, $z \in U$, and (2.2) becomes

$$p(z) + \frac{1}{\delta} z p'(z) \prec h(z), \quad z \in U.$$

Using Lemma 1.1, we have

$$p(z) \prec q(z), \quad z \in U, \quad \text{i.e.,} \quad \left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta} \prec q(z) = \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z h(t) t^{\frac{\delta}{n}-1}\,dt, \quad z \in U,$$

and q is the best dominant. $\square$

Corollary 2.3 *Let* $h(z) = \frac{1+(2\beta-1)z}{1+z}$ *be a convex function in U, where* $0 \leq \beta < 1$.

If $\alpha, \delta, \lambda \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination

$$\left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta-1} \left(RD_{\lambda,\alpha}^m f(z)\right)' \prec h(z), \quad z \in U, \tag{2.3}$$

then

$$\left(\frac{RD_{\lambda,\alpha}^m f(z)}{z}\right)^{\delta} \prec q(z), \quad z \in U,$$

where q is given by $q(z) = (2\beta - 1) + \frac{2(1-\beta)\delta}{nz^{\frac{\delta}{n}}} \int_0^z \frac{t^{\frac{\delta}{n}-1}}{1+t}\,dt$, $z \in U$. The function q is convex, and it is the best dominant.

Proof Following the same steps as in the proof of Theorem 2.2 and considering $p(z) = (\frac{RD^m_{\lambda,\alpha}f(z)}{z})^\delta$, the differential subordination (2.3) becomes

$$p(z) + \frac{z}{\delta}p'(z) \prec h(z) = \frac{1 + (2\beta - 1)z}{1 + z}, \quad z \in U.$$

By using Lemma 1.1, for $\gamma = \delta$, we have $p(z) \prec q(z)$, *i.e.*,

$$\begin{aligned}\left(\frac{RD^m_{\lambda,\alpha}f(z)}{z}\right)^\delta \prec q(z) &= \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z h(t)t^{\frac{\delta}{n}-1}\,dt \\ &= \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z t^{\frac{\delta}{n}-1} \frac{1 + (2\beta - 1)t}{1 + t}\,dt \\ &= \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z \left[(2\beta - 1)t^{\frac{\delta}{n}-1} + 2(1 - \beta)\frac{t^{\frac{\delta}{n}-1}}{1 + t}\right] dt \\ &= (2\beta - 1) + \frac{2(1 - \beta)\delta}{nz^{\frac{\delta}{n}}} \int_0^z \frac{t^{\frac{\delta}{n}-1}}{1 + t}\,dt, \quad z \in U.\end{aligned}$$

□

Remark 2.1 For $n = 1$, $\lambda = \frac{1}{2}$, $\alpha = 2$, $\delta = 1$, we obtain the same example as in [13, Example 4.2.1, p.125].

Theorem 2.4 *Let g be a convex function such that $g(0) = 1$, and let h be the function $h(z) = g(z) + \frac{nz}{\delta}g'(z)$, $z \in U$.*

If $\alpha, \lambda, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and the differential subordination

$$z\frac{\delta + 1}{\delta} \frac{RD^n_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2} + \frac{z^2}{\delta} \frac{RD^n_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2} \left[\frac{(RD^n_{\lambda,\alpha}f(z))'}{RD^n_{\lambda,\alpha}f(z)} - 2\frac{(RD^{n+1}_{\lambda,\alpha}f(z))'}{RD^{n+1}_{\lambda,\alpha}f(z)}\right] \prec h(z), \quad z \in U \tag{2.4}$$

holds, then

$$z\frac{RD^m_{\lambda,\alpha}f(z)}{(RD^{m+1}_{\lambda,\alpha}f(z))^2} \prec g(z), \quad z \in U,$$

and this result is sharp.

Proof For $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^\infty a_j z^j$, we have

$$RD^m_{\lambda,\alpha}f(z) = z + \sum_{j=n+1}^\infty \left\{\alpha\left[1 + (j - 1)\lambda\right]^m + (1 - \alpha)C^m_{m+j-1}\right\}a_j z^j, \quad z \in U.$$

Consider $p(z) = z\frac{RD^m_{\lambda,\alpha}f(z)}{(RD^{m+1}_{\lambda,\alpha}f(z))^2}$, and we obtain

$$p(z) + \frac{z}{\delta}p'(z) = z\frac{\delta + 1}{\delta} \frac{RD^n_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2} + \frac{z^2}{\delta} \frac{RD^n_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2} \left[\frac{(RD^n_{\lambda,\alpha}f(z))'}{RD^n_{\lambda,\alpha}f(z)} - 2\frac{(RD^{n+1}_{\lambda,\alpha}f(z))'}{RD^{n+1}_{\lambda,\alpha}f(z)}\right].$$

Relation (2.4) becomes

$$p(z)+\frac{z}{\delta}p'(z)\prec h(z)=g(z)+\frac{nz}{\delta}g'(z),\quad z\in U.$$

By using Lemma 1.2, we have

$$p(z)\prec g(z),\quad z\in U,\quad \textit{i.e.},\quad z\frac{RD^{m}_{\lambda,\alpha}f(z)}{(RD^{m+1}_{\lambda,\alpha}f(z))^2}\prec g(z),\quad z\in U.$$ □

Theorem 2.5 *Let h be a holomorphic function, which satisfies the inequality* $\operatorname{Re}(1+\frac{zh''(z)}{h'(z)})>-\frac{1}{2}$, $z\in U$, *and* $h(0)=1$.

If $\alpha,\lambda,\delta\geq 0$, $n,m\in\mathbb{N}$, $f\in\mathcal{A}_n$ *and satisfies the differential subordination*

$$z\frac{\delta+1}{\delta}\frac{RD^{n}_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2}+\frac{z^2}{\delta}\frac{RD^{n}_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2}\left[\frac{(RD^{n}_{\lambda,\alpha}f(z))'}{RD^{n}_{\lambda,\alpha}f(z)}-2\frac{(RD^{n+1}_{\lambda,\alpha}f(z))'}{RD^{n+1}_{\lambda,\alpha}f(z)}\right]$$
$$\prec h(z),\quad z\in U,\tag{2.5}$$

then

$$z\frac{RD^{m}_{\lambda,\alpha}f(z)}{(RD^{m+1}_{\lambda,\alpha}f(z))^2}\prec q(z),\quad z\in U,$$

where $q(z)=\frac{\delta}{nz^{\frac{\delta}{n}}}\int_0^z h(t)t^{\frac{\delta}{n}-1}\,dt$. *The function q is convex, and it is the best dominant.*

Proof Let $p(z)=z\frac{RD^{m}_{\lambda,\alpha}f(z)}{(RD^{m+1}_{\lambda,\alpha}f(z))^2}$, $z\in U$, $p\in\mathcal{H}[1,n]$.

Differentiating, we obtain $p(z)+\frac{z}{\delta}p'(z)=z\frac{\delta+1}{\delta}\frac{RD^{n}_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2}+\frac{z^2}{\delta}\frac{RD^{n}_{\lambda,\alpha}f(z)}{(RD^{n+1}_{\lambda,\alpha}f(z))^2}[\frac{(RD^{n}_{\lambda,\alpha}f(z))'}{RD^{n}_{\lambda,\alpha}f(z)}-2\frac{(RD^{n+1}_{\lambda,\alpha}f(z))'}{RD^{n+1}_{\lambda,\alpha}f(z)}]$, $z\in U$, and (2.5) becomes

$$p(z)+\frac{z}{\delta}p'(z)\prec h(z),\quad z\in U.$$

Using Lemma 1.1, we have

$$p(z)\prec q(z),\quad z\in U,\quad \textit{i.e.},$$
$$z\frac{RD^{m}_{\lambda,\alpha}f(z)}{(RD^{m+1}_{\lambda,\alpha}f(z))^2}\prec q(z)=\frac{\delta}{nz^{\frac{\delta}{n}}}\int_0^z h(t)t^{\frac{\delta}{n}-1}\,dt,\quad z\in U,$$

and q is the best dominant. □

Theorem 2.6 *Let g be a convex function such that* $g(0)=1$, *and let h be the function* $h(z)=g(z)+\frac{nz}{\delta}g'(z)$, $z\in U$.

If $\alpha,\lambda,\delta\geq 0$, $n,m\in\mathbb{N}$, $f\in\mathcal{A}_n$ *and the differential subordination*

$$z^2\frac{\delta+2}{\delta}\frac{(RD^{n}_{\lambda,\alpha}f(z))'}{RD^{n}_{\lambda,\alpha}f(z)}+\frac{z^3}{\delta}\left[\frac{(RD^{n}_{\lambda,\alpha}f(z))''}{RD^{n}_{\lambda,\alpha}f(z)}-\left(\frac{(RD^{n}_{\lambda,\alpha}f(z))'}{RD^{n}_{\lambda,\alpha}f(z)}\right)^2\right]\prec h(z),\quad z\in U\tag{2.6}$$

holds, then

$$z^2 \frac{(RD^m_{\lambda,\alpha} f(z))'}{RD^m_{\lambda,\alpha} f(z)} \prec g(z), \quad z \in U.$$

This result is sharp.

Proof Let $p(z) = z^2 \frac{(RD^m_{\lambda,\alpha} f(z))'}{RD^m_{\lambda,\alpha} f(z)}$. We deduce that $p \in \mathcal{H}[0,n]$.

Differentiating, we obtain $p(z) + \frac{z}{\delta} p'(z) = z^2 \frac{\delta+2}{\delta} \frac{(RD^n_{\lambda,\alpha} f(z))'}{RD^n_{\lambda,\alpha} f(z)} + \frac{z^3}{\delta}[\frac{(RD^n_{\lambda,\alpha} f(z))''}{RD^n_{\lambda,\alpha} f(z)} - (\frac{(RD^n_{\lambda,\alpha} f(z))'}{RD^n_{\lambda,\alpha} f(z)})^2]$, $z \in U$.

Using the notation in (2.6), the differential subordination becomes

$$p(z) + \frac{1}{\delta} z p'(z) \prec h(z) = g(z) + \frac{nz}{\delta} g'(z).$$

By using Lemma 1.2, we have

$$p(z) \prec g(z), \quad z \in U, \quad \text{i.e.,} \quad z^2 \frac{(RD^m_{\lambda,\alpha} f(z))'}{RD^m_{\lambda,\alpha} f(z)} \prec g(z), \quad z \in U,$$

and this result is sharp. □

Theorem 2.7 *Let h be an holomorphic function, which satisfies the inequality* $\operatorname{Re}(1 + \frac{zh''(z)}{h'(z)}) > -\frac{1}{2}$, *$z \in U$, and $h(0) = 1$.*

If $\alpha, \lambda, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination

$$z^2 \frac{\delta+2}{\delta} \frac{(RD^n_{\lambda,\alpha} f(z))'}{RD^n_{\lambda,\alpha} f(z)} + \frac{z^3}{\delta} \left[\frac{(RD^n_{\lambda,\alpha} f(z))''}{RD^n_{\lambda,\alpha} f(z)} - \left(\frac{(RD^n_{\lambda,\alpha} f(z))'}{RD^n_{\lambda,\alpha} f(z)} \right)^2 \right] \prec h(z), \quad z \in U, \tag{2.7}$$

then

$$z^2 \frac{(RD^m_{\lambda,\alpha} f(z))'}{RD^m_{\lambda,\alpha} f(z)} \prec q(z), \quad z \in U,$$

where $q(z) = \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z h(t) t^{\frac{\delta}{n}-1}\, dt$. The function q is convex, and it is the best dominant.

Proof Let $p(z) = z^2 \frac{(RD^m_{\lambda,\alpha} f(z))'}{RD^m_{\lambda,\alpha} f(z)}$, $z \in U$, $p \in \mathcal{H}[0,n]$.

Differentiating, we obtain $p(z) + \frac{z}{\delta} p'(z) = z^2 \frac{\delta+2}{\delta} \frac{(RD^n_{\lambda,\alpha} f(z))'}{RD^n_{\lambda,\alpha} f(z)} + \frac{z^3}{\delta}[\frac{(RD^n_{\lambda,\alpha} f(z))''}{RD^n_{\lambda,\alpha} f(z)} - (\frac{(RD^n_{\lambda,\alpha} f(z))'}{RD^n_{\lambda,\alpha} f(z)})^2]$, $z \in U$, and (2.7) becomes

$$p(z) + \frac{1}{\delta} z p'(z) \prec h(z), \quad z \in U.$$

Using Lemma 1.1, we have

$$p(z) \prec q(z), \quad z \in U, \quad \text{i.e.,} \quad z^2 \frac{(RD^m_{\lambda,\alpha} f(z))'}{RD^m_{\lambda,\alpha} f(z)} \prec q(z) = \frac{\delta}{nz^{\frac{\delta}{n}}} \int_0^z h(t) t^{\frac{\delta}{n}-1}\, dt, \quad z \in U,$$

and q is the best dominant. □

Theorem 2.8 *Let g be a convex function such that* $g(0) = 1$, *and let h be the function* $h(z) = g(z) + nzg'(z)$, $z \in U$.

If $\alpha, \lambda \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and the differential subordination*

$$1 - \frac{RD^m_{\lambda,\alpha} f(z) \cdot (RD^m_{\lambda,\alpha} f(z))''}{[(RD^m_{\lambda,\alpha} f(z))']^2} \prec h(z), \quad z \in U \tag{2.8}$$

holds, then

$$\frac{RD^m_{\lambda,\alpha} f(z)}{z(RD^m_{\lambda,\alpha} f(z))'} \prec g(z), \quad z \in U.$$

This result is sharp.

Proof Let $p(z) = \frac{RD^m_{\lambda,\alpha} f(z)}{z(RD^m_{\lambda,\alpha} f(z))'}$. We deduce that $p \in \mathcal{H}[1, n]$.

Differentiating, we obtain $1 - \frac{RD^m_{\lambda,\alpha} f(z) \cdot (RD^m_{\lambda,\alpha} f(z))''}{[(RD^m_{\lambda,\alpha} f(z))']^2} = p(z) + zp'(z)$, $z \in U$.

Using the notation in (2.8), the differential subordination becomes

$$p(z) + zp'(z) \prec h(z) = g(z) + nzg'(z).$$

By using Lemma 1.2, we have

$$p(z) \prec g(z), \quad z \in U, \quad i.e., \quad \frac{RD^m_{\lambda,\alpha} f(z)}{z(RD^m_{\lambda,\alpha} f(z))'} \prec g(z), \quad z \in U,$$

and this result is sharp. □

Theorem 2.9 *Let h be a holomorphic function, which satisfies the inequality* $\operatorname{Re}(1 + \frac{zh''(z)}{h'(z)}) > -\frac{1}{2}$, $z \in U$, *and* $h(0) = 1$.

If $\alpha, \lambda \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$1 - \frac{RD^m_{\lambda,\alpha} f(z) \cdot (RD^m_{\lambda,\alpha} f(z))''}{[(RD^m_{\lambda,\alpha} f(z))']^2} \prec h(z), \quad z \in U, \tag{2.9}$$

then

$$\frac{RD^m_{\lambda,\alpha} f(z)}{z(RD^m_{\lambda,\alpha} f(z))'} \prec q(z), \quad z \in U,$$

where $q(z) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t) t^{\frac{1}{n}-1}\, dt$. *The function q is convex, and it is the best dominant.*

Proof Let $p(z) = \frac{RD^m_{\lambda,\alpha} f(z)}{z(RD^m_{\lambda,\alpha} f(z))'}$, $z \in U$, $p \in \mathcal{H}[0, n]$.

Differentiating, we obtain $1 - \frac{RD^m_{\lambda,\alpha} f(z) \cdot (RD^m_{\lambda,\alpha} f(z))''}{[(RD^m_{\lambda,\alpha} f(z))']^2} = p(z) + zp'(z)$, $z \in U$, and (2.9) becomes

$$p(z) + zp'(z) \prec h(z), \quad z \in U.$$

Using Lemma 1.1, we have

$$p(z) \prec q(z), \quad z \in U, \quad i.e., \quad \frac{RD^m_{\lambda,\alpha}f(z)}{z(RD^m_{\lambda,\alpha}f(z))'} \prec q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}\,dt, \quad z \in U,$$

and q is the best dominant. □

Corollary 2.10 *Let* $h(z) = \frac{1+(2\beta-1)z}{1+z}$ *be a convex function in* U, *where* $0 \le \beta < 1$.
If $\alpha, \lambda \ge 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$1 - \frac{RD^m_{\lambda,\alpha}f(z) \cdot (RD^m_{\lambda,\alpha}f(z))''}{[(RD^m_{\lambda,\alpha}f(z))']^2} \prec h(z), \quad z \in U, \tag{2.10}$$

then

$$\frac{RD^m_{\lambda,\alpha}f(z)}{z(RD^m_{\lambda,\alpha}f(z))'} \prec q(z), \quad z \in U,$$

where q *is given by* $q(z) = (2\beta-1) + \frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z \frac{t^{\frac{1}{n}-1}}{1+t}\,dt$, $z \in U$. *The function* q *is convex, and it is the best dominant.*

Proof Following the same steps as in the proof of Theorem 2.9 and considering $p(z) = \frac{RD^m_{\lambda,\alpha}f(z)}{z(RD^m_{\lambda,\alpha}f(z))'}$, the differential subordination (2.10) becomes

$$p(z) + zp'(z) \prec h(z) = \frac{1+(2\beta-1)z}{1+z}, \quad z \in U.$$

By using Lemma 1.1 for $\gamma = 1$, we have $p(z) \prec q(z)$, *i.e.*,

$$\begin{aligned} \frac{RD^m_{\lambda,\alpha}f(z)}{z(RD^m_{\lambda,\alpha}f(z))'} \prec q(z) &= \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}\,dt \\ &= \frac{1}{nz^{\frac{1}{n}}}\int_0^z \frac{1+(2\beta-1)t}{1+t}t^{\frac{1}{n}-1}\,dt \\ &= \frac{1}{nz^{\frac{1}{n}}}\int_0^z t^{\frac{1}{n}-1}\left[(2\beta-1) + \frac{2(1-\beta)}{1+t}\right]dt \\ &= (2\beta-1) + \frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z \frac{t^{\frac{1}{n}-1}}{1+t}\,dt, \quad z \in U. \end{aligned}$$

□

Example 2.1 Let $h(z) = \frac{1-z}{1+z}$ be a convex function in U with $h(0) = 1$ and $\mathrm{Re}(\frac{zh''(z)}{h'(z)} + 1) > -\frac{1}{2}$.
Let $f(z) = z + z^2$, $z \in U$. For $n = 1$, $m = 1$, $\lambda = \frac{1}{2}$, $\alpha = 2$, we obtain $RD^1_{\frac{1}{2},2}f(z) = -R^1f(z) + 2D^1_{\frac{1}{2}}f(z) = -zf'(z) + 2(\frac{1}{2}f(z) + \frac{1}{2}zf'(z)) = f(z) = z + z^2$, $z \in U$.
Then $(RD^1_{\frac{1}{2},2}f(z))' = f'(z) = 1 + 2z$,

$$\frac{RD^1_{\frac{1}{2},2}f(z)}{z(RD^1_{\frac{1}{2},2}f(z))'} = \frac{z+z^2}{z(1+2z)} = \frac{1+z}{1+2z},$$

$$1 - \frac{RD^1_{\frac{1}{2},2}f(z) \cdot (RD^1_{\frac{1}{2},2}f(z))''}{[(RD^1_{\frac{1}{2},2}f(z))']^2} = 1 - \frac{(z+z^2)\cdot 2}{(1+2z)^2} = \frac{2z^2+2z+1}{(1+2z)^2}.$$

We have $q(z) = \frac{1}{z}\int_0^z \frac{1-t}{1+t}\,dt = -1 + \frac{2\ln(1+z)}{z}$.

Using Theorem 2.9, we obtain

$$\frac{2z^2 + 2z + 1}{(1+2z)^2} \prec \frac{1-z}{1+z}, \quad z \in U,$$

induce

$$\frac{1+z}{1+2z} \prec -1 + \frac{2\ln(1+z)}{z}, \quad z \in U.$$

Theorem 2.11 *Let g be a convex function such that $g(0) = 0$, and let h be the function $h(z) = g(z) + nzg'(z)$, $z \in U$.*

If $\alpha, \lambda \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and the differential subordination

$$\bigl[\bigl(RD^m_{\lambda,\alpha}f(z)\bigr)'\bigr]^2 + RD^m_{\lambda,\alpha}f(z) \cdot \bigl(RD^m_{\lambda,\alpha}f(z)\bigr)'' \prec h(z), \quad z \in U \tag{2.11}$$

holds, then

$$\frac{RD^m_{\lambda,\alpha}f(z) \cdot (RD^m_{\lambda,\alpha}f(z))'}{z} \prec g(z), \quad z \in U.$$

This result is sharp.

Proof Let $p(z) = \frac{RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'}{z}$. We deduce that $p \in \mathcal{H}[0,n]$.

Differentiating, we obtain $[(RD^m_{\lambda,\alpha}f(z))']^2 + RD^m_{\lambda,\alpha}f(z) \cdot (RD^m_{\lambda,\alpha}f(z))'' = p(z) + zp'(z)$, $z \in U$.

Using the notation in (2.11), the differential subordination becomes

$$p(z) + zp'(z) \prec h(z) = g(z) + nzg'(z).$$

By using Lemma 1.2, we have

$$p(z) \prec g(z), \quad z \in U, \quad \textit{i.e.}, \quad \frac{RD^m_{\lambda,\alpha}f(z) \cdot (RD^m_{\lambda,\alpha}f(z))'}{z} \prec g(z), \quad z \in U,$$

and this result is sharp. □

Theorem 2.12 *Let h be a holomorphic function, which satisfies the inequality* $\operatorname{Re}(1 + \frac{zh''(z)}{h'(z)}) > -\frac{1}{2}$, *$z \in U$, and $h(0) = 0$.*

If $\alpha, \lambda \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination

$$\bigl[\bigl(RD^m_{\lambda,\alpha}f(z)\bigr)'\bigr]^2 + RD^m_{\lambda,\alpha}f(z) \cdot \bigl(RD^m_{\lambda,\alpha}f(z)\bigr)'' \prec h(z), \quad z \in U, \tag{2.12}$$

then

$$\frac{RD^m_{\lambda,\alpha}f(z) \cdot (RD^m_{\lambda,\alpha}f(z))'}{z} \prec q(z), \quad z \in U,$$

where $q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}\,dt$. The function q is convex, and it is the best dominant.

Proof Let $p(z) = \frac{RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'}{z}$, $z \in U$, $p \in \mathcal{H}[0,n]$.

Differentiating, we obtain $[(RD^m_{\lambda,\alpha}f(z))']^2 + RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'' = p(z) + zp'(z)$, $z \in U$, and (2.12) becomes

$$p(z) + zp'(z) \prec h(z), \quad z \in U.$$

Using Lemma 1.1, we have

$$p(z) \prec q(z), \quad z \in U, \quad \text{i.e.,}$$

$$\frac{RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'}{z} \prec q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}\,dt, \quad z \in U,$$

and q is the best dominant. □

Corollary 2.13 *Let* $h(z) = \frac{1+(2\beta-1)z}{1+z}$ *be a convex function in* U, *where* $0 \le \beta < 1$.

If $\alpha, \lambda \ge 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\left[\left(RD^m_{\lambda,\alpha}f(z)\right)'\right]^2 + RD^m_{\lambda,\alpha}f(z)\cdot\left(RD^m_{\lambda,\alpha}f(z)\right)'' \prec h(z), \quad z \in U, \tag{2.13}$$

then

$$\frac{RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'}{z} \prec q(z), \quad z \in U,$$

where q *is given by* $q(z) = (2\beta-1) + \frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z \frac{t^{\frac{1}{n}-1}}{1+t}\,dt$, $z \in U$. *The function* q *is convex, and it is the best dominant.*

Proof Following the same steps as in the proof of Theorem 2.12 and considering $p(z) = \frac{RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'}{z}$, the differential subordination (2.13) becomes

$$p(z) + zp'(z) \prec h(z) = \frac{1+(2\beta-1)z}{1+z}, \quad z \in U.$$

By using Lemma 1.1 for $\gamma = 1$, we have $p(z) \prec q(z)$, *i.e.*,

$$\begin{aligned}
\frac{RD^m_{\lambda,\alpha}f(z)\cdot(RD^m_{\lambda,\alpha}f(z))'}{z} \prec q(z) &= \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}\,dt \\
&= \frac{1}{nz^{\frac{1}{n}}}\int_0^z \frac{1+(2\beta-1)t}{1+t}t^{\frac{1}{n}-1}\,dt \\
&= \frac{1}{nz^{\frac{1}{n}}}\int_0^z t^{\frac{1}{n}-1}\left[(2\beta-1) + \frac{2(1-\beta)}{1+t}\right]dt \\
&= (2\beta-1) + \frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z \frac{t^{\frac{1}{n}-1}}{1+t}\,dt, \quad z \in U.
\end{aligned}$$

□

Example 2.2 Let $h(z) = \frac{1-z}{1+z}$ be a convex function in U with $h(0) = 1$ and $\operatorname{Re}(\frac{zh''(z)}{h'(z)} + 1) > -\frac{1}{2}$.

Let $f(z) = z + z^2$, $z \in U$. For $n = 1$, $m = 1$, $\lambda = \frac{1}{2}$, $\alpha = 2$, we obtain $RD^1_{\frac{1}{2},2}f(z) = -R^1f(z) + 2D^1_{\frac{1}{2}}f(z) = -zf'(z) + 2(\frac{1}{2}f(z) + \frac{1}{2}zf'(z)) = f(z) = z + z^2$, $z \in U$.

Then $(RD^1_{\frac{1}{2},2}f(z))' = f'(z) = 1 + 2z$,

$$\frac{RD^1_{\frac{1}{2},2}f(z)\cdot(RD^1_{\frac{1}{2},2}f(z))'}{z} = \frac{(z+z^2)(1+2z)}{z} = 2z^2 + 3z + 1,$$
$$\bigl[\bigl(RD^1_{\frac{1}{2},2}f(z)\bigr)'\bigr]^2 + RD^1_{\frac{1}{2},2}f(z)\cdot\bigl(RD^1_{\frac{1}{2},2}f(z)\bigr)'' = (1+2z)^2 + (z+z^2)\cdot 2 = 6z^2 + 6z + 1.$$

We have $q(z) = \frac{1}{z}\int_0^z \frac{1-t}{1+t}\,dt = -1 + \frac{2\ln(1+z)}{z}$.

Using Theorem 2.12, we obtain

$$6z^2 + 6z + 1 \prec \frac{1-z}{1+z}, \quad z \in U,$$

induce

$$2z^2 + 3z + 1 \prec -1 + \frac{2\ln(1+z)}{z}, \quad z \in U.$$

Theorem 2.14 *Let g be a convex function such that $g(0) = 0$, and let h be the function $h(z) = g(z) + \frac{nz}{1-\delta}g'(z)$, $z \in U$.*

If $\alpha, \lambda \geq 0$, $\delta \in (0,1)$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and the differential subordination

$$\left(\frac{z}{RD^m_{\lambda,\alpha}f(z)}\right)^\delta \frac{RD^{m+1}_{\lambda,\alpha}f(z)}{1-\delta}\left(\frac{(RD^{m+1}_{\lambda,\alpha}f(z))'}{RD^{m+1}_{\lambda,\alpha}f(z)} - \delta\frac{(RD^m_{\lambda,\alpha}f(z))'}{RD^m_{\lambda,\alpha}f(z)}\right) \prec h(z), \quad z \in U \tag{2.14}$$

holds, then

$$\frac{RD^{m+1}_{\lambda,\alpha}f(z)}{z}\cdot\left(\frac{z}{RD^m_{\lambda,\alpha}f(z)}\right)^\delta \prec g(z), \quad z \in U.$$

This result is sharp.

Proof Let $p(z) = \frac{RD^{m+1}_{\lambda,\alpha}f(z)}{z}\cdot(\frac{z}{RD^m_{\lambda,\alpha}f(z)})^\delta$. We deduce that $p \in \mathcal{H}[1,n]$.

Differentiating, we obtain $(\frac{z}{RD^m_{\lambda,\alpha}f(z)})^\delta \frac{RD^{m+1}_{\lambda,\alpha}f(z)}{1-\delta}(\frac{(RD^{m+1}_{\lambda,\alpha}f(z))'}{RD^{m+1}_{\lambda,\alpha}f(z)} - \delta\frac{(RD^m_{\lambda,\alpha}f(z))'}{RD^m_{\lambda,\alpha}f(z)}) = p(z) + \frac{1}{1-\delta}zp'(z)$, $z \in U$.

Using the notation in (2.14), the differential subordination becomes

$$p(z) + \frac{1}{1-\delta}zp'(z) \prec h(z) = g(z) + \frac{nz}{1-\delta}g'(z).$$

By using Lemma 1.2, we have

$$p(z) \prec g(z), \quad z \in U, \quad \textit{i.e.}, \quad \frac{RD^{m+1}_{\lambda,\alpha}f(z)}{z}\cdot\left(\frac{z}{RD^m_{\lambda,\alpha}f(z)}\right)^\delta \prec g(z), \quad z \in U,$$

and this result is sharp. □

Theorem 2.15 *Let h be a holomorphic function, which satisfies the inequality* $\operatorname{Re}(1 + \frac{zh''(z)}{h'(z)}) > -\frac{1}{2}$, *$z \in U$, and $h(0) = 1$.*

If $\alpha, \lambda \geq 0$, $\delta \in (0,1)$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination

$$\left(\frac{z}{RD_{\lambda,\alpha}^{m} f(z)}\right)^{\delta} \frac{RD_{\lambda,\alpha}^{m+1} f(z)}{1-\delta}\left(\frac{(RD_{\lambda,\alpha}^{m+1} f(z))'}{RD_{\lambda,\alpha}^{m+1} f(z)} - \delta\frac{(RD_{\lambda,\alpha}^{m} f(z))'}{RD_{\lambda,\alpha}^{m} f(z)}\right) \prec h(z), \quad z \in U, \tag{2.15}$$

then

$$\frac{RD_{\lambda,\alpha}^{m+1} f(z)}{z} \cdot \left(\frac{z}{RD_{\lambda,\alpha}^{m} f(z)}\right)^{\delta} \prec q(z), \quad z \in U,$$

where $q(z) = \frac{1-\delta}{nz^{\frac{1-\delta}{n}}} \int_0^z h(t) t^{\frac{1-\delta}{n}-1}\,dt$. *The function q is convex, and it is the best dominant.*

Proof Let $p(z) = \frac{RD_{\lambda,\alpha}^{m+1} f(z)}{z} \cdot (\frac{z}{RD_{\lambda,\alpha}^{m} f(z)})^{\delta}$, $z \in U$, $p \in \mathcal{H}[0,n]$.

Differentiating, we obtain $(\frac{z}{RD_{\lambda,\alpha}^{m} f(z)})^{\delta} \frac{RD_{\lambda,\alpha}^{m+1} f(z)}{1-\delta}(\frac{(RD_{\lambda,\alpha}^{m+1} f(z))'}{RD_{\lambda,\alpha}^{m+1} f(z)} - \delta\frac{(RD_{\lambda,\alpha}^{m} f(z))'}{RD_{\lambda,\alpha}^{m} f(z)}) = p(z) + \frac{1}{1-\delta} zp'(z)$, $z \in U$, and (2.15) becomes

$$p(z) + \frac{1}{1-\delta} zp'(z) \prec h(z), \quad z \in U.$$

Using Lemma 1.1, we have

$$p(z) \prec q(z), \quad z \in U, \quad \textit{i.e.},$$
$$\frac{RD_{\lambda,\alpha}^{m+1} f(z)}{z} \cdot \left(\frac{z}{RD_{\lambda,\alpha}^{m} f(z)}\right)^{\delta} \prec q(z) = \frac{1-\delta}{nz^{\frac{1-\delta}{n}}} \int_0^z h(t) t^{\frac{1-\delta}{n}-1}\,dt, \quad z \in U,$$

and q is the best dominant. □

Competing interests
The author declares that she has no competing interests.

Author's contributions
The author drafted the manuscript, read and approved the final manuscript.

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