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Impulsive stabilization of delay difference equations and its application in Nicholson's blowflies model

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Abstract

In this article, we consider the impulsive stabilization of delay difference equations. By employing the Lyapunov function and Razumikhin technique, we establish the criteria of exponential stability for impulsive delay difference equations. As an application, by using the results we obtained, we deal with the exponential stability of discrete impulsive delay Nicholson's blowflies model. At last, an example is given to illustrate the efficiency of our results.

Mathematics Subject Classification 2000: 39A30; 39A60; 39A10; 92B05.

Keywords: impulsive, difference equation, exponential stability, stabilization, Nicholson's blowflies model

Introduction

Discrete systems exist in the word widely and most of them are described by the difference equations. The properties of difference equations, especially the stability and stabilization, were studied by many researchers, see [1-6] and the references therein.

As well known, in the practice, many systems are subject to short-term disturbances, these disturbances are often described by impulses in the modeling process, therefore the impulsive systems arise in many scientific fields and there are many works were reported on impulsive systems [7-16]. In those works, the stability study for the impulsive system is one of the research focuses, see [11-16].

In the study of stability, the Lyapunov function and Razumikhin method were used by many authors, see, for example, [6,17]. In [6], the Razumikhin technique was extended to the discrete systems. Although the stability of impulsive delay difference equations has been studied in some articles, for example, see [18], there are few article concerning on impulsive stabilization of delay difference equations. From the article [19], we know that the continuity is crucial in the proof of the stabilization theorem under the continuous situation. However, under the discrete situation, there is no continuity to be utilized. The loss of continuity puts difficulties in the way to get the stabilization theorem. The main aim of this article is to establish the criteria of impulsive stabilization for delay difference equations, using the Lyapunov function and Razumikhin method.

Biological models were studied by many authors, see [20-25] and the references therein. The stability of the positive equilibrium is a hot topic to be studied. In this article, we also study the stabilization of an impulsive delay difference Nicholson's blowflies model. We take an unstable difference Nicholson's blowflies equation without impulses, then the impulsive effects are adopted and the criterion of stability is established for the impulsive Nicholson's blowflies model.

The rest of this article is organized as follows. In Section 2, we introduce our notations and definitions. Then in Section 3, we present a theorem of impulsive stabilization for delay difference equations. In Section 4, by using our result, we deal with the discrete impulsive delay Nicholson's blowflies equation. In Section 5, an example is given to illustrate the efficiency of our results.

Preliminaries

Let $\mathbb{R}$ denote the field of real numbers and $\mathbb{R}^n$ denote the n-dimensional Euclidean space. $\mathbb{N}$ and $\mathbb{Z}$ represent the natural numbers and the integer numbers respectively. For some positive integer m, $N_{-m} = \{-m, ..., -1, 0\}$. Given a positive integer m, for any function ϕ: $N_{-m} \to \mathbb{R}^n$, we define $\|\varphi\|_m = \max_{\theta \in N_{-m}}\{|\varphi(\theta)|\}$, where $|\cdot|$ presents the Euclidean norm.

We consider the following impulsive delay difference system:

$$\begin{cases} x(n+1) = f(n, x(n-m), x(n-m+1), \ldots, x(n)), n \neq \eta_k - 1, \\ x(\eta_k) = \beta_k x(\eta_k - 1), \end{cases} \tag{1}$$

where $x(n) \in \mathbb{R}^n$, $f: \mathbb{N} \times \underbrace{\mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{m+1} \to \mathbb{R}^n$. β_k is a constant for any $k \in \mathbb{N}$. The impulsive moments $\{\eta_k\}_1^\infty$ are natural numbers and satisfy $0 = \eta_0 < \eta_1 < \cdots < \eta_k < \cdots$, $\eta_k \to \infty$ as $k \to \infty$.

The following initial values are imposed on system (1):

$$x(s) = \varphi(s), s \in N_{-m}, \tag{2}$$

where ϕ: $[-m, 0] \to \mathbb{R}^n$ satisfies $\| \phi \|_m < \infty$.

We assume $f(n, 0, 0, ..., 0) \equiv 0$, then systems (1) admits the trivial solution. We also assume that for any initial values $x(s) = \phi(s)$, $s \in N_{-m}$, system (1) has a unique solution, denoted by $x(n, \phi)$.

Definition 1. [6] The trivial solution of (1) is said to be globally exponentially stable, if for any solution $x(n, \phi)$ with the initial data $x(n) = \phi(n)$, $n \in N_{-m}$, there exist constants $\gamma > 0$ and $M > 0$ such that

$$|x(n, \varphi)| \leq M\|\varphi\|_m e^{-\gamma n}, \ \forall n \in N_{-m} \cup \mathbb{N}. \tag{3}$$

Impulsive stabilization of delay difference equations

In this section, we present the stabilization theorem of impulsive delay difference equations. By using the Razumikhin technique, we obtain the sufficient conditions to guarantee the exponential stability of system (1). Moreover, another criterion of exponential stability for system (1) is given, which does not depend on the Lyapunov function but just depends on the system function f, impulsive moments $\{\eta_k\}$ and the impulsive gain $\{\beta_k\}$. Some techniques we used in the proof of the stabilization theorem are motivated by [19].

Theorem 2. *Assume there exist a positive function $V(n, x)$ and positive constants c_1, c_2, p, λ, α, $\alpha > 1$, such that*

C_1: $c_1|x|^p \le V(n, x) \le c_2 |x|^p$, *for all* $n \in N_{-m} \cup \mathbb{N}$ *and* $x \in \mathbb{R}^n$.

C_2: *If* $n \ne \eta_k - 1$, *for any function* $\phi: N_{-m} \cup \mathbb{N} \to \mathbb{R}^n$, *the following inequality holds*

$$V(n+1, f(n, \varphi)) \le (1+\lambda)V(n, \varphi(n))$$

whenever $qV(n+1, \phi(n+1)) \ge V(n+s, \phi(n+s))$ *for all* $s \in N_{-m}$, *where* $q \ge e^{2\lambda\alpha}$.

C_3: $V(\eta_k, \beta_k(\phi(\eta_k - 1))) \le d_k V(\eta_k - 1, \phi(\eta_k - 1))$, *where* $d_k > 0$.

C_4: $\eta_{k+1} - \eta_k \le \alpha$, $\ln d_k + \alpha\lambda < -\lambda(\eta_{k+1} - \eta_k)$.

Then, for any initial data $x(n) = \phi(n)$, $n \in N_{-m}$, *there exists a positive constant* C, *such that*

$$|x(n, \varphi)| \le C\|\varphi\|_m e^{-\frac{\lambda}{p}n},$$

that is, the trivial solution of system (1) *is exponentially stable.*

Proof. For the sake of simplicity, we write $V(n) = V(n, x(n))$.

Choose $M > 1$, such that

$$(1+\lambda)c_2 \|\varphi\|_m^p \le M\|\varphi\|_m^p e^{-\lambda\eta_1}e^{-\alpha\lambda} < M\|\varphi\|_m^p e^{-\lambda\eta_1} \le qc_2\|\varphi\|_m^p. \tag{4}$$

We claim that for any $n \in [\eta_k, \eta_{k+1})$, $k \in \mathbb{N}$,

$$V(n) \le M\|\varphi\|_m^p e^{-\lambda\eta_{k+1}}. \tag{5}$$

First, we will show, when $n \in [0, \eta_1)$,

$$V(n) \le M\|\varphi\|_m^p e^{-\lambda\eta_1}. \tag{6}$$

Obviously, when $n \in N_{-m}$, $V(n) \le M\|\varphi\|_m^p e^{-\lambda\eta_1}$.

If (6) is not true, then there must be an $\bar{n} \in [0, \eta_1 - 1)$ and an $n^* \ge 0$ such that

$$V(\bar{n}+1) > M\|\varphi\|_m^p e^{-\lambda\eta_1}, \quad V(n) \le M\|\varphi\|_m^p e^{-\lambda\eta_1}, \quad n \le \bar{n},$$

and

$$V(n^*) \le c_2\|\varphi\|_m^p, \quad c_2\|\varphi\|_m^p < V(n) \le M\|\varphi\|_m^p e^{-\lambda\eta_1}, \quad n^* < n \le \bar{n}. \tag{7}$$

It should be pointed out there may be a case $n^* = \bar{n}$, that is, there no n satisfies the second segment of (7). If it is true, then for any $n \le \bar{n}$, we have

$$V(n) \le c_2\|\varphi\|_m^p. \tag{8}$$

Obviously, for any $s \in N_{-m}$,

$$qV(\bar{n}+1) > qM\|\varphi\|_m^p e^{-\lambda\eta_1} > qc_2\|\varphi\|_m^p \ge V(\bar{n}+s).$$

From C_2 we get

$$V(\bar{n}+1) \le (1+\lambda)V(\bar{n}),$$

that is

$$\begin{aligned} V(\bar{n}) &\ge \frac{1}{1+\lambda}V(\bar{n}+1) > \frac{1}{1+\lambda}M\|\varphi\|_m^p e^{-\lambda\eta_1} \\ &= \frac{e^{\alpha\lambda}}{1+\lambda}M\|\varphi\|_m^p e^{-\lambda\eta_1}e^{-\alpha\lambda} \\ &> M\|\varphi\|_m^p e^{-\lambda\eta_1}e^{-\alpha\lambda} \ge c_2\|\varphi\|_m^p, \end{aligned}$$

which contradicts with (8), then there must be an n such that the second segment of (7) holds.

When $n \in [n^* + 1, \bar{n}]$, from (7),

$$V(n+s) \le M \|\varphi\|_m^p e^{-\lambda\eta_1} < qc_2 \|\varphi\|_m^p < qV(n).$$

By virtue of condition C_2, when $n \in [n^* + 1, \bar{n}]$,

$$V(n) \le (1+\lambda)V(n-1). \tag{9}$$

From the definitions of $\bar{n}$ and n^*, we have $V(\bar{n}+1) \ge V(\bar{n}+s)$ and $V(n^* + 1) \ge V(n^* + s)$, then we get

$$qV(\bar{n}+1) \ge V(\bar{n}+s), \quad s \in N_{-m},$$

and

$$qV(n^*+1) \ge V(n^*+s), \quad s \in N_{-m}.$$

Using condition C_2 and inequality (9), we obtain

$$\begin{aligned} V(\bar{n}+1) &\le (1+\lambda)V(\bar{n}) \le (1+\lambda)^{\bar{n}-n^*} V(n^*+1) \\ &\le (1+\lambda)^{\alpha} V(n^*) < e^{\alpha\lambda} c_2 \|\varphi\|_m^p. \end{aligned}$$

Since $V(\bar{n}+1) > M \|\varphi\|_m^p e^{-\lambda\eta_1}$, we get

$$M \|\varphi\|_m^p e^{-\lambda\eta_1} < e^{\alpha\lambda} c_2 \|\varphi\|_m^p,$$

which is in contradiction with (4), then (6) holds, that is (5) holds for $k = 1$.

Now we assume (5) holds for $k = 1, 2, ..., h - 1$, i.e. when $n \in [\eta_{k-1}, \eta_k)$, $k = 1, 2, ..., h$,

$$V(n) \le M \|\varphi\|_m^p e^{-\lambda\eta_k}. \tag{10}$$

From condition C_3 and condition C_4,

$$\begin{aligned} V(\eta_h) &\le d_h V(\eta_h - 1) \le d_h M \|\varphi\|_m^p e^{-\lambda\eta_h} \\ &\le M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda} \le M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}}. \end{aligned} \tag{11}$$

Now we will show, when $n \in [\eta_h, \eta_{h+1})$,

$$V(n) \le M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}}. \tag{12}$$

If (12) doesn't hold, there must be an $\bar{n} \in (\eta_h, \eta_{h+1} - 1)$ and an $n^* \in [\eta_h, \bar{n}]$, such that

$$V(\bar{n}+1) > M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}}, \quad V(n) \le M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}}, \quad n \in [\eta_h, \bar{n}],$$

and

$$V(n^*) \le M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda}, \quad V(n) > M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda}, \quad n^* < n \le \bar{n}. \tag{13}$$

Now we claim $n^* < \bar{n}$. If it is not true, then $n^* = \bar{n}$. Since $qV(\bar{n}+1) \ge V(\bar{n}+s)$, $s \in N_{-m}$, from condition C_2, we get $V(\bar{n}+1) \le (1+\lambda)V(\bar{n})$, that is

$$V(n^*) = V(\bar{n}) \geq \frac{1}{1+\lambda} V(\bar{n}+1) \geq \frac{e^{\lambda\alpha}}{1+\lambda} M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda} > M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda},$$

which is in conflict with (13).

For $n \in [n^*+1, \bar{n}]$ and $s \in N_{-m}$,

$$V(n+s) \leq M \|\varphi\|_m^p e^{-\lambda\eta_h} = e^{\lambda(\eta_{h+1}-\eta_h)} M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} \leq e^{2\lambda\alpha} M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda} \leq qV(n).$$

Using condition C_2, we have

$$V(n) \leq (1+\lambda)V(n-1), \quad n \in [n^*+1, \bar{n}],$$

and, obviously,

$$qV(\bar{n}+1) \geq V(\bar{n}),$$

then by virtue of condition C_2, we obtain

$$V(\bar{n}+1) \leq (1+\lambda)V(\bar{n}). \tag{14}$$

Using the definition of $V(n^*)$, we can easily get

$$qV(n^*+1) > V(n^*+s), \ s \in N_{-m}.$$

Then, by virtue of condition C_2 we have

$$V(n^*+1) \leq (1+\lambda)V(n^*). \tag{15}$$

Consequently,

$$\begin{aligned} V(\bar{n}+1) &\leq (1+\lambda)V(\bar{n}) \leq (1+\lambda)^{\bar{n}-n^*} V(n^*+1) \\ &\leq (1+\lambda)^{\bar{n}-n^*+1} V(n^*) \leq (1+\lambda)^{\alpha} V(n^*) \\ &< e^{\alpha\lambda} M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} e^{-\alpha\lambda} \\ &= M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} < V(\bar{n}+1), \end{aligned}$$

which is a contradiction. Then (5) holds for $k = h + 1$.

By induction, we know (5) holds for any $n \in [\eta_k, \eta_{k+1})$, $k \in \mathbb{N}$.

From condition C_1, for any $n \in [\eta_k, \eta_{k+1})$, $k \in \mathbb{N}$

$$c_1 |x(n, \varphi)|^p \leq V(n) \leq M \|\varphi\|_m^p e^{-\lambda\eta_{h+1}} \leq M \|\varphi\|_m^p e^{-\lambda n},$$

that is

$$|x(n, \varphi)| \leq \left(\frac{M}{c_1}\right)^{1/p} \|\varphi\|_m e^{-\frac{\lambda}{p} n},$$

which is the assertion. □

Now we are on the position to state a corollary, which is another criterion of exponential stability for system (1). This criterion does not dependent on the Lyapunov function but just dependents on the system function, impulsive moments and impulsive gain.

Corollary 3. *Assume that system* (1) *satisfies*

(1) for any $n \in \mathbb{N}$*, there exist positive constants* $u(n)$ *and* $a_j(n)$, $j = 0, 1, ..., m$*, such that*

$$|f(n, x(n-m), x(n-m+1), \ldots, x(n))| \le u(n)|x(n)| + \sum_{j=0}^{m} a_j(n)|x(n-j)|$$

and $\mu_0 = \sup_{n\in\mathbb{N}}\{u(n)\}$, $\mu = \sup_{n\in\mathbb{N}}\left\{\sum_{j=0}^{m} a_j(n)\right\}$ *are finite numbers.*

(2) there exist positive constant λ*, integer* $\alpha > 1$ *and constant* q*, satisfying* $q \ge e^{2\lambda\alpha}$*, such that* $\mu q(\mu_0 + \mu) < 1$ *and*

$$0 < \frac{\mu_0^2 + \mu_0\mu}{1 - q\mu(\mu_0 + \mu)} - 1 \le \lambda.$$

(3) $\eta_{k+1} - \eta_k \le \alpha$ *and* $\ln d_k + \lambda(\eta_{k+1} - \eta_k) \le -\lambda\alpha$ *where* $d_k = \beta_k^2$, $k \in \mathbb{N}$.

Then, for any initial data $\phi(s)$, $s \in N_{-m}$*, the solution* $x(n, \phi)$ *of system* (1) *satisfies*

$$|x(n, \varphi)| \le \|\varphi\|_m e^{-\frac{\lambda}{2}n},$$

that is, the trivial solution of (1) *is globally exponentially stable.*

Proof. Let $c_1 = c_2 = 1$, $p = 2$, $V(n) = |x(n)|^2$ in Theorem 2. Under this situation, it is sufficient to verify the condition C_2 of Theorem 2. Using condition (1), Hölder inequality and the assumption $|x_{n+j}|^2 \le q|x_{n+1}|^2$, for $j \in N_{-m}$, if $n \ne \eta_k - 1$, we can obtain

$$\begin{aligned}
|x(n+1)|^2 &= |f(n, x(n-m), x(n-m+1), \ldots, x(n))|^2 \\
&\le \left(u(n)|x(n)| + \sum_{j=0}^{m} a_j(n)|x(n-j)|\right)^2 \\
&= u^2(n)|x(n)|^2 + 2u(n)|x(n)|\left(\sum_{j=0}^{m} a_j(n)|x(n-j)|\right) + \left(\sum_{j=0}^{m} a_j(n)|x(n-j)|\right)^2 \\
&\le u^2(n)|x(n)|^2 + u(n)\sum_{j=0}^{m} a_j(n)(|x(n-j)|^2 + |x(n)|^2) \\
&\quad + \left(\sum_{j=0}^{m} (a_j(n))^{\frac{1}{2}}(a_j(n))^{\frac{1}{2}}|x(n-j)|\right)^2 \\
&\le u^2(n)|x(n)|^2 + u(n)\sum_{j=0}^{m} a_j(n)(q|x(n+1)|^2 + |x(n)|^2) \\
&\quad + \left(\sum_{j=0}^{m} a_j(n)\right)\left(\sum_{j=0}^{m} a_j(n)q|x(n+1)|^2\right) \\
&\le u(n)\left(u(n) + \sum_{j=0}^{m} a_j(n)\right)|x(n)|^2 + q\left(\sum_{j=0}^{m} a_j(n)\right)(u(n) + a(n))|x(n+1)|^2 \\
&\le \mu_0(\mu_0 + \mu)|x(n)|^2 + q\mu(\mu_0 + \mu)|x(n+1)|^2.
\end{aligned}$$

From condition (2) we have $q\mu(\mu_0 + \mu) < 1$, this yields

$$|x(n+1)|^2 - |x(n)|^2 \leq \left(\frac{(\mu_0 + \mu)\mu_0}{1 - q\mu(\mu_0 + \mu)} - 1 \right) |x(n)|^2 \leq \lambda |x(n)|^2.$$

That is,

$$V(n+1) - V(n) \leq \lambda V(n).$$

This completes the proof. □

Application to discrete impulsive delay Nicholson's blowflies model

Consider the discrete Nicholson's blowflies model with delay (see [24,25]):

$$x(n+1) - x(n) = -cx(n) + ax(n-m)e^{-bx(n-m)}, \quad n = 0, 1, \ldots, \tag{16}$$

where $c \in (0, 1)$, $a, b \in (0; +\infty)$ and $m \in \mathbb{N}$, together with the initial values

$$x(n) = \varphi(n), \quad n \in N_{-m},$$

where $\phi(n) > 0$, $n \in N_{-m}$.

In view of the application of system (16) in practice, we only take an interest in the positive value of (16). When $c < a$, there is a unique positive equilibrium

$$u^* = \frac{1}{b} \ln \frac{a}{c}.$$

In [24,25], the authors studied the fold bifurcation and Neimark-Sacker bifurcation. For the convenience, we present the result in [25] as follows:

Lemma 4. *Suppose that $c < a$ is satisfied and denotes*

$$a^* = c \exp \left(1 + \frac{((1-c)^2 + 1 - 2(1-c)\cos\theta)^{\frac{1}{2}}}{c} \right),$$

where θ is the solution of $\frac{\sin(m\theta)}{\sin((m+1)\theta)} = \frac{1}{c}$, and $\theta \in (0, \frac{\pi}{m+1})$,

(1) If $a < a^*$, then u^* is asymptotically stable.

(2) If $a > a^*$, then u^* is unstable.

Here, we assume that $a > a^*$ and consider a discrete impulsive Nicholson's blowflies model with delay:

$$\begin{cases} x(n+1) - x(n) = -cx(n) + ax(n-m)e^{-bx(n-m)}, & n \neq \eta_k - 1, \\ x(\eta_k) = u^* + \beta_k(x(\eta_k - 1) - u^*), \\ x(n) = \varphi(n), \quad n \in N_{-m} \end{cases} \tag{17}$$

where $\beta_k \in \mathbb{R}$, η_k, $k = 1, 2, \ldots$, are the instances of impulse effect, satisfying $0 < \eta_1 < \eta_2 < \ldots < \eta_k < \ldots$, and $\eta_k \to \infty$ as $k \to +\infty$. We suppose there exists a positive constant α such that $\eta_{k+1} - \eta_k \leq \alpha$.

Substituting $y_n = x_n - u^*$ into (17) yields

$$\begin{cases} y(n+1) = (1-c)y(n) + c(y(n-m) + u^*)e^{-by(n-m)} - cu^*, \; n \neq \eta_k - 1, \\ y(\eta_k) = \beta_k y(\eta_k - 1), \\ y(n) = \varphi(n) - u^*, & n \in N_{-m}. \end{cases} \tag{18}$$

Definition 5. We call the equilibrium u^* of system (17) is exponentially stable, if the trivial solution of system (18) is exponentially stable.

It is easy to get that $(-u^*, +\infty)$ is an invariant set of system (18). For $\{y(n)\} \subset (-u^*, +\infty)$,

$$\begin{aligned} &|f(n, y(n), \dots, y(n-m))| \\ &\quad = |(1-c)y(n) + c(y(n-m)+u^*)e^{-by(n-m)} - cu^*| \\ &\quad \le (1-c)|y(n)| + c|(y(n-m)+u^*)e^{-by(n-m)} - cu^*| \\ &\quad = (1-c)|y(n)| + c|e^{-b\xi}(1-b(\xi+u))||y(n-m)| \\ &\quad \le (1-c)|y(n)| + ce^{bu^*}|y(n-m)| \\ &\quad = (1-c)|y(n)| + a|y(n-m)|, \end{aligned} \tag{19}$$

Where $\xi \in (-u^*, y(n-m)]$.

By using Corollary 3, inequality (19) and noting $\eta_{k+1} - \eta_k \le \alpha$, we can get the following corollary:

Corollary 6. *Assume there exist constants* $\lambda > 0$, *integer* $\alpha > 1$ *and* $q \ge e^{2\lambda\alpha}$, *such that the following inequalities hold*

(1) $aq(1 - c + a) < 1$ *and* $0 < \frac{(1-c)^2 + a(1-c)}{1-aq(1-c+a)} - 1 \le \lambda$.

(2) $\ln \beta_k^2 + \lambda(\eta_{k+1} - \eta_k) \le -\lambda\alpha$.

Then, the positive equilibrium u^* *of* (17) *is exponentially stable.*

Corollary 7. *Suppose that* $0 < a(1 - c + a) < 1$ *in system* (17). *Given a positive constant* λ *and an integer* $\alpha > 1$ *satisfying* $\lambda < -\frac{1}{2\alpha}\ln(a(1-c+a))$, $\eta_{k+1} - \eta_k < \alpha$, $k = 1, 2, \dots$, *and*

$$0 < \frac{(1-c)^2 + a(1-c)}{1 - ae^{2\alpha\lambda}(1-c+a)} - 1 \le \lambda.$$

If there exist constants $\{\beta_k\}_{k=1}^{\infty}$, *such that*

$$\ln \beta_k^2 < -2\lambda\alpha,$$

then, the positive equilibrium u^* *of* (17) *is exponentially stable.*

Proof. Taking $q = e^{2\lambda\alpha}$, noting $\eta_{k+1} - \eta_k \le \alpha$ and by virtue of Corollary 6, we get the assertion directly. □

Remark 8. Corollary 7 tells us, for any positive constant λ satisfying $\lambda < -\frac{1}{2\alpha}\ln(a(1-c+a))$, we can take an impulsive strategy $\{\eta_k\}_{k=1}^{\infty}$ and $\{\beta_k\}_{k=1}^{\infty}$, such that the equilibrium u^* is exponentially stable, the exponential rate is less than $-\frac{\lambda}{2}$.

Numerical experiments

We take $a = 0.03$, $b = 0.5$, $c = 0.001$, $m = 1000$ in the system of (16), the equilibrium of Equation (16) is $u^* = 6.8024$ and it is unstable [24,25] (see Figure 1), where the initial values are $\phi(n) \equiv 6$.

We adopt the impulsive control as follows:

Choose $\eta_k = 3k$, and then choose $\beta_k = e^{-0.3}$, take $\lambda = 0.1$ and $q = 2$.

The conditions of Corollary 6 are satisfied, then the positive equilibrium point of (16) is exponentially stable (see Figure 2), where the initial values are also $\phi(n) \equiv 6$.

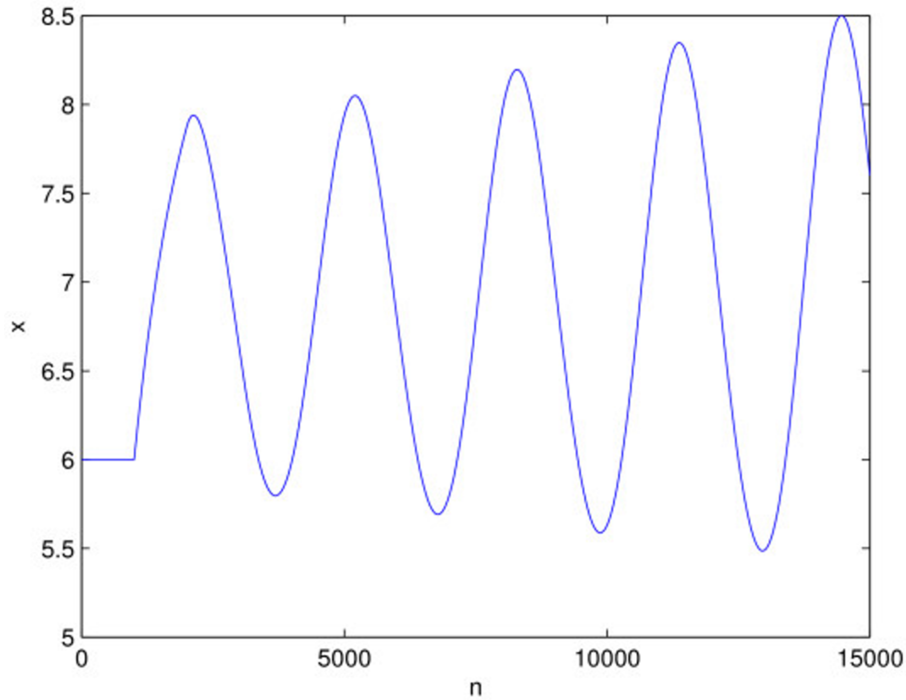

Figure 1 Instability of the equilibrium $u^* = 6.8024$ **(no impulses)**.

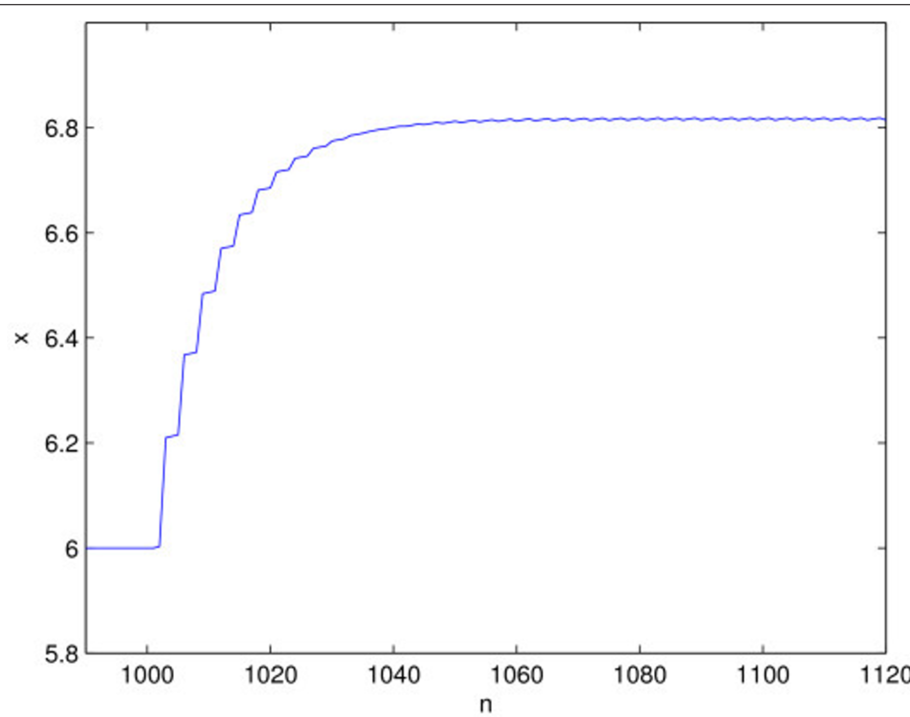

Figure 2 Stability of the equilibrium $u^* = 6.8024$ **(with impulsive effect)**.

Conclusion

In this article, we established some global exponential stability criteria for impulsive delay difference systems by employing the Lyapunov function and Razumikhin technique. Using our result, we dealt with the discrete impulsive Nicholson's blowflies model. We obtained the sufficient conditions of exponential stability for the positive equilibrium of this model. At last, we presented an example to illustrated the e ciency of our results.

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Authors' contributions
Both authors contributed equally to the manuscript.

Competing interests
The authors declare that they have no competing interests.

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