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Output feedback stabilization of nonlinear discrete-time systems with time-delay

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Abstract

This article considers both the static output feedback stabilization issue and output-feedback guaranteed cost controller design of a class of discrete-time nonlinear systems with time-delay. First, by static output feedback controller, the new sufficient conditions for static output feedback stabilization of a class of discrete-time nonlinear systems with time-delay are presented. Then, we establish the new delay-independent sufficient conditions for existence of the guaranteed cost control by static output-feedback controller in terms of matrix inequalities. Finally, two examples are given to show the effectiveness of our proposed approaches.

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1. Introduction

Time delay exists commonly in dynamic systems due to measurement, transmission and transport lags, computational delays, or unmodeled inertia of system components, which has been generally regarded as a main source of instability and poor performance. Therefore, considerable attention has been devoted to the problem of analysis and synthesis for time-delayed systems, and many research results have been reported in the literature. To mention a few, the stability analysis result is reported in [1,2], the stabilization problem for switched nonlinear time-delay systems is solved in [3], and the model filtering problems are solved in [4], and the design problem of a hybrid output feedback controller is also considered in [5].

The static output feedback problem for linear and nonlinear systems is an important problem not yet completely solved and continuously investigated by many people. In practice, it is not always possible to have full access to the state vector and only the partial information through a measured output is available. Introducing all of the results is not easy because there exist various unconnected approaches. However, among the proposed results, we can distinguish stability conditions expressed in terms of linear matrix inequalities for discrete-time switched linear systems with average dwell time [6], constructive approaches based on the resolution of Riccati equations [7], linear matrix inequality approach to static output-feedback stabilization of discrete-time networked control systems [8] or optimization techniques [9,10], pole or eigenstructure assignment techniques [11,12].

Recently, much effort has been directed towards finding a feedback controller in order to guarantee robust stability, see [13,14]. On the other hand, when controlling a real plant, it is also desirable to design a control systems which is not only asymptotically stable, but also guarantees an adequate level of performance index. One way to address the robust performance problem is to consider a linear quadratic cost function. This approach is the so-called guaranteed cost control [15,16]. In recent years, with the development of robust control theory and H_∞ control theory, the robust guaranteed cost control approach to the design of state feedback control laws for uncertain systems has been a subject of intensive research [17]. Quadratic guaranteed cost control for linear systems with norm-bounded uncertainty was dealt with in [18]. However, all this research has been done on uncertain system without time delay or continuous-time delay systems. Little attention has been paid towards discrete-time systems with delay.

This article is concerned with both the static output feedback stabilization and output-feedback guaranteed cost controller design for a class of discrete-time nonlinear systems with time-delay. The new sufficient conditions for static output feedback stabilization of a class of discrete-time nonlinear systems with time-delay are presented. Then, sufficient LMI conditions for guaranteed cost control by static output feedback are given. Finally, examples are given to show the effectiveness of our proposed approaches.

The rest of the article is organized as follows. In Section 2, we present the main results concerning the static output feedback stabilization problem for a class of nonlinear discrete-time systems with time-delay. In Section 3, we deal with the problem of guaranteed cost control via static output feedback for a class of nonlinear discrete-time systems with time-delay. Two numerical examples are given in Section 4 to illustrate the proposed results. Finally, we draw some conclusions in Section 5.

The following notations will be used throughout this article. R is the set of all real numbers. Z_+ is the set of all non-negative integers. Z^+ is the set of all positive integers. R^n denotes the n-dimensional Euclidean space. $R^{n\times m}$ is the set of all $(n\times m)$-dimensional real matrices. I denotes an identity matrix with appropriate dimension. The superscript 'T' represents the matrix transposition. If a matrix is invertible, the superscript '-1' represents the matrix inverse. $X > 0(X \geq 0)$ means that X is a real symmetric and positive-definite (semi-definite) matrix. The notation $||\cdot||$ refers to the Euclidean norm. For an arbitrary matrix B and two symmetric matrices A and C, the symmetric term in a symmetric matrix is denoted by an asterisk, i.e., $\begin{bmatrix} A & B \\ * & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$.

2. Static output feedback

Consider the following nonlinear discrete-time systems with time-delay described by

$$\begin{aligned} x_{k+1} &= Ax_k + A_d x_{k-d} + Bu_k + f(x_k) + g(x_{k-d}), \\ x_k &= \phi_k, \quad -d \leq k \leq 0, \\ y_k &= Cx_k + C_d x_{k-d}, \end{aligned} \tag{2.1}$$

where $x_k \in R^n$ is the system state; $u_k \in R^m$ is the input, $y_k \in R^p$ is the measured output; A, A_d, C, and C_d are known real matrices with appropriate dimensions. d is a positive integer; $f = f(x_k): R^n \to R^n$ and $g = g(x_{k-d}): R^n \to R^n$ are nonlinear functions satisfying $f(0) = 0$, $g(0) = 0$ and

$$\frac{\partial f(x_k)}{\partial x_k} \in \mathrm{Co}\left\{F_1, F_2, \ldots, F_{v_1}\right\}, \quad \frac{\partial g(x_{k-d})}{\partial x_{k-d}} \in \mathrm{Co}\left\{T_1, T_2, \ldots, T_{v_2}\right\}, \tag{2.2}$$

where the symbol "Co" stands for the convex hull, $(F_i)_{1\leq i\leq v_1}$ and $(T_i)_{1\leq i\leq v_2}$ are the associated convex hull matrices. We say that the Jacobian $\frac{\partial f(x_k)}{\partial x_k}$ belongs to a convex polytopic set defined as

$$\Xi_1 = \left\{ \Xi_1(\beta_1) = \sum_{i=1}^{v_1} \beta_{1i} F_i, \ \sum_{i=1}^{v_1} \beta_{1i} = 1, \beta_{1i} \geq 0 \right\}, \tag{2.3}$$

and $\frac{\partial g(x_{k-d})}{\partial x_{k-d}}$ belongs to a convex polytopic set defined as

$$\Xi_2 = \left\{ \Xi_2(\beta_2) = \sum_{i=1}^{v_2} \beta_{2i} T_i, \ \sum_{i=1}^{v_2} \beta_{2i} = 1, \beta_{2i} \geq 0 \right\}. \tag{2.4}$$

Our first objective is to give sufficient linear matrix inequality conditions for stabilization of system (2.1) by a static output controller u_k=Ky_k We introduce the following key lemmas which will be used in setting the proofs of the next statements.

Lemma 2.1 [19]. Given the matrices *X, Y*, and *Z* of appropriate dimensions where $X=X^{\mathrm{T}}>0$, and $Z=Z^{\mathrm{T}}>0$, then the following linear matrix inequality holds:

$$\begin{pmatrix} -X & Y^{\mathrm{T}} \\ Y & -Z^{-1} \end{pmatrix} < 0,$$

if there exists a positive constant α such that

$$\begin{pmatrix} -X & \alpha Y^{\mathrm{T}} & 0 \\ \alpha Y & -2\alpha I & Z \\ 0 & Z & -Z \end{pmatrix} < 0.$$

Lemma 2.2 [20]. For any pair of symmetric positive definite constant matrix $G\in R^{n\times n}$ and scalar $r>0$, if there exists a vector function $\phi[0,r]\rightarrow R^n$ such that integrals $\int_0^r \varphi^{\mathrm{T}}(s)G\varphi(s)ds$ and $\int_0^r \varphi(s)ds$ are well definite, then the following inequality holds:

$$r\int_0^r \varphi^{\mathrm{T}}(s)G\varphi(s)ds \geq \left(\int_0^r \varphi(s)ds\right)^{\mathrm{T}} G\left(\int_0^r \varphi(s)ds\right).$$

Theorem 2.3. System (2.1) satisfying (2.3) and (2.4) is globally asymptotically stable under the action of the static output feedback $u_k = \left(\tilde{K}/\alpha\right) y_k$ provided that there exist a scalar $\alpha>0$, and a real matrix $\tilde{K}$, and positive definite matrices $P > 0$ and $Q > 0$ such that the following linear matrix inequalities hold:

$$\begin{aligned} &\begin{pmatrix} Q-P & 0 & \alpha(A^{\mathrm{T}}+2F_i^{\mathrm{T}})+C^{\mathrm{T}}\tilde{K}^{\mathrm{T}}B^{\mathrm{T}} & 0 \\ 0 & -Q & \alpha A_d^{\mathrm{T}}+C_d^{\mathrm{T}}\tilde{K}^{\mathrm{T}}B^{\mathrm{T}} & 0 \\ \alpha(A+2F_i)+B\tilde{K}C & \alpha A_d+B\tilde{K}C_d & -2\alpha I & P \\ 0 & 0 & P & -P \end{pmatrix} < 0, \quad 1\leq i\leq v_1, \\ &\begin{pmatrix} Q-P & 0 & \alpha A^{\mathrm{T}}+C^{\mathrm{T}}\tilde{K}^{\mathrm{T}}B^{\mathrm{T}} & 0 \\ 0 & -Q & \alpha(A_d^{\mathrm{T}}+2T_i^{\mathrm{T}})+C_d^{\mathrm{T}}\tilde{K}^{\mathrm{T}}B^{\mathrm{T}} & 0 \\ \alpha A+B\tilde{K}C & \alpha(A_d+2T_i)+B\tilde{K}C_d & -2\alpha I & P \\ 0 & 0 & P & -P \end{pmatrix} < 0, \quad 1\leq i\leq v_2, \end{aligned} \tag{2.5}$$

where ν_1 and ν_2 are the numbers of the convex hull matrices of the Jacobian of $f(x_k)$ and $g(x_k)$, respectively.

Proof. Consider the discrete-time nonlinear-time system (2.1) under the action of the feedback $u_k = Ky_k$. Using the mean-value theorem, the closed-loop dynamics can be written as

$$x_{k+1} = (A+BKC)x_k+(A_d+BKC_d)x_{k-d}+\int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} x_k ds+\int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} x_{k-d} ds. \tag{2.6}$$

Choosing the Lyapunov-Krasovskii functional candidate as

$$V_k = x_k^{\mathrm{T}} P x_k + \sum_{i=1}^{d} (x_{k-i}^{\mathrm{T}} Q x_{k-i}), \tag{2.7}$$

we have

$$\begin{aligned}
V_{k+1} - V_k &= x_{k+1}^{\mathrm{T}} P x_{k+1} + \sum_{i=1}^{d} (x_{k+1-i}^{\mathrm{T}} Q x_{k+1-i}) - x_k^{\mathrm{T}} P x_k - \sum_{i=1}^{d} (x_{k-i}^{\mathrm{T}} Q x_{k-i}) \\
&= x_{k+1}^{\mathrm{T}} P x_{k+1} + x_k^{\mathrm{T}} Q x_k - x_k^{\mathrm{T}} P x_k - x_{k-d}^{\mathrm{T}} Q x_{k-d} \\
&= \left((A + BKC)x_k + (A_d + BKC_d)x_{k-d} + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} x_k ds + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} x_{k-d} ds\right)^{\mathrm{T}} \\
&\quad \times P\left((A + BKC)x_k + (A_d + BKC_d)x_{k-d} + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} x_k ds + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} x_{k-d} ds\right) \\
&\quad + x_k^{\mathrm{T}} Q x_k - x_k^{\mathrm{T}} P x_k - x_{k-d}^{\mathrm{T}} Q x_{k-d} \\
&= \left[x_k^{\mathrm{T}}\left(A + BKC + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} ds\right)^{\mathrm{T}} + x_{k-d}^{\mathrm{T}}\left(A_d + BKC_d + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} ds\right)^{\mathrm{T}}\right] \\
&\quad \times P\left[\left(A + BKC + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} ds\right) x_k + \left(A_d + BKC_d + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} ds\right) x_{k-d}\right] \\
&\quad + x_k^{\mathrm{T}} Q x_k - x_k^{\mathrm{T}} P x_k - x_{k-d}^{\mathrm{T}} Q x_{k-d} \\
&= \left(x_k^{\mathrm{T}}\ x_{k-d}^{\mathrm{T}}\right)\begin{pmatrix} \left(A + BKC + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} ds\right)^{\mathrm{T}} \\ \left(A_d + BKC_d + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} ds\right)^{\mathrm{T}} \end{pmatrix} \\
&\quad \times P\left(A + BKC + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} ds \quad A_d + BKC_d + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} ds\right)\begin{pmatrix} x_k \\ x_{k-d} \end{pmatrix} \\
&\quad + \left(x_k^{\mathrm{T}}\ x_{k-d}^{\mathrm{T}}\right)\begin{pmatrix} Q-P & 0 \\ 0 & -Q \end{pmatrix}\begin{pmatrix} x_k \\ x_{k-d} \end{pmatrix}.
\end{aligned}$$

Using Lemma 2.2, we have

$$\begin{aligned}
V_{k+1} - V_k \le \int_0^1 &\Bigg\{\left(x_k^{\mathrm{T}}\ x_{k-d}^{\mathrm{T}}\right)\begin{pmatrix} \left(A + BKC + \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k}\right)^{\mathrm{T}} \\ \left(A_d + BKC_d + \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}}\right)^{\mathrm{T}} \end{pmatrix} \\
&\times P\left(A + BKC + \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} \quad A_d + BKC_d + \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}}\right)\begin{pmatrix} x_k \\ x_{k-d} \end{pmatrix}\Bigg\} ds \\
&+ \left(x_k^{\mathrm{T}}\ x_{k-d}^{\mathrm{T}}\right)\begin{pmatrix} Q-P & 0 \\ 0 & -Q \end{pmatrix}\begin{pmatrix} x_k \\ x_{k-d} \end{pmatrix}.
\end{aligned}$$

Then, we can write that V_{k+1}-V_k<0, if the following holds

$$\begin{aligned}
\int_0^1 &\Bigg\{\begin{pmatrix} (A + BKC + \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k})^{\mathrm{T}} \\ (A_d + BKC_d + \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}})^{\mathrm{T}} \end{pmatrix} P \\
&\times \left(A + BKC + \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} \quad A_d + BKC_d + \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}}\right) + \begin{pmatrix} Q-P & 0 \\ 0 & -Q \end{pmatrix}\Bigg\} ds < 0,
\end{aligned} \tag{2.8}$$

which is equivalent by the Schur complement lemma to the following matrix inequality

$$\begin{aligned}
&\int_0^1 \begin{pmatrix} Q-P & 0 & A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}+\left(\frac{\partial f(\alpha_k)}{\partial \alpha_k}\right)^{\mathrm{T}}\Big|_{\alpha_k=(1-s)x_k} \\ * & -Q & A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}+\left(\frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\right)^{\mathrm{T}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} \\ * & * & -P^{-1} \end{pmatrix} ds \\
&= \int_0^1 \begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+\left(\frac{\partial f(\alpha_k)}{\partial \alpha_k}\right)^{\mathrm{T}}\Big|_{\alpha_k=(1-s)x_k} \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & * & -P^{-1}/2 \end{pmatrix} ds \\
&\quad + \int_0^1 \begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+\left(\frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\right)^{\mathrm{T}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} \\ * & * & -P^{-1}/2 \end{pmatrix} ds < 0.
\end{aligned} \tag{2.9}$$

Since $\frac{\partial f(\alpha_k)}{\partial \alpha_k}$ and $\frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}$ are norm-bounded and continuous for all $k \in Z_+$ and all the matrices involved in (2.9) are real, then the integration in (2.9) is well defined. Using the fact that

$$\begin{aligned}
&\begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+\left(\frac{\partial f(\alpha_k)}{\partial \alpha_k}\right)^{\mathrm{T}}\Big|_{\alpha_k=(1-s)x_k} \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & * & -P^{-1}/2 \end{pmatrix} \\
&\in Co\left\{ \begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+F_i^{\mathrm{T}} \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & * & -P^{-1}/2 \end{pmatrix}, \quad 1 \le i \le v_1 \right\},
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
&\begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+\left(\frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\right)^{\mathrm{T}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} \\ * & * & -P^{-1}/2 \end{pmatrix} \\
&\in Co\left\{ \begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+T_i^{\mathrm{T}} \\ * & * & -P^{-1}/2 \end{pmatrix}, \quad 1 \le i \le v_2 \right\},
\end{aligned}$$

after replacing the Jacobian matrix by its convex hull matrices, sufficient conditions for fulfilling (2.9) are

$$\begin{aligned}
&\begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+F_i^{\mathrm{T}} \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & * & -P^{-1}/2 \end{pmatrix} < 0, \quad 1 \le i \le v_1, \\
&\begin{pmatrix} (Q-P)/2 & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2 \\ * & -Q/2 & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)/2+T_i^{\mathrm{T}} \\ * & * & -P^{-1}/2 \end{pmatrix} < 0, \quad 1 \le i \le v_2.
\end{aligned} \tag{2.11}$$

(2.11) is equivalent to

$$\begin{pmatrix} (Q-P) & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)+2F_i^{\mathrm{T}} \\ * & -Q & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right) \\ * & * & -P^{-1} \end{pmatrix} < 0, \quad 1 \le i \le v_1,$$
$$\begin{pmatrix} (Q-P) & 0 & \left(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right) \\ * & -Q & \left(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}\right)+2T_i^{\mathrm{T}} \\ * & * & -P^{-1} \end{pmatrix} < 0, \quad 1 \le i \le v_2. \tag{2.12}$$

Then, from the result of Lemma 2.1, we can get (2.12) are satisfied if conditions (2.5) are verified. This ends the proof.

Similar to the proof of Theorem 2.3, we can easily get the following corollary for system (2.1).

Corollary 2.4. Consider system (2.1) with $g(x_{k-d})=0$ System (2.1) satisfying (2.3) is globally asymptotically stable under the action of the static output feedback $u_k = \left(\tilde{K}/\alpha\right) y_k$ provided that there exist a scalar $\alpha > 0$ and a real matrix $\tilde{K}$, and positive definite matrices $P > 0$ and $Q > 0$ such that the following linear matrix inequalities hold:

$$\begin{pmatrix} Q-P & 0 & \alpha(A^{\mathrm{T}}+F_i^{\mathrm{T}})+C^{\mathrm{T}}\tilde{K}^{\mathrm{T}}B^{\mathrm{T}} & 0 \\ 0 & -Q & \alpha A_d^{\mathrm{T}}+C_d^{\mathrm{T}}\tilde{K}^{\mathrm{T}}B^{\mathrm{T}} & 0 \\ \alpha(A+F_i)+B\tilde{K}C & \alpha A_d+B\tilde{K}C_d & -2\alpha I & P \\ 0 & 0 & P & -P \end{pmatrix} < 0, \quad 1 \le i \le v_1, \tag{2.13}$$

Remark 2.5. In [21], the stabilization problem for a class of discrete-time linear systems with time-delay is investigated by state feedback. But, the state vector is not often available for feedback. In this article, the static output feedback is used and the system is nonlinear. Compare to [21], the results obtained in this article have a greater range of applications.

3. Guaranteed cost control via static output feedback

In this section, we consider the optimal control problem of system (2.1) under the feedback $u_k = Ky_k$ Our objective is to find the gain K such that for all initial conditions $\varphi_k \in R^n, -d \le k \le 0$

$$J_\infty = \sum_{k=0}^{\infty}\left(x_k^{\mathrm{T}}Ux_k + u_k^{\mathrm{T}}Ru_k\right) < x_0^{\mathrm{T}}Px_0 + \sum_{i=1}^{d}\left(x_{-i}^{\mathrm{T}}Qx_{-i}\right), \tag{3.1}$$

where $U = U^{\mathrm{T}} \ge 0$ and $R = R^{\mathrm{T}} > 0$ are some prescribed real matrices and $P = P^{\mathrm{T}}, Q = Q^{\mathrm{T}}$ are positive definite matrices to be determined.

Theorem 3.1. Consider the system (2.1) and let $U^{\mathrm{T}} \ge 0$, and $R = R^{\mathrm{T}} > 0$ be given symmetric real matrices. If there exist scalars $\alpha_1 > 0$ and $\alpha_2 > 0$, and a matrix K, and symmetric and positive definite matrices $P > 0$ and $Q > 0$ such that the following matrix inequalities hold:

$$\begin{pmatrix} Q-P+U+C^{\mathrm{T}}K^{\mathrm{T}}RKC & C^{\mathrm{T}}K^{\mathrm{T}}RKC_d & \alpha_1(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}+2F_i^{\mathrm{T}}) & 0 \\ C_d^{\mathrm{T}}K^{\mathrm{T}}RKC & -Q+C_d^{\mathrm{T}}K^{\mathrm{T}}RKC_d & \alpha_1(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}) & 0 \\ \alpha_1(A+BKC+2F_i) & \alpha_1(A_d+BKC_d) & -2\alpha_1 I & P \\ 0 & 0 & P & -P \end{pmatrix} < 0, \quad 1 \le i \le v_1,$$
$$\begin{pmatrix} Q-P+U+C^{\mathrm{T}}K^{\mathrm{T}}RKC & C^{\mathrm{T}}K^{\mathrm{T}}RKC_d & \alpha_2(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}) & 0 \\ C_d^{\mathrm{T}}K^{\mathrm{T}}RKC & -Q+C_d^{\mathrm{T}}K^{\mathrm{T}}RKC_d & \alpha_2(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}+2T_i^{\mathrm{T}}) & 0 \\ \alpha_2(A+BKC) & \alpha_2(A_d+BKC_d+2T_i) & -2\alpha_2 I & P \\ 0 & 0 & P & -P \end{pmatrix} < 0, \quad 1 \le i \le v_2, \tag{3.2}$$

then system (2.1) is asymptotically stable under the static output feedback $u_k = Ky_k$, and

$$J_\infty = \sum_{k=0}^{\infty} \left(x_k^{\mathrm{T}} U x_k + u_k^{\mathrm{T}} R u_k\right) < x_0^{\mathrm{T}} P x_0 + \sum_{i=1}^{d} \left(x_{-i}^{\mathrm{T}} Q x_{-i}\right),$$

for all initial conditions $\varphi_k \in R^n$-$d \le k \le 0$.

Proof. Choose the Lyapunov-Krasovskii functional candidate as

$$V_k = x_k^{\mathrm{T}} P x_k + \sum_{i=1}^{d} \left(x_{k-i}^{\mathrm{T}} Q x_{k-i}\right).$$

For a given natural number N, we have

$$\begin{aligned}
\mathrm{H}_N &= \sum_{k=0}^{N} \left(x_k^{\mathrm{T}} U x_k + u_k^{\mathrm{T}} R u_k\right) - V_0 \\
&= \sum_{k=0}^{N} \left(x_k^{\mathrm{T}} U x_k + u_k^{\mathrm{T}} R u_k + V_{k+1} - V_k\right) - V_{N+1} \\
&\le \sum_{k=0}^{N} \left(x_k^{\mathrm{T}} U x_k + (Cx_k + C_d x_{k-d})^{\mathrm{T}} K^{\mathrm{T}} R K (Cx_k + C_d x_{k-d}) + V_{k+1} - V_k\right) \\
&= \sum_{k=0}^{N} \left\{ \left(x_k^T\ x_{k-d}^T\right) \begin{pmatrix} \left(A + BKC + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} ds\right)^{\mathrm{T}} \\ \left(A_d + BKC_d + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} ds\right)^{\mathrm{T}} \end{pmatrix} \right. \\
&\quad \times P \left(A + BKC + \int_0^1 \frac{\partial f(\alpha_k)}{\partial \alpha_k}\Big|_{\alpha_k=(1-s)x_k} ds \quad A_d + BKC_d + \int_0^1 \frac{\partial g(\tau_{k-d})}{\partial \tau_{k-d}}\Big|_{\tau_{k-d}=(1-s)x_{k-d}} ds \right) \begin{pmatrix} x_k \\ x_{k-d} \end{pmatrix} \\
&\quad \left. + \left(x_k^{\mathrm{T}}\ x_{k-d}^{\mathrm{T}}\right) \begin{pmatrix} Q - P + U + C^{\mathrm{T}} K^{\mathrm{T}} R K C & C^{\mathrm{T}} K^{\mathrm{T}} R K C_d \\ C_d^{\mathrm{T}} K^{\mathrm{T}} R K C & -Q + C_d^{\mathrm{T}} K^{\mathrm{T}} R K C_d \end{pmatrix} \begin{pmatrix} x_k \\ x_{k-d} \end{pmatrix} \right\}.
\end{aligned}$$

This implies that if

$$\begin{aligned}
&\begin{pmatrix} Q - P + U + C^{\mathrm{T}} K^{\mathrm{T}} R K C & C^{\mathrm{T}} K^{\mathrm{T}} R K C_d & A^{\mathrm{T}} + C^{\mathrm{T}} K^{\mathrm{T}} B^{\mathrm{T}} + 2F_i^{\mathrm{T}} \\ * & -Q + C_d^{\mathrm{T}} K^{\mathrm{T}} R K C_d & A_d^{\mathrm{T}} + C_d^{\mathrm{T}} K^{\mathrm{T}} B^{\mathrm{T}} \\ * & * & -P^{-1} \end{pmatrix} < 0, \quad 1 \le i \le \nu_1, \\
&\begin{pmatrix} Q - P + U + C^{\mathrm{T}} K^{\mathrm{T}} R K C & C^{\mathrm{T}} K^{\mathrm{T}} R K C_d & A^{\mathrm{T}} + C^{\mathrm{T}} K^{\mathrm{T}} B^{\mathrm{T}} \\ * & -Q + C_d^{\mathrm{T}} K^{\mathrm{T}} R K C_d & A_d^{\mathrm{T}} + C_d^{\mathrm{T}} K^{\mathrm{T}} B^{\mathrm{T}} + 2T_i^{\mathrm{T}} \\ * & * & -P^{-1} \end{pmatrix} < 0, \quad 1 \le i \le \nu_2,
\end{aligned} \tag{3.3}$$

is verified then the static output feedback $u_k = Ky_k$ minimizes the criterion (3.1). From result of Lemma 2.1, we can get that (3.3) are satisfied if conditions (3.2) are verified. So, we get

$$J_\infty = \sum_{k=0}^{\infty} \left(x_k^{\mathrm{T}} U_1 x_k + u_k^{\mathrm{T}} R u_k\right) < x_0^{\mathrm{T}} P x_0 + \sum_{i=1}^{d} \left(x_{-i}^{\mathrm{T}} Q x_{-i}\right).$$

This ends the proof.

Corollary 3.2. Consider the system (2.1) and let $U^{\mathrm{T}} \ge 0$ and $R = R^{\mathrm{T}} > 0$ be given symmetric real matrices. Given scalars $\alpha_1 > 0$ and $\alpha_2 > 0$, if there exist a matrix K, and symmetric and positive definite matrices $P > 0$ and $Q > 0$ such that the following linear matrix inequalities hold:

$$\begin{pmatrix} Q-P+U & 0 & \alpha_1(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}+2F_i^{\mathrm{T}}) & 0 & C^{\mathrm{T}}K^{\mathrm{T}} & C^{\mathrm{T}}K^{\mathrm{T}} \\ * & -Q & \alpha_1(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}) & 0 & C_d^{\mathrm{T}}K^{\mathrm{T}} & C_d^{\mathrm{T}}K^{\mathrm{T}} \\ * & * & -2\alpha_1 I & P & 0 & 0 \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -2R^{-1} & 0 \\ * & * & * & * & * & -2R^{-1} \end{pmatrix} < 0, \quad 1 \le i \le v_1,$$
$$\begin{pmatrix} Q-P+U & 0 & \alpha_2(A^{\mathrm{T}}+C^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}) & 0 & C^{\mathrm{T}}K^{\mathrm{T}} & C^{\mathrm{T}}K^{\mathrm{T}} \\ * & -Q & \alpha_2(A_d^{\mathrm{T}}+C_d^{\mathrm{T}}K^{\mathrm{T}}B^{\mathrm{T}}+2T_i^{\mathrm{T}}) & 0 & C_d^{\mathrm{T}}K^{\mathrm{T}} & C_d^{\mathrm{T}}K^{\mathrm{T}} \\ * & * & -2\alpha_2 I & P & 0 & 0 \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -2R^{-1} & 0 \\ * & * & * & * & * & -2R^{-1} \end{pmatrix} < 0, \quad 1 \le i \le v_2, \tag{3.4}$$

then system (2.1) is asymptotically stable under the static output feedback $u_k = Ky_k$, and

$$J_\infty = \sum_{k=0}^{\infty} \left(x_k^{\mathrm{T}} U x_k + u_k^{\mathrm{T}} R u_k\right) < x_0^{\mathrm{T}} P x_0 + \sum_{i=1}^{d} \left(x_{-i}^{\mathrm{T}} Q x_{-i}\right), \tag{3.5}$$

for all initial conditions $\varphi_k \in R^n$,-d $\le k \le 0$.

Proof. By Theorem 3.1 and the Schur complement lemma, the condition of the corollary follows readily.

Remark 3.3. The study [19] considered the static output feedback and guaranteed cost control for a class of discrete-time nonlinear systems. But, the time-delay system did not deal with in [19]. In this article, we investigated the static output feedback stabilization and output-feedback guaranteed cost controller design for a class of discrete-time nonlinear systems with time-delay. So, the results obtained in this article have a greater range of applications.

4. Illustrative examples

Example 1. Consider the following system

$$\begin{aligned} x_{k+1} = &\begin{pmatrix} -0.15 & -0.01 & 0 \\ 0.05 & 0.06 & 0 \\ 0 & 0 & -0.01 \end{pmatrix} x_k + \begin{pmatrix} -0.03 & -0.01 & 0 \\ 0.05 & -0.08 & 0 \\ 0 & -0.01 & -0.03 \end{pmatrix} x_{k-d} \\ &+ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} u_k + \begin{pmatrix} 0 \\ \dfrac{x_k^{(1)}}{7\left(1 + x_k^{(1)^2}\right)} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \sin x_{k-d}^{(2)} \\ 0 \\ 0 \end{pmatrix}, \end{aligned} \tag{4.1}$$

$$y_k = \begin{pmatrix} 0.01 & 0 & 0 \end{pmatrix} x_k + \begin{pmatrix} 0 & 0.01 & 0 \end{pmatrix} x_{k-d},$$

where $x_k^{\mathrm{T}} = \left(x_k^{(1)}\ x_k^{(2)}\ x_k^{(3)}\right)$.

One can easily verify that the vertices of the Jacobian of the system nonlinearity are

$$F_1 = \begin{pmatrix} 0 & 0 & 0 \\ -1/56 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1/7 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_1 = \begin{pmatrix} 0 & -1/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Let $K = \frac{1}{\alpha}\tilde{K} = 1, \quad \alpha = 0.01, \quad d = 1$ After solving the LMIs (2.5) with respect to their variables, we find

$$P = \begin{pmatrix} 0.0102 & 0.0002 & 0 \\ 0.0002 & 0.0160 & 0 \\ 0 & 0 & 0.0153 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.0050 & 0 & 0 \\ 0 & 0.0121 & 0 \\ 0 & 0 & 0.0085 \end{pmatrix}.$$

Under the action of the static output feedback, $u_k = y_k$, the state response of closed-loop system (4.1) with $x(-1) = (1\ -1\ 1)^{\mathrm{T}}$, $x(0) = (-1\ 1\ -1)^{\mathrm{T}}$ is shown in Figure 1, from which one can see that the state vector is globally asymptotically stable.

Example 2. Consider the following system

$$\begin{aligned} x_{k+1} = & \begin{pmatrix} -0.30 & -0.1 & 0 \\ 0.2 & 0.06 & 0 \\ 0 & 0 & -0.1 \end{pmatrix} x_k + \begin{pmatrix} -0.1 & -0.1 & 0 \\ 0.2 & -0.01 & 0 \\ 0 & -0.12 & -0.3 \end{pmatrix} x_{k-d} \\ & + \begin{pmatrix} 0 \\ 1.2 \\ 0 \end{pmatrix} u_k + \begin{pmatrix} 0 \\ \dfrac{x_k^{(1)}}{7\left(1 + x_k^{(1)^2}\right)} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}\sin x_{k-d}^{(2)} \\ 0 \\ 0 \end{pmatrix}, \end{aligned} \tag{4.2}$$

$$y_k = \begin{pmatrix} 0.05 & 0 & 0 \end{pmatrix} x_k + \begin{pmatrix} 0 & 0.05 & 0 \end{pmatrix} x_{k-d},$$

where $x_k^{\mathrm{T}} = \begin{pmatrix} x_k^{(1)} & x_k^{(2)} & x_k^{(3)} \end{pmatrix}$.

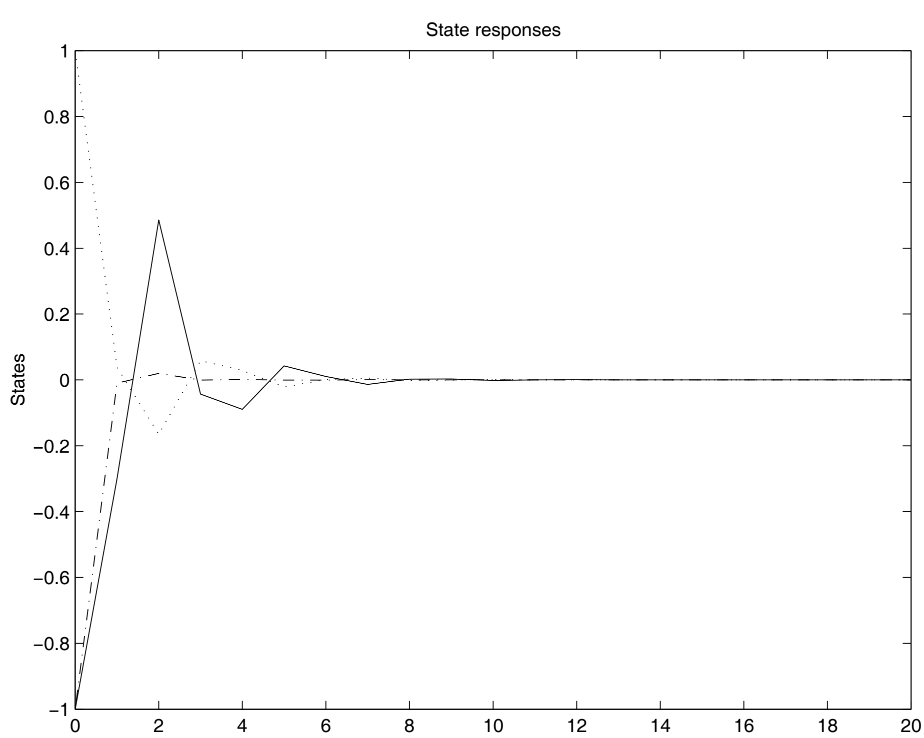

Figure 1 State trajectories of *x*(*k*) in system (4.1).

One can easily verify that the vertices of the Jacobian of the system nonlinearity are

$$F_1 = \begin{pmatrix} 0 & 0 & 0 \\ -1/56 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1/7 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_1 = \begin{pmatrix} 0 & -1/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 1/2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Let

$$\alpha_1 = 4, \quad \alpha_2 = 4, \quad R = 5, \quad U = \begin{pmatrix} 1.5 & 0.2 & 2 \\ 0.2 & 2 & -0.6 \\ 2 & -0.6 & 5 \end{pmatrix}. \tag{4.3}$$

Using LMI Toolbox for (3.4), it is computed that there is a feasible solution and one set of feasible solution is given by

$$P = \begin{pmatrix} 6.8449 & 0.6611 & 0.2678 \\ 0.6611 & 6.2166 & 0.2628 \\ 0.2678 & 0.2628 & 7.0728 \end{pmatrix}, \quad Q = \begin{pmatrix} 2.3875 & 0.1161 & -0.9952 \\ 0.1161 & 2.5594 & 0.6901 \\ -0.9952 & 0.6901 & 1.5301 \end{pmatrix}, \quad K = 1.1517. \tag{4.4}$$

According to Corollary 3.2, then system (4.2) is asymptotically stable under the static output feedback $u_k = 1.1517y_k$, and

$$J_\infty = \sum_{k=0}^{\infty} \left(x_k^{\mathrm{T}} U x_k + u_k^{\mathrm{T}} R u_k \right) < x_0^{\mathrm{T}} P x_0 + \sum_{i=1}^{d} \left(x_{-i}^{\mathrm{T}} Q x_{-i} \right),$$

for all initial conditions $\varphi_k \in R^n$,-d $\le k \le 0$, where U, R, P, and Q are given by (4.3) and (4.4).

5. Conclusion

Both the problems of the static output feedback stabilization and output-feedback guaranteed cost controller design for a class of discrete-time nonlinear systems with time-delay are investigated in this article. The new static output feedback stabilization conditions are proposed, which are independent of the time delay. We develop a quadratic guaranteed cost control method for stabilization via static output feedback. Two numerical examples are provided to show the applicability of the developed results.

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Authors' contributions

YD carried out the main part of this manuscript. JW participated discussion and gave the examples. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests

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