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Almost periodic solutions of a single-species system with feedback control on time scales

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Abstract
This paper is concerned with a single-species system with feedback control on time scales. Based on the theory of calculus on time scales, by using the properties of almost periodic functions and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the existence of a unique globally attractive positive almost periodic solution of the system are obtained. Finally, an example and numerical simulations are presented to illustrate the feasibility and effectiveness of the results.

Keywords: permanence; almost periodic solution; global attractivity; time scale

1 Introduction

In the past few years, different types of ecosystems with periodic coefficients have been studied extensively; see, for example, [1–5] and the references therein. However, if the various constituent components of the temporally nonuniform environment are with incommensurable (nonintegral multiples) periods, then one has to consider the environment to be almost periodic since there is no *a priori* reason to expect the existence of periodic solutions. Therefore, if we consider the effects of the environmental factors (*e.g.*, seasonal effects of weather, food supplies, mating habits and harvesting), the assumption of almost periodicity is more realistic, more important and more general. Almost periodicity of different types of ecosystems has received more recently researchers' special attention; see [6–10] and the references therein.

However, in the natural world, there are many species whose developing processes are both continuous and discrete. Hence, using the only differential equation or difference equation cannot accurately describe the law of their development; see, for example, [11, 12]. Therefore, there is a need to establish correspondent dynamic models on new time scales.

To the best of the authors' knowledge, there are few papers published on the existence of an almost periodic solution of ecosystems on time scales.

Motivated by the above, in the present paper, we shall study an almost periodic single-species system with feedback control on time scales as follows:

$$\begin{cases} x^{\Delta}(t) = r(t)x(t)[1 - \frac{x(t)}{a(t)+d(t)x(t)} - b(t)x(\sigma(t)) - c(t)y(t)], \\ y^{\Delta}(t) = -\eta(t)y(t) + g(t)x(t), \end{cases} \tag{1.1}$$

where $t \in \mathbb{T}$, $\mathbb{T}$ is an almost time scale. All the coefficients $r(t)$, $a(t)$, $b(t)$, $c(t)$, $d(t)$, $\eta(t)$, $g(t)$ are continuous, almost periodic functions.

For convenience, we introduce the notation

$$f^u = \sup_{t\in\mathbb{T}} f(t), \qquad f^l = \inf_{t\in\mathbb{T}} f(t),$$

where f is a positive and bounded function. Throughout this paper, we assume that the coefficients of almost periodic system (1.1) satisfy

$$\min\{r^l, a^l, b^l, c^l, d^l, \eta^l, g^l\} > 0, \qquad \max\{r^u, a^u, b^u, c^u, d^u, \eta^u, g^u\} < +\infty.$$

The initial condition of system (1.1) is in the form

$$x(t_0) = x_0, \qquad y(t_0) = y_0, \quad t_0 \in \mathbb{T}, x_0 > 0, y_0 > 0. \tag{1.2}$$

The aim of this paper is, by using the properties of almost periodic functions and constructing a suitable Lyapunov functional, to obtain sufficient conditions for the existence of a unique globally attractive positive almost periodic solution of system (1.1).

In this paper, the time scale $\mathbb{T}$ considered is unbounded above, and for each interval $\mathbb{I}$ of $\mathbb{T}$, we denote $\mathbb{I}_{\mathbb{T}} = \mathbb{I} \cap \mathbb{T}$.

2 Preliminaries

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\}, \qquad \mu(t) = \sigma(t) - t.$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T}\backslash\{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is right-dense continuous provided it is continuous at a right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$.

For the basic theories of calculus on time scales, one can see [13].

A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $p : \mathbb{T} \to \mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}, \mathbb{R})$. Define the set $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0, \forall t \in \mathbb{T}\}$.

If r is a regressive function, then the generalized exponential function e_r is defined by

$$e_r(t,s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}\big(r(\tau)\big)\Delta\tau\right\}$$

for all $s, t \in \mathbb{T}$, with the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\mathrm{Log}(1+hz)}{h} & \text{if } h \neq 0, \\ z & \text{if } h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, define

$$p \oplus q = p + q + \mu pq, \qquad \ominus p = -\frac{p}{1 + \mu p}, \qquad p \ominus q = p \oplus (\ominus q).$$

Lemma 2.1 (see [13]) *If $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then*

(i) $e_0(t, s) \equiv 1$ *and* $e_p(t, t) \equiv 1$;
(ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;
(iii) $e_p(t, s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s, t)$;
(iv) $e_p(t, s)e_p(s, r) = e_p(t, r)$;
(v) $\frac{e_p(t,s)}{e_q(t,s)} = e_{p\ominus q}(t, s)$;
(vi) $(e_p(t, s))^\Delta = p(t)e_p(t, s)$.

Lemma 2.2 (see [14]) *Assume that $a > 0$, $b > 0$ and $-a \in \mathcal{R}^+$. Then*

$$y^\Delta(t) \geq (\leq)\, b - ay(t), \quad y(t) > 0, t \in [t_0, \infty)_\mathbb{T}$$

implies

$$y(t) \geq (\leq)\, \frac{b}{a}\left[1 + \left(\frac{ay(t_0)}{b} - 1\right)e_{(-a)}(t, t_0)\right], \quad t \in [t_0, \infty)_\mathbb{T}.$$

Lemma 2.3 (see [14]) *Assume that $a > 0$, $b > 0$. Then*

$$y^\Delta(t) \leq (\geq)\, y(t)\big(b - ay\big(\sigma(t)\big)\big), \quad y(t) > 0, t \in [t_0, \infty)_\mathbb{T}$$

implies

$$y(t) \leq (\geq)\, \frac{b}{a}\left[1 + \left(\frac{b}{ay(t_0)} - 1\right)e_{\ominus b}(t, t_0)\right], \quad t \in [t_0, \infty)_\mathbb{T}.$$

Let $\mathbb{T}$ be a time scale with at least two positive points, one of them being always one: $1 \in \mathbb{T}$. There exists at least one point $t \in \mathbb{T}$ such that $0 < t \neq 1$. Define the natural logarithm function on the time scale $\mathbb{T}$ by

$$L_\mathbb{T}(t) = \int_1^t \frac{1}{\tau}\Delta\tau, \quad t \in \mathbb{T} \cap (0, +\infty).$$

Lemma 2.4 (see [15]) *Assume that $x : \mathbb{T} \to \mathbb{R}^+$ is strictly increasing and $\widetilde{\mathbb{T}} := x(\mathbb{T})$ is a time scale. If $x^\Delta(t)$ exists for $t \in \mathbb{T}^k$, then*

$$\frac{\Delta}{\Delta t}L_\mathbb{T}\big(x(t)\big) = \frac{x^\Delta(t)}{x(t)}.$$

Lemma 2.5 (see [13]) *Assume that $f, g : \mathbb{T} \to \mathbb{R}$ are differentiable at $t \in \mathbb{T}^k$, then $fg : \mathbb{T} \to \mathbb{R}$ is differentiable at t with*

$$(fg)^\Delta(t) = f^\Delta(t)g(t) + f\big(\sigma(t)\big)g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g\big(\sigma(t)\big).$$

Definition 2.1 (see [16]) A time scale $\mathbb{T}$ is called an almost periodic time scale if

$$\Pi = \{\tau \in \mathbb{R} : t \pm \tau \in \mathbb{T}, \forall t \in \mathbb{T}\}.$$

Definition 2.2 (see [16]) Let $\mathbb{T}$ be an almost periodic time scale. A function $f : \mathbb{T} \to \mathbb{R}$ is called an almost periodic function if the ε-translation set of f

$$E\{\varepsilon, f\} = \{\tau \in \Pi : |f(t+\tau) - f(t)| < \varepsilon, \forall t \in \mathbb{T}\}$$

is a relatively dense set in $\mathbb{T}$ for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists a constant $l(\varepsilon) > 0$ such that in any interval of length $l(\varepsilon)$, there exists at least a $\tau \in E\{\varepsilon, f\}$ such that

$$|f(t+\tau) - f(t)| < \varepsilon, \quad \forall t \in \mathbb{T}.$$

τ is called the ε-translation number of f, and $l(\varepsilon)$ is called the inclusion length of $E\{\varepsilon, f\}$.

For relevant definitions and the properties of almost periodic functions, see [16–18]. Similar to the proof of Corollary 1.2 in [18], we can get the following lemma.

Lemma 2.6 *Let $\mathbb{T}$ be an almost periodic time scale. If $f(t)$, $g(t)$ are almost periodic functions, then, for any $\varepsilon > 0$, $E\{\varepsilon, f\} \cap E\{\varepsilon, g\}$ is a nonempty relatively dense set in $\mathbb{T}$; that is, for any given $\varepsilon > 0$, there exists a constant $l(\varepsilon) > 0$ such that in any interval of length $l(\varepsilon)$, there exists at least a $\tau \in E\{\varepsilon, f\} \cap E\{\varepsilon, g\}$ such that*

$$|f(t+\tau) - f(t)| < \varepsilon, \qquad |g(t+\tau) - g(t)| < \varepsilon, \quad \forall t \in \mathbb{T}.$$

Remark 2.1 Lemma 2.6 is a special case of Theorem 3.22 in [16].

3 Main results

Assume that the coefficients of (1.1) satisfy

(H_1) $1 - \frac{M_1}{a^l} - c^u M_2 > 0$.

Lemma 3.1 *Let $(x(t), y(t))$ be any positive solution of system* (1.1) *with initial condition* (1.2). *If* (H_1) *holds, then system* (1.1) *is permanent, that is, any positive solution $(x(t), y(t))$ of system* (1.1) *satisfies*

$$m_1 \le \liminf_{t \to +\infty} x(t) \le \limsup_{t \to +\infty} x(t) \le M_1, \tag{3.1}$$

$$m_2 \le \liminf_{t \to +\infty} y(t) \le \limsup_{t \to +\infty} y(t) \le M_2, \tag{3.2}$$

especially if $m_1 \le x_0 \le M_1$, $m_2 \le y_0 \le M_2$, then

$$m_1 \le x(t) \le M_1, \qquad m_2 \le y(t) \le M_2, \quad t \in [t_0, +\infty)_{\mathbb{T}},$$

where

$$M_1 = \frac{1}{b^l}, \qquad M_2 = \frac{g^u M_1}{\eta^l}, \qquad m_1 = \frac{1 - \frac{M_1}{a^l} - c^u M_2}{b^u}, \qquad m_2 = \frac{g^l m_1}{\eta^u}.$$

Proof Assume that $(x(t), y(t))$ is any positive solution of system (1.1) with initial condition (1.2). From the first equation of system (1.1), we have

$$x^{\Delta}(t) \leq r^u x(t)\bigl(1 - b^l x\bigl(\sigma(t)\bigr)\bigr). \tag{3.3}$$

By Lemma 2.3, we can get

$$\limsup_{t\to+\infty} x(t) \leq \frac{1}{b^l} := M_1.$$

Then, for an arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_1 > 0$ such that

$$x(t) < M_1 + \varepsilon, \quad \forall t \in [T_1, +\infty]_{\mathbb{T}}.$$

From the second equation of system (1.1), when $t \in [T_1, +\infty)_{\mathbb{T}}$,

$$y^{\Delta}(t) < -\eta^l y(t) + g^u(M_1 + \varepsilon).$$

Let $\varepsilon \to 0$, then

$$y^{\Delta}(t) \leq -\eta^l y(t) + g^u M_1. \tag{3.4}$$

By Lemma 2.2, we can get

$$\limsup_{t\to+\infty} y(t) = \frac{g^u M_1}{\eta^l} := M_2.$$

Then, for an arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_2 > T_1$ such that

$$y(t) < M_2 + \varepsilon, \quad \forall t \in [T_2, +\infty]_{\mathbb{T}}.$$

On the other hand, from the first equation of system (1.1), when $t \in [T_2, +\infty)_{\mathbb{T}}$,

$$x^{\Delta}(t) > r^l x(t)\left[1 - \frac{M_1 + \varepsilon}{a^l} - b^u x\bigl(\sigma(t)\bigr) - c^u(M_2 + \varepsilon)\right].$$

Let $\varepsilon \to 0$, then

$$x^{\Delta}(t) \geq r^l x(t)\left[1 - \frac{M_1}{a^l} - b^u x\bigl(\sigma(t)\bigr) - c^u M_2\right]. \tag{3.5}$$

By Lemma 2.3, we can get

$$\liminf_{t\to+\infty} x(t) = \frac{1 - \frac{M_1}{a^l} - c^u M_2}{b^u} := m_1.$$

Then, for an arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_3 > T_2$ such that

$$x(t) > m_1 - \varepsilon, \quad \forall t \in [T_3, +\infty]_{\mathbb{T}}.$$

From the second equation of system (1.1), when $t \in [T_3, +\infty)_{\mathbb{T}}$,

$$y^{\Delta}(t) > -\eta^u y(t) + g^l(m_1 - \varepsilon).$$

Let $\varepsilon \to 0$, then

$$y^{\Delta}(t) \geq -\eta^u y(t) + g^l m_1. \tag{3.6}$$

By Lemma 2.2, we can get

$$\liminf_{t\to+\infty} y(t) = \frac{g^l m_1}{\eta^u} := m_2.$$

Then, for arbitrarily small positive constant $\varepsilon > 0$, there exists a $T_4 > T_3$ such that

$$y(t) > m_2 - \varepsilon, \quad \forall t \in [T_4, +\infty]_{\mathbb{T}}.$$

In special case, if $m_1 \leq x_0 \leq M_1$, $m_2 \leq y_0 \leq M_2$, by Lemma 2.2 and Lemma 2.3, it follows from (3.3)-(3.6) that

$$m_1 \leq x(t) \leq M_1, \qquad m_2 \leq y(t) \leq M_2, \quad t \in [t_0, +\infty)_{\mathbb{T}},$$

This completes the proof. □

Let $S(\mathbb{T})$ be a set of all solutions $(x(t), y(t))$ of system (1.1) satisfying $m_1 \leq x(t) \leq M_1$, $m_2 \leq y(t) \leq M_2$ for all $t \in \mathbb{T}$.

Lemma 3.2 $S(\mathbb{T}) \neq \emptyset$.

Proof By Lemma 3.1, we see that for any $t_0 \in \mathbb{T}$ with $m_1 \leq x_0 \leq M_1$, $m_2 \leq y_0 \leq M_2$, system (1.1) has a solution $(x(t), y(t))$ satisfying $m_1 \leq x(t) \leq M_1$, $m_2 \leq y(t) \leq M_2$, $t \in [t_0, +\infty)_{\mathbb{T}}$. Since $r(t), a(t), b(t), c(t), d(t), \eta(t), g(t), \sigma(t)$ are almost periodic, it follows from Lemma 2.6 that there exists a sequence $\{t_n\}$, $t_n \to +\infty$ as $n \to +\infty$ such that $r(t+t_n) \to r(t)$, $a(t+t_n) \to a(t)$, $b(t+t_n) \to b(t)$, $c(t+t_n) \to c(t)$, $d(t+t_n) \to d(t)$, $\eta(t+t_n) \to \eta(t)$, $g(t+t_n) \to g(t)$, $\sigma(t+t_n) \to \sigma(t)$ as $n \to +\infty$ uniformly on $\mathbb{T}$.

We claim that $\{x(t+t_n)\}$ and $\{y(t+t_n)\}$ are uniformly bounded and equi-continuous on any bounded interval in $\mathbb{T}$.

In fact, for any bounded interval $[\alpha, \beta]_{\mathbb{T}} \subset \mathbb{T}$, when n is large enough, $\alpha + t_n > t_0$, then $t + t_n > t_0$, $\forall t \in [\alpha, \beta]_{\mathbb{T}}$. So, $m_1 \leq x(t+t_n) \leq M_1$, $m_2 \leq y(t+t_n) \leq M_2$ for any $t \in [\alpha, \beta]_{\mathbb{T}}$, that is, $\{x(t+t_n)\}$ and $\{y(t+t_n)\}$ are uniformly bounded. On the other hand, $\forall t_1, t_2 \in [\alpha, \beta]_{\mathbb{T}}$, from the mean value theorem of differential calculus on time scales, we have

$$\left|x(t_1+t_n) - x(t_2+t_n)\right| \leq r^u M_1 \left(1 + \frac{M_1}{a^l} + b^u M_1 + c^u M_2\right) |t_1 - t_2|, \tag{3.7}$$

$$\left|y(t_1+t_n) - y(t_2+t_n)\right| \leq \left(\eta^u M_2 + g^u M_1\right) |t_1 - t_2|. \tag{3.8}$$

Inequalities (3.7) and (3.8) show that $\{x(t+t_n)\}$ and $\{y(t+t_n)\}$ are equi-continuous on $[\alpha, \beta]_{\mathbb{T}}$. By the arbitrariness of $[\alpha, \beta]_{\mathbb{T}}$, the conclusion is valid.

By the Ascoli-Arzela theorem, there exists a subsequence of $\{t_n\}$, we still denote it as $\{t_n\}$, such that

$$x(t+t_n) \to p(t), \qquad y(t+t_n) \to q(t),$$

as $n \to +\infty$ uniformly in t on any bounded interval in $\mathbb{T}$. For any $\theta \in \mathbb{T}$, we can assume that $t_n + \theta \geq t_0$ for all n. Let $t \geq 0$, integrating both equations of system (1.1) from $t_n + \theta$ to $t + t_n + \theta$, we have

$$\begin{aligned} & x(t+t_n+\theta) - x(t_n+\theta) \\ &\quad = \int_{t_n+\theta}^{t+t_n+\theta} r(s)x(s)\left(1 - \frac{x(s)}{a(s)+d(s)x(s)} - b(s)x\big(\sigma(s)\big) - c(s)y(s)\right)\Delta s \\ &\quad = \int_{\theta}^{t+\theta} r(s+t_n)x(s+t_n)\left(1 - \frac{x(s+t_n)}{a(s+t_n)+d(t+t_n)x(s+t_n)}\right. \\ &\qquad \left. - b(s+t_n)x\big(\sigma(s+t_n)\big) - c(s+t_n)y(s+t_n)\right)\Delta s, \end{aligned}$$

and

$$\begin{aligned} & y(t+t_n+\theta) - y(t_n+\theta) \\ &\quad = \int_{t_n+\theta}^{t+t_n+\theta} \big[-\eta(s)y(s) + g(s)x(s)\big]\Delta s \\ &\quad = \int_{\theta}^{t+\theta} \big[-\eta(s+t_n)y(s+t_n) + g(s+t_n)x(s+t_n)\big]\Delta s. \end{aligned}$$

Using the Lebesgue dominated convergence theorem, we have

$$\begin{aligned} p(t+\theta) - p(\theta) &= \int_{\theta}^{t+\theta} r(s)x(s)\left(1 - \frac{x(s)}{a(s)+d(s)x(s)} - b(s)x\big(\sigma(s)\big) - c(s)y(s)\right)\Delta s, \\ q(t+\theta) - q(\theta) &= \int_{\theta}^{t+\theta} \big[-\eta(s)y(s) + g(s)x(s)\big]\Delta s. \end{aligned}$$

This means that $(p(t), q(t))$ is a solution of system (1.1), and by the arbitrariness of θ, $(p(t), q(t))$ is a solution of system (1.1) on $\mathbb{T}$. It is clear that

$$m_1 \leq p(t) \leq M_1, \qquad m_2 \leq q(t) \leq M_2, \quad \forall t \in \mathbb{T}.$$

This completes the proof. □

Lemma 3.3 *In addition to condition* (H_1), *assume further that the coefficients of system* (1.1) *satisfy the following conditions*:

(H_2) $\frac{r^l a^l}{[a^u + d^u M_1]^2} - g^u > 0$;
(H_3) $\eta^l - r^u c^u > 0$.

Then system (1.1) *is globally attractive*.

Proof Let $z_1(t) = (x_1(t), y_1(t))$ and $z_2(t) = (x_2(t), y_2(t))$ be any two positive solutions of system (1.1). It follows from (3.1)-(3.2) that for a sufficiently small positive constant ε_0 ($0 < \varepsilon_0 < \min\{m_1, m_2\}$), there exists a $T > 0$ such that

$$m_1 - \varepsilon_0 < x_i(t) < M_1 + \varepsilon_0, \qquad m_2 - \varepsilon_0 < y_i(t) < M_2 + \varepsilon_0, \quad t \in [T, +\infty)_{\mathbb{T}}, i = 1, 2, \tag{3.9}$$

and

$$\frac{r^l a^l}{[a^u + d^u(M_1 + \varepsilon_0)]^2} - g^u > 0. \tag{3.10}$$

Since $x_i(t)$, $i = 1, 2$, are positive, bounded and differentiable functions on $\mathbb{T}$, then there exists a positive, bounded and differentiable function $m(t)$, $t \in \mathbb{T}$, such that $x_i(t)(1 + m(t))$, $i = 1, 2$, are strictly increasing on $\mathbb{T}$. By Lemma 2.4 and Lemma 2.5, we have

$$\begin{aligned} \frac{\Delta}{\Delta t} L_{\mathbb{T}}\big(x_i(t)\big[1 + m(t)\big]\big) &= \frac{x_i^{\Delta}(t)[1 + m(t)] + x_i(\sigma(t))m^{\Delta}(t)}{x_i(t)[1 + m(t)]} \\ &= \frac{x_i^{\Delta}(t)}{x_i(t)} + \frac{x_i(\sigma(t))m^{\Delta}(t)}{x_i(t)[1 + m(t)]}, \quad i = 1, 2. \end{aligned}$$

Here, we can choose a function $m(t)$ such that $\frac{|m^{\Delta}(t)|}{1+m(t)}$ is bounded on $\mathbb{T}$, that is, there exist two positive constants $\zeta > 0$ and $\xi > 0$ such that $0 < \zeta < \frac{|m^{\Delta}(t)|}{1+m(t)} < \xi$, $\forall t \in \mathbb{T}$.

Set

$$V(t) = e_{-\delta}(t, T)\big(\big|L_{\mathbb{T}}\big(x_1(t)\big(1 + m(t)\big)\big) - L_{\mathbb{T}}\big(x_2(t)\big(1 + m(t)\big)\big)\big| + \big|y_1(t) - y_2(t)\big|\big),$$

where $\delta \geq 0$ is a constant (if $\mu(t) = 0$, then $\delta = 0$; if $\mu(t) > 0$, then $\delta > 0$). It follows from the mean value theorem of differential calculus on time scales for $t \in [T, +\infty)_{\mathbb{T}}$ that

$$\begin{aligned} \frac{1}{M_1 + \varepsilon_0}\big|x_1(t) - x_2(t)\big| &\leq \big|L_{\mathbb{T}}\big(x_1(t)\big(1 + m(t)\big)\big) - L_{\mathbb{T}}\big(x_2(t)\big(1 + m(t)\big)\big)\big| \\ &\leq \frac{1}{m_1 - \varepsilon_0}\big|x_1(t) - x_2(t)\big|. \end{aligned} \tag{3.11}$$

Let $\gamma = \min\{(m_1 - \varepsilon_0)(\frac{r^l a^l}{[a^u + d^u(M_1+\varepsilon_0)]^2} - g^u), \eta^l - r^u c^u\}$. We divide the proof into two cases.

Case I. If $\mu(t) > 0$, set $\delta > \max\{(r^u b^u + \frac{\xi}{m_1})M_1, \gamma\}$ and $1 - \mu(t)\delta < 0$. Calculating the upper right derivatives of $V(t)$ along the solution of system (1.1), it follows from (3.9)-(3.11), (H_2) and (H_3) that for $t \in [T, +\infty)_{\mathbb{T}}$,

$$\begin{aligned} &D^+ V(t) \\ &\quad = e_{-\delta}(t, T)\operatorname{sgn}\big(x_1(t) - x_2(t)\big)\left[\frac{x_1^{\Delta}(t)}{x_1(t)} - \frac{x_2^{\Delta}(t)}{x_2(t)} + \frac{m^{\Delta}(t)}{1 + m(t)}\left(\frac{x_1(\sigma(t))}{x_1(t)} - \frac{x_2(\sigma(t))}{x_2(t)}\right)\right] \\ &\qquad - \delta e_{-\delta}(t, T)\big|L_{\mathbb{T}}\big(x_1\big(\sigma(t)\big)\big(1 + m\big(\sigma(t)\big)\big)\big) - L_{\mathbb{T}}\big(x_2\big(\sigma(t)\big)\big(1 + m\big(\sigma(t)\big)\big)\big)\big| \\ &\qquad + e_{-\delta}(t, T)\operatorname{sgn}\big(y_1(t) - y_2(t)\big)\big(y_1^{\Delta}(t) - y_2^{\Delta}(t)\big) - \delta e_{-\delta}(t, T)\big|y_1\big(\sigma(t)\big) - y_2\big(\sigma(t)\big)\big| \\ &\quad = e_{-\delta}(t, T)\operatorname{sgn}\big(x_1(t) - x_2(t)\big)\left[r(t)\left(-\frac{a(t)(x_1(t) - x_2(t))}{(a(t) + d(t)x_1(t))(a(t) + d(t)x_2(t))}\right.\right. \end{aligned}$$

$$
\begin{aligned}
&- b(t)\big(x_1\big(\sigma(t)\big) - x_2\big(\sigma(t)\big)\big) - c(t)\big(y_1(t) - y_2(t)\big)\bigg) \\
&+ \frac{m^\Delta(t)}{1+m(t)} \frac{x_1(\sigma(t))x_2(t) - x_1(t)x_2(\sigma(t))}{x_1(t)x_2(t)}\bigg] \\
&- \delta e_{-\delta}(t,T)\big|L_{\mathbb{T}}\big(x_1\big(\sigma(t)\big)\big(1+m\big(\sigma(t)\big)\big)\big) - L_{\mathbb{T}}\big(x_2\big(\sigma(t)\big)\big(1+m\big(\sigma(t)\big)\big)\big)\big| \\
&+ e_{-\delta}(t,T)\operatorname{sgn}\big(y_1(t)-y_2(t)\big)\big[-\eta(t)\big(y_1(t)-y_2(t)\big) + g(t)\big(x_1(t)-x_2(t)\big)\big] \\
&- \delta e_{-\delta}(t,T)\big|y_1\big(\sigma(t)\big) - y_2\big(\sigma(t)\big)\big| \\
={}& e_{-\delta}(t,T)\operatorname{sgn}\big(x_1(t)-x_2(t)\big)\bigg[-\frac{r(t)a(t)(x_1(t)-x_2(t))}{(a(t)+d(t)x_1(t))(a(t)+d(t)x_2(t))} \\
&- r(t)b(t)\big(x_1\big(\sigma(t)\big) - x_2\big(\sigma(t)\big)\big) - r(t)c(t)\big(y_1(t)-y_2(t)\big) \\
&+ \frac{m^\Delta(t)}{1+m(t)} \frac{x_1(\sigma(t))(x_2(t)-x_1(t)) + x_1(t)(x_1(\sigma(t)) - x_2(\sigma(t)))}{x_1(t)x_2(t)}\bigg] \\
&- \delta e_{-\delta}(t,T)\big|L_{\mathbb{T}}\big(x_1\big(\sigma(t)\big)\big(1+m\big(\sigma(t)\big)\big)\big) - L_{\mathbb{T}}\big(x_2\big(\sigma(t)\big)\big(1+m\big(\sigma(t)\big)\big)\big)\big| \\
&+ e_{-\delta}(t,T)\operatorname{sgn}\big(y_1(t)-y_2(t)\big)\big[-\eta(t)\big(y_1(t)-y_2(t)\big) + g(t)\big(x_1(t)-x_2(t)\big)\big] \\
&- \delta e_{-\delta}(t,T)\big|y_1\big(\sigma(t)\big) - y_2\big(\sigma(t)\big)\big| \\
\le{}& -e_{-\delta}(t,T)\bigg[\frac{r(t)a(t)}{(a(t)+d(t)x_1(t))(a(t)+d(t)x_2(t))} - g(t) \\
&+ \frac{|m^\Delta(t)|}{1+m(t)} \frac{x_1(\sigma(t))}{x_1(t)x_2(t)}\bigg]\big|x_1(t)-x_2(t)\big| \\
&- e_{-\delta}(t,T)\bigg[\frac{\delta}{M_1+\varepsilon_0} - r(t)b(t) - \frac{|m^\Delta(t)|}{1+m(t)}\frac{1}{x_2(t)}\bigg]\big|x_1\big(\sigma(t)\big) - x_2\big(\sigma(t)\big)\big| \\
&- e_{-\delta}(t,T)\big(\eta(t) - r(t)c(t)\big)\big|y_1(t)-y_2(t)\big| \\
&- \delta e_{-\delta}(t,T)\big|y_1\big(\sigma(t)\big) - y_2\big(\sigma(t)\big)\big| \\
\le{}& -e_{-\delta}(t,T)\bigg(\frac{r^l a^l}{[a^u + d^u(M_1+\varepsilon_0)]^2} - g^u\bigg)\big|x_1(t)-x_2(t)\big| \\
&- e_{-\delta}(t,T)\big(\eta^l - r^u c^u\big)\big|y_1(t)-y_2(t)\big| \\
\le{}& -e_{-\delta}(t,T)\bigg[(m_1-\varepsilon_0)\bigg(\frac{r^l a^l}{[a^u + d^u(M_1+\varepsilon_0)]^2} - g^u\bigg)\big|L_{\mathbb{T}}\big(x_1(t)\big(1+m(t)\big)\big) \\
&- L_{\mathbb{T}}\big(x_2(t)\big(1+m(t)\big)\big)\big| + \big(\eta^l - c^u\big)\big|y_1(t)-y_2(t)\big|\bigg] \\
\le{}& -\gamma e_{-\delta}(t,T)\big(\big|L_{\mathbb{T}}\big(x_1(t)\big(1+m(t)\big)\big) - L_{\mathbb{T}}\big(x_2(t)\big(1+m(t)\big)\big)\big| + \big|y_1(t)-y_2(t)\big|\big) \\
={}& -\gamma V(t).
\end{aligned}
\tag{3.12}
$$

By the comparison theorem and (3.12), we have

$$
V(t) \le e_{-\gamma}(t,T)V(T) < 2\bigg(\frac{M_1+\varepsilon_0}{m_1-\varepsilon_0} + M_2 + \varepsilon_0\bigg)e_{-\gamma}(t,T),
$$

that is,

$$\begin{aligned}
&e_{-\delta}(t,T)\bigl(\bigl|L_{\mathbb{T}}\bigl(x_1(t)\bigl(1+m(t)\bigr)\bigr)-L_{\mathbb{T}}\bigl(x_2(t)\bigl(1+m(t)\bigr)\bigr)\bigr|+\bigl|y_1(t)-y_2(t)\bigr|\bigr)\\
&\quad<2\biggl(\frac{M_1+\varepsilon_0}{m_1-\varepsilon_0}+M_2+\varepsilon_0\biggr)e_{-\gamma}(t,T),
\end{aligned}$$

then

$$\begin{aligned}
&\frac{1}{M_1+\varepsilon_0}\bigl|x_1(t)-x_2(t)\bigr|+\bigl|y_1(t)-y_2(t)\bigr|\\
&\quad<2\biggl(\frac{M_1+\varepsilon_0}{m_1-\varepsilon_0}+M_2+\varepsilon_0\biggr)e_{(-\gamma)\ominus(-\delta)}(t,T).
\end{aligned}\tag{3.13}$$

Since $1-\mu(t)\delta<0$ and $0<\gamma<\delta$, then $(-\gamma)\ominus(-\delta)<0$. It follows from (3.13) that

$$\lim_{t\to+\infty}\bigl|x_1(t)-x_2(t)\bigr|=0,\qquad \lim_{t\to+\infty}\bigl|y_1(t)-y_2(t)\bigr|=0.$$

Case II. If $\mu(t)=0$, set $\delta=0$, then $\sigma(t)=t$ and $e_{-\delta}(t,T)=1$. Calculating the upper right derivatives of $V(t)$ along the solution of system (1.1), it follows from (3.9)-(3.11), (H_2) and (H_3) that for $t\in[T,+\infty)_{\mathbb{T}}$,

$$\begin{aligned}
&D^+V(t)\\
&\quad=\operatorname{sgn}\bigl(x_1(t)-x_2(t)\bigr)\biggl(\frac{x_1^\Delta(t)}{x_1(t)}-\frac{x_2^\Delta(t)}{x_2(t)}\biggr)+\operatorname{sgn}\bigl(y_1(t)-y_2(t)\bigr)\bigl(y_1^\Delta(t)-y_2^\Delta(t)\bigr)\\
&\quad=\operatorname{sgn}\bigl(x_1(t)-x_2(t)\bigr)r(t)\biggl[-\biggl(\frac{a(t)}{(a(t)+d(t)x_1(t))(a(t)+d(t)x_2(t))}+b(t)\biggr)\bigl(x_1(t)-x_2(t)\bigr)\\
&\qquad-c(t)\bigl(y_1(t)-y_2(t)\bigr)\biggr]\\
&\qquad+\operatorname{sgn}\bigl(y_1(t)-y_2(t)\bigr)\bigl[-\eta(t)\bigl(y_1(t)-y_2(t)\bigr)+g(t)\bigl(x_1(t)-x_2(t)\bigr)\bigr]\\
&\quad\le-\biggl(\frac{r(t)a(t)}{(a(t)+d(t)x_1(t))(a(t)+d(t)x_2(t))}+r(t)b(t)-g(t)\biggr)\bigl|x_1(t)-x_2(t)\bigr|\\
&\qquad-\bigl(\eta(t)-r(t)c(t)\bigr)\bigl|y_1(t)-y_2(t)\bigr|\\
&\quad\le-\biggl((m_1-\varepsilon_0)\biggl(\frac{r^la^l}{[a^u+d^u(M_1+\varepsilon_0)]^2}+r^lb^l-g^u\biggr)\bigl|L_{\mathbb{T}}\bigl(x_1(t)\bigl(1+m(t)\bigr)\bigr)\\
&\qquad-L_{\mathbb{T}}\bigl(x_2(t)\bigl(1+m(t)\bigr)\bigr)\bigr|+\bigl(\eta^l-r^uc^u\bigr)\bigl|y_1(t)-y_2(t)\bigr|\biggr)\\
&\quad\le-\widehat{\gamma}\bigl(\bigl|L_{\mathbb{T}}\bigl(x_1(t)\bigl(1+m(t)\bigr)\bigr)-L_{\mathbb{T}}\bigl(x_2(t)\bigl(1+m(t)\bigr)\bigr)\bigr|+\bigl|y_1(t)-y_2(t)\bigr|\bigr)\\
&\quad\le-\gamma V(t),
\end{aligned}\tag{3.14}$$

where $\widehat{\gamma}=\min\{(m_1-\varepsilon_0)(\frac{r^la^l}{[a^u+d^u(M_1+\varepsilon_0)]^2}+r^lb^l-g^u),\eta^l-r^uc^u\}$. By the comparison theorem and (3.14), we have

$$V(t)\le e_{-\gamma}(t,T)V(T)<2\biggl(\frac{M_1+\varepsilon_0}{m_1-\varepsilon_0}+M_2+\varepsilon_0\biggr)e_{-\gamma}(t,T),$$

that is,

$$\begin{aligned}&\bigl|L_{\mathbb{T}}\bigl(x_1(t)\bigl(1+m(t)\bigr)\bigr)-L_{\mathbb{T}}\bigl(x_2(t)\bigl(1+m(t)\bigr)\bigr)\bigr|+\bigl|y_1(t)-y_2(t)\bigr|\\&\quad<2\left(\frac{M_1+\varepsilon_0}{m_1-\varepsilon_0}+M_2+\varepsilon_0\right)e_{-\gamma}(t,T),\end{aligned}$$

then

$$\begin{aligned}&\frac{1}{M_1+\varepsilon_0}\bigl|x_1(t)-x_2(t)\bigr|+\bigl|y_1(t)-y_2(t)\bigr|\\&\quad<2\left(\frac{M_1+\varepsilon_0}{m_1-\varepsilon_0}+M_2+\varepsilon_0\right)e_{-\gamma}(t,T).\end{aligned}\tag{3.15}$$

It follows from (3.15) that

$$\lim_{t\to+\infty}\bigl|x_1(t)-x_2(t)\bigr|=0,\qquad\lim_{t\to+\infty}\bigl|y_1(t)-y_2(t)\bigr|=0.$$

From the above discussion, we can see that system (1.1) is globally attractive. This completes the proof. □

Theorem 3.1 *Assume that conditions* (H_1)-(H_3) *hold, then system* (1.1) *has a unique globally attractive positive almost periodic solution.*

Proof By Lemma 3.2, there exists a bounded positive solution $u(t)=(u_1(t),u_2(t))\in S(\mathbb{T})$, then there exists a sequence $\{t'_k\}$, $\{t'_k\}\to+\infty$ as $k\to+\infty$, such that $(u_1(t+t'_k),u_2(t+t'_k))$ is a solution of the following system:

$$\begin{cases}x^{\Delta}(t)=r(t+t'_k)x(t)[1-\frac{x(t)}{a(t+t'_k)+d(t+t'_k)x(t)}-b(t+t'_k)x(\sigma(t+t'_k))-c(t+t'_k)y(t)],\\ y^{\Delta}(t)=-\eta(t+t'_k)y(t)+g(t+t'_k)x(t).\end{cases}$$

From the above discussion and Lemma 2.1, we have that not only $\{u_i(t+t'_k)\}$, $i=1,2$, but also $\{u_i^{\Delta}(t+t'_k)\}$, $i=1,2$, are uniformly bounded, thus $\{u_i(t+t'_k)\}$, $i=1,2$, are uniformly bounded and equi-continuous. By the Ascoli-Arzela theorem, there exists a subsequence of $\{u_i(t+t_k)\}\subseteq\{u_i(t+t'_k)\}$ such that for any $\varepsilon>0$, there exists a $N(\varepsilon)>0$ with the property that if $m,k>N(\varepsilon)$ then

$$\bigl|u_i(t+t_m)-u_i(t+t_k)\bigr|<\varepsilon,\quad i=1,2.$$

It shows that $u_i(t)$, $i=1,2$, are asymptotically almost periodic functions, then $\{u_i(t+t_k)\}$, $i=1,2$, are the sum of an almost periodic function $q_i(t+t_k)$, $i=1,2$, and a continuous function $p_i(t+t_k)$, $i=1,2$, defined on $\mathbb{T}$, that is,

$$u_i(t+t_k)=p_i(t+t_k)+q_i(t+t_k),\quad\forall t\in\mathbb{T},$$

where

$$\lim_{k\to+\infty}p_i(t+t_k)=0,\qquad\lim_{k\to+\infty}q_i(t+t_k)=q_i(t),$$

$q_i(t)$ is an almost periodic function. It means that $\lim_{k\to+\infty}u_i(t+t_k)=q_i(t)$, $i=1,2$.

On the other hand,

$$\begin{aligned}\lim_{k\to+\infty} u_i^{\Delta}(t+t_k) &= \lim_{k\to+\infty}\lim_{h\to 0}\frac{u_i(t+t_k+h)-u_i(t+t_k)}{h}\\ &= \lim_{h\to 0}\lim_{k\to+\infty}\frac{u_i(t+t_k+h)-u_i(t+t_k)}{h}\\ &= \lim_{h\to 0}\frac{q_i(t+h)-q_i(t)}{h}.\end{aligned}$$

So, the limit $q_i(t)$, $i=1,2$, exists.

Next, we shall prove that $(q_1(t),q_2(t))$ is an almost solution of system (1.1).

From the properties of an almost periodic function, there exists a sequence $\{t_n\}$, $t_n\to+\infty$ as $n\to+\infty$, such that $r(t+t_n)\to r(t)$, $a(t+t_n)\to a(t)$, $b(t+t_n)\to b(t)$, $c(t+t_n)\to c(t)$, $d(t+t_n)\to d(t)$, $\eta(t+t_n)\to\eta(t)$, $g(t+t_n)\to g(t)$, $\sigma(t+t_n)\to\sigma(t)$ as $n\to+\infty$ uniformly on $\mathbb{T}$.

It is easy to know that $u_i(t+t_n)\to q_i(t)$, $i=1,2$ as $n\to+\infty$, then we have

$$\begin{aligned} q_1^{\Delta}(t) &= \lim_{n\to+\infty} u_1^{\Delta}(t+t_n)\\ &= \lim_{n\to+\infty} r(t+t_n)u_1(t+t_n)\biggl[1-\frac{u_1(t+t_n)}{a(t+t_n)+d(t+t_n)u_1(t+t_n)}\\ &\quad -b(t+t_n)u_1\bigl(\sigma(t+t_n)\bigr)-c(t+t_n)u_2(t+t_n)\biggr]\\ &= r(t)q_1(t)\biggl[1-\frac{q_1(t)}{a(t)+d(t)q_1(t)}-b(t)q_1\bigl(\sigma(t)\bigr)-c(t)q_2(t)\biggr],\\ q_2^{\Delta}(t) &= \lim_{n\to+\infty} u_2^{\Delta}(t+t_n)\\ &= \lim_{n\to+\infty}\bigl[-\eta(t+t_n)u_2(t+t_n)+g(t+t_n)x(t+t_n)\bigr]\\ &= -\eta(t)q_2(t)+g(t)q_1(t).\end{aligned}$$

This proves that $(q_1(t),q_2(t))$ is a positive almost periodic solution of system (1.1). Together with Lemma 3.3, system (1.1) has a unique globally attractive positive almost periodic solution. This completes the proof. □

4 Example and simulations

Consider the following system on time scales:

$$\begin{cases} x^{\Delta}(t)=(0.8+0.2\sin\sqrt{2}t)x(t)[1-\frac{x(t)}{(4.5+0.5\sin t)+0.01x(t)}-x(\sigma(t))-0.2y(t)],\\ y^{\Delta}(t)=-(0.4+0.1\cos\sqrt{3}t)y(t)+(0.015+0.005\sin\sqrt{2}t)x(t).\end{cases}\tag{4.1}$$

By a direct calculation, we can get

$$\begin{aligned}&r^u=1,\quad r^l=0.6,\quad a^u=5,\quad a^l=4,\\ &b^u=b^l=1,\quad c^u=c^l=0.2,\quad d^u=d^l=0.01,\\ &\eta^u=0.5,\quad \eta^l=0.3,\quad g^u=0.02,\quad g^l=0.01,\quad M_1=1,\quad M_2=0.0667,\end{aligned}$$

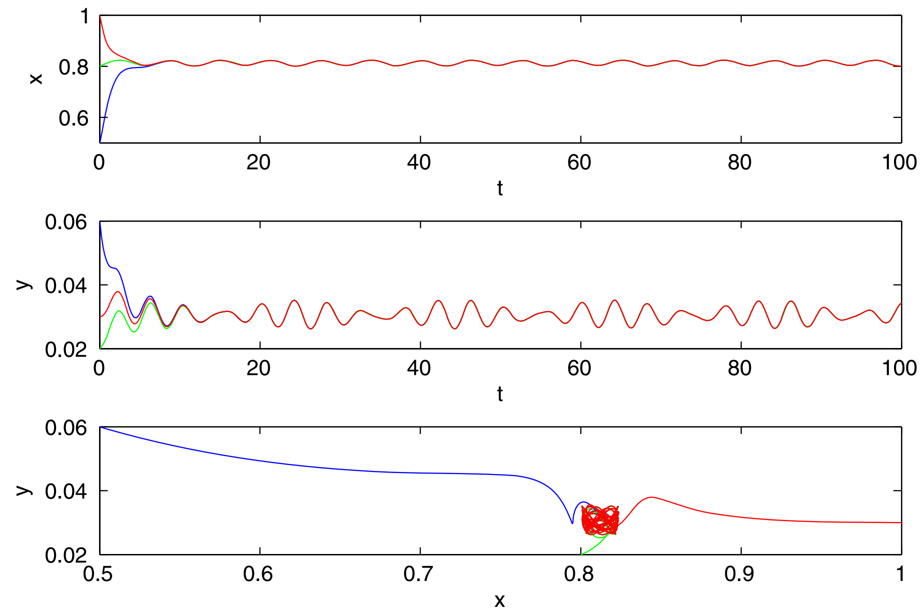

Figure 1 $\mathbb{T} = \mathbb{R}$. **Dynamics behavior of system (4.1) with initial condition** $(x(0), y(0)) = \{(0.5, 0.06); (0.8, 0.02); (1, 0.03)\}$.

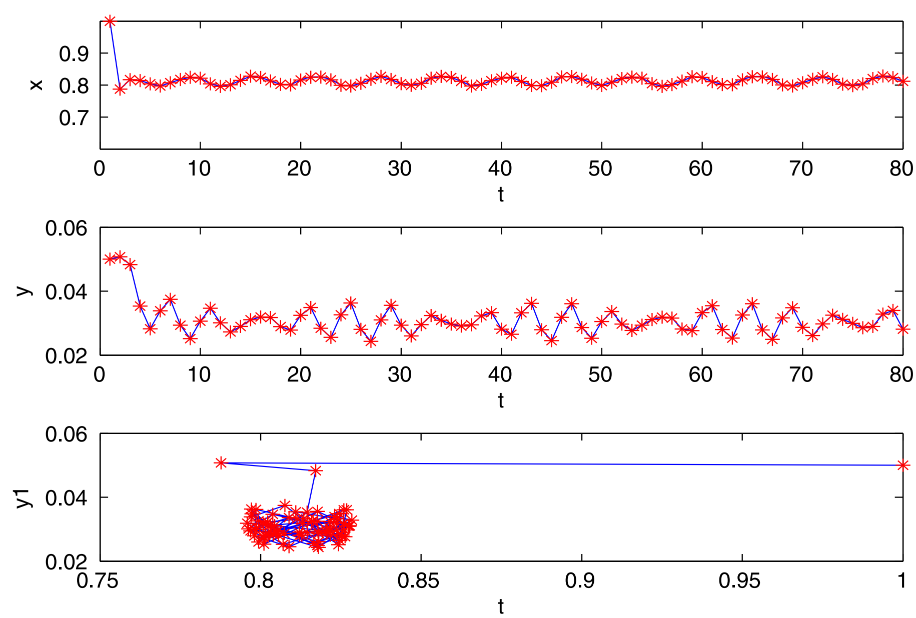

Figure 2 $\mathbb{T} = \mathbb{Z}$. **Dynamics behavior of system (4.1) with initial condition** $(x(1), y(1)) = (1, 0.05)$.

then

$$1 - \frac{M_1}{a^l} - c^u M_2 = 0.7367 > 0, \qquad \frac{r^l a^l}{[a^u + d^u M_1]^2} - g^u = 0.0756 > 0,$$

$$\eta^l - r^u c^u = 0.1000 > 0,$$

that is, conditions (H_1)-(H_3) hold. According to Theorem 3.1, system (4.1) has a unique globally attractive positive almost periodic solution. For dynamic simulations of system (4.1) with $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$, see Figures 1 and 2, respectively.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. The authors read and approved the final manuscript.

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