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Corrigendum to "Oscillation behavior of third-order neutral Emden-Fowler delay xdynamic equations on time scales" [Adv. Difference Equ., 2010, 1-23 (2010)]

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Abstract

In this article, we revise results obtained by Han et al.

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1. Introduction

Emden-Fowler type dynamic equations have some applications in the real world; see the background details introduced by Hilger [1]. Hence [2] studied a class of third-order Emden- Fowler neutral dynamic equations

$$\left(r(t)(x(t)-a(t)x(\tau(t)))^{\Delta^2}\right)^{\Delta}+p(t)x^{\gamma}(\delta(t))=0 \tag{1.1}$$

on a time scale $\mathbb{T}$ with sup $\mathbb{T}=\infty$, where the authors assume the following hypotheses hold.

(A_1) $\gamma>0$ is the quotient of odd positive integers;
(A_2) r and p are positive real-valued rd-continuous functions defined on $\mathbb{T}$ such that $r^{\Delta}(t)\geq 0$;
(A_3) a is a positive real-valued rd-continuous function defined on $\mathbb{T}$ such that $0<a(t)\leq a_0<1$ and $\lim_{t\to\infty}a(t)=a_1$;
(A_4) the functions $\tau:\mathbb{T}\to\mathbb{T}$ and $\delta:\mathbb{T}\to\mathbb{T}$ are rd-continuous functions such that $\tau(t)\leq t$, $\delta(t)\leq t$, and $\lim_{t\to\infty}\tau(t)=\lim_{t\to\infty}\delta(t)=\infty$.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Since we are interested in oscillatory behavior, we suppose that the time scale under consideration is not bounded above and is a time scale interval of the form $[t_0,\infty)_{\mathbb{T}}:=[t_0,\infty)\cap\mathbb{T}$. For some concepts related to the notion of time scales; see [3]. Regarding the oscillation properties of (1.1) with $a(t)=0$, Saker [4-7] established some types of criteria, e.g., Hille-Nehari-type and Philos-type.

To establish oscillation criteria for (1.1), [2] obtained various oscillation theorems by using some lemmas, one of which we present below for the convenience of the reader.

Lemma 1.1. (See [2, Lemma 2.1]). Let $z(t) := x(t) - a(t)x(\tau(t))$. Assume that (A_1)-(A_4) hold and

$$(H)\ \textit{there exists}\ \{c_k\}_{k\in\mathbb{N}_0} \subset \mathbb{T}\ \textit{such that}\ \lim_{k\to\infty} c_k = \infty\ \textit{and}\ \tau(c_{k+1}) = c_k.$$

Assume also that x is an eventually positive solution of (1.1). If

$$\int_{t_0}^{\infty} \frac{\Delta t}{r(t)} = \infty, \tag{1.2}$$

then there are only the following three cases for $t \in [t_1, \infty)_{\mathbb{T}}$, where $t_1 \in [t_0, \infty)_{\mathbb{T}}$ sufficiently large:

Case (i). $z(t) > 0,\ z^{\Delta}(t) > 0, z^{\Delta^2}(t) > 0,\ z^{\Delta^3}(t) < 0;$

Case (ii). $z(t) > 0,\ z^{\Delta}(t) > 0,\ z^{\Delta^2}(t) > 0,\ z^{\Delta^3}(t) < 0; \lim_{t\to\infty} x(t) = 0;$

Case (iii).

$z(t) > 0,\ z^{\Delta}(t) < 0,\ z^{\Delta^2}(t) > 0,\ z^{\Delta^3}(t) < 0, \lim_{t\to\infty} z(t) = l \geq 0,\ \lim_{t\to\infty} x(t) = l/(1-a) \geq 0.$

We note that there exists a mistake in the above statements. First, the case (ii) does not occur since $z^{\Delta} > 0$ and $z^{\Delta^2} > 0$ imply that $\lim_{t\to\infty} z(t) = \infty$, and so $z > 0$ eventually. Second, the restrictive assumption (H) can be omitted. Hence the purpose of this article is to revise the related results in [2].

2. Revised results

Now we use notation z as in Lemma 1.1 and present the following new lemmas.

Lemma 2.1. Let (1.2), (A_1), (A_2), and (A_4) hold with (A_3) replaced by $(A3^*)$ a is a positive real-valued rd-continuous function defined on $\mathbb{T}$

such that $0 < a(t) \leq a_0 < 1$. Suppose that x is an eventually positive solution of (1.1). Then there are only the following three cases eventually:

Case (1). $z > 0,\ z^{\Delta} > 0,\ z^{\Delta^2} > 0,\ \left(rz^{\Delta^2}\right)^{\Delta} < 0;$

Case (2). $z > 0,\ z^{\Delta} < 0,\ z^{\Delta^2} > 0,\ \left(rz^{\Delta^2}\right)^{\Delta} < 0;$

Case (3). $z < 0,\ z^{\Delta} < 0,\ z^{\Delta^2} > 0,\ \left(rz^{\Delta^2}\right)^{\Delta} < 0.$

Proof. Assume that x is an eventually positive solution of (1.1). Then, we have by (1.1) that $(rz^{\Delta^2})^{\Delta} < 0$, and hence rz^{Δ^2} is decreasing and of one sign. The condition $rz^{\Delta^2} < 0$ implies that there exist a $t_1 \in [t_0, \infty)_{\mathbb{T}}$ and a constant $M > 0$ such that

$$r(t)z^{\Delta^2}(t) \leq -M,\ \textit{for}\ t \in [t_1,\ \infty)_{\mathbb{T}},$$

which yields

$$z^{\Delta^2}(t) \leq -\frac{M}{r(t)},\ \textit{for}\ t \in [t_1,\ \infty)_{\mathbb{T}}.$$

Integrating from t_1 to t and letting $t \to \infty$, we have by (1.2) that

$$\lim_{t\to\infty} z^{\Delta}(t) = -\infty.$$

Hence there exist a $t_2 \in [t_1, \infty)_{\mathbb{T}}$ and a constant $M_1 > 0$ such that

$$z^{\Delta}(t) \le -M_1, \ \textit{for} \, t \in [t_2, \infty)_{\mathbb{T}}.$$

Integrating the above inequality from t_2 to t and letting $t \to \infty$, we have

$$\lim_{t\to\infty} z(t) = -\infty,$$

which yields $z < 0$ eventually. Then, we get

$$z < 0, \ z^{\Delta} < 0, \ z^{\Delta^2} < 0, \ (rz^{\Delta^2})^{\Delta} < 0. \tag{2.1}$$

From (2.1) we have that $\lim_{t\to\infty} z(t) = -\infty$. Next we claim that x is bounded and (2.1) does not occur. If not, there exists a sequence $\{t_m\}_{m\in\mathbb{N}} \in [t_0, \infty)_{\mathbb{T}}$ with $t_m \to \infty$ as $m \to \infty$ such that

$$x(t_m) = \max\{x(s) \ : \ t_0 \le s \le t_m\} \ \textit{and} \ \lim_{m\to\infty} x(t_m) = \infty.$$

It follows from $\tau(t) \le t$ that

$$z(t_m) = x(t_m) - a(t_m)x(\tau(t_m)) \ge (1 - a_0)x(t_m),$$

which implies that $\lim_{m\to\infty} z(t_m) = \infty$, this contradicts the fact that $\lim_{t\to\infty} z(t) = -\infty$. Hence x is bounded, and so (2.1) does not hold.

If $z^{\Delta} > 0$ and $z^{\Delta^2} > 0$, then $z > 0$. Thus, for $z^{\Delta^2} > 0$ only the cases (1), (2), and (3) may occur. The proof is complete. □

Lemma 2.2. Let $0 < a(t) \le a_0 < 1$. If case (3) holds, then $\lim_{t\to\infty} x(t) = 0$.

Proof. Assume that (3) holds. Then $\lim_{t\to\infty} z(t) \le 0$. Next we claim that x is bounded. Similar as in the proof of Lemma 2.1, we have that $\lim_{m\to\infty} z(t_m) = \infty$ which contradicts the fact that $\lim_{t\to\infty} z(t) \le 0$. Thus, x is bounded. Hence we can suppose that $\limsup_{t\to\infty} x(t) = x_1$, where $0 \le x_1 < \infty$. Then, there exists a sequence $\{t_k\}_{k\in\mathbb{N}} \in [t_0, \infty)_{\mathbb{T}}$ with $t_k \to \infty$ as $k \to \infty$ such that $\lim_{k\to\infty} x(t_k) = x_1$. Next we show that $\lim_{t\to\infty} x(t) = 0$. If not, then $x_1 > 0$. Pick $\varepsilon = x_1(1 - a_0)/(2a_0)$, we find that $x(\tau(t_k)) < x_1 + \varepsilon$ eventually. Moreover,

$$0 = \lim_{k\to\infty} z(t_k) \ge \lim_{k\to\infty} (x(t_k) - a_0(x_1 + \varepsilon)) = \frac{x_1(1 - a_0)}{2} > 0.$$

This is a contradiction. The proof is complete. □

3. Discussions

In this article, we establish Lemmas 2.1 and 2.2 which improve Lemma 1.1 used in [2]. Using these lemmas and methods given in [2,4-7], one can renew those results of [2] and present some other new results. In particular, new results only require that $0 < a(t) \le a_0 < 1$ rather than (H), $0 < a(t) \le a_0 < 1$, and $\lim_{t\to\infty} a(t) = a_1$. The details are left to the reader.

To achieve new results, we are forced to require that $0 < a(t) \le a_0 < 1$. The question regarding the oscillatory properties of (1.1) without this assumption remains open at the moment.

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Authors' contributions
TJ and ST framed and solved the problem. TL modified and made necessary changes in the proof of the results. All authors read and approved the final manuscript.

Competing interests
The authors declared that they have no competing interests.

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