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Finite-time H_∞ control of a switched discrete-time system with average dwell time

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Abstract

For switched discrete-time systems, switching behavior always affects the finite-time stability property, which was neglected by most previous research. This paper investigates the problem of H_∞ control for a switched discrete-time system with average dwell time. Based on the results on finite-time boundned and average dwell time, sufficient conditions for finite-time bounded and finite-time H_∞ control under arbitrary switching are derived, and the closed-loop system trajectory stays within a prescribed bound. Finally, an example is given to illustrate the efficiency of the proposed method.

Keywords: H_∞ finite-time stability; switched discrete-time system; Lyapunov-Krasovskii function; H_∞ control

1 Introduction

During the last few decades, the problem of H_∞ control for continuous time systems with time delays has been extensively investigated [1, 2]. In the discrete-time context, there is rapidly growing interest in H_∞ control for a discrete-time system due to its being frequently encountered in many practical engineering systems such as chemical, electronics, process control systems and networked control systems [3–5]. Furthermore, in the state feedback case, the augmentation approach generally leads to a static output feedback control problem, which is non-convex [6]. Control of stochastic systems is a research topic of both practical and theoretical importance, which has received much attention [7, 8]. Many results related to stochastic systems have been reported in the literature. For instance, an optimal stochastic linear-quadratic control problem was investigated by a stochastic algebraic Riccati equation approach in infinite-time horizon in [9], where the diffusion term in dynamics depends on both the state and the control variables. It is important to invest H_∞ control problem.

In recent years, there has been increasing interest in the analysis of hybrid and switched systems due to their significance both in theory and applications. Switched linear control systems, as an important class of hybrid systems, comprise a collection of linear subsystems described by differential/difference equations and a switching law to specify the switching among these subsystems. A switched system is a type of a hybrid system which is a combination of discrete and continuous dynamical systems. These systems arise as models for phenomena which cannot be described by exclusively continuous or exclusively discrete processes. Moreover, control design remains open for the switched systems

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that exhibit switching jumps and subsystem models. Most recently, on the basis of Lyapunov functions and other analysis tools, the control design for linear switched systems has been further investigated and many valuable results have been obtained; for a recent survey on this topic and related questions one can refer to [10–25].

It well known that most of the existing literature has focused on Lyapunov asymptotic stability for switched systems, the behavior of which is over an infinite time interval. On the other hand, many concerned problems are the practical ones which described system state which does not exceed some bound over a time interval. To deal with the above problem, in 1961, Dorato proposed the concept of finite-time stability in [26].

Over the years, many research efforts have been devoted to the study of finite-time stability (FTS) of systems. In the study of the transient behavior of systems, FTS concerns the stability of a system over a finite interval of time and plays an important role. It is worth pointing out that finite-time stability and Lyapunov asymptotic stability are different concepts, and they are independent of each other. Therefore, it is important to emphasize the distinction between classical Lyapunov stability and finite-time stability. The problem of finite-time stability has been accordingly studied in the literature [26–40].

Recently, some papers related to finite-time stability for switched systems can be found. For example, based on the average dwell time technique, the problem of finite-time boundedness for a switched linear system with time-delay was investigated in [17], and [39] discussed the static-state feedback and dynamic output feedback finite-time stabilization. But to the best of our knowledge, the finite-time H_∞ control problems for switched discrete-time systems with average dwell time have not been studied, and this motivates us to consider this interesting and challenging problem.

The main contribution of this paper lies in that we present a novel approach to finite-time stability of a switched system. Moreover, several sufficient conditions ensuring finite-time stability and boundness are proposed with different information from what we know about the switching signal. It is shown that less conservative results can be derived when more information about the switching signal is available. By selecting the appropriate Lyapunov-Krasovskii functional, the sufficient conditions are derived to guarantee finite-time boundness of the systems. The finite-time boundness (FTB) criteria can be tackled in the form of LMIs. Finally, an example is used to illustrate the effectiveness of the developed techniques.

Notations: Throughout this paper, we let $P > 0$ ($P \geq 0$, $P < 0$, $P \leq 0$) denote a symmetric positive definite matrix P (positive-semi definite, negative definite and negative-semi definite). For any symmetric matrix P, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space and $\mathcal{R}^{n\times m}$ refers to the set of all $n \times m$ real matrices. The identity matrix of order n is denoted as I_n. * represents the elements below the main diagonal of a symmetric matrix. The superscripts $\top$ and -1 stand for matrix transposition and matrix inverse, respectively.

2 Preliminaries

In this section, we give a mathematical description of the problem under the study, followed by a definition of the average dwell time for a discrete switched system.

Consider the following switched discrete-time system:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + C_{\sigma(k)}\omega(k), \\ z(k) = L_{\sigma(k)}x(k) + D_{\sigma(k)}u(k), \end{cases} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the discrete state vector of the system, $u(k) \in \mathcal{R}^l$ is the control input, $z(k) \in \mathcal{R}^m$ is the controlled output, $\omega(k) \in \mathcal{R}^q$ is the noise signal which satisfies

$$\sum_{k=0}^{N} \omega^{\mathsf{T}}(k)\omega(k) < d. \tag{2}$$

$\sigma(k) : \mathbb{Z}^+ \to \mathcal{N} = \{1, 2, \dots, N\}$ is called a switching law or switching signal, which is a piecewise constant function of discrete-time k and takes its values in the finite set $\mathcal{N}$. $N > 0$ is the number of subsystems. For simplicity, at any arbitrary discrete-time $k \in \mathbb{Z}^+$, the switching signal $\sigma(k)$ is denoted by σ. Matrices $A_{\sigma(k)}$, $B_{\sigma(k)}$, $C_{\sigma(k)}$, $D_{\sigma(k)}$ and $L_{\sigma(k)}$ are constant real matrices with appropriate dimensions for all $\sigma(k) = i \in \mathcal{N}$. We denote the matrices associated with $A_{\sigma(k)} = A_i$, $B_{\sigma(k)} = B_i$, $C_{\sigma(k)} = C_i$, $D_{\sigma(k)} = D_i$ and $L_{\sigma(k)} = L_i$.

Definition 2.1 [17] For any $k \geq k_0$ and any switching signal $\sigma(l)$, $k_0 \leq l < k$, let $N_\sigma(k_0, k)$ denote the number of switchings of $\sigma(k)$. If

$$N_\sigma(k_0, k) \leq N_0 + \frac{k - k_0}{T_a} \tag{3}$$

holds for $N_0 \geq 0$ and $T_a > 0$, then T_a is called the average dwell time and N_0 is the chatter bound.

Remark 1 The concept of average dwell time has been modified to fit the discrete-time ones in some existing literature [26–31]. The definition of average dwell time in Definition 2.1 is borrowed from these existing results. For simplicity, but without loss of generality, we choose $N_0 = 0$ in what follows.

Definition 2.2 [27] The discrete-time linear system

$$x(k+1) = Ax(k), \quad k \in N,$$

is said to be finite-time stable (FTS) with respect to (c_1, c_2, R, N), where $R > 0$ is a positive definite matrix, $0 < c_1 < c_2$ and $N \in \mathcal{N}$, if $x^{\mathsf{T}}(0)Rx(0) \leq c_1 \Rightarrow x^{\mathsf{T}}(k)Rx(k) < c_2$, $\forall k \in \{1, 2, \dots, N\}$.

Definition 2.3 [27] The discrete-time linear system

$$x(k+1) = Ax(k) + G\omega(k), \quad k \in N,$$

subject to an exogenous disturbance $\omega(k)$ satisfying (2), is said to be finite-time bounded (FTB) with respect to (c_1, c_2, R, d, N), where $R > 0$ is a positive definite matrix, $0 < c_1 < c_2$ and $N \in \mathcal{N}$, if $x^{\mathsf{T}}(0)Rx(0) \leq c_1 \Rightarrow x^{\mathsf{T}}(k)Rx(k) < c_2$, $\forall k \in \{1, 2, \dots, N\}$.

Definition 2.4 [4] For $\gamma > 0$, $0 \leq c_1 < c_2$, R is a positive definite matrix, system (4) is said to be H_∞ finite-time bounded with respect to $(c_1, c_2, d, \gamma, R, N)$, the following condition should be satisfied:

$$\sum_{k=0}^{N} z^{\mathsf{T}}(k)z(k) < \gamma^2 \sum_{k=0}^{N} \omega^{\mathsf{T}}(k)\omega(k). \tag{4}$$

Under zero initial condition, it holds for all nonzero ω: $\sum_{k=0}^{N} \omega^{\mathsf{T}}(k)\omega(k) < d$.

In this paper, applying state feedback control $u(k) = K_{\sigma(k)}x(k)$ to (1), we get

$$\begin{cases} x(k+1) = (A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)})x(k) + C_{\sigma(k)}\omega(k), \\ z(k) = (L_{\sigma(k)} + D_{\sigma(k)}K_{\sigma(k)})x(k). \end{cases} \tag{5}$$

3 Finite-time stability analysis

Theorem 3.1 *Consider switched system* (5) *for given positive scalars* c_1 *and* c_2 *with* $c_1 < c_2$, $\mu > 0$, $\tau_a > 0$, $\alpha > 0$. *Let* $P_i = R^{-\frac{1}{2}}\bar{P}_i R^{-\frac{1}{2}}$ *for all admissible* $\omega(k)$ *subject to condition* (2), *if there exist symmetric positive definite matrices* P_i, Q_i, $1 \le i \le N$, *and* $\lambda_1 = \lambda_{\min}(\bar{P}_i)$, $\lambda_2 = \lambda_{\max}(\bar{P}_i)$, $\lambda_3 = \lambda_{\max}(Q_i)$, *such that the linear matrix inequalities*

$$\begin{bmatrix} -(1+\alpha)P_i & 0 & A_i^{\mathsf{T}}P_j \\ * & -Q_i & C_i^{\mathsf{T}}P_j \\ * & * & -P_j \end{bmatrix} < 0, \tag{6}$$

$$P_i \le \mu P_j, \quad \forall i \in \mathcal{N}, \tag{7}$$

$$\begin{bmatrix} -P_i^{-1} & \lambda_1 P_i^{-1}R \\ * & -\lambda_1 R \end{bmatrix} < 0, \qquad \begin{bmatrix} -\lambda_2 R & I \\ * & -P_i^{-1} \end{bmatrix} < 0, \quad \forall i \in \mathcal{N}, \tag{8}$$

$$\frac{\lambda_1 c_2}{(1+\alpha)^N(\lambda_2 c_1 + \lambda_3 d)} > 1 \tag{9}$$

hold, and the average dwell time of the switched discrete-time signal $\sigma(k)$ *satisfies*

$$\tau_a > \tau_a^* = \frac{N \ln \mu}{\ln(\lambda_1 c_2) - N\ln(1+\alpha) - \ln(\lambda_2 c_1 + \lambda_3 d)}. \tag{10}$$

Then switched discrete-time system (5) *with* $z(k) = 0$ *is finite-time bounded with respect to* (c_1, c_2, d, R, N).

Proof We consider the following Lyapunov-Krasovskii functional:

$$V\bigl(x_k, \sigma(k)\bigr) = x^{\mathsf{T}}(k)P_{\sigma(k)}x(k). \tag{11}$$

Taking the difference between the Lyapunov function candidates for two consecutive time instants yields

$$\begin{aligned} &\Delta V\bigl(x_k, \sigma(k)\bigr) \\ &\quad = V\bigl(x_{k+1}, \sigma(k+1)\bigr) - V\bigl(x_k, \sigma(k)\bigr) \\ &\quad = x^{\mathsf{T}}(k)\bigl[(A_i + B_iK_i)^{\mathsf{T}}P_jA_i - P_i\bigr]x(k) + 2x^{\mathsf{T}}(k)A_i^{\mathsf{T}}P_jC_i\omega(k) \\ &\qquad + \omega^{\mathsf{T}}(k)C_i^{\mathsf{T}}P_jC_i\omega(k) \\ &\quad = \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} A_i^{\mathsf{T}}P_jA_i - P_i & A_i^{\mathsf{T}}P_jC_i \\ * & C_i^{\mathsf{T}}P_jC_i \end{bmatrix} \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}. \end{aligned} \tag{12}$$

From condition (6), we can obtain

$$\Delta V\big(x_k,\sigma(k)\big) < \alpha V\big(x_k,\sigma(k)\big) + \omega^{\mathsf{T}}(k)Q_i\omega(k). \tag{13}$$

Equations (9) and (11) yield

$$\begin{aligned}
V\big(x_k,\sigma(k)\big) &< (1+\alpha)V\big(x_{k-1},\sigma(k-1)\big) \\
&\quad + \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\omega^{\mathsf{T}}(k-1)\omega(k-1) \\
&< (1+\alpha)^2 V\big(x_{k-2},\sigma(k-2)\big) \\
&\quad + \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}(1+\alpha)\omega^{\mathsf{T}}(k-2)\omega(k-2) \\
&\quad + \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\omega^{\mathsf{T}}(k-1)\omega(k-1) < \cdots \\
&< (1+\alpha)^{k-k_l} V\big(x_{k_l},\sigma(k_l)\big) \\
&\quad + \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\sum_{s=k_l}^{k-1}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s).
\end{aligned} \tag{14}$$

Noticing (7), we know that

$$V\big(x_k,\sigma(k)\big) < \mu V\big(x_k,\sigma(k)\big). \tag{15}$$

Thus

$$\begin{aligned}
V\big(x_k,\sigma(k)\big) &< (1+\alpha)^{k-k_l} V\big(x_{k_l},\sigma(k_l)\big) \\
&\quad + \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\sum_{s=k_l}^{k-1}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&< \mu(1+\alpha)^{k-k_{l-1}} V\big(x_{k_{l-1}},\sigma(k_{l-1})\big) \\
&\quad + \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\sum_{s=k_l}^{k-1}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&\quad + \mu \sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\sum_{s=k_{l-1}}^{k_l}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&< \cdots \\
&< \mu^{N_\sigma(0,k)}(1+\alpha)^k V\big(x_0,\sigma(0)\big) \\
&\quad + \mu^{N_\sigma(0,k)}\sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\sum_{s=0}^{k_1}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&\quad + \mu^{N_\sigma(k_1,k)}\sup_{i\in\mathcal{N}}\big\{\lambda_{\max}(Q_i)\big\}\sum_{s=k_1}^{k_2}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&\quad + \cdots
\end{aligned}$$

$$
\begin{aligned}
&+ \sup_{i\in\mathcal{N}}\bigl\{\lambda_{\max}(Q_i)\bigr\}\sum_{s=k_l}^{k-1}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&< \mu^{N_\sigma(0,k)}(1+\alpha)^k V\bigl(x_0,\sigma(0)\bigr) + \mu^{N_\sigma(0,k)}\sup_{i\in\mathcal{N}}\bigl\{\lambda_{\max}(Q_i)\bigr\} \\
&\quad \times \sum_{s=0}^{k-1}(1+\alpha)^{k-1-s}\omega^{\mathsf{T}}(s)\omega(s) \\
&< \mu^{N_\sigma(0,k)}(1+\alpha)^k\Bigl\{V\bigl(x_0,\sigma(0)\bigr) + \sup_{i\in\mathcal{N}}\bigl\{\lambda_{\max}(Q_i)\bigr\}d\Bigr\}.
\end{aligned}
\tag{16}
$$

Define $P_i = R^{-\frac{1}{2}}\bar{P}_i R^{-\frac{1}{2}}$, $i\in\mathcal{N}$, then $\forall k\in\{1,2,\dots,N\}$, we have

$$
V\bigl(x_k,\sigma(k)\bigr) = x^{\mathsf{T}}(k)R^{\frac{1}{2}}\bar{P}_i R^{\frac{1}{2}}x(k) \geq \inf_{i\in\mathcal{N}}\bigl\{\lambda_{\min}(\bar{P}_i)\bigr\}x^{\mathsf{T}}(k)Rx(k).
\tag{17}
$$

Using the fact $\alpha \geq 0$ and $x^{\mathsf{T}}(0)Rx(0)\leq c_1$, for $\forall i\in\mathcal{N}$, we have

$$
V\bigl(x_0,\sigma(0)\bigr) \leq \sup_{i\in\mathcal{N}}\bigl\{\lambda_{\max}(\bar{P}_i)\bigr\}x^{\mathsf{T}}(0)Rx(0) \leq \sup_{i\in\mathcal{N}}\bigl\{\lambda_{\max}(\bar{P}_i)\bigr\}c_1.
\tag{18}
$$

Then we can obtain

$$
\begin{aligned}
&x^{\mathsf{T}}(k)Rx(k) \\
&\quad < \frac{\mu^{N_\sigma(0,k)}(1+\alpha)^N\{\sup_{i\in\mathcal{N}}\{\lambda_{\max}(\bar{P}_i)\}c_1 + \sup_{i\in\mathcal{N}}\{\lambda_{\max}(Q_i)\}d\}}{\inf_{i\in\mathcal{N}}\{\lambda_{\min}(\bar{P}_i)\}} \\
&\quad < \frac{\mu^{\frac{N}{\tau_a}}(1+\alpha)^N\{\sup_{i\in\mathcal{N}}\{\lambda_{\max}(\bar{P}_i)\}c_1 + \sup_{i\in\mathcal{N}}\{\lambda_{\max}(Q_i)\}d\}}{\inf_{i\in\mathcal{N}}\{\lambda_{\min}(\bar{P}_i)\}}.
\end{aligned}
\tag{19}
$$

From (8), we have

$$
\lambda_1 R < P_i < \lambda_2 R \quad\Rightarrow\quad \lambda_1 I < \bar{P}_i < \lambda_2 I.
\tag{20}
$$

Define $\sup_{i\in\mathcal{N}}\{\lambda_{\max}(Q_i)\} = \lambda_3$, since $\sup_{i\in\mathcal{N}}\{\lambda_{\max}(\bar{P}_i)\}\leq\lambda_2$, $\inf_{i\in\mathcal{N}}\{\lambda_{\min}(\bar{P}_i)\}\geq\lambda_1$ and by condition (19), we can obtain

$$
x^{\mathsf{T}}(k)Rx(k) < \frac{\mu^{\frac{N}{\tau_a}}(1+\alpha)^N(\lambda_2 c_1 + \lambda_3 d)}{\lambda_1}.
\tag{21}
$$

When $\mu = 1$, which is the trivial case, from (21), $x^{\mathsf{T}}(k)Rx(k) < c_2$. When $\mu > 1$, from (9), $\ln(\lambda_1 c_2) - N\ln(1+\alpha) - \ln(\lambda_2 c_1 + \lambda_3 d) > 0$. By virtue of (10), we have

$$
\frac{N}{\tau_a} < \frac{\ln(\lambda_1 c_2) - N\ln(1+\alpha) - \ln(\lambda_2 c_1 + \lambda_3 d)}{\ln\mu}.
\tag{22}
$$

Substituting (22) into (21) yields

$$
\begin{aligned}
&x^{\mathsf{T}}(k)Rx(k) \\
&\quad < \frac{\mu^{\frac{\ln(\lambda_1 c_2) - N\ln(1+\alpha) - \ln(\lambda_2 c_1 + \lambda_3 d)}{\ln\mu}}(1+\alpha)^N(\lambda_2 c_1 + \lambda_3 d)}{\lambda_1} = c_2.
\end{aligned}
\tag{23}
$$

Thus we can conclude that switched discrete-time system (5) with $u(k) = 0$ is finite-time bounded with respect to (c_1, c_2, d, R, N). The proof is completed. □

4 Finite-time H_∞ performance analysis

Theorem 4.1 *Consider switched discrete-time system* (5). *If there exist symmetric positive definite matrices P_i, $1 \le i \le N$, and positive scalars $\kappa > 0$, $\alpha > 0$, $0 \le c_1 < c_2$, $d > 0$, $\alpha > 0$, such that $\forall i \in \mathcal{N}$, the linear matrix inequalities*

$$\begin{bmatrix} -(1+\alpha)P_i & 0 & (A_i + B_iK_i)^\mathsf{T} P_j & (L_i + D_iK_i)^\mathsf{T} \\ * & -\frac{\gamma^2 I}{(1+\mu)^N} & C_i^\mathsf{T} P_j & 0 \\ * & * & -P_j & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{24}$$

$$P_i \le \mu P_j, \quad \forall i \in \mathcal{N}, \tag{25}$$

$$\begin{bmatrix} -P_i^{-1} & P_i^{-1}R \\ * & -R \end{bmatrix} < 0, \qquad \begin{bmatrix} -\kappa R & I \\ * & -P_i^{-1} \end{bmatrix} < 0, \quad \forall i \in \mathcal{N}, \tag{26}$$

$$\begin{bmatrix} -c_2 & \sqrt{c_1} \\ * & -(1+\alpha)^N \kappa \end{bmatrix} < 0, \tag{27}$$

hold, and the average dwell time of the switched discrete-time signal $\sigma(k)$ satisfies

$$\tau_a > \tau_a^* = \frac{N \ln \mu}{\ln c_2 - \ln(\gamma^2 d)}. \tag{28}$$

Then switched system (5) *is finite-time bounded with H_∞ performance level γ for any switching discrete-time signal with respect to $(c_1, c_2, N, d, R, \sigma)$.*

Proof We will show the H_∞ performance of system (5), from Theorem 3.1, we have

$$\begin{aligned} &\Delta V\bigl(x_k, \sigma(k)\bigr) \\ &\quad = \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}^\mathsf{T} \begin{bmatrix} (A_i + B_iK_i)^\mathsf{T} P_j (A_i + B_iK_i) - P_i & (A_i + B_iK_i)^\mathsf{T} P_j C_i \\ * & C_i^\mathsf{T} P_j C_i \end{bmatrix} \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}. \end{aligned} \tag{29}$$

Define

$$J(k) = z^\mathsf{T}(k) z(k) - \frac{\gamma^2}{(1+\mu)^N} \omega^\mathsf{T}(k)\omega(k). \tag{30}$$

From (12) it follows

$$\begin{aligned} &\Delta V\bigl(x_k, \sigma(k)\bigr) + J(k) \\ &\quad = \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}^\mathsf{T} \begin{bmatrix} \Xi_{1,1} & (A_i + B_iK_i)^\mathsf{T} P_j C_i \\ * & C_i^\mathsf{T} P_j C_i \end{bmatrix} \begin{bmatrix} x(k) \\ \omega(k) \end{bmatrix}, \end{aligned} \tag{31}$$

where

$$\Xi_{1,1} = (A_i + B_iK_i)^\mathsf{T} P_j (A_i + B_iK_i)(L_i + D_iK_i)^\mathsf{T} (L_i + D_iK_i) - P_i.$$

From (24) and the Schur complement, we have

$$\Delta V\big(x_k,\sigma(k)\big) + J(k) < \mu V\big(x_k,\sigma(k)\big). \tag{32}$$

It follows from (32) that

$$\Delta V\big(x_k,\sigma(k)\big) < \alpha V\big(x_k,\sigma(k)\big) - J(k), \tag{33}$$

which means

$$\begin{aligned} & V\big(x_k,\sigma(k)\big) \\ &\quad < (1+\alpha)V\big(x_{k-1},\sigma(k-1)\big) + J(k-1) \\ &\quad < (1+\alpha)^2 V\big(x_{k-2},\sigma(k-2)\big) \\ &\qquad + J(k-1) + (1+\alpha)J(k-2) \\ &\quad < \cdots < (1+\alpha)^k V\big(x_0,\sigma(0)\big) - \sum_{s=0}^{k-1}(1+\alpha)^{k-s-1}J(s). \end{aligned} \tag{34}$$

Under the zero-initial condition, we have $V(x_0,\sigma(0)) = 0$, and due to the fact $V(x_k, \sigma(k)) \geq 0$, it yields

$$\sum_{s=0}^{k-1}(1+\alpha)^{k-s-1}J(s) \geq 0, \tag{35}$$

which means

$$\begin{aligned} & (1+\alpha)^N \frac{\gamma^2}{(1+\alpha)^N} \sum_{s=0}^{k-1} \omega^{\mathsf{T}}(s)\omega(s) \geq \sum_{s=0}^{k-1} z^{\mathsf{T}}(s)z(s) \\ &\quad \Rightarrow \quad \gamma^2 \sum_{s=0}^{N} \omega^{\mathsf{T}}(s)\omega(s) \geq \sum_{s=0}^{N} z^{\mathsf{T}}(s)z(s). \end{aligned} \tag{36}$$

According to Definition 2.4, we know that Theorem 4.1 holds. This completes the proof. □

5 Finite-time H_∞ control design

Theorem 5.1 *Consider finite-time switched discrete-time system* (1) *and a given scalar* $\gamma > 0$. *Then there exists a switched* H_∞ *control in the form of* $u(k) = K_{\sigma(k)}x(k)$ *such that switched discrete-time system* (5) *is finite-time bounded with* H_∞ *performance level* γ, *if there exist symmetric positive-definite matrixes* P_i, X_i, Y_i *and* Z_i *such that* $\forall i \in \mathcal{N}$,

$$\begin{bmatrix} -(1+\alpha)Y_i & 0 & Y_iA_i^{\mathsf{T}} + X_i^{\mathsf{T}}B_i^{\mathsf{T}} & Y_iL_i^{\mathsf{T}} + X_i^{\mathsf{T}}D_i^{\mathsf{T}} \\ * & -\frac{\gamma^2 I}{(1+\alpha)^N} & C_i^{\mathsf{T}} & 0 \\ * & * & -Z_j & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{37}$$

$$P_i \leq \mu P_j, \quad \forall i \in \mathcal{N}, \tag{38}$$

$$\begin{bmatrix} -Y_i & Y_iR \\ * & -R \end{bmatrix} < 0, \qquad \begin{bmatrix} -\kappa R & I \\ * & -Y_i \end{bmatrix} < 0, \quad \forall i \in \mathcal{N}, \tag{39}$$

$$\frac{c_2}{(1+\alpha)^N \kappa c_1 + \gamma^2 d} > 1, \tag{40}$$

hold, and the average dwell time of the switched discrete-time signal $\sigma(k)$ *satisfies*

$$\tau_a > \tau_a^* = \max\left\{\frac{N \ln \mu}{\ln c_2 - \ln(\gamma^2 d)}, \frac{\ln \mu}{\alpha}\right\}. \tag{41}$$

Then the set of state feedback controllers is given by

$$X_i = K_i P_i^{-1}, \quad \forall i, j \in \mathcal{N}. \tag{42}$$

Proof Pre- and post-multiplying (24) with $\operatorname{diag}\{P_i^{-1}, I, P_i^{-1}, I\}$ and $\operatorname{diag}\{P_i^{-1}, I, P_j^{-1}, I\}$, respectively, then (24) is transformed into

$$\begin{bmatrix} -(1+\alpha)P_i^{-1} & 0 & P_i^{-1}A_i^\intercal + P_i^{-1}K_i^\intercal B_i^\intercal & P_i^{-1}L_i^\intercal + P_i^{-1}K_i^\intercal D_i^\intercal \\ * & -\frac{\gamma^2 I}{(1+\alpha)^N} & C_i^\intercal & 0 \\ * & * & -P_j^{-1} & 0 \\ * & * & * & -I \end{bmatrix} < 0. \tag{43}$$

Denote

$$Y_i = P_i^{-1}, \qquad Z_j = P_j^{-1}, \qquad X_i = K_i P_i^{-1}, \quad \forall i, j \in \mathcal{N}.$$

Therefore, we can obtain (37). The proof is completed. □

Remark 2 In our paper, finite-time stability and Lyapunov asymptotic stability are independent concepts: a system which is finite-time stable maybe not Lyapunov asymptotically stable. On the contrary, a Lyapunov asymptotically stable system could be not finite-time stable, and during the transients, its state exceeds the prescribed bounds.

Remark 3 In many actual applications, the minimum value of $\gamma_{\min}^2$ is of interest. In Theorem 4.1, as for finite-time stability and boundness, once the state bound c_2 is not ascertained, the minimum value $c_{2\min}$ is of interest. With fixed α and μ, defining $\lambda_1 = 1$, $\lambda_2 = \kappa$, we then can formulate the following optimization problem to get the minimum value $c_{2\min}$:

$$\begin{aligned} &\min c_2 \\ &\text{s.t. (26), (27) and (28).} \end{aligned}$$

Therefore, the optimal value of $\rho(\theta)$ can be derived through the convex combination of $\gamma_{\min}^2$ and $c_{2\min}$, *i.e.*, denote $0 \le \vartheta \le 1$, $\rho(\vartheta) = \vartheta \gamma_{\min}^2 + (1-\vartheta)c_{2\min}$, which can be obtained through

$$\begin{aligned} &\min \rho(\vartheta) \\ &\text{s.t. (26), (27) and (28).} \end{aligned}$$

The optimized controller gains $X_i = K_i P_i^{-1}$, $\forall i,j \in \mathcal{N}$ can be derived by optimization procedure $\min_{\kappa \geq 1} \kappa$ subject to (39) and (40) with fixed γ and minimum c_2.

Remark 4 In this paper, if we can find a feasible solution with the parameter $\mu = 0$, through the discussion above, we know that the designed controller can ensure both finite-time and asymptotical stability of the delayed switched system. While in most situations we obtain controller with $\mu > 0$, and only finite-time stability can be established. Therefore, in real applications, asymptotical stabilizing controller for each subsystem should be designed to ensure asymptotical stability, which can be easily obtained by existing results for a non-switched system.

6 Illustrative example

Example 1 Consider a finite-time stabilization of the switched system as follows:

$$A_1 = \begin{bmatrix} 1.499 & -1.019 & -0.601 \\ 0.138 & -1.385 & -1.172 \\ -1.587 & 0.955 & -0.577 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.836 & 0.302 & -0.949 \\ 0.853 & 0.416 & 0.542 \\ 0.477 & 0.043 & -0.821 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -1.072 & 0.981 & 0.252 \\ -1.074 & -1.759 & -0.610 \\ 0.870 & -0.148 & 0.761 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.254 & 0.677 & -0.240 \\ -0.153 & 0.849 & 0.921 \\ -0.557 & 0.469 & -0.907 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.144 & -0.844 & 0.442 \\ 1.396 & 1.943 & -0.804 \\ -0.190 & -0.247 & 0.288 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.467 & 1.205 & -1.991 \\ 0.005 & -1.404 & 0.028 \\ 0.326 & 0.206 & 1.931 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.941 & 0.528 & 1.615 \\ -0.007 & 1.057 & -0.816 \\ -0.338 & 0.015 & -1.998 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -0.748 & -0.847 & -0.492 \\ -0.199 & 1.337 & -0.969 \\ -0.373 & -1.616 & -0.474 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.666 & -0.821 & -1.855 \\ 0.478 & 0.101 & 1.800 \\ -1.041 & 1.084 & -0.878 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0.6477 & -0.517 & -0.606 \\ 1.050 & 0.024 & 0.272 \\ 1.323 & -0.162 & 0.359 \end{bmatrix}.$$

In this paper, disturbance $\sum_{k=0}^{\infty} \omega^{\mathsf{T}}(k)\omega(k) < 1$. The control objective is to find a feedback controller ensuring system (5) is finite-time bounded with respect to (c_1, c_2, d, R, N) and minimum value of $\gamma_{\min}^2 + c_{2\min}$. Choose $c_1 = 2$, $d = 8$ and $\alpha = 3.35$. According to Theorem 5.1, the optimal value of $\gamma_{\min}^2 + c_{2\min}$ depends on parameter μ. Through LMI then we see that the feasible solution is $\gamma = 1.72$, $c_2 = 61.512$ and $\mu = 3.75$.

The state feedback controllers are given as

$$K_1 = \begin{bmatrix} 23.366 & 119.322 & -109.468 \\ -454.469 & 333.654 & 406.088 \\ 478.929 & -296.609 & -78.267 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 48.041 & 155.125 & 690.152 \\ -467.431 & -18.232 & 38.023 \\ -900.225 & 92.460 & 291.209 \end{bmatrix}.$$

According to (28), for any switching signal $\sigma(k)$ with average dwell time $\tau_a > \tau_a^* = 2.768$, system (5) is finite-time stochastically bounded with respect to the above parameters. Figure 1 shows the switching signal $\sigma(k)$ with average dwell time $\tau_a = 2.8$.

Choose the initial condition $[-0.4, 0.3]$, then switched discrete-time system (5) is finite-time bounded. The state responses of a filtering error system is shown in Figure 1. It can be seen that the designed filter meets the specified requirement. The state trajectory of system (5) is shown in Figure 2, where the initial state $x(0) = [0.4 - 0.2]$. From Figure 2, it is easy to see that system (5) is finite-time bounded.

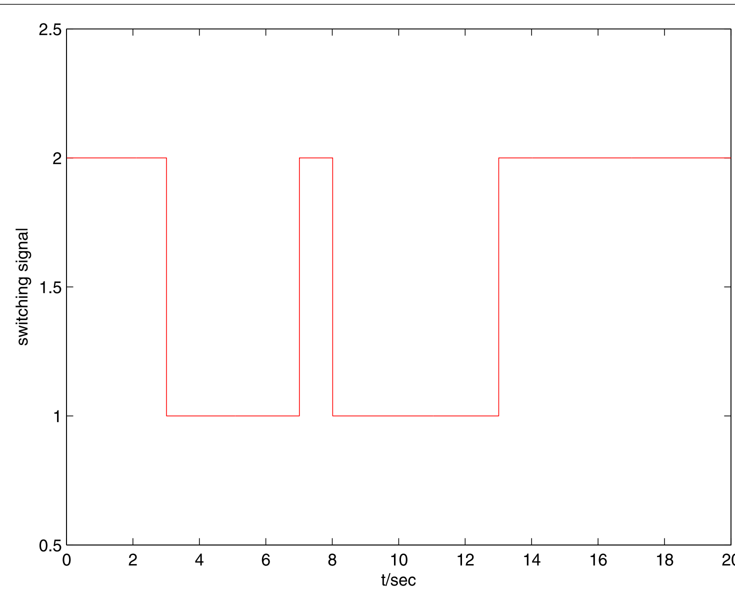

Figure 1 Switching signal $\sigma(k)$.

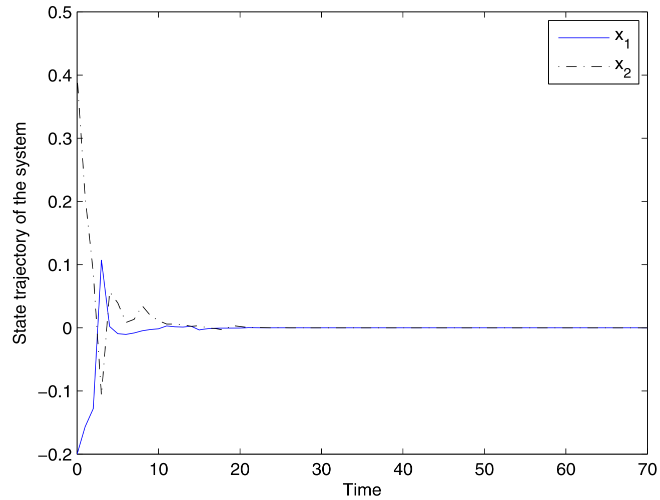

Figure 2 State trajectory of system (5).

7 Conclusions

In this paper, we have examined the problems of finite-time H_∞ control of a switched discrete-time system with average dwell time. Based on the analysis result, the static state feedback control of finite-time boundness is given. Although the derived result is not in an LMIs form, we can turn it into the LMIs feasibility problem by fixing some parameters. A numerical example has also been given to demonstrate the effectiveness of the proposed approach. It should be noted that one of future research topics would be to investigate the problems of synchronous or asynchronous estimation for the switched neural network under the dwell time over a finite-time horizon.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors have equal contributions.

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